All-Charm TetraQuarks at Hadron Colliders
A High-Precision Fragmentation Perspective

Francesco Giovanni Celiberto, UAH Madrid



QCD@LHC 2026

Oxford - October 1st

[Phys. Rev. D Letters 111 (2025) 11, L111501 [arXiv:2504.03949]]

[Phys. Rev. D 113 (2026) 5, 054014 [arXiv:2511.18984]]

[Phys. Rev. D 113 (2026) 11, 114037 [arXiv:2604.11646]]

\+ Work in Progress [TQHL2.0]

Madrid UAH

talento cm Programa de atracción de talento investigador Comunidad de Madrid

EUGLOH

QCD Hydrogen Atoms

I

¿ Why studying (Heavy) Exotics ?

JETHAD
v0.5.5

Precision & Exploration

¿ What is an Exotic Hadron ?

- Baryons & Mesons → composite states → $|q_1 q_2 q_3\rangle$ & $|q_1 \bar{q}_2\rangle$
- Ordinary Hadrons ← simplest valence-quark color-singlet states
- QCD color neutrality → TetraQuarks, PentaQuarks, Hybrids allowed

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- First discovery of a (crypto-)exotic at Belle (🇯🇵) in 2003 → $X(3872)$

[Yesterday's overview talk by J. He 🔗; today's talks by Y. Zhang 🔗, C. Chen 🔗, C. Thomas 🔗, A. Di Florio 🔗]

Interpretation of Exotic States

Compact MultiQuark



Interpretation of Exotic States

Compact MultiQuark





Hybrid

Interpretation of Exotic States

diQuark + AntidiQuark



Compact MultiQuark





Hybrid

Interpretation of Exotic States

Compact MultiQuark



diQuark + AntidiQuark





Hybrid



Molecule of Mesons

Interpretation of Exotic States

Compact MultiQuark



diQuark + AntidiQuark



HadroQuarkonium





Hybrid



Molecule of Mesons

All-Charm TetraQuarks: Experimental Highlights

EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH (CERN)

CERN-EP-2025-118
2025/06/10

CMS-BPH-24-002

Determination of the spin and parity of all-charm tetraquarks

The CMS Collaboration*

arXiv:2506.07944v1 [hep-ex] 9 Jun 2025

Abstract

The traditional quark model accounts for the existence of baryons, such as protons and neutrons, which consist of three quarks, as well as mesons, composed of a quark-antiquark pair. Only recently has substantial evidence started to accumulate for exotic states composed of four or five quarks and antiquarks. The exact nature of their internal structure remains uncertain. This paper reports the first measurement of quantum numbers of the recently discovered family of three all-charm tetraquarks, using data collected by the CMS experiment at the Large Hadron Collider from 2016 to 2018. The angular analysis techniques developed for the discovery and characterization of the Higgs boson have been applied to the new exotic states. The quantum numbers for parity P and charge conjugation C symmetries are found to be $+1$. The spin J of these exotic states is consistent with $2\hbar$, while $0\hbar$ and $1\hbar$ are excluded at 95% and 99% confidence level, respectively. The $J^{PC} = 2^{++}$ assignment implies particular configurations of constituent spins and orbital angular momenta, which constrain the possible internal structure of these tetraquarks.

Submitted to Nature



All-Charm TetraQuarks: Experimental Highlights

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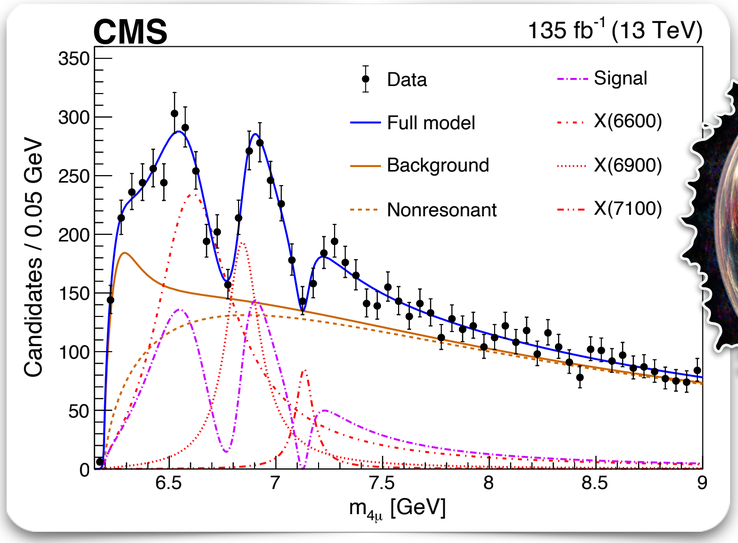
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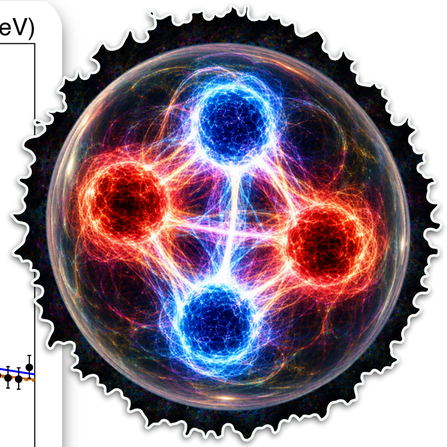
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[Talk by A. Di Florio 🔗, this session]

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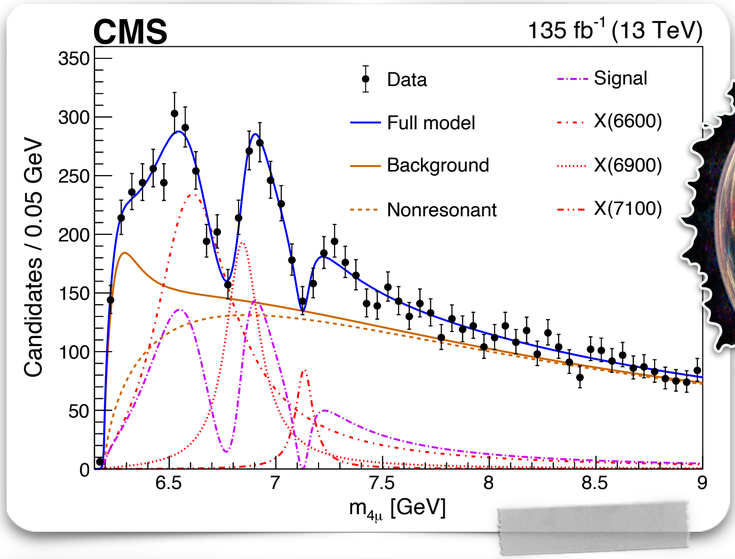
The CMS Collaboration*

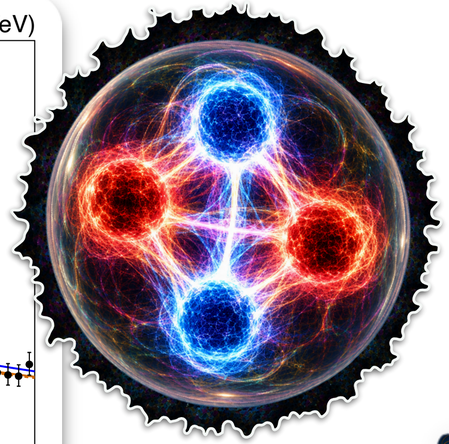
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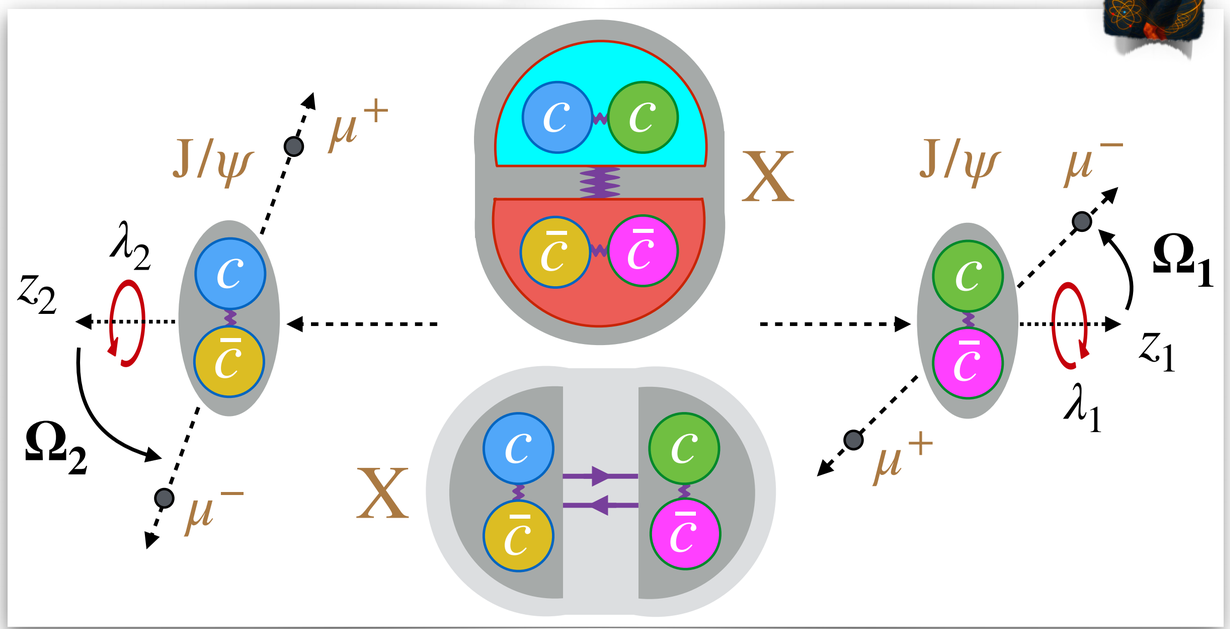
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All-Charm TetraQuarks: Experimental Highlights

Article

Determination of the spin and parity of all-charm tetraquarks

https://doi.org/10.1038/s41586-025-09711-7

Received: 9 June 2025

Accepted: 6 October 2025

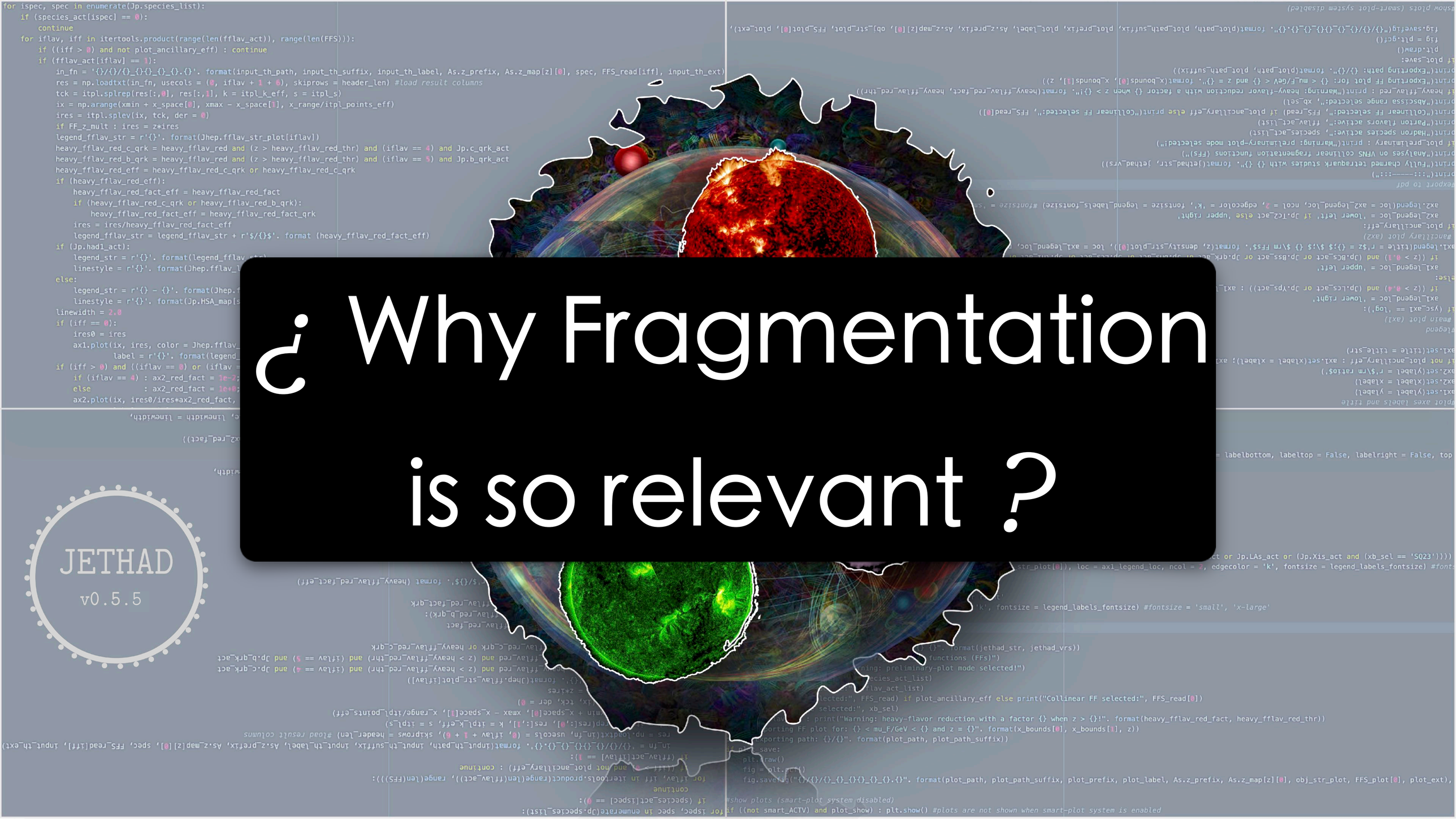
Published online: 3 December 2025

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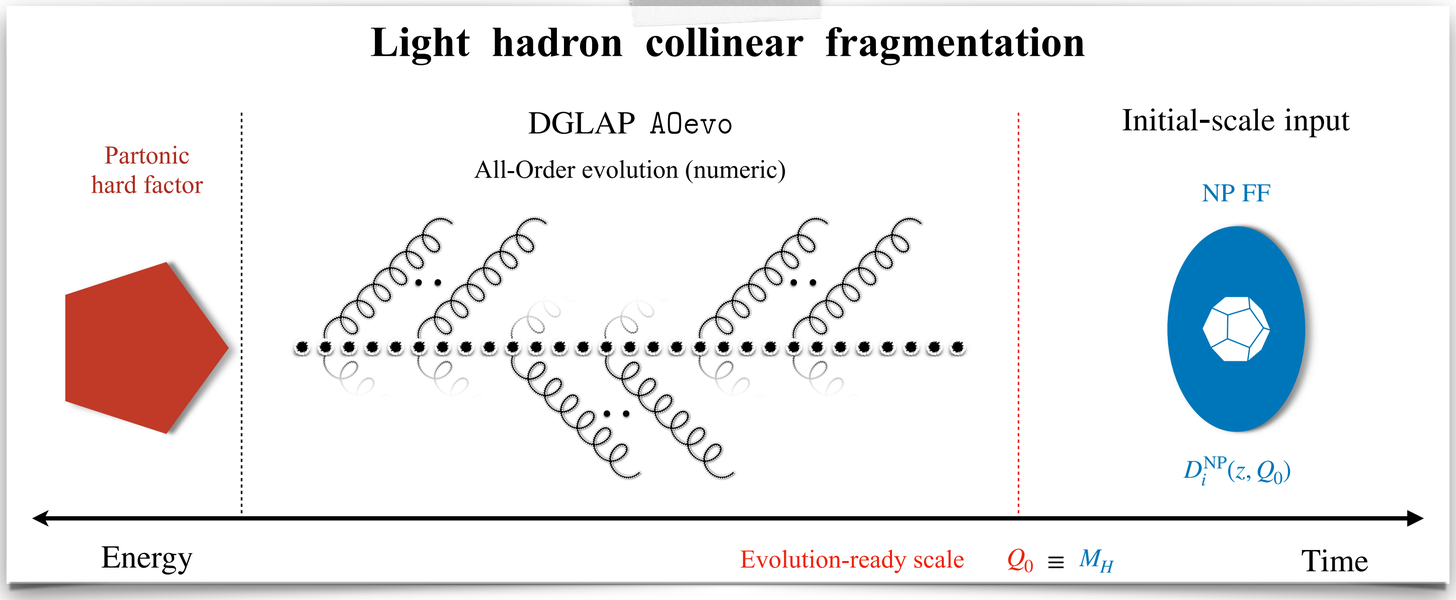
The CMS Collaboration*✉

The traditional quark model[1,2] accounts for the existence of baryons, such as protons and neutrons, which consist of three quarks, as well as mesons, composed of a quark–antiquark pair. Only recently has substantial evidence started to accumulate for exotic states composed of four or five quarks and antiquarks[3]. The exact nature of their internal structure remains uncertain[4–29]. Here we report the first measurement of quantum numbers of the recently discovered family of three all-charm tetraquarks[30–32], using data collected by the CMS experiment at the Large Hadron Collider from 2016 to 2018 (refs. 33,34). The angular analysis techniques developed for the discovery and characterization of the Higgs boson[35–37] have been applied to the new exotic states. Here we show that the quantum numbers for parity P and charge conjugation C symmetries are found to be +1. The spin J of these exotic states is determined to be consistent with $2\hbar$, while $0\hbar$ and $1\hbar$ are excluded at 95% and 99% confidence levels, respectively. The $J^{PC} = 2^{++}$ assignment implies particular configurations of constituent spins and orbital angular momenta, which constrain the possible internal structure of these tetraquarks.


¿ Why Fragmentation
is so relevant ?
JETHAD
v0.5.5

Fragmentation in QCD (with Heavy Flavors)

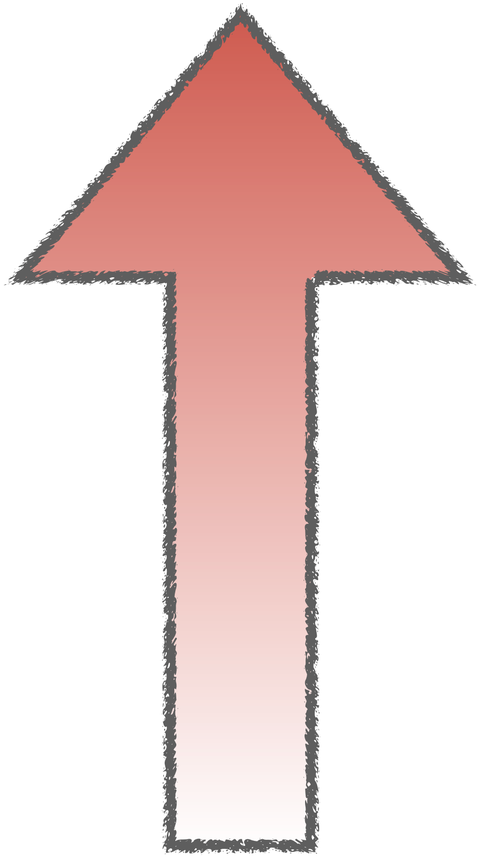
PRECISION CONTROL

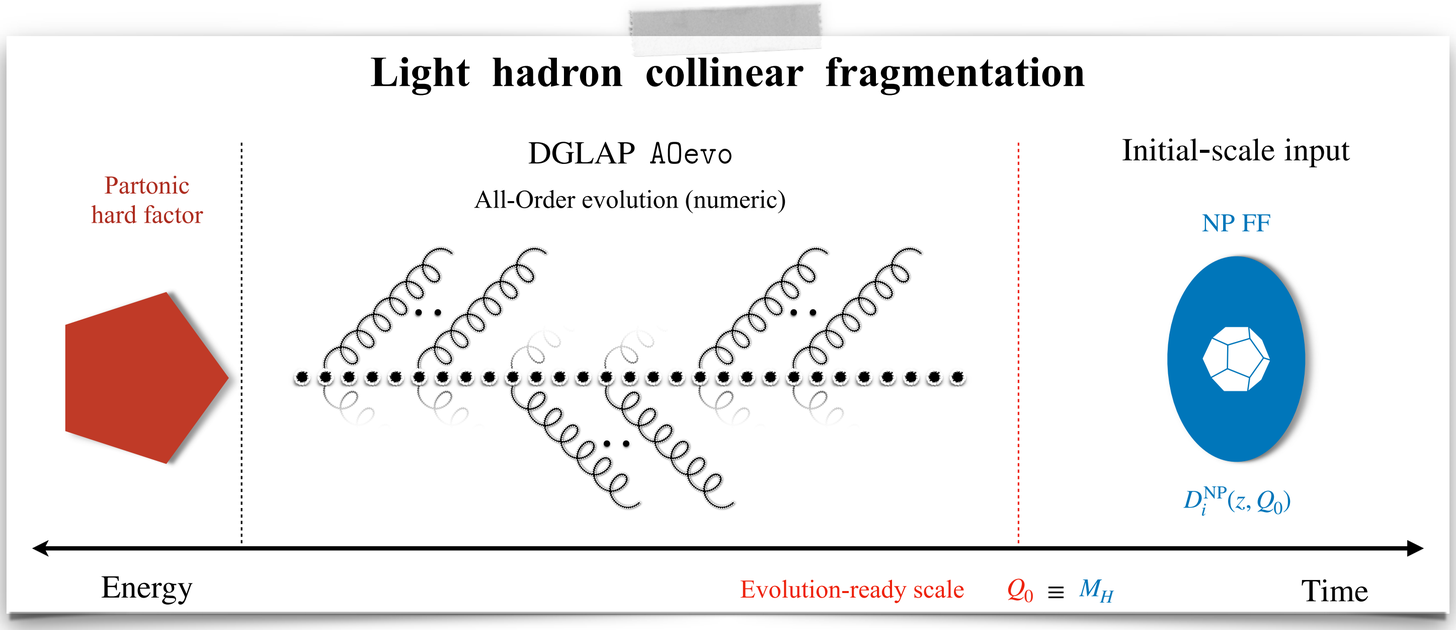


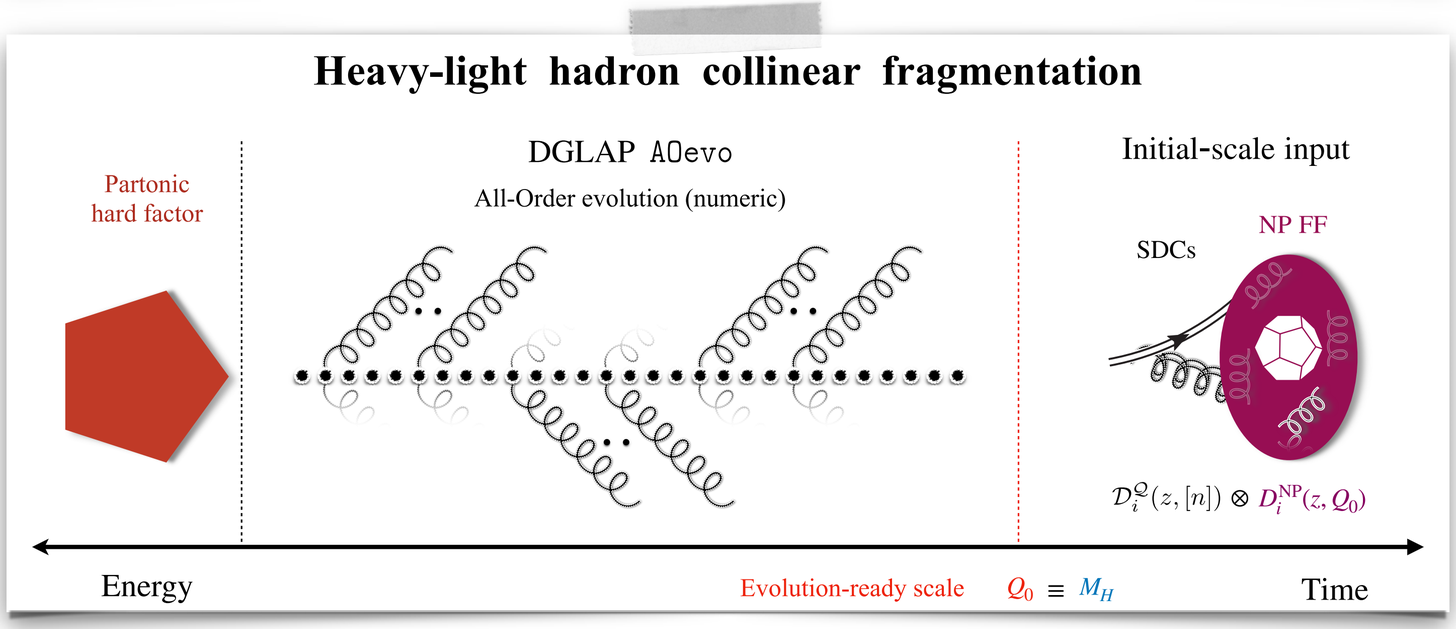
DISCOVERY POTENTIAL

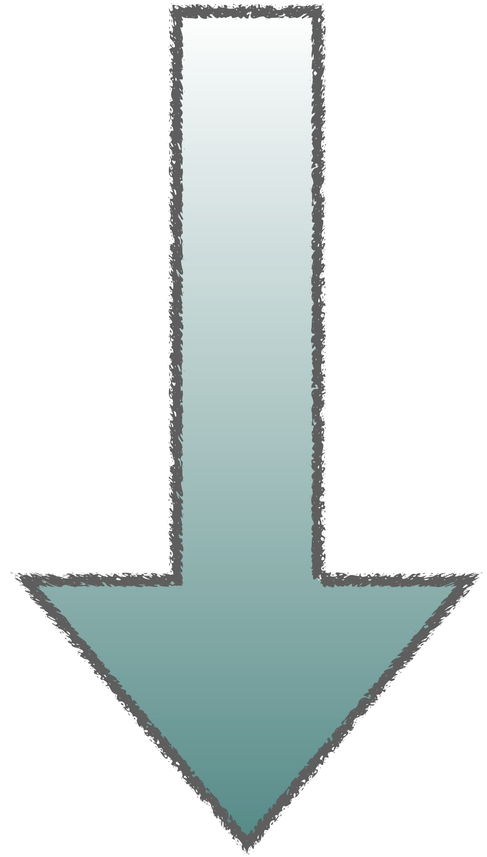
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PRECISION CONTROL





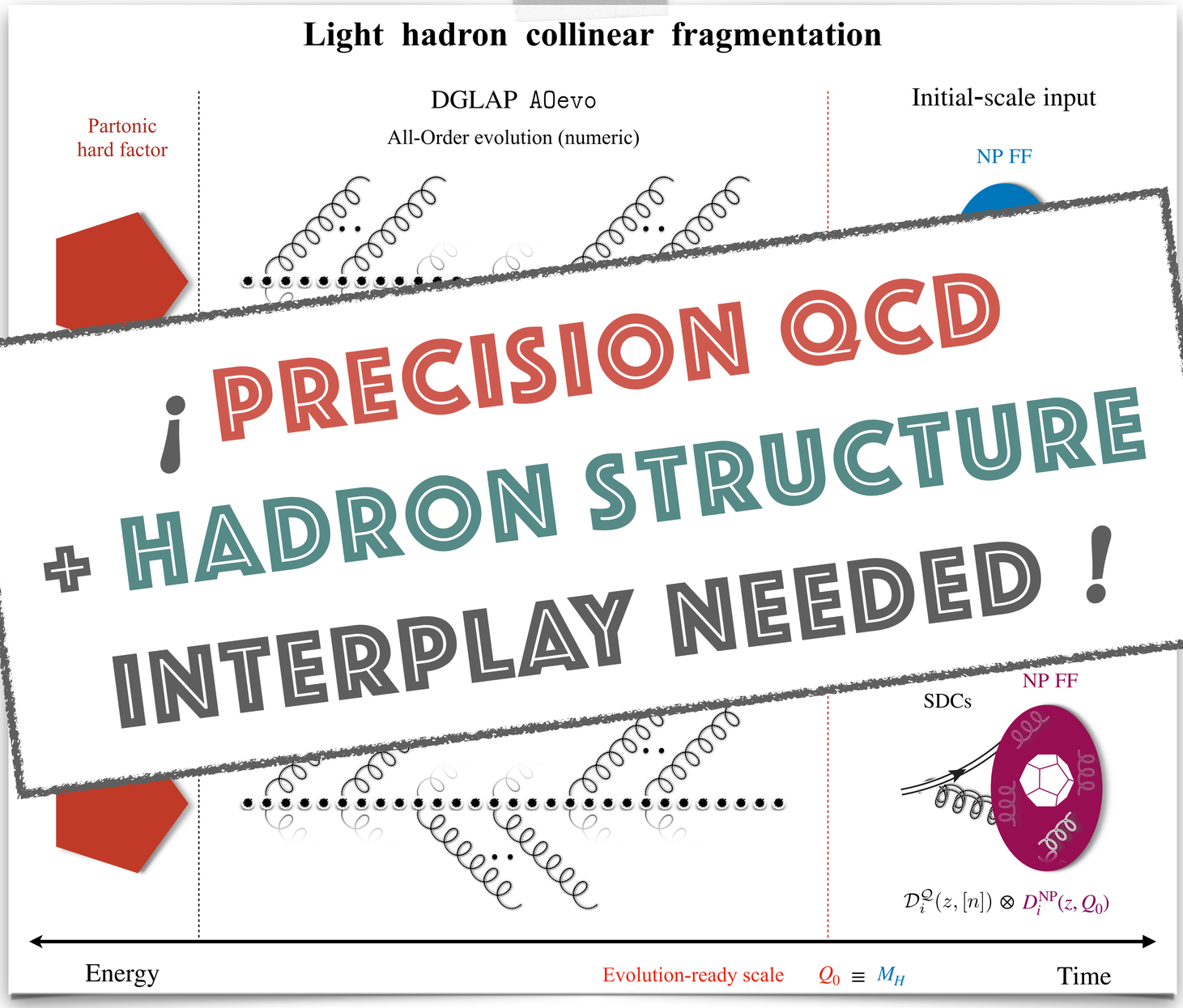




DISCOVERY POTENTIAL

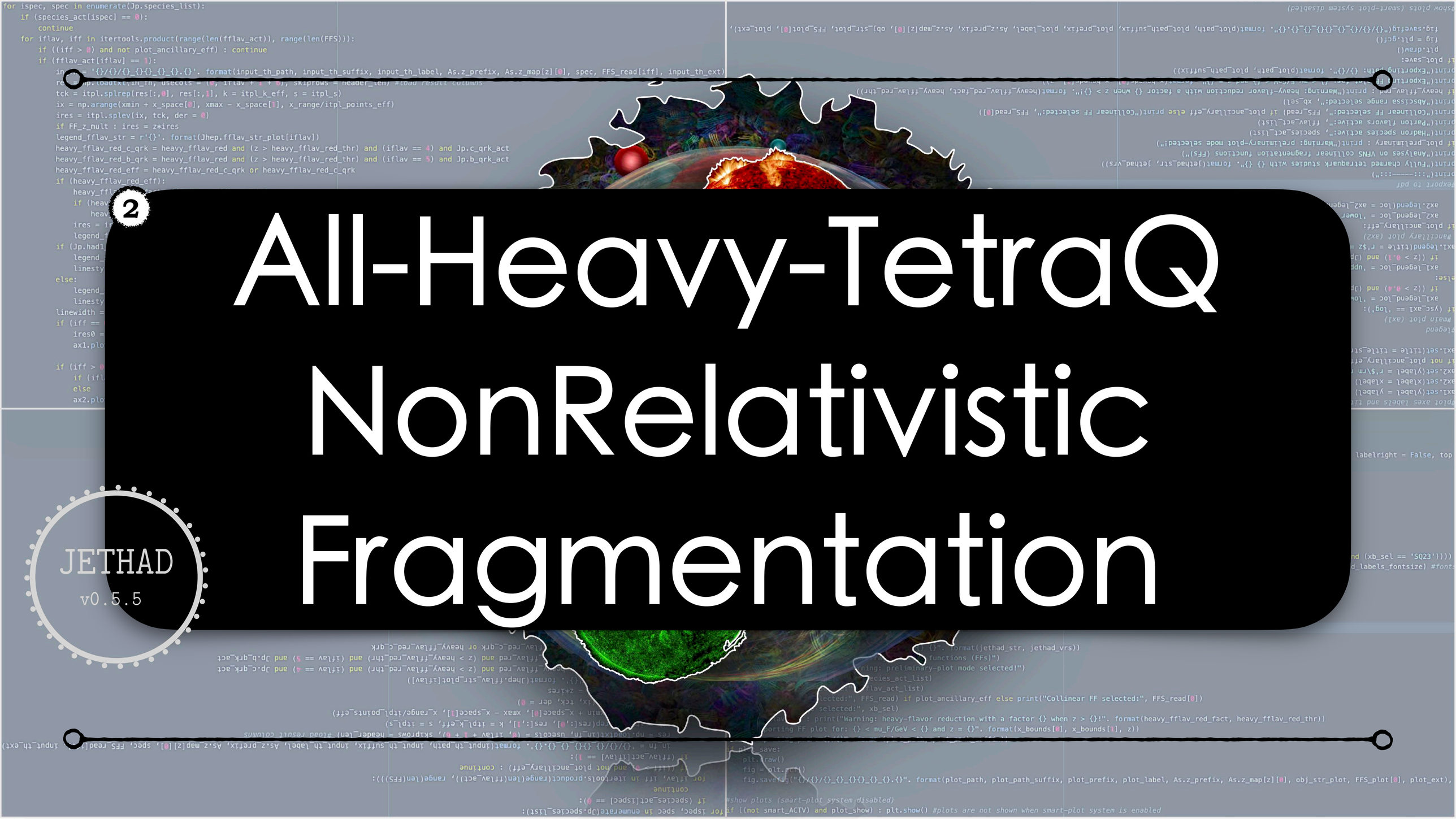
Fragmentation in QCD (with Heavy Flavors)

PRECISION CONTROL



¡ PRECISION QCD + HADRON STRUCTURE INTERPLAY NEEDED !

DISCOVERY POTENTIAL


2
All-Heavy-TetraQ
NonRelativistic
Fragmentation
JETHAD
v0.5.5

Heavy-Flavor Non-Relativistic evolution

HADRONIC STRUCTURE

QUARKONIUM THEORY

Heavy-Flavor Non-Relativistic evolution

HADRONIC STRUCTURE

QUARKONIUM THEORY

PRECISION QCD

COLLINEAR FACTORIZATION

Heavy-Flavor Non-Relativistic evolution

HADRONIC STRUCTURE

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PRECISION QCD

COLLINEAR FACTORIZATION

Guiding principle ⇒ ¡ Use the Best of the Two Worlds as much as we can !

Interpretation

Heavy-Flavor Non-Relativistic evolution

HADRONIC STRUCTURE

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Evolution

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Interpretation

Evolution

Uncertainties

Heavy-Flavor Non-Relativistic evolution

HF-NRevo

Interpretation | Evolution | Uncertainties

- NRQCD ⇒ proxy for initial-scale FF inputs
- A heavy-flavor-scheme based vision
- Low vs large p_T ⇒ FFNS vs VFNS fragmentation

Heavy-Flavor Non-Relativistic evolution

HF-NRevo

Interpretation | Evolution | Uncertainties

- Proper DGLAP collinear evolution
- Thresholds at work for all partons
- Two-step strategy $\Rightarrow$ `EDevo` $\otimes$ `AOevo`

Heavy-Flavor Non-Relativistic evolution

HF-NRevo

Interpretation

Evolution

Uncertainties

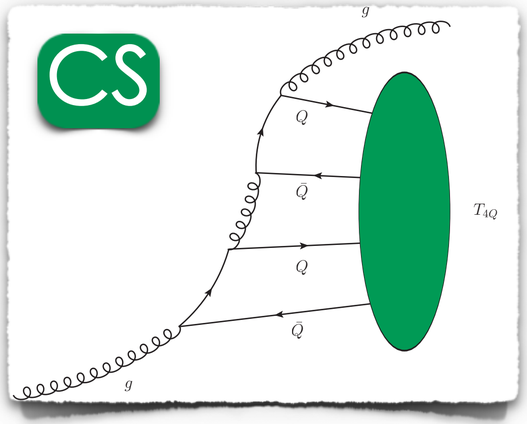
- Initial-scale MHOUs ⇒ first systematic analysis
- Idea ⇔ ThCovM 🔗🔗 & MCscales 🔗 MHOUs
- Future ⇐ extraction of NP LDMEs from data

TetraQuark Fragmentation from NRQCD & symJETHAD

LO

Gluon

CS

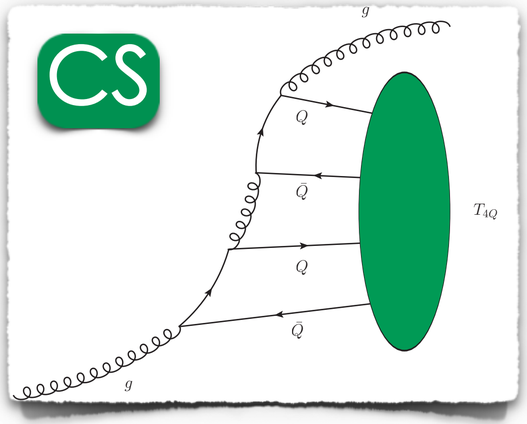


[F. Feng, Y. Huang et al., Phys. Rev. D 106 (2022) 11, 114029]

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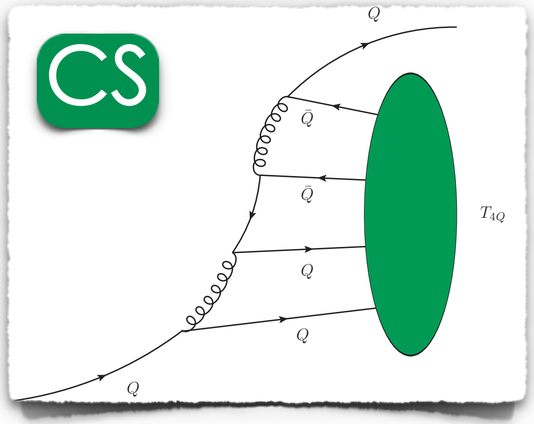
LO

Gluon



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Constituent heavy quark

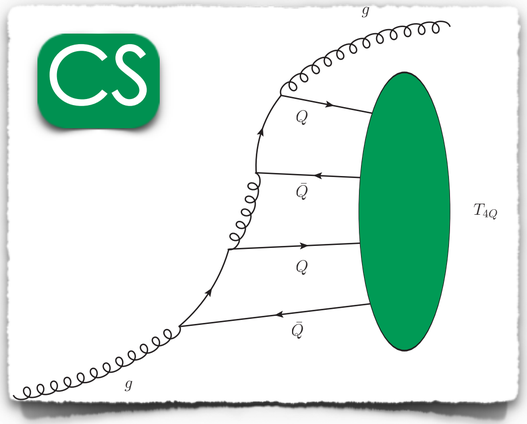


[X.-W. Bai, F. Feng et al., JHEP 09 (2024) 002

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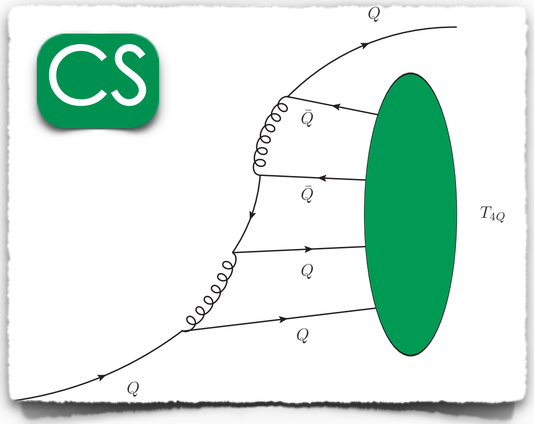
LO

Gluon



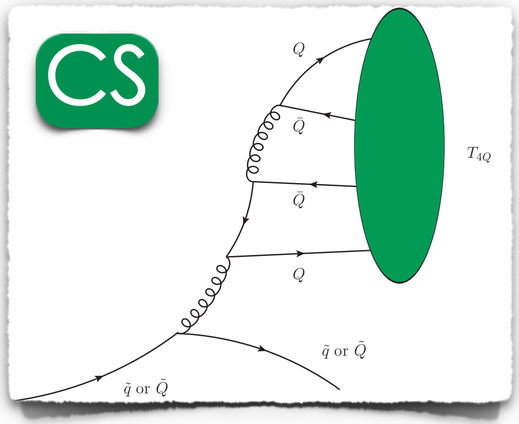
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Constituent heavy quark



[X.-W. Bai, F. Feng et al., JHEP 09 (2024) 002

Non-constituent heavy or light quark

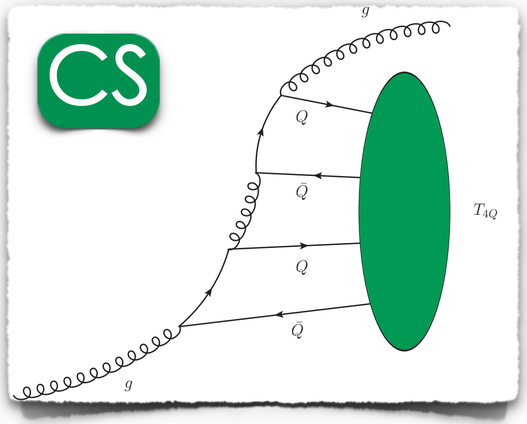


[X.-W. Bai, Y. Huang et al., Phys. Rev. D 111 (2025) 5, 054006]

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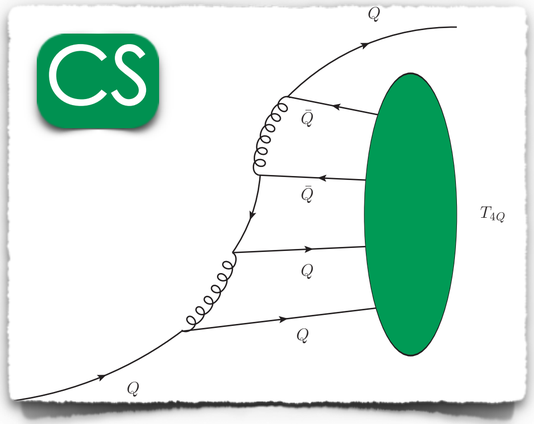
LO

Gluon



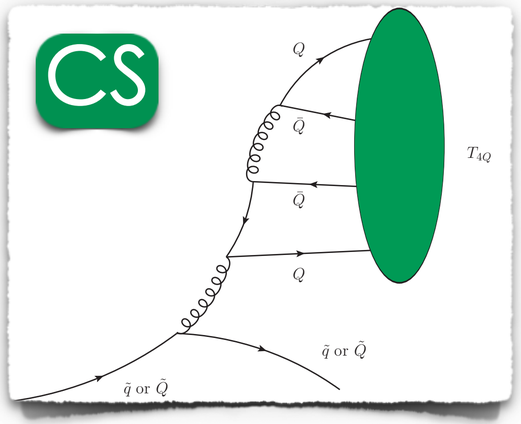
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Non-constituent heavy or light quark



[X.-W. Bai, Y. Huang et al., Phys. Rev. D 111 (2025) 5, 054006]

$J^{PC} = 0^{++}$

$J^{PC} = 1^{+-}$

$J^{PC} = 2^{++}$

TetraQuark Fragmentation from NRQCD & symJETHAD

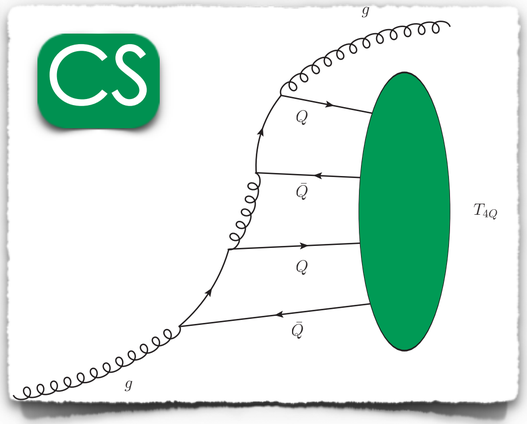
LO

2024

TQ4Q1.0

Gluon

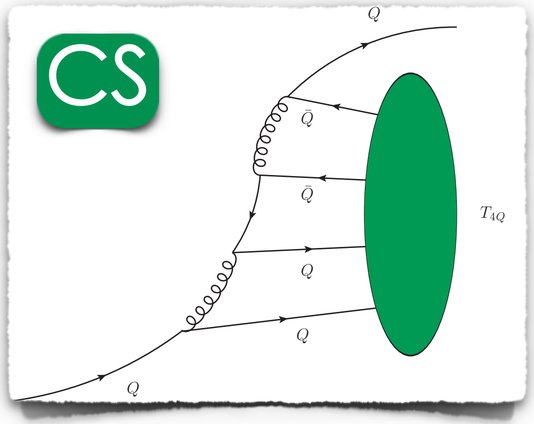
CS



[F. Feng, Y. Huang et al., Phys. Rev. D 106 (2022) 11, 114029]

Constituent heavy quark

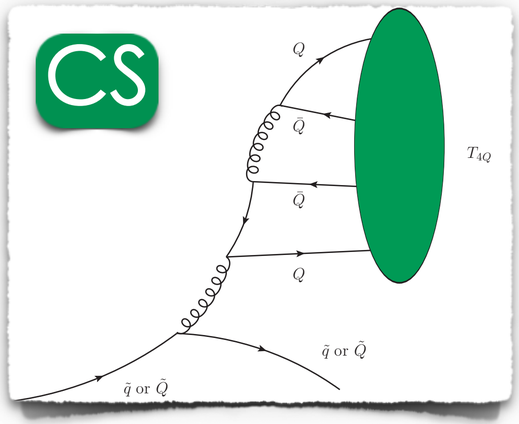
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Non-constituent heavy or light quark

CS



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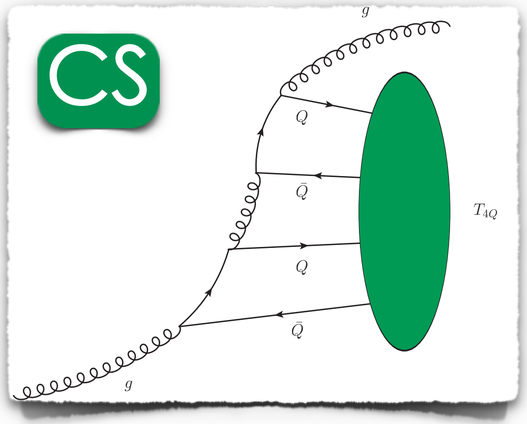
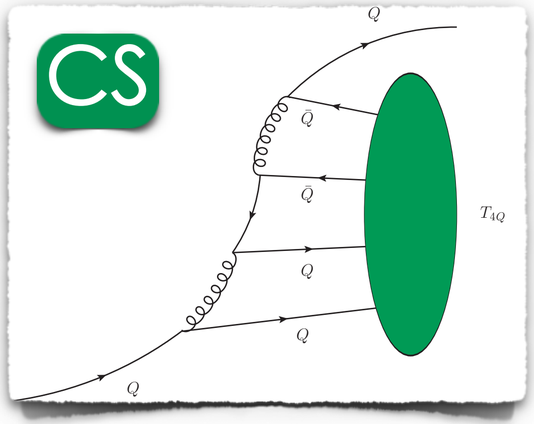
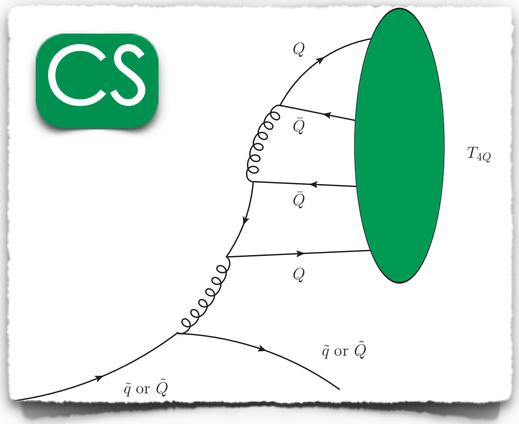
$J^{PC} = 0^{++}$	TQ4Q1.0+ ✓	¡! Suzuki [TQ4Q1.0]	¡! Not implemented
$J^{PC} = 1^{+-}$	Landau-Yang LO: evolution only ✗	¡! Not implemented	C-parity LO: evolution only ✗
$J^{PC} = 2^{++}$	TQ4Q1.0+ ✓	¡! Suzuki [TQ4Q1.0]	¡! Not implemented

TetraQuark Fragmentation from NRQCD & symJETHAD

LO

2024 → 2025

TQ4Q1.1

	Gluon	Constituent heavy quark	Non-constituent heavy or light quark
	CS  [F. Feng, Y. Huang et al., Phys. Rev. D 106 (2022) 11, 114029]	CS  [X.-W. Bai, F. Feng et al., JHEP 09 (2024) 002]	CS  [X.-W. Bai, Y. Huang et al., Phys. Rev. D 111 (2025) 5, 054006]
$J^{PC} = 0^{++}$	TQ4Q1.0+ ✓	TQ4Q1.1+ ✓	¡! Not implemented
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$J^{PC} = 2^{++}$	TQ4Q1.0+ ✓	TQ4Q1.1+ ✓	¡! Not implemented

TetraQuark Fragmentation from NRQCD & symJETHAD

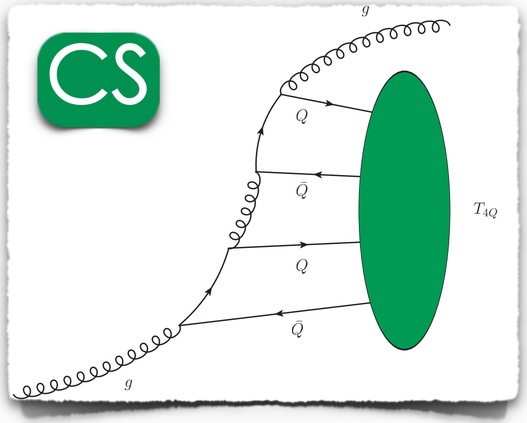
LO

2024 → 2025 → 2026

TQ4Q2.0

Gluon

NEW



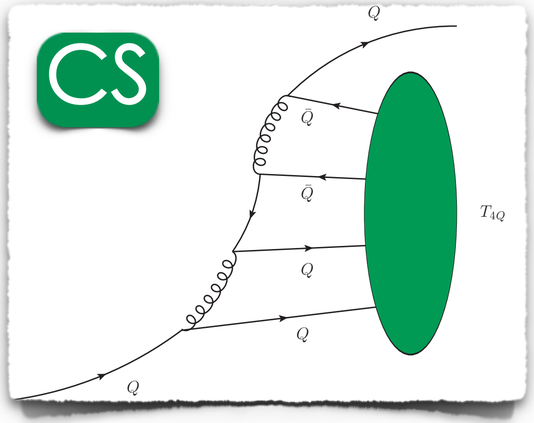
[F. Feng, Y. Huang et al., Phys. Rev. D 106 (2022) 11, 114029]

$J^{PC} = 0^{++}$ TQ4Q1.0+ ✓

$J^{PC} = 1^{+-}$ Landau-Yang LO: evolution only ✗

$J^{PC} = 2^{++}$ TQ4Q1.0+ ✓

Constituent heavy quark



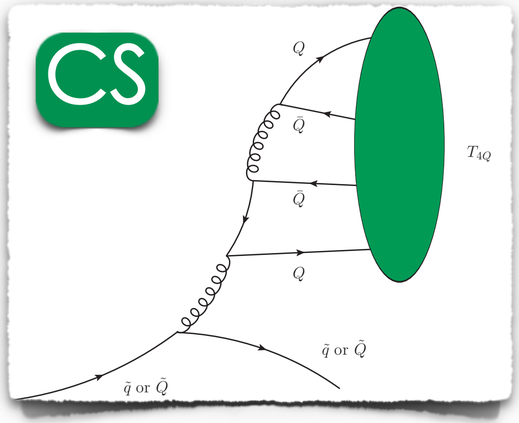
[X.-W. Bai, F. Feng et al., JHEP 09 (2024) 002

TQ4Q1.1+ ✓

TQ4Q1.1+ ✓

TQ4Q1.1+ ✓

Non-constituent heavy or light quark



[X.-W. Bai, Y. Huang et al., Phys. Rev. D 111 (2025) 5, 054006]

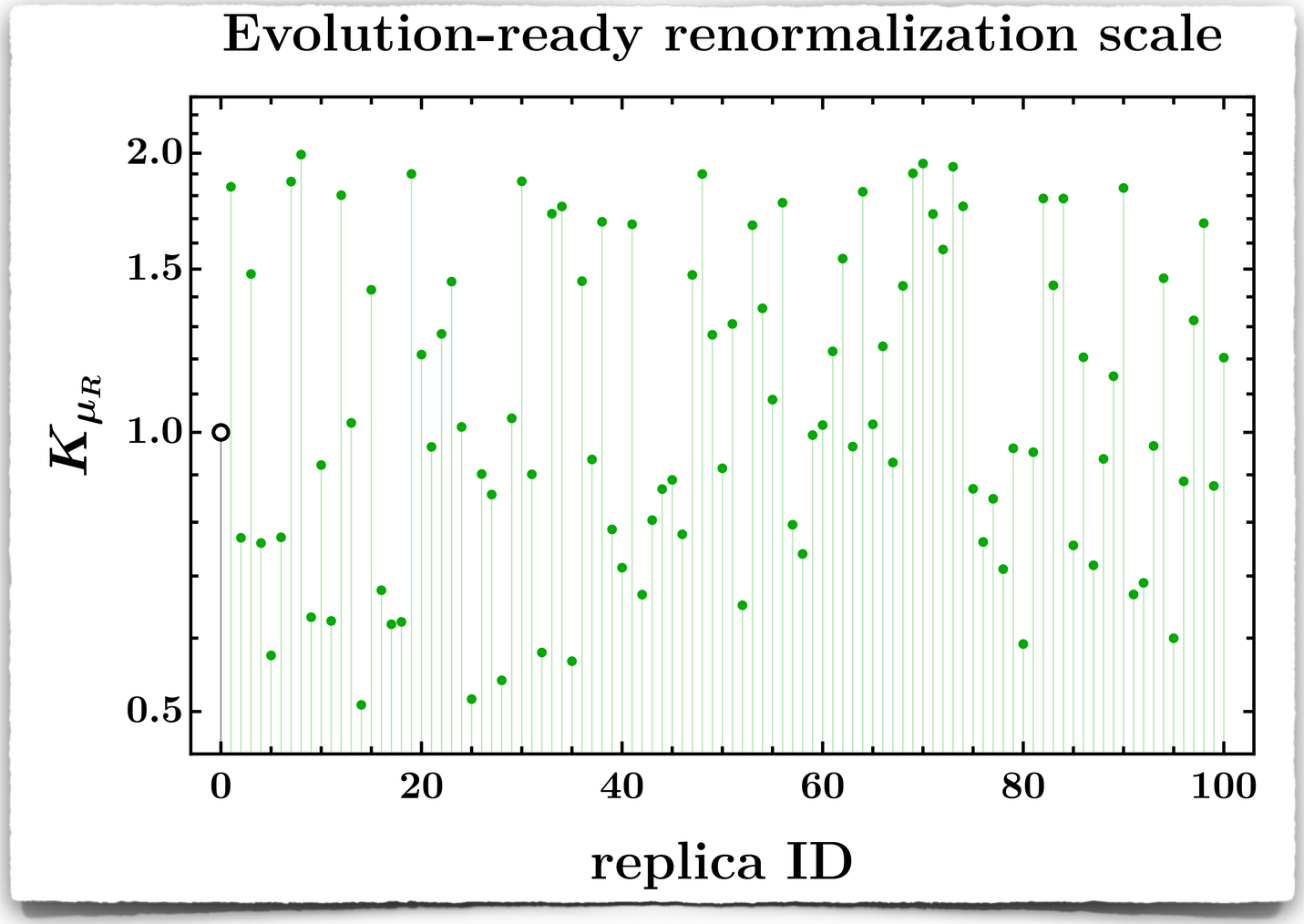
NEW TQ4Q2.0+ ✓

C-parity LO: evolution only ✗

NEW TQ4Q2.0+ ✓

TQ4Q2.0: F-MHOU Replica Analysis

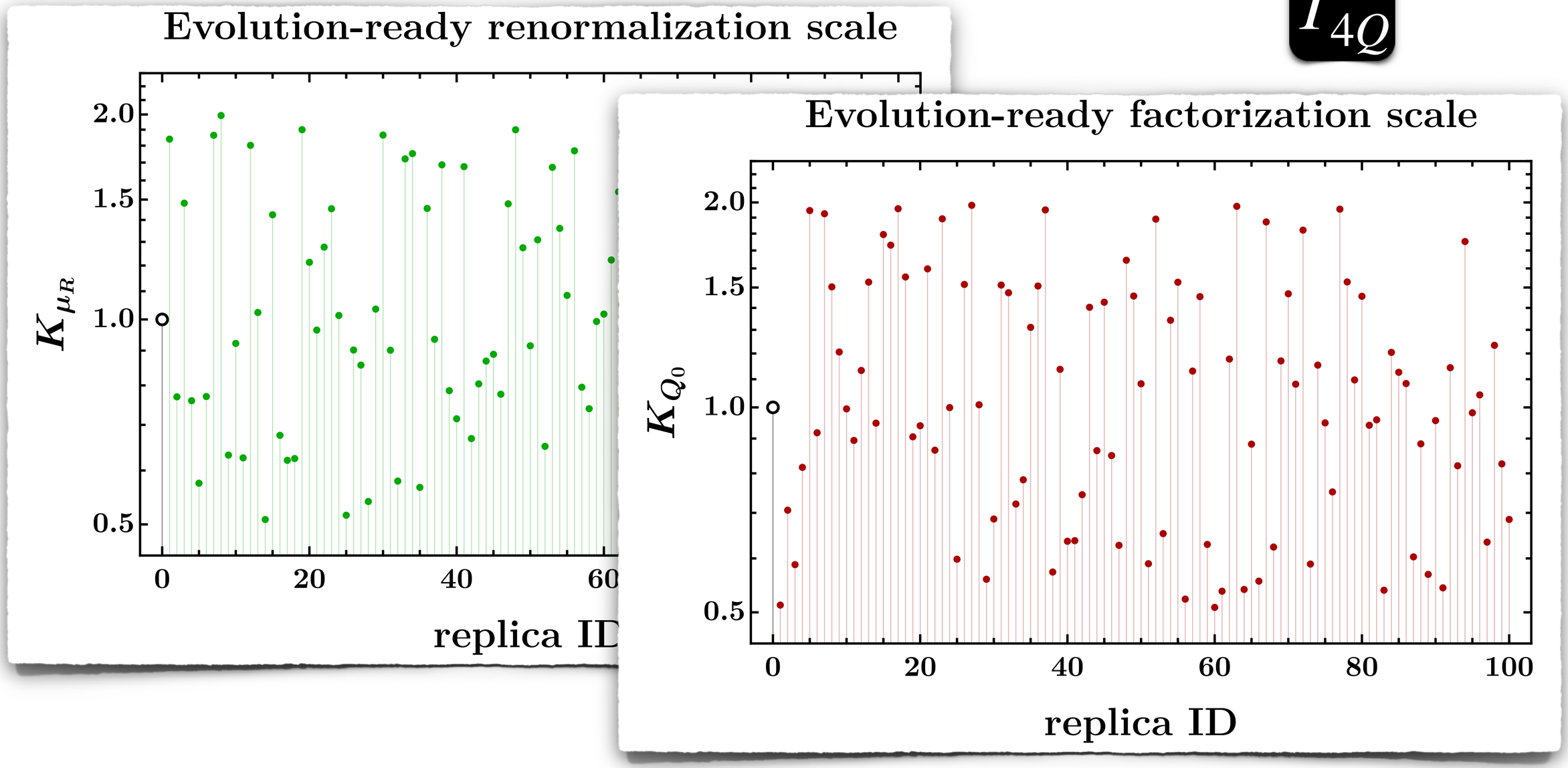
T_{4Q}



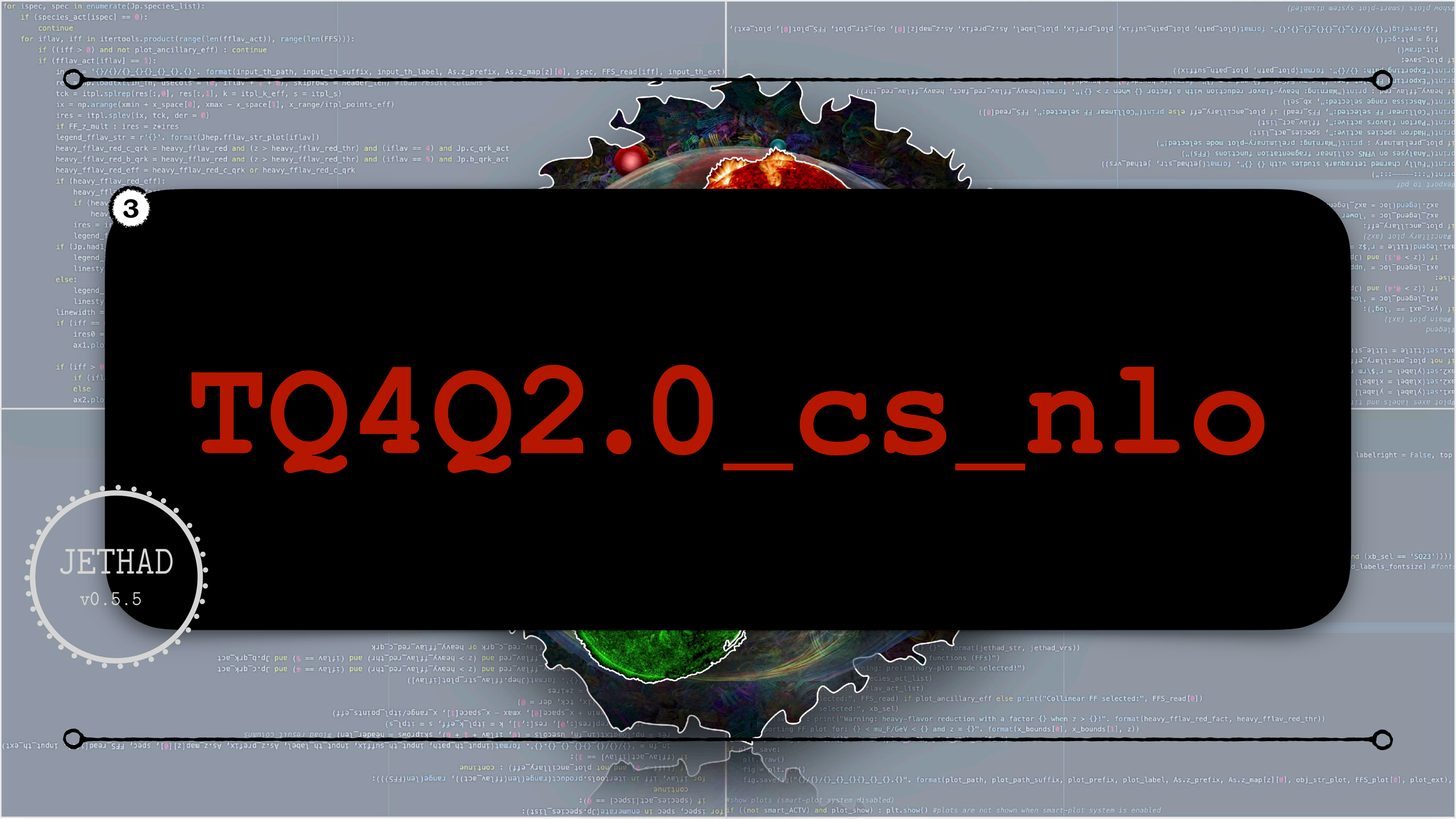
[F. G. C., Phys. Rev. D 113 (2026) 11, 114037 [arXiv:2604.11646]]

TQ4Q2.0: F-MHOU Replica Analysis

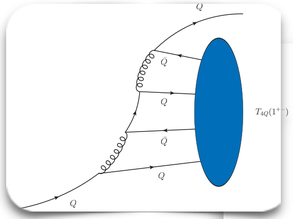
T_{4Q}

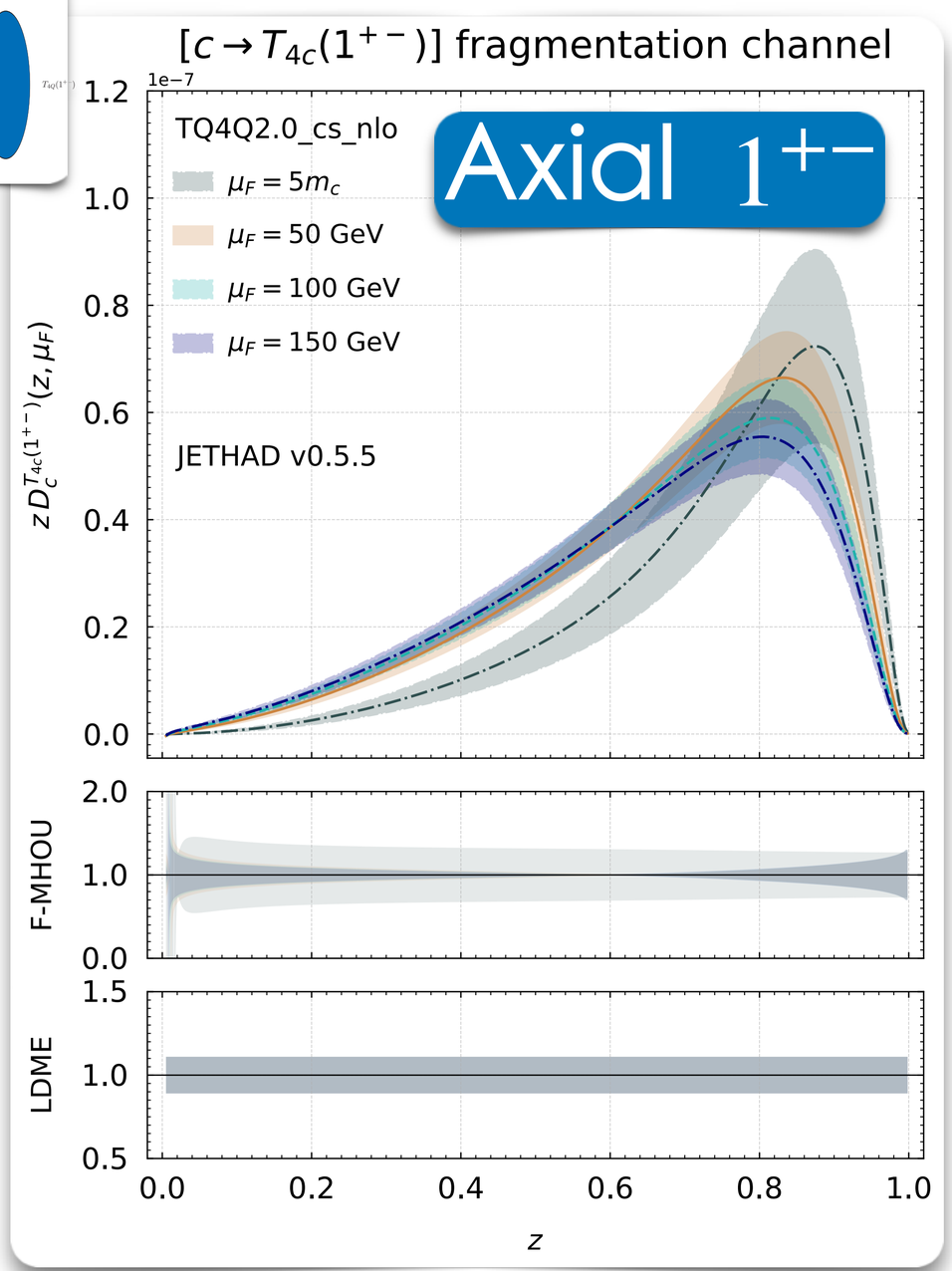


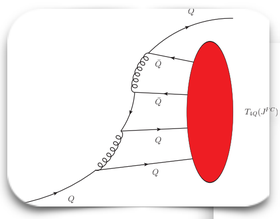
[F. G. C., Phys. Rev. D 113 (2026) 11, 114037 [arXiv:2604.11646]]

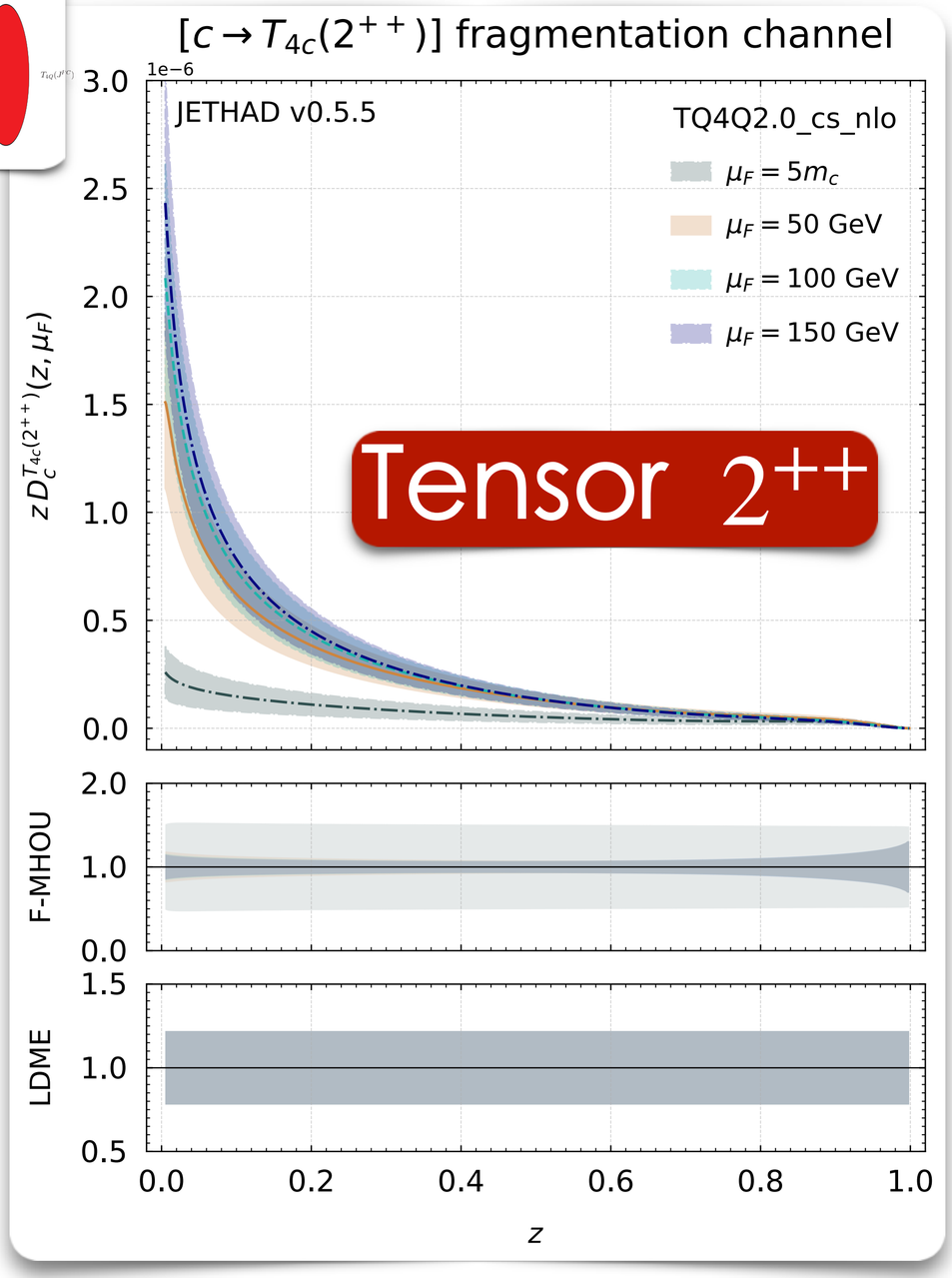

3
TQ4Q2.0_cs_nlo
JETHAD
v0.5.5

TQ4Q2.0: Charm Fragmentation to Charmoniumlike

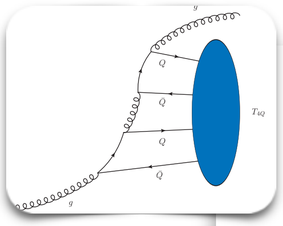


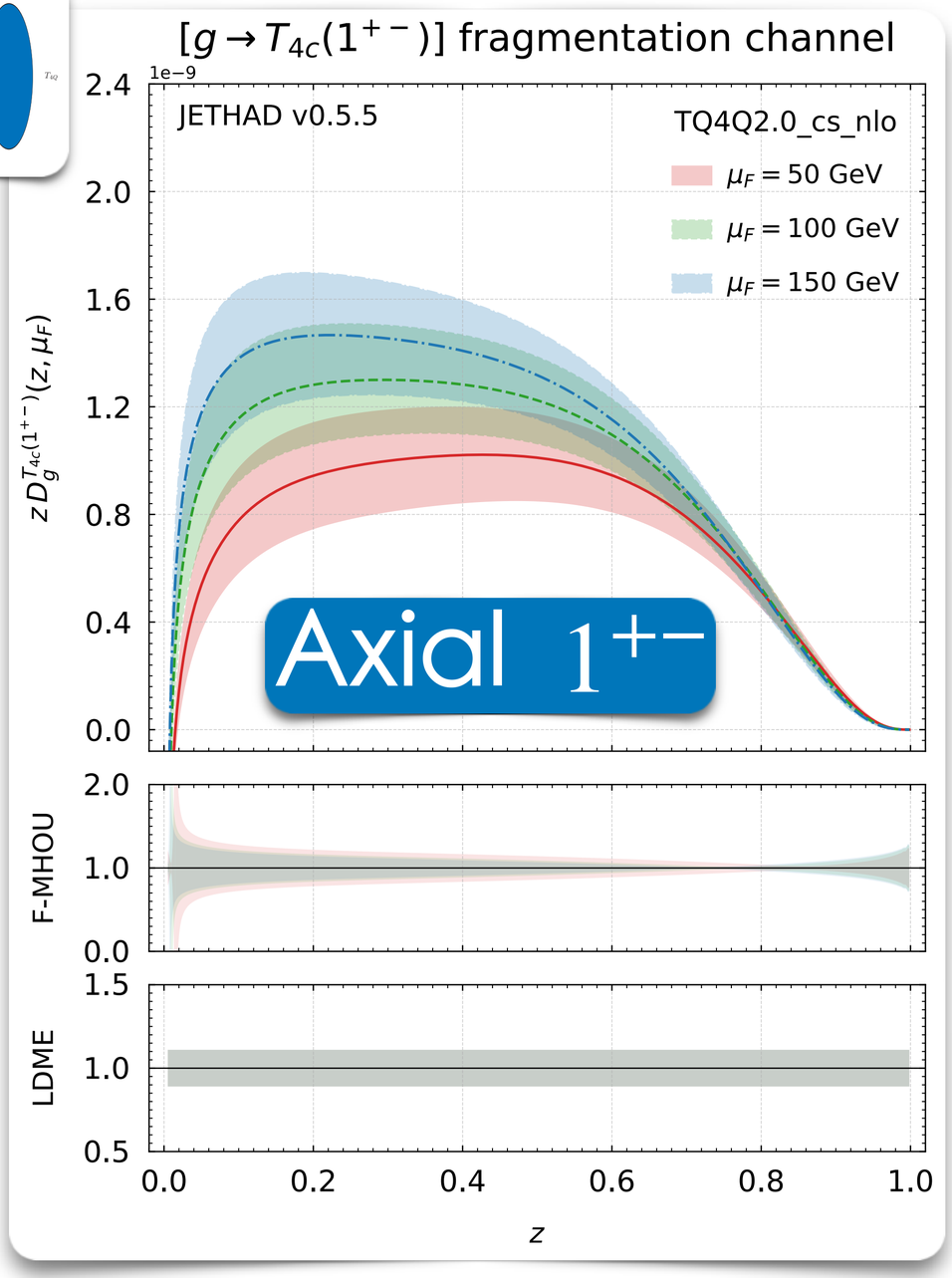


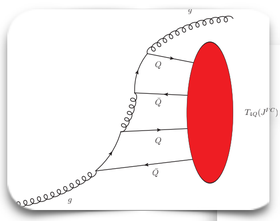


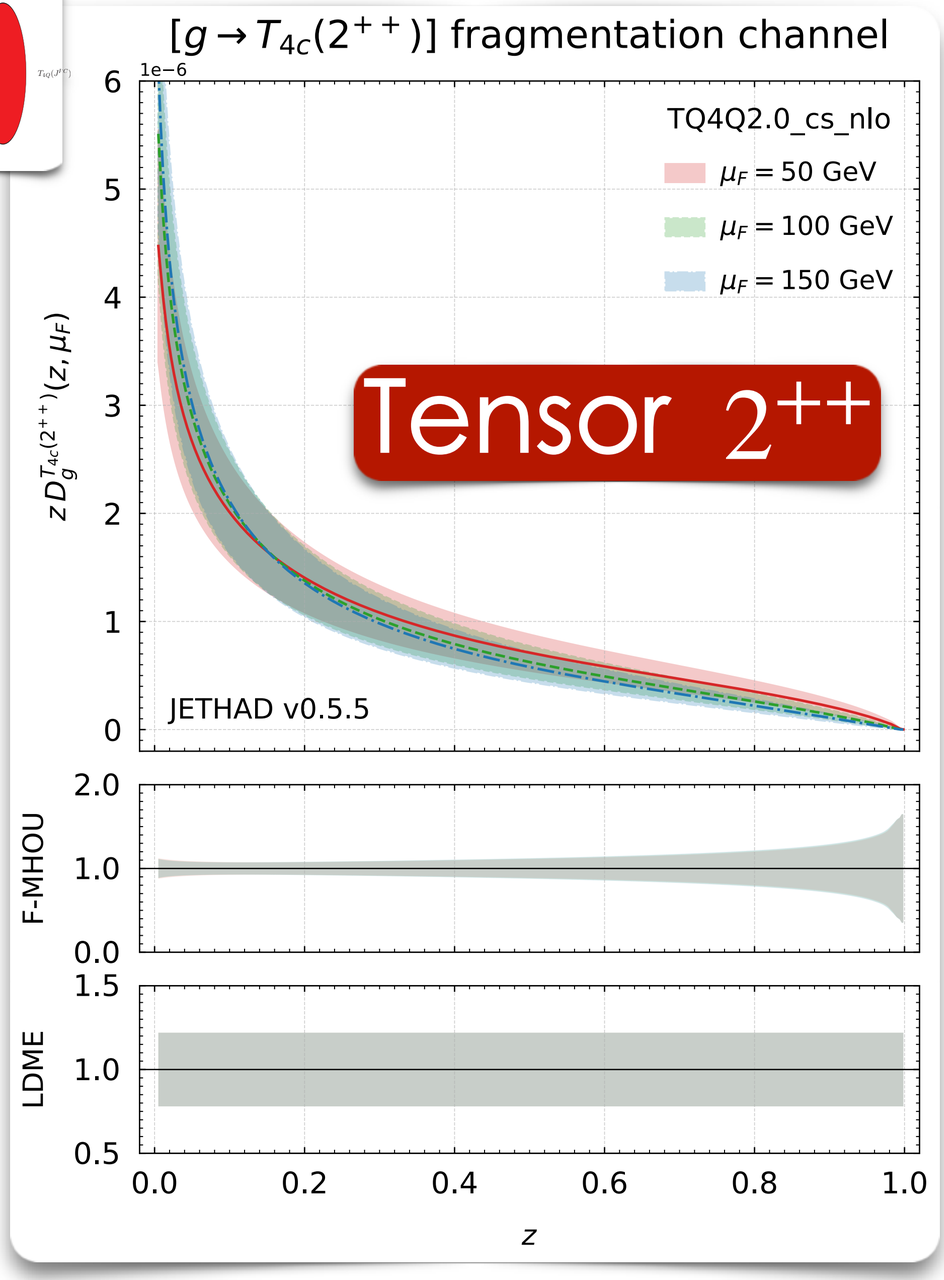


TQ4Q2.0: Gluon Fragmentation to Charmoniumlike

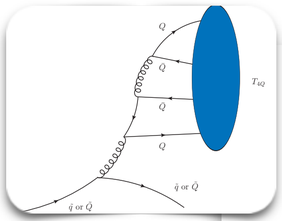


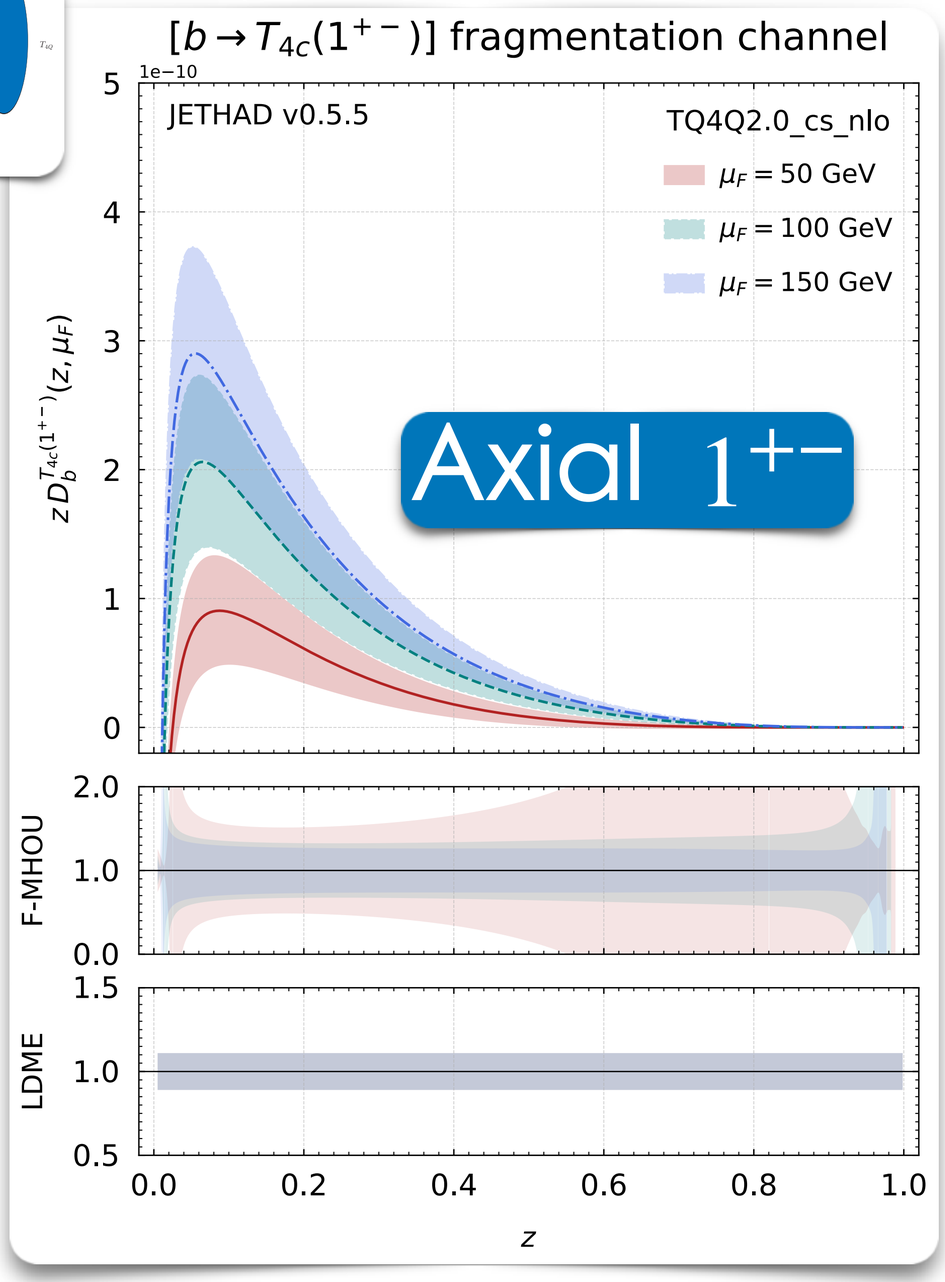


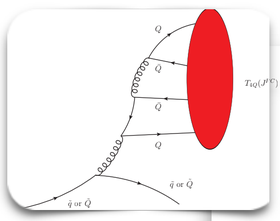


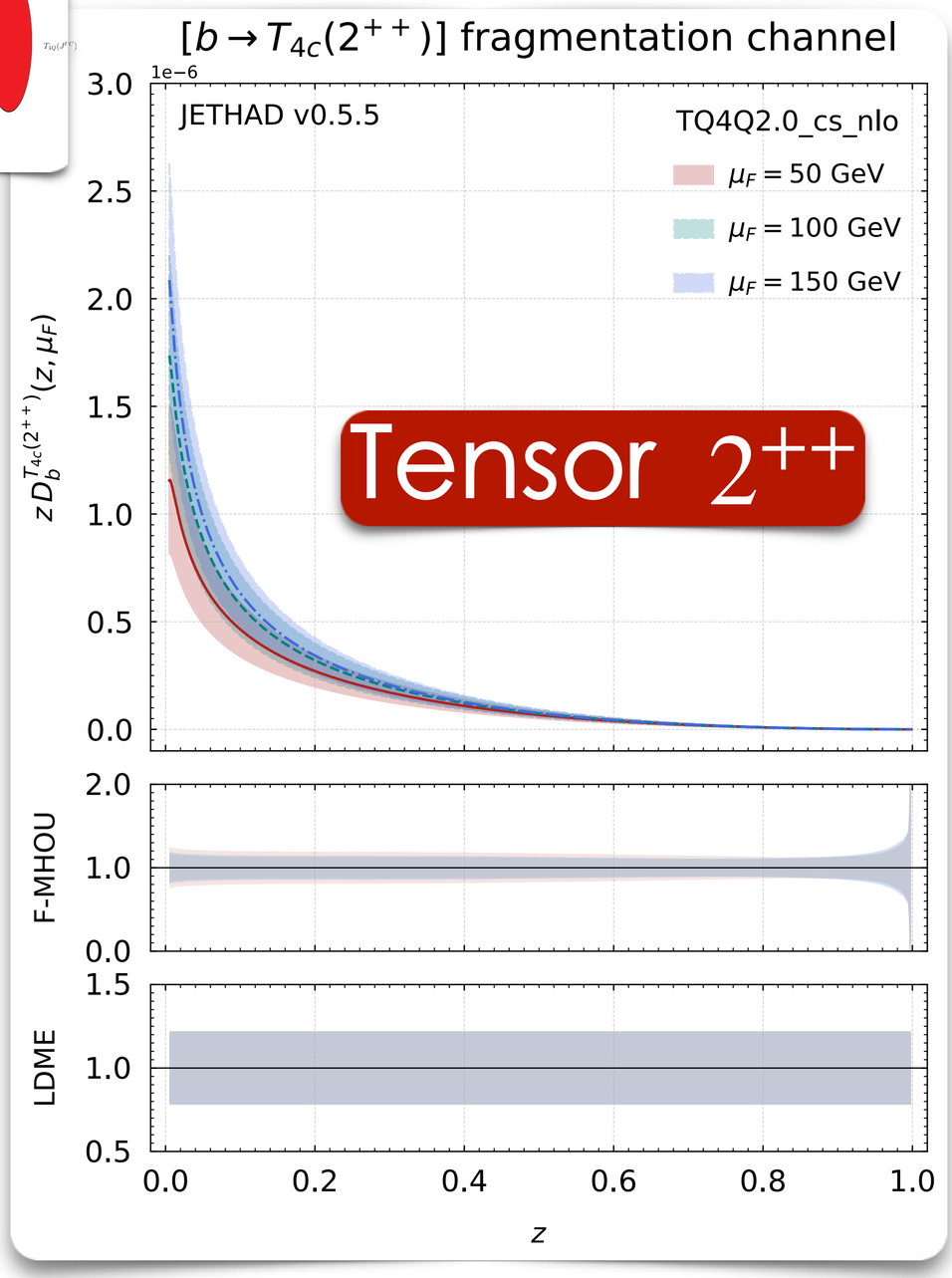


TQ4Q2.0: Bottom Fragmentation to Charmoniumlike

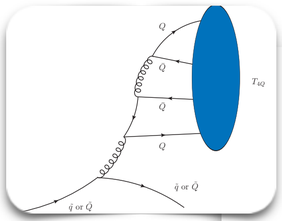


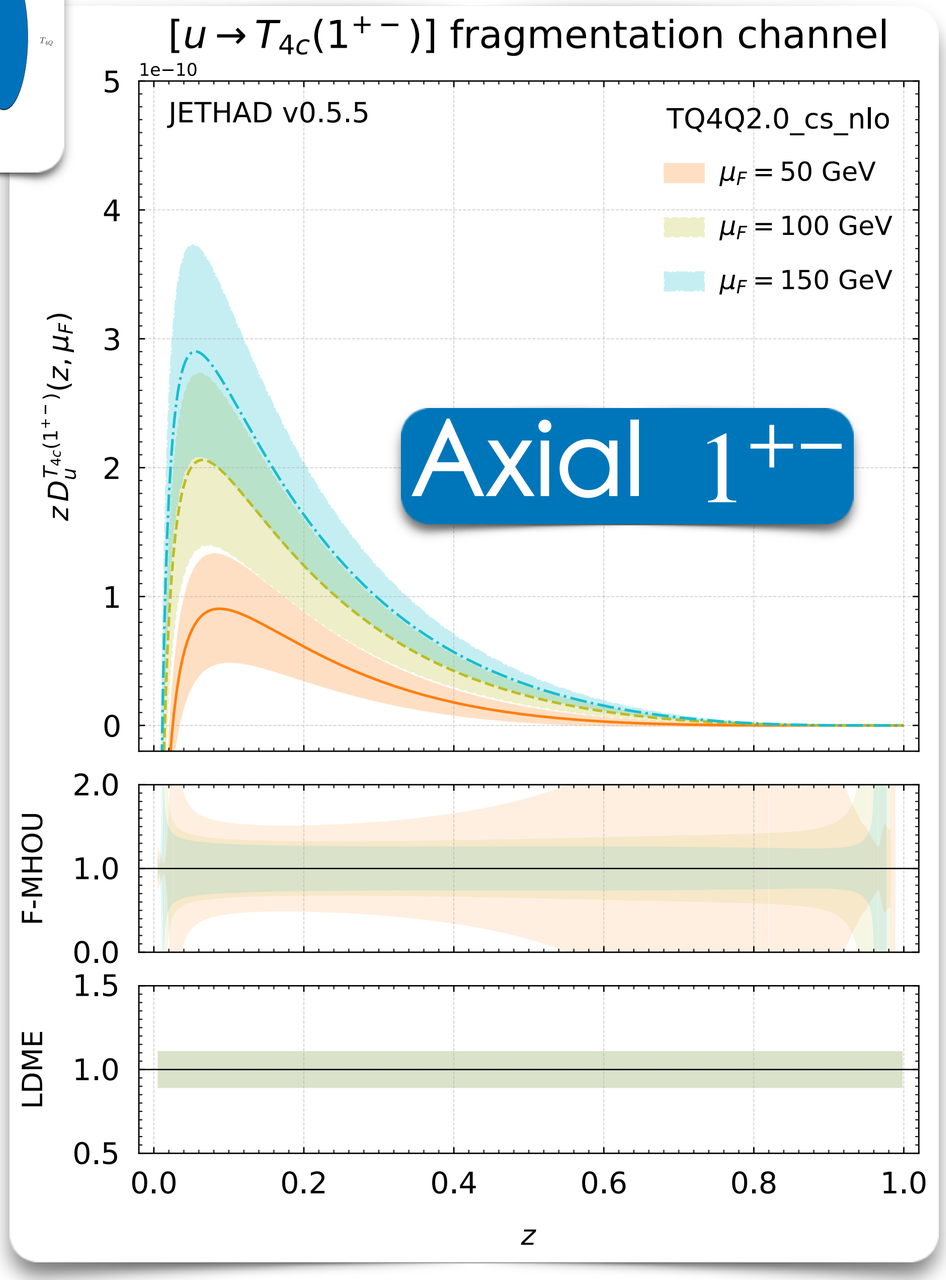


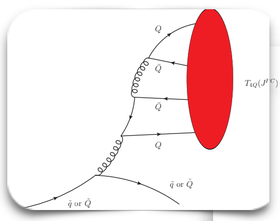


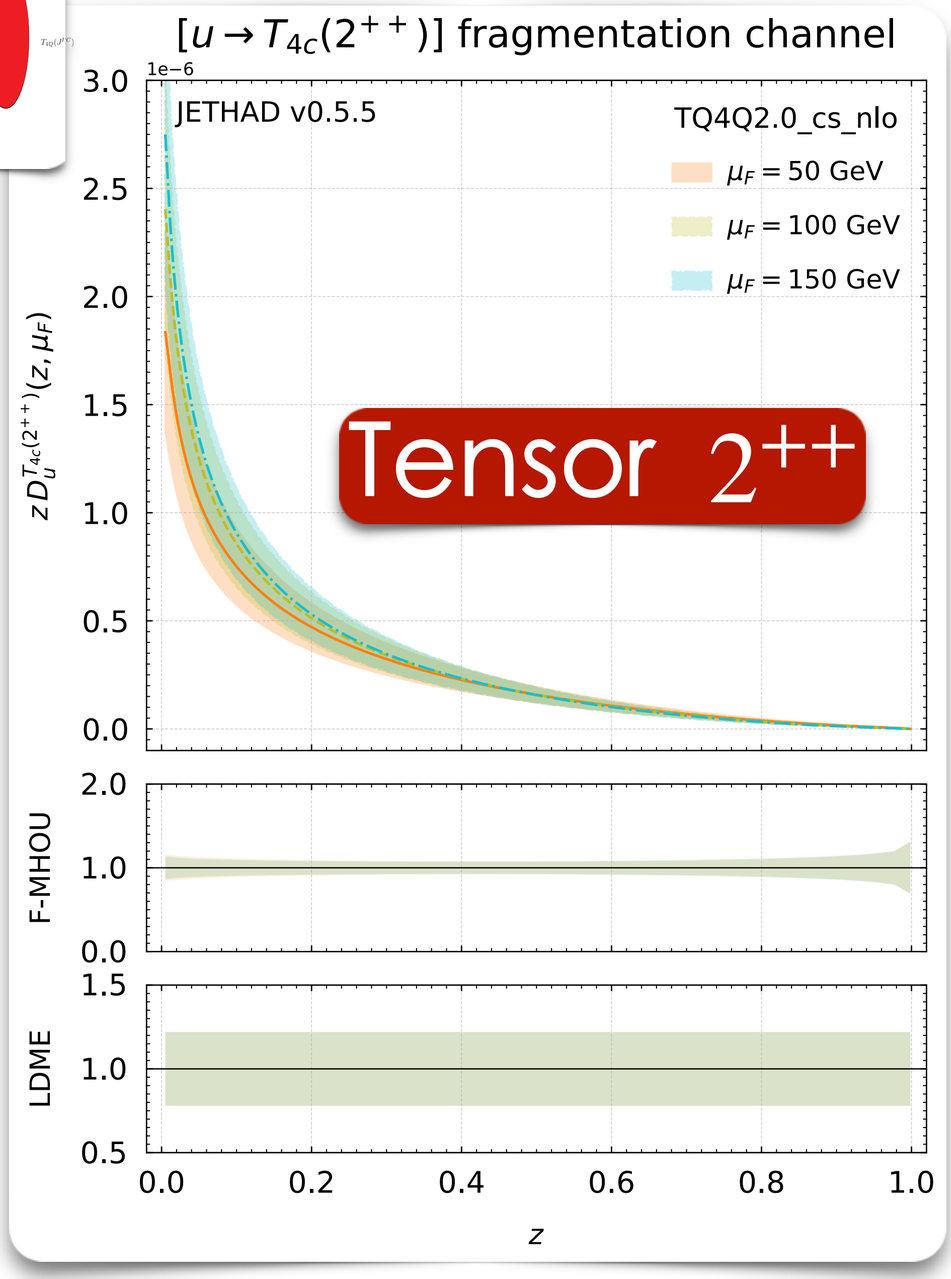


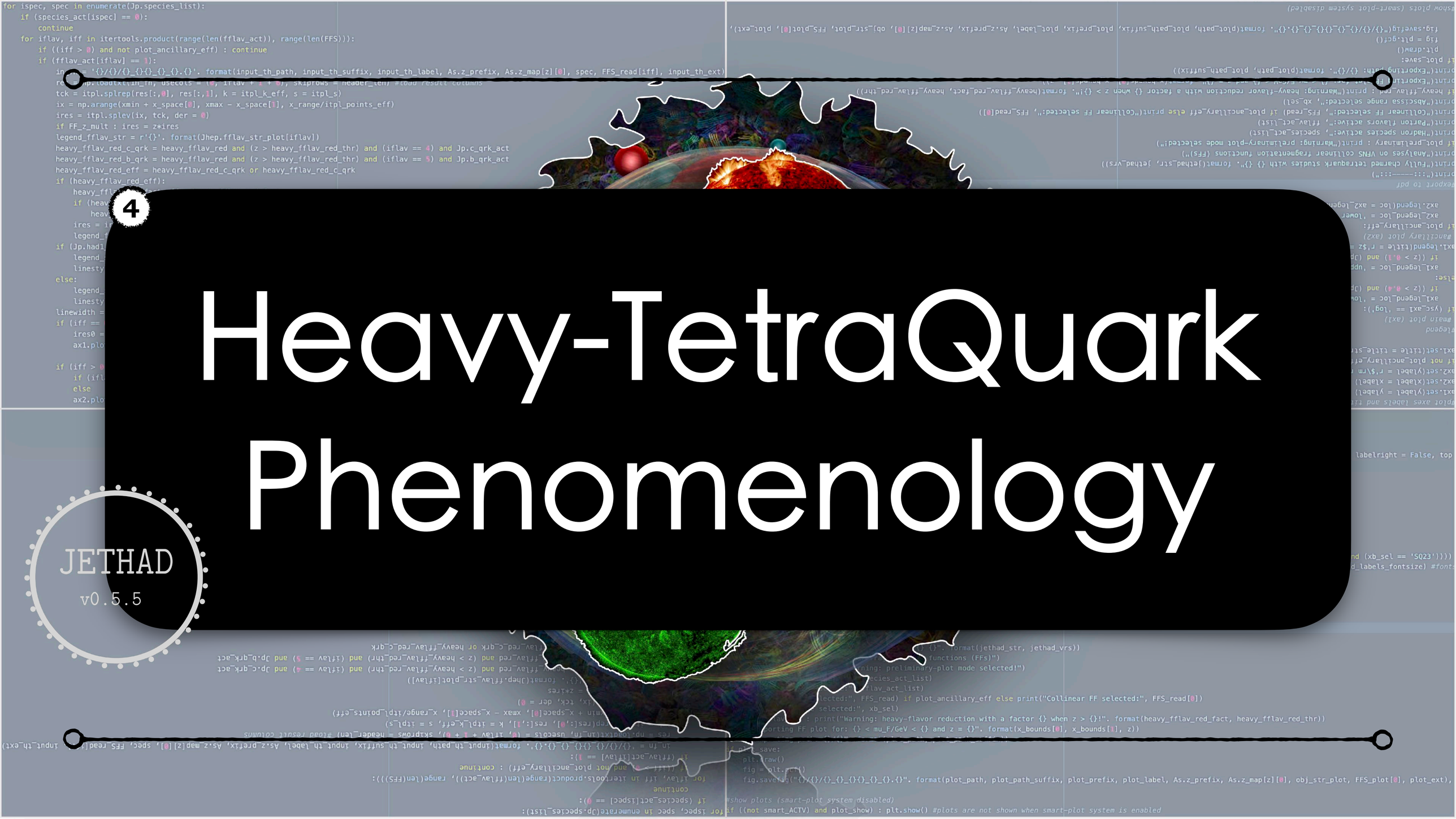
TQ4Q2.0: Light-Quark Fragmentation to Charmoniumlike










4
Heavy-TetraQuark
Phenomenology
JETHAD
v0.5.5

TetraQuark-Jet Systems with Resummation: Event Yields

Expected events yields [13 TeV Run 2]

T_{4Q}	J^{PC}	TQ4Q2.0	TQ4Q1.1	Δ (%)
T_{4c}	0^{++}	2016031 ± 399168	1768764 ± 309078	+13.98%
T_{4c}	1^{+-}	8619 ± 1770	8619 ± 1612	+0%
T_{4c}	2^{++}	3071882 ± 602910	2721362 ± 480942	+12.88%
T_{4b}	0^{++}	6046 ± 1168	5151 ± 945	+17.38%
T_{4b}	1^{+-}	17 ± 3	17 ± 2	+0%
T_{4b}	2^{++}	8095 ± 1552	7108 ± 828	+13.88%

TABLE IV: Expected event yields for all-heavy tetraquark production at $\sqrt{s} = 13$ TeV, obtained by integrating the NLL-resummed ΔY distributions over the rapidity range $|\Delta Y| < 6.5$. The yields assume an integrated luminosity of $\mathcal{L}^{(\mathrm{CMS})} = 138.6$ fb^{-1}, corresponding to the total dataset collected by CMS during Run 2 [308, 309]. The last column shows the relative variation $\Delta = (R - 1) \times 100$, with $R =$ TQ4Q2.0/TQ4Q1.1. Uncertainties are propagated from the ΔY distributions and reflect the combined effect of scale variations, FF evolution, and LDME inputs.



Pile-up effects can degrade Experimental discrimination

Expected events yields [100 TeV FCC]

T_{4Q}	J^{PC}	TQ4Q2.0	TQ4Q1.1	Δ (%)
T_{4c}	0^{++}	22140238 ± 4433814	19465674 ± 3373524	+13.74%
T_{4c}	1^{+-}	23463 ± 1094	23463 ± 1053	+0%
T_{4c}	2^{++}	32784967 ± 6421338	29144313 ± 5467493	+12.49%
T_{4b}	0^{++}	23424 ± 1982	20237 ± 1552	+15.75%
T_{4b}	1^{+-}	50 ± 2	50 ± 3	+0%
T_{4b}	2^{++}	27894 ± 1972	24493 ± 1323	+13.89%

TABLE V: Expected event yields for all-heavy tetraquark production at $\sqrt{s} = 100$ TeV, obtained by integrating the NLL-resummed ΔY distributions over the rapidity range $|\Delta Y| < 6.5$. The yields assume an integrated luminosity of $\mathcal{L}^{(\mathrm{CMS})} = 138.6$ fb^{-1}, corresponding to the full Run 2 CMS dataset [308, 309]. This value is conservatively adopted at $\sqrt{s} = 100$ TeV to enable a baseline comparison of energy scaling, independently of specific FCC projections. The last column shows the relative variation $\Delta = (R - 1) \times 100$, with $R =$ TQ4Q2.0/TQ4Q1.1. Uncertainties are propagated from the ΔY distributions and reflect the combined effect of scale variations, FF evolution, and LDME inputs.

TetraQuark-Jet Systems with Resummation: Event Yields

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TABLE IV: Expected event yields for a… tetraquark production at $\sqrt{s} = 13$ TeV, obta… integrating the NLL-resummed ΔY distributio… the rapidity range $|\Delta Y| < 6.5$. The yields assume an integrated luminosity of $\mathcal{L}^{(\mathrm{CMS})} = 138.6\ \mathrm{fb}^{-1}$, corresponding to the total dataset collected by CMS during Run 2 [308, 309]. The last column shows the relative variation $\Delta = (R - 1) \times 100$, with $R =$ TQ4Q2.0/TQ4Q1.1. Uncertainties are propagated from the ΔY distributions and reflect the combined effect of scale variations, FF evolution, and LDME inputs.

Δ(%)



Pile-up effects can degrade Experimental discrimination

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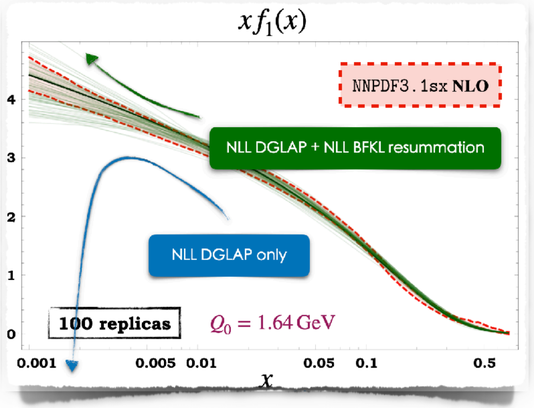
TABLE V: Expected event yields for all-h… tetraquark production at $\sqrt{s} = 100$ TeV, obtaine… integrating the NLL-resummed ΔY distributions … the rapidity range $|\Delta Y| < 6.5$. The yields assume an integrated luminosity of $\mathcal{L}^{(\mathrm{CMS})} = 138.6\ \mathrm{fb}^{-1}$, corresponding to the full Run 2 CMS dataset [308, 309]. This value is conservatively adopted at $\sqrt{s} = 100$ TeV to enable a baseline comparison of energy scaling, independently of specific FCC projections. The last column shows the relative variation $\Delta = (R - 1) \times 100$, with $R =$ TQ4Q2.0/TQ4Q1.1. Uncertainties are propagated from the ΔY distributions and reflect the combined effect of scale variations, FF evolution, and LDME inputs.

Δ(%)

All-Charm TetraQuarks & Intrinsic Charm

Quakonium Studies: Challenges

Forward Onia ⇒ Gluon PDF & TMD Positivity



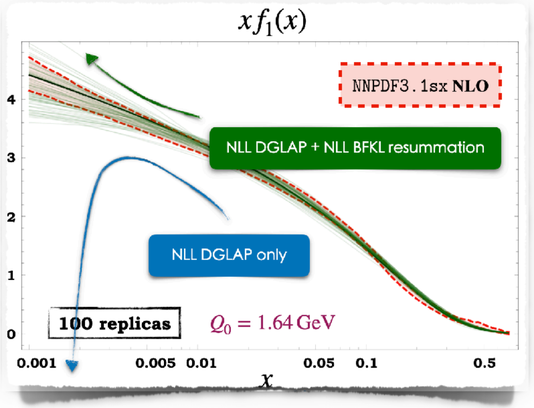
Low x and low Q^2

- [G. Altarelli et al. (1998)]
- [A. Candido et al. (2020)]
- [J. Collins et al. (2022)]
- [A. Candido et al. (2023)]

All-Charm TetraQuarks & Intrinsic Charm

Quakonium Studies: Challenges

Forward Onia ⇒ Gluon PDF & TMD Positivity

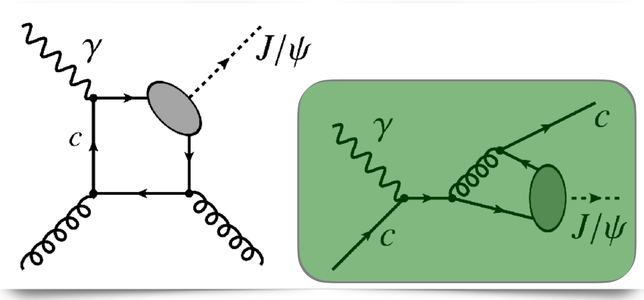


Low x and low Q^2

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🔗 [A. Candido et al. (2020)]
🔗 [J. Collins et al. (2022)]
🔗 [A. Candido et al. (2023)]

$\gamma + g \rightarrow J/\psi + c$ ⇒ Valence Intrinsic Charm @EIC

Large x
Moderate to large p_T

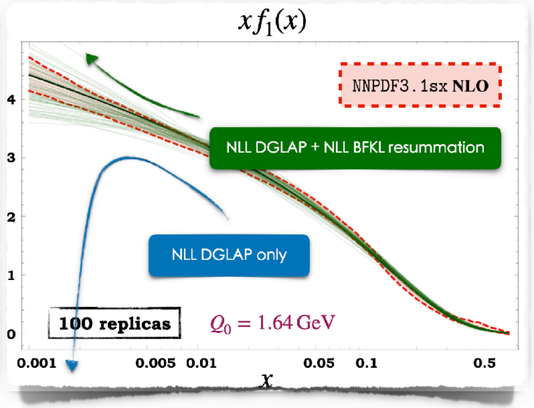


(J/ψ @EIC) 🔗 [C. Flore, J.-P. Lansberg, H.-S. Shao, Y. Yedelkina (2020)]
(Intrinsic Charm + valence) studies 🔗🔗 [NNPDF Collaboration (2022, 2023)]
🔗🔗 [CT Collaboration (2018, 2023)]

All-Charm TetraQuarks & Intrinsic Charm

Quakonium Studies: Challenges

Forward Onia ⇒ Gluon PDF & TMD Positivity

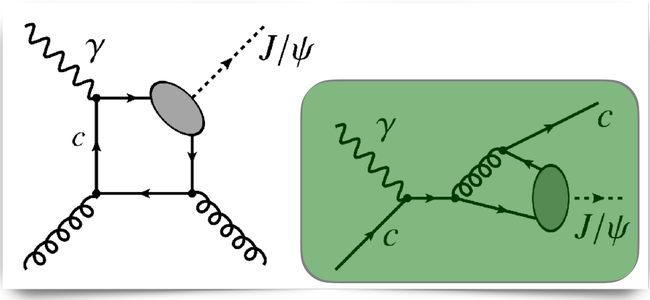


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Moderate to large p_T



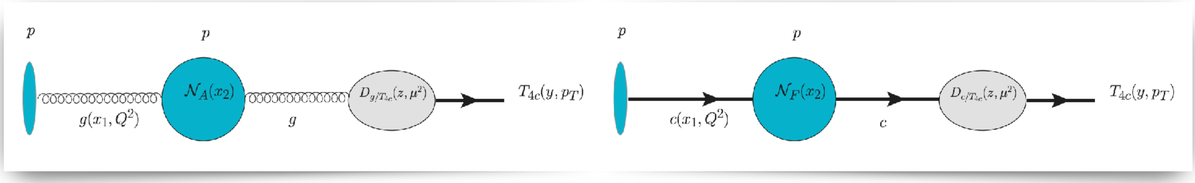
(J/ψ @EIC) 🔗 [C. Flore, J.-P. Lansberg, H.-S. Shao, Y. Yedelkina (2020)]
(Intrinsic Charm + valence) studies 🔗🔗 [NNPDF Collaboration (2022, 2023)]
🔗🔗 [CT Collaboration (2018, 2023)]

TetraQuark Explorations: Assets

$\{T_{4c}\} \ni X(6900)$ ⇒ Charm PDF @HL-LHC, FCC

Forward 0^{++}, 1^{+-}, 2^{++} within the CGC

🔗 [F.G. C, A.V. Giannini, V.P. Gonçalves, Y.N. Lima (2025)]

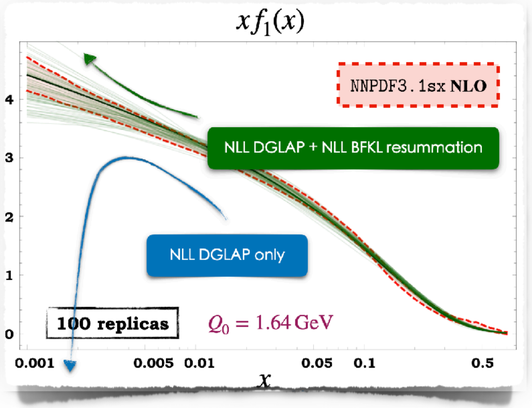


$$\sigma(pp \to T_{4c}X) \propto g(x_1, Q^2) \otimes \mathcal{N}_A(x_2) \otimes D_{g/T_{4c}} + c(x_1, Q^2) \otimes \mathcal{N}_F(x_2) \otimes D_{c/T_{4c}}$$

All-Charm TetraQuarks & Intrinsic Charm

Quakonium Studies: Challenges

Forward Onia ⇒ Gluon PDF & TMD Positivity

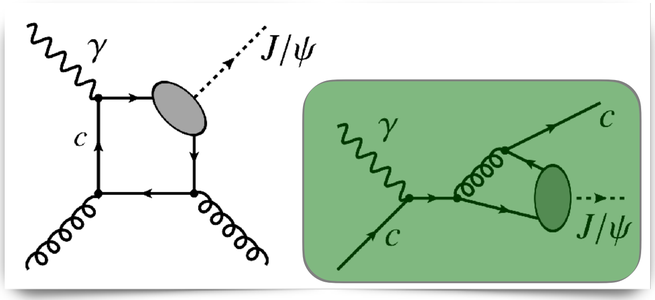


Low x and low Q^2

🔗 [G. Altarelli et al. (1998)]
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🔗 [J. Collins et al. (2022)]
🔗 [A. Candido et al. (2023)]

$\gamma + g \to J/\psi + c$ ⇒ Valence Intrinsic Charm @EIC

Large x
Moderate to large p_T



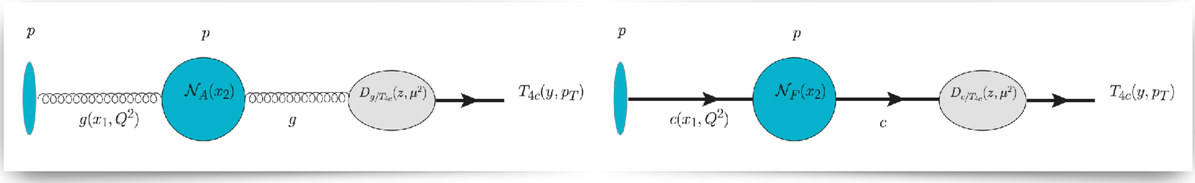
(J/ψ @EIC) 🔗 [C. Flore, J.-P. Lansberg, H.-S. Shao, Y. Yedelkina (2020)]
(Intrinsic Charm + valence) studies 🔗🔗 [NNPDF Collaboration (2022, 2023)]
🔗🔗 [CT Collaboration (2018, 2023)]

TetraQuark Explorations: Assets

$\{T_{4c}\} \ni X(6900)$ ⇒ Charm PDF @HL-LHC, FCC

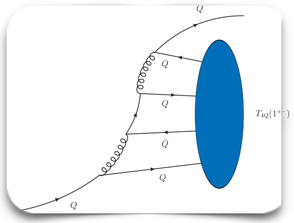
Forward 0^{++}, 1^{+-}, 2^{++} within the CGC

🔗 [F.G. C, A.V. Giannini, V.P. Gonçalves, Y.N. Lima (2025)]



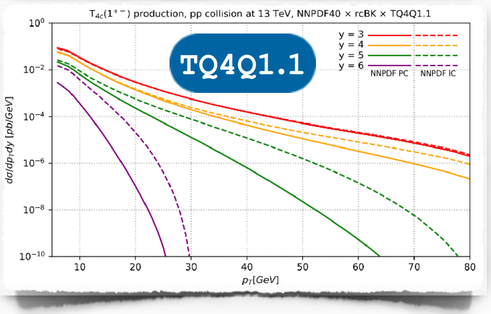
$$\sigma(pp \to T_{4c}X) \propto g(x_1, Q^2) \otimes \mathcal{N}_A(x_2) \otimes D_{g/T_{4c}} + c(x_1, Q^2) \otimes \mathcal{N}_F(x_2) \otimes D_{c/T_{4c}}$$

Forward $T_{4c}(1^{+-})$ ⇒ Intrinsic Charm @pp, pA



1^{+-} FFs

dilute-dense



A Multimodal Perspective

Final Remarks

JETHAD
v0.5.5

A Portal to Exotic Matter

Exotic Matter State-Driven MultiModal Fragmentation

Quantum Superposition



$+ e^{-i\vartheta_1 \Delta t}$



$+ e^{-i\vartheta_2 \Delta t}$



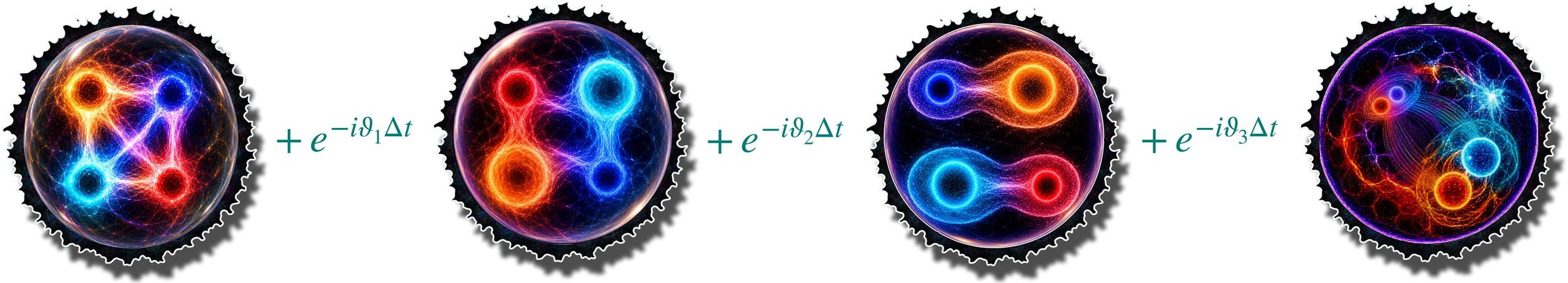
$+ e^{-i\vartheta_3 \Delta t}$



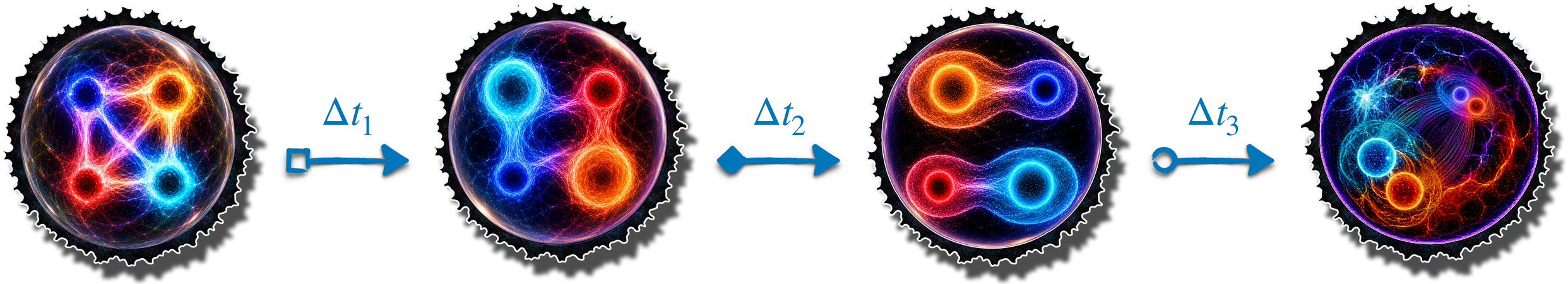
Compact | Diquark | Molecule | Hadro-onium

Exotic Matter State-Driven MultiModal Fragmentation

Quantum Superposition



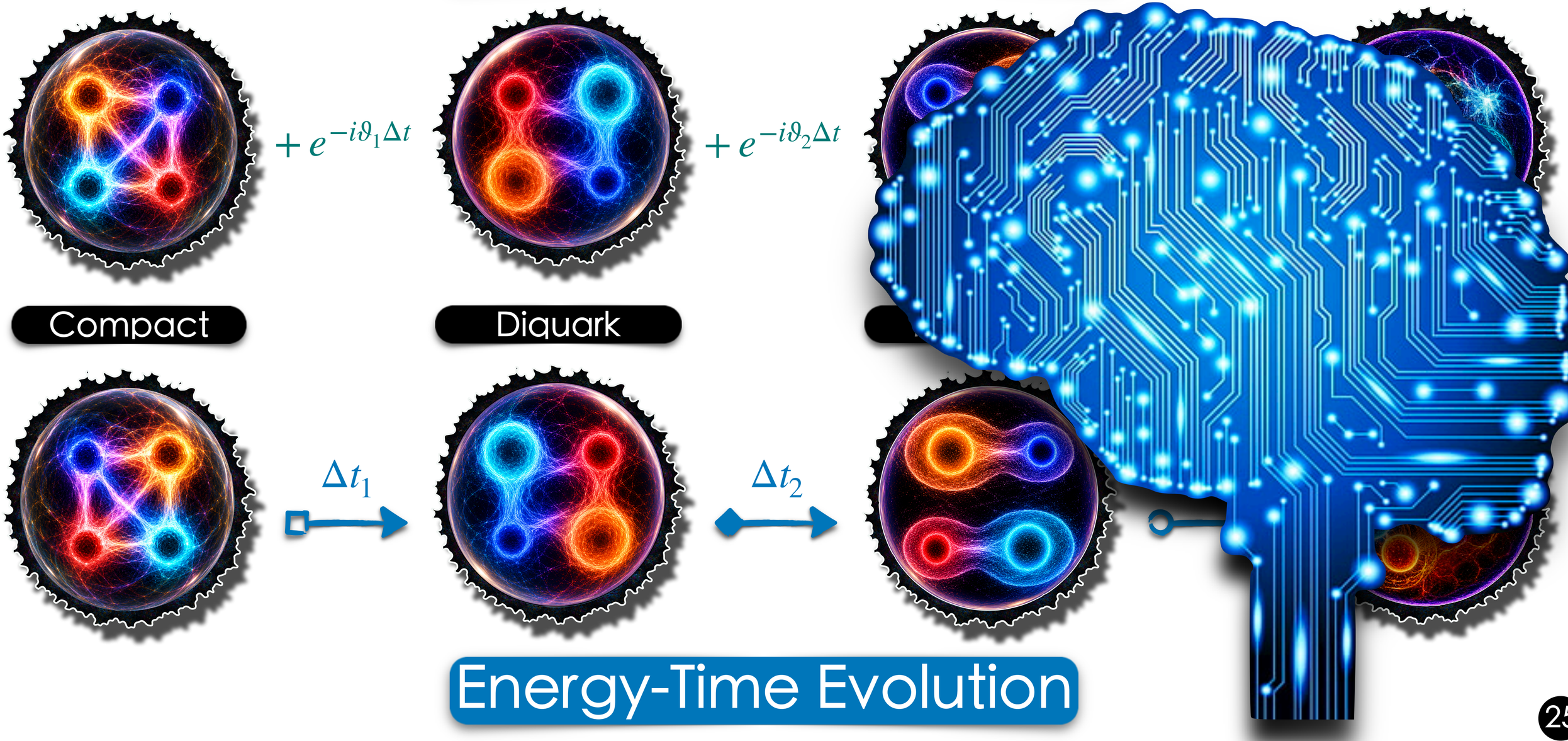
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Energy-Time Evolution

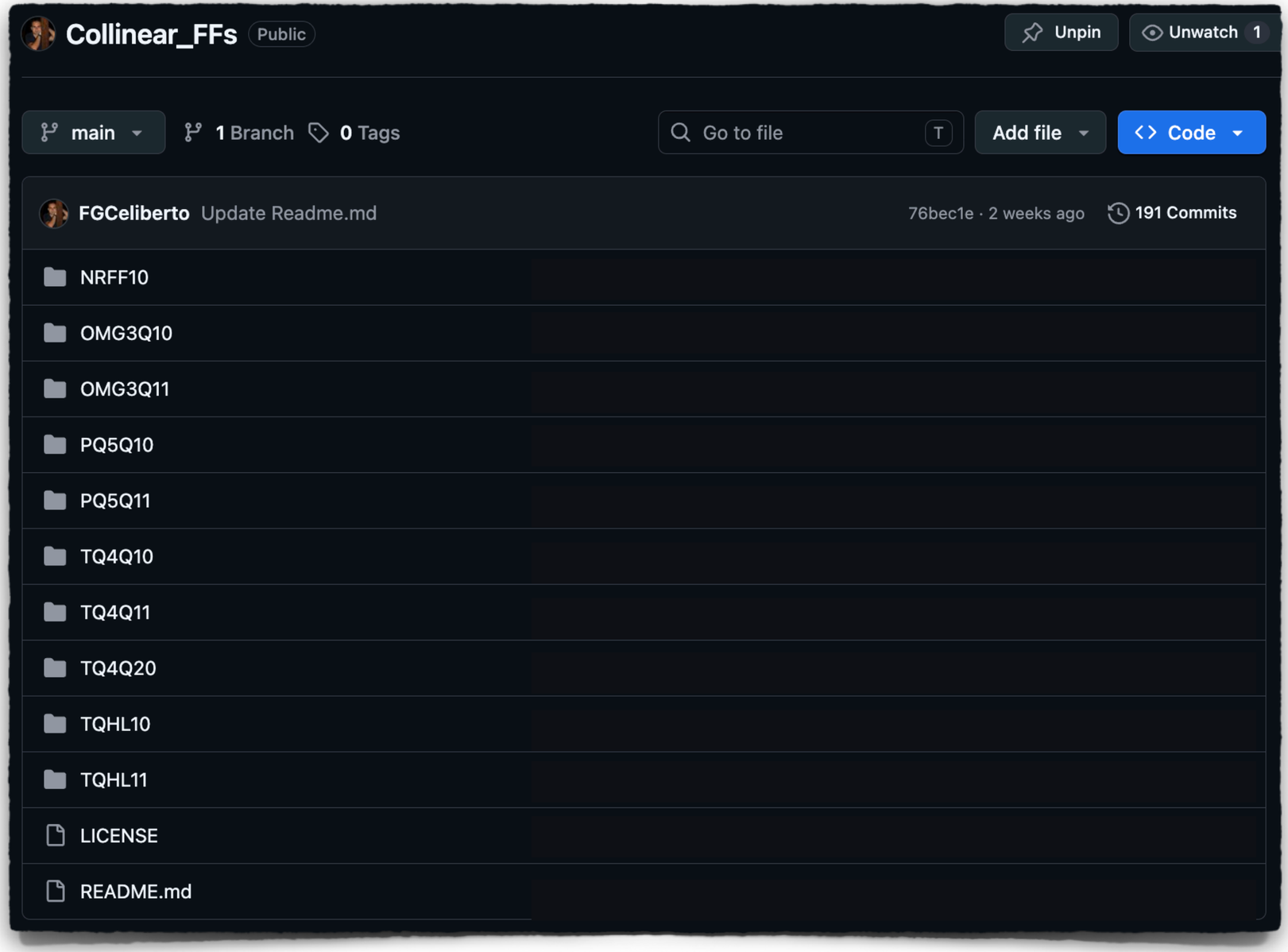
Exotic Matter State-Driven MultiModal Fragmentation

Quantum Superposition



Energy-Time Evolution

Heavy-Flavor Fragmentation from HF-NRevo



Heavy-Flavor Fragmentation from **HF-NRevo**

Collinear_FFs Public

main | 1 Branch | 0 Tags | Go to file

FGCeliberto Update Readme.md

- NRFF10
- OMG3Q10
- OMG3Q11
- PQ5Q10
- PQ5Q11
- TQ4Q10
- TQ4Q11
- TQ4Q20
- TQHL10
- TQHL11
- LICENSE
- README.md

README | CC0-1.0 license

Collinear_FFs

Collections of collinear fragmentation functions (FFs) given as LHPDF grids.

Available FF families to date:

1. NRFFx.y (NonRelativistic Fragmentation Functions)

   NRFF1.0 (NLO, 2 sets, central value only, LDME uncertainties)

2. OMG3Qx.y (OMeGa baryons with 3 heavy Quarks VFNS FFs)

   OMG3Q1.0 (NLO, 2 sets, central value only, diquark-like)

   OMG3Q1.1 (NLO, 2 sets, F-MHOU + F-NPWF uncertainties, replica 0->8, diquark-like)

3. TQHLx.y (TetraQuarks with Heavy and Light flavors)

   TQHL1.0 (NLO, 4 sets, central value only)

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   TQ4Q2.0 (NLO, 6 sets, replica-like MHOU uncertainties)

5. PQ5Qx.y (PentaQuarks with 5 heavy Quarks)

   PQ5Q1.0 (NLO, 2 sets, replica 0 = direct multiquark, replica 1 = diquark-like)

   PQ5Q1.1 (NLO, 1 set, F-MHOU + F-NPWF uncertainties, replica 0->8 = direct multicharm, replica 9->17 = diquark-like)

🔗 https://github.com/ FGCeliberto/Collinear_FFs/

Heavy-Flavor Fragmentation from **HF-NRevo**

Collinear_FFs Public

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NEW

TQHL2.0 ¡to be released soon!

🔗 https://github.com/ FGCeliberto/Collinear_FFs/

Toward **ExoFF**

Toward ExoFF



Exotic hadrons ⇒ precision QCD & production mechanisms

Toward **ExoFF**



Exotic hadrons ⇒ precision QCD & production mechanisms



HF-NRevo ⇒ consistent evolution, uncertainty-aware FFs

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- From theory-driven benchmarks to state-oriented fragmentation

Toward **ExoFF**

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- **HF-NRevo** ⇒ consistent evolution, uncertainty-aware FFs
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- From theory-driven benchmarks to state-oriented fragmentation
- Road to a MULTIMODAL **ExoFF** framework

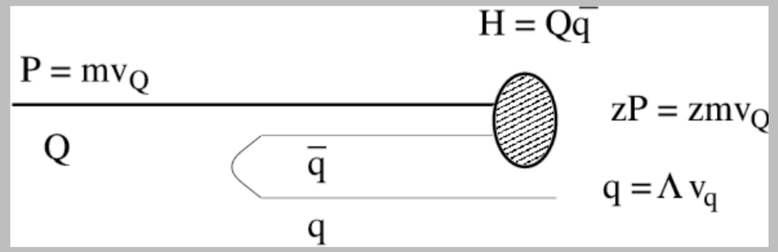
EXTRAS

Highlights on Heavy-Flavor Fragmentation

We observe hadrons, whose momentum is different from that of the parent heavy quark. We usually write this as

$$\text{FF(hadron)} = \text{FF(quark)} \otimes \text{FF(quark}\rightarrow\text{hadron)}$$

What are the characteristics of FF(quark→hadron)?



The heavy quark has momentum $P = mv_Q$, the light quarks have momentum $q = \Lambda v_q$, with Λ a hadronic mass scale

For the binding we need $v_Q \simeq v_q = v$. We have then

$$P = zP + q \qquad mv = zmv + \Lambda v$$

and therefore

$$\langle z \rangle \simeq 1 - \frac{\Lambda}{m}$$

Hence, FF(quark→hadron) is peaked near $z = 1$ and binding effects scale linearly in the heavy quark mass (Bjorken/Suzuki ($\sim$ 1977))

(NB. more rigorous derivations of this result are available)

Quarkonia as Tools: Precision & Exploration

QUARKONIA

QCD Hydrogen Atoms

Precision determinations of QCD running coupling

$\Upsilon(1S)$ hadronic inclusive decays

[CLEO Collaboration (2006)]
[N. Brambilla et al. (2007)]
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Forward inclusive reactions

Gluon PDF positivity at low x and low Q^2

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Quarkonia in nuclear matter

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Quarkonia as Tools: Precision & Exploration

Portals to exotic matter and BSM physics

Hadroquarkonium

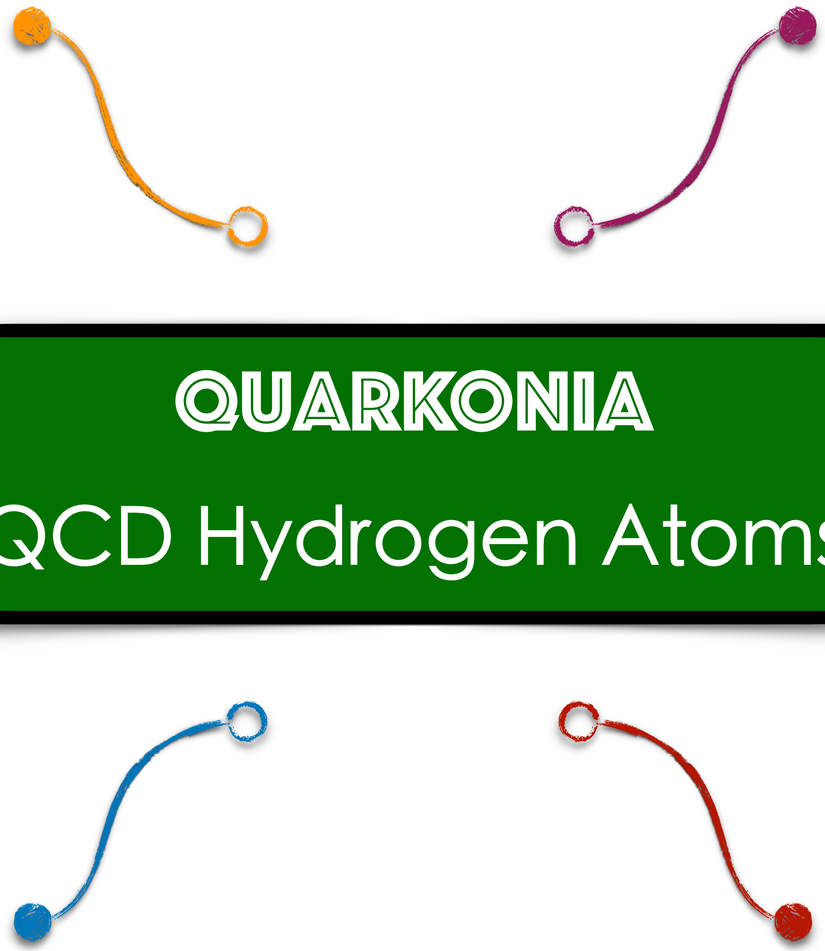
EW boson rare decays

[S. Dubynskiy et al. (2008)]
[F.-K. Guo et al. (2018)]
[J. Ferretti et al. (2020)]
[CMS Collaboration (2019)]

[E. G. Ferreiro (2015)]
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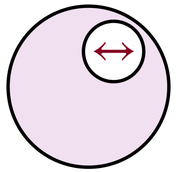
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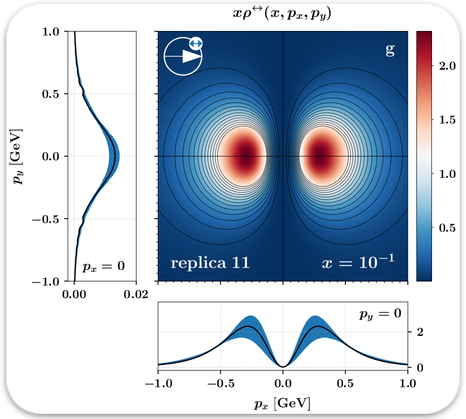
Quarkonium Studies: Assets & Challenges

Hadronic structure: Assets



EIC, LHCb, LHCspin

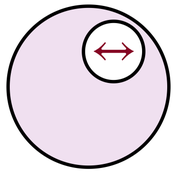
Boer-Mulders



🔗 [A. Bacchetta, F.G. C., M. Radici, P. Taels (EPJC 2020)]

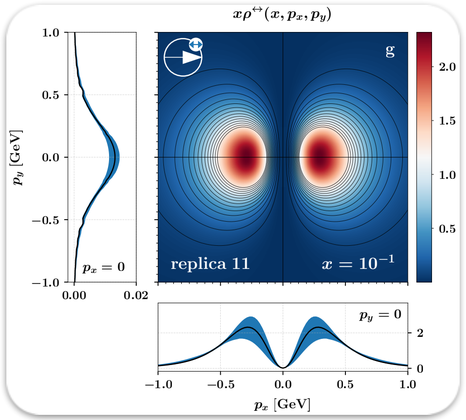
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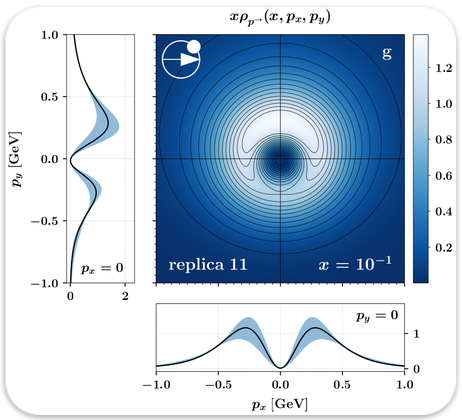
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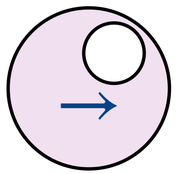
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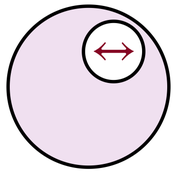


EIC, LHCspin

Sivers

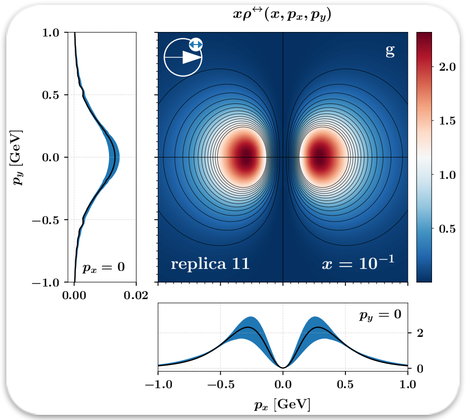
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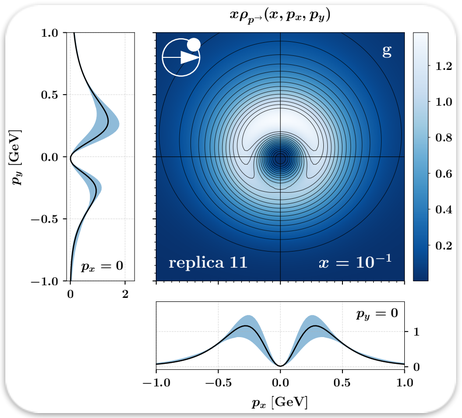


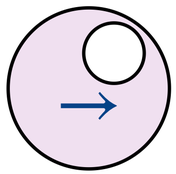
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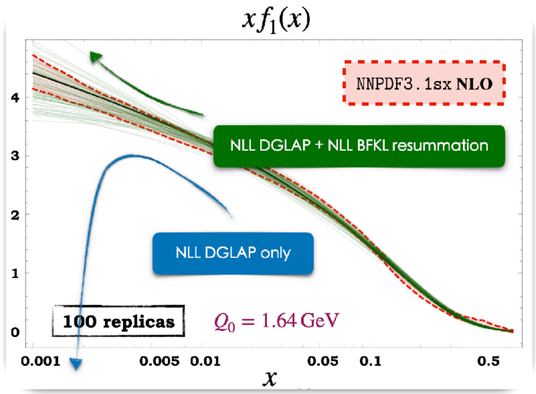


EIC, LHCspin

Sivers

Precision QCD: Challenges

Forward Onia ⇒ Gluon PDF & TMD Positivity

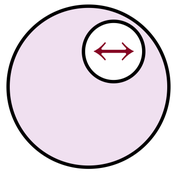


Low x and low Q^2

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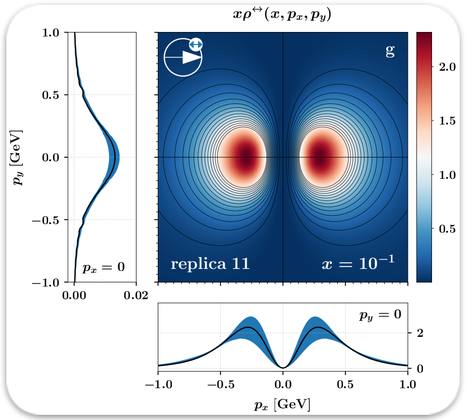
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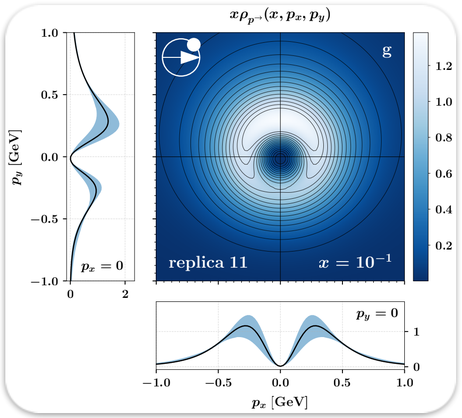


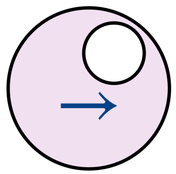
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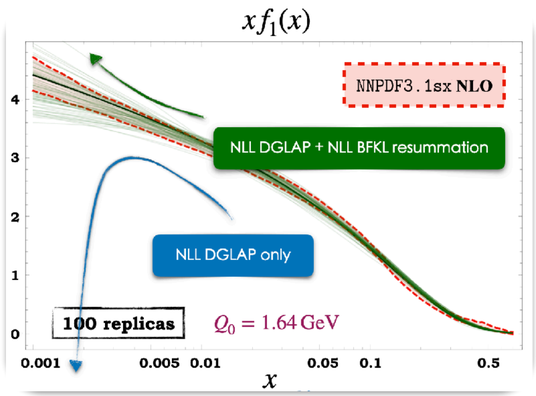


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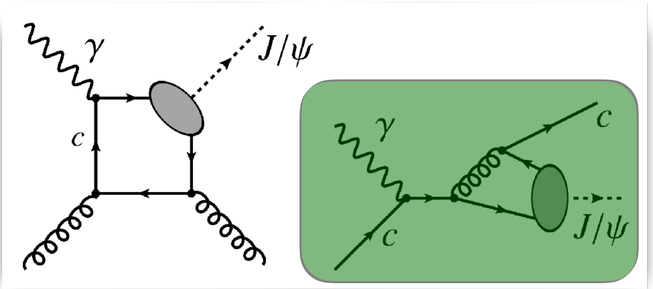


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$\gamma + g \rightarrow J/\psi + c$ ⇒ Valence Intrinsic Charm @EIC

Large x
Moderate to large p_T



(J/ψ @EIC) 🔗 [C. Flore, J.-P. Lansberg, H.-S. Shao, Y. Yedelkina (2020)]
(Intrinsic Charm + valence) studies 🔗🔗 [NNPDF Collaboration (2022, 2023)]

Highlights of Non-Relativistic QCD

NRQCD → effective field theory → nonrelativistic (NR) $Q\bar{Q}$ system

Heavy-quark spinor fields as NR DOFs in the NQRCD Lagrangian

$$\mathcal{L}_{\mathrm{NRQCD}} = \psi^\dagger \left(iD_0 + \frac{\vec{D}^2}{2m_Q} \right) \psi + \chi^\dagger \left(iD_0 - \frac{\vec{D}^2}{2m_Q} \right) \chi + \mathcal{L}_{\mathrm{light}} + \delta\mathcal{L}$$

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[cross section] = [short-distance coeffs.] ⊗ [long-distance matrix elements]

$$d\sigma(H + X) = \sum_n d\hat{\sigma}(Q\overline{Q}\,[n] + X) \left\langle \mathcal{O}^H\,[n] \right\rangle$$

Highlights of Non-Relativistic QCD

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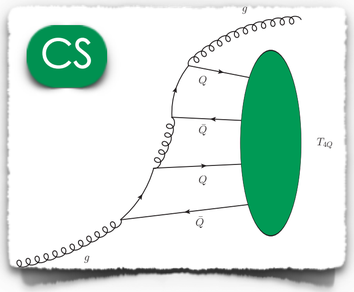
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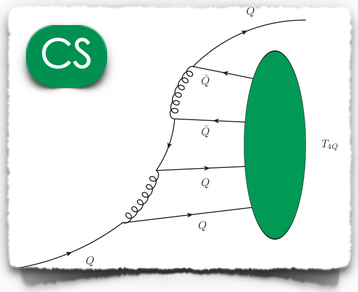
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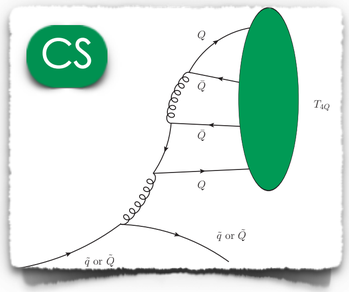
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$$|\mathcal{Q}\rangle = \mathcal{O}(1)\,|Q\bar{Q}[^3S_1^{(1)}]\rangle + \mathcal{O}(v)\,|Q\bar{Q}[^3P_J^{(8)}g]\rangle + \mathcal{O}(v^2)\,|Q\bar{Q}[^1S_0^{(8)}g]\rangle + \mathcal{O}(v^2)\,|Q\bar{Q}[^3S_1^{(1,8)}gg]\rangle + \mathcal{O}(v^2)\,|Q\bar{Q}[^3D_J^{(1,8)}gg]\rangle + \ldots .$$

HF-NRevo: Initial-Scale NRCQD Inputs





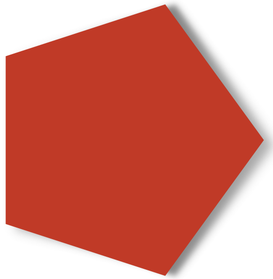


Scalar $[J^{PC} = 0^{++}]$

Tensor $[J^{PC} = 2^{++}]$

HF-NRevo for $T_{4c}(0^{++}, 2^{++})$ collinear fragmentation

Partonic hard factor



DGLAP AOevo

All-Order evolution (numeric)

DGLAP EDevo

Expanded Decoupled evolution

NRQCD

Color-composite LDMEs

SDCs

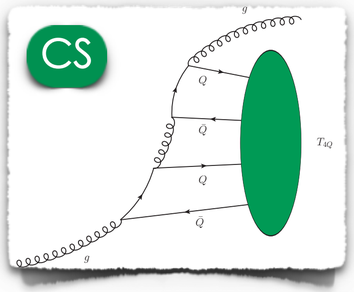
$\mathcal{D}_i^{\mathcal{Q}}(z,[n]) \otimes \langle \mathcal{O}^{\mathcal{Q}}([n]) \rangle$

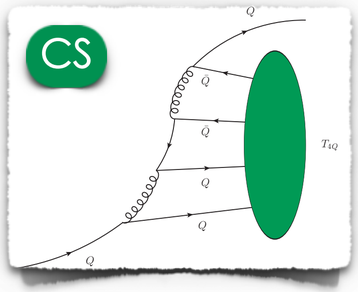
Energy

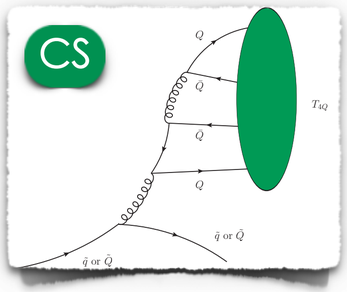
Evolution-ready scale $Q_0 \equiv m_b + 4m_c \quad 5m_c \quad m_{\tilde{q}} + 4m_c$

Time

HF-NRevo: Initial-Scale NRCQD Inputs





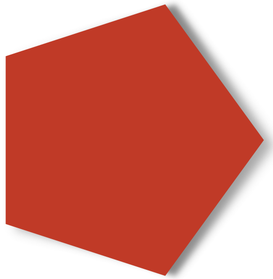


Scalar $[J^{PC} = 0^{++}]$

Tensor $[J^{PC} = 2^{++}]$

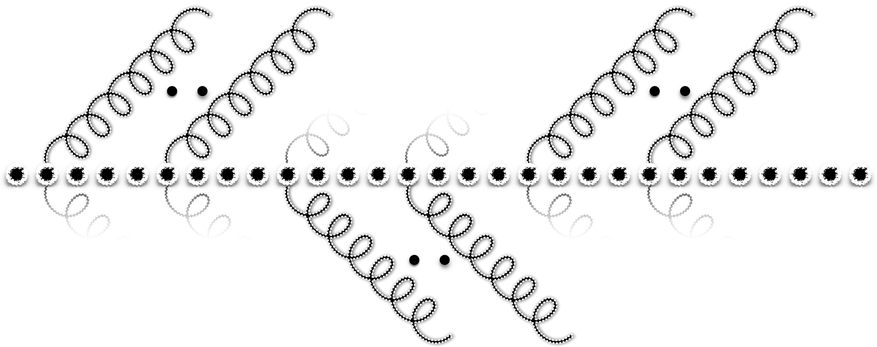
HF-NRevo for $T_{4c}(0^{++}, 2^{++})$ collinear fragmentation

Partonic hard factor



DGLAP AOevo

All-Order evolution (numeric)



DGLAP EDevo

Expanded Decoupled evolution

NRQCD

Color-composite LDMEs

SDCs

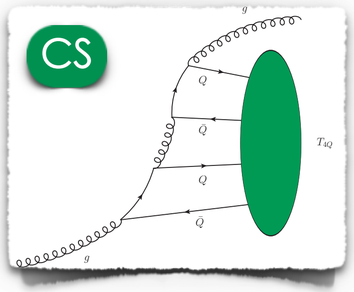
$\mathcal{D}_i^{\mathcal{Q}}(z,[n]) \otimes \langle \mathcal{O}^{\mathcal{Q}}([n]) \rangle$

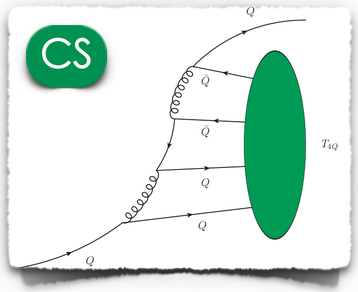
Energy

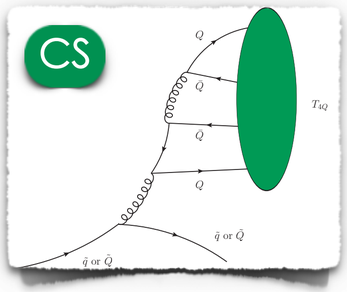
Evolution-ready scale $Q_0 \equiv m_b + 4m_c \quad 5m_c \quad m_{\tilde{q}} + 4m_c$

Time

HF-NRevo: Initial-Scale NRCQD Inputs





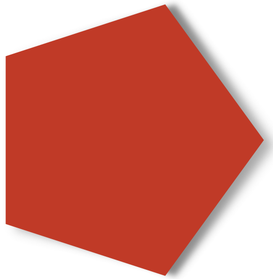


Scalar $[J^{PC} = 0^{++}]$

Tensor $[J^{PC} = 2^{++}]$

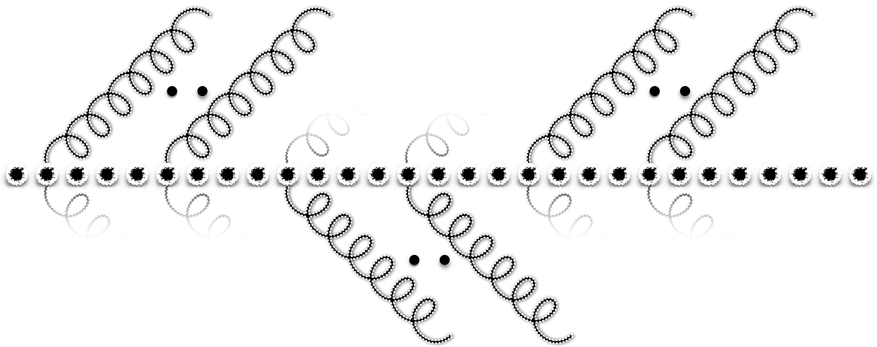
HF-NRevo for $T_{4c}(0^{++}, 2^{++})$ collinear fragmentation

Partonic hard factor



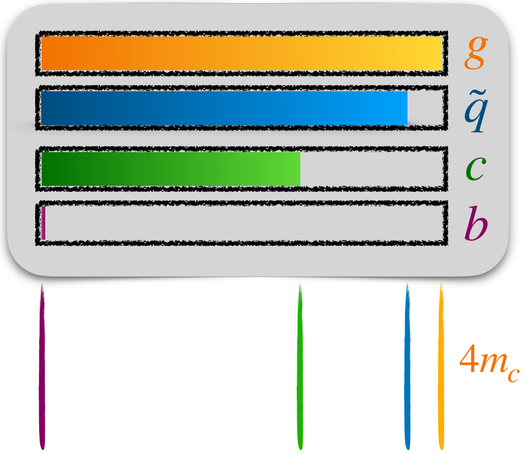
DGLAP AOevo

All-Order evolution (numeric)



DGLAP EDevo

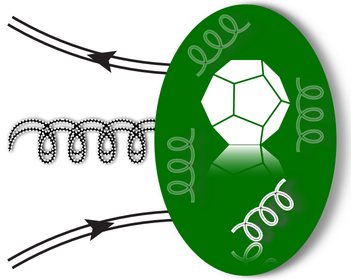
Expanded Decoupled evolution



NRQCD

Color-composite LDMEs

SDCs



$\mathcal{D}_i^{\mathcal{Q}}(z,[n]) \otimes \langle \mathcal{O}^{\mathcal{Q}}([n]) \rangle$

Energy

Evolution-ready scale $Q_0 \equiv m_b + 4m_c$ $5m_c$ $m_{\tilde{q}} + 4m_c$

Time

HIDDEN-HEAVY TETRAQUARK MULTIMODAL FRAGMENTATION

TQHL2.0: Toward a State-Driven Fragmentation

Table 1: Scalar (0^{++}) hidden-heavy-flavor tetraquark candidates considered as representative case studies within the multimodal TQHL2.0 fragmentation framework. Masses are quoted as central values with uncertainties in parentheses. Inline references indicate key experimental observations and phenomenological interpretations relevant to each state.

State	Mass [MeV]	Status	Leading Fock	Comments
$X(3860)$	3862(44)	Candidate	$\lvert c\bar{c}\,q\bar{q}\rangle$	Non-strange 0^{++} candidate close to the $D\bar{D}$ threshold [1]. Useful as a controlled test case for assessing $[u \leftrightarrow d]$ quark mixing effects in hidden-heavy tetraquarks, with $q = u, d$.
$X(3960)$	3960(40)	Observed	$\lvert c\bar{c}\,s\bar{s}\rangle$	Observed in the $D_s^+ D_s^-$ channel by LHCb [2, 3]. Its flavor structure is clean, with no $[u \leftrightarrow d]$ quark mixing. Compatible with both compact [4] and molecular [5] interpretations.
$X_b(s)$	10 700(100)	Hypothetical	$\lvert b\bar{b}\,s\bar{s}\rangle$	Bottom counterpart of $X(3960)$ with clean heavy–strange content. Predicted in several tetraquark models [6–8]. Additional constraints may arise from studies of $X_0(4140)$ [9].

[F. G. C., in preparation]

MultiModal Fragmentation to TetraQuarks

Mode 1
Direct MultiQuark
Scalar S-wave *X*

TQHL1.0 (All-charm) 🔗 [F. G. C., A. Papa, Phys. Lett. B 848 (2024) 138406, arXiv:2308.01539]

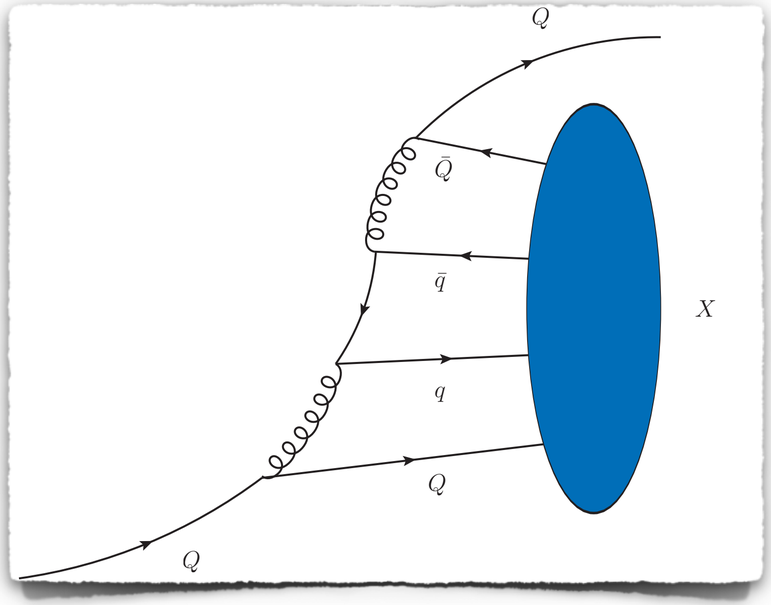
TQHL1.0 (All-bottom, Best Paper Award) 🔗 [F. G. C., Symmetry 16 (2024) 5, 550, arXiv:2403.15639]

TQHL1.1 (All-heavy) 🔗 [F. G. C., G. Gatto, Phys. Rev. D. 111 (2025) 3, 034037, arXiv:2412.10549]

TQHL2.0 (All-heavy, physical states, uncertainties) [F. G. C., in preparation]

MultiModal Fragmentation to TetraQuarks

Mode 1
Direct MultiQuark
Scalar S-wave *X*



Initial-scale input: 🔗🔗 Suzuki model

TQHL2.0 FFs: $(Q \to X) \otimes$ **APFEL++**

F-MHOUs [P] + F-NPWF [NP]

TQHL1.0 (All-charm) 🔗 [F. G. C., A. Papa, Phys. Lett. B 848 (2024) 138406, arXiv:2308.01539]

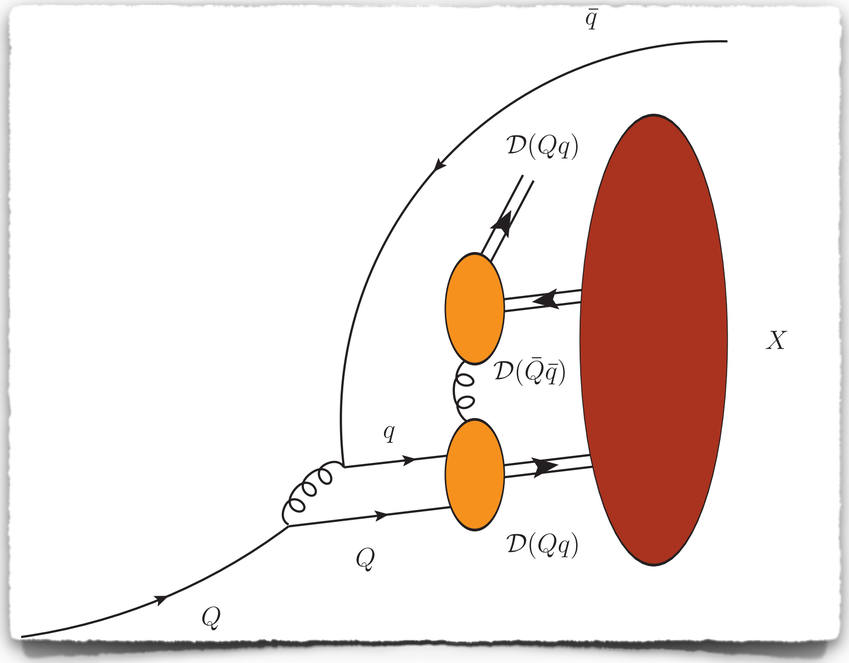
TQHL1.0 (All-bottom, Best Paper Award) 🔗 [F. G. C., Symmetry 16 (2024) 5, 550, arXiv:2403.15639]

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TQHL2.0 (All-heavy, physical states, uncertainties) [F. G. C., in preparation]

MultiModal Fragmentation to TetraQuarks

Mode 2
Scalar DiQuark
Scalar S-wave *X*



Initial-scale input: 🔗🔗 Suzuki model

TQHL2.0 FFs: $(Q \to X) \otimes$ **APFEL++**

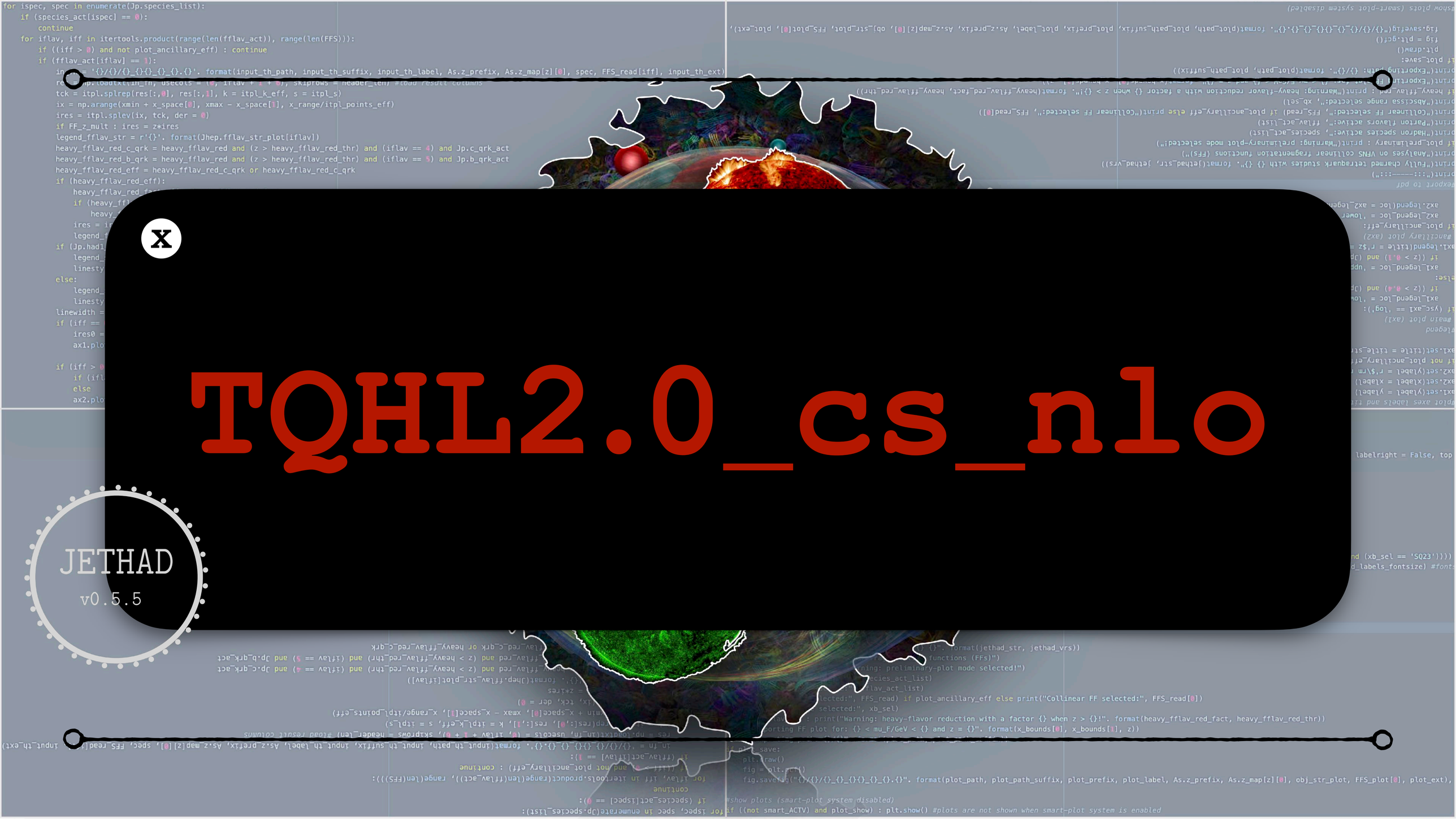
F-MHOUs [P] + F-NPWF [NP]

TQHL1.0 (Hidden-charm) 🔗 [F. G. C., A. Papa, Phys. Lett. B 848 (2024) 138406, arXiv:2308.01539]

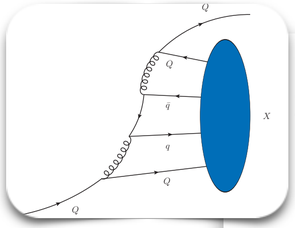
TQHL1.0 (Hidden-bottom, Best Paper Award) 🔗 [F. G. C., Symmetry 16 (2024) 5, 550, arXiv:2403.15639]

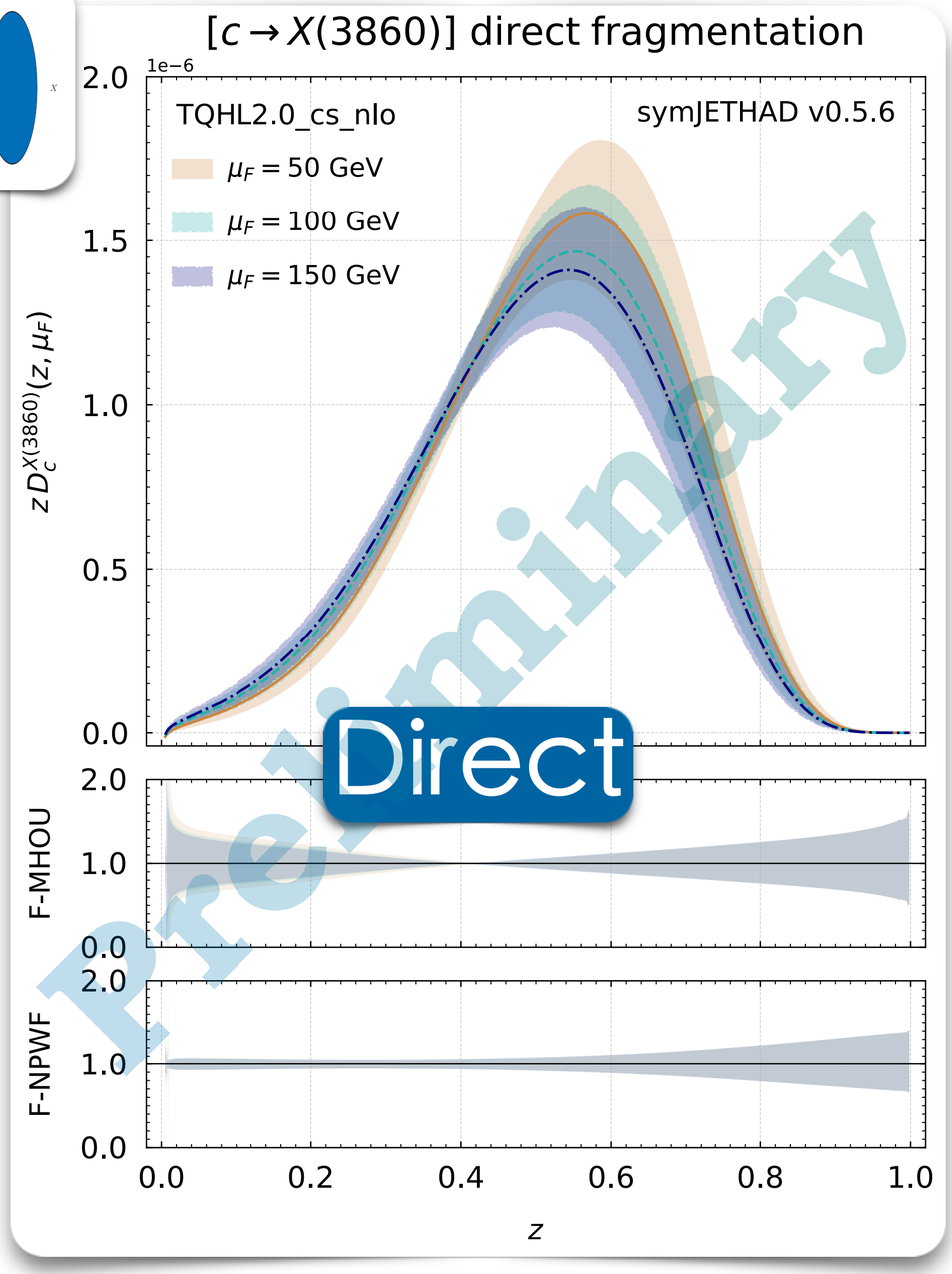
TQHL1.1 (Hidden-heavy) 🔗 [F. G. C., G. Gatto, Phys. Rev. D. 111 (2025) 3, 034037, arXiv:2412.10549]

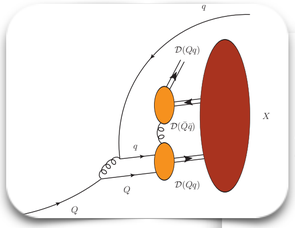
TQHL2.0 (Hidden-heavy, physical states, uncertainties) [F. G. C., in preparation]

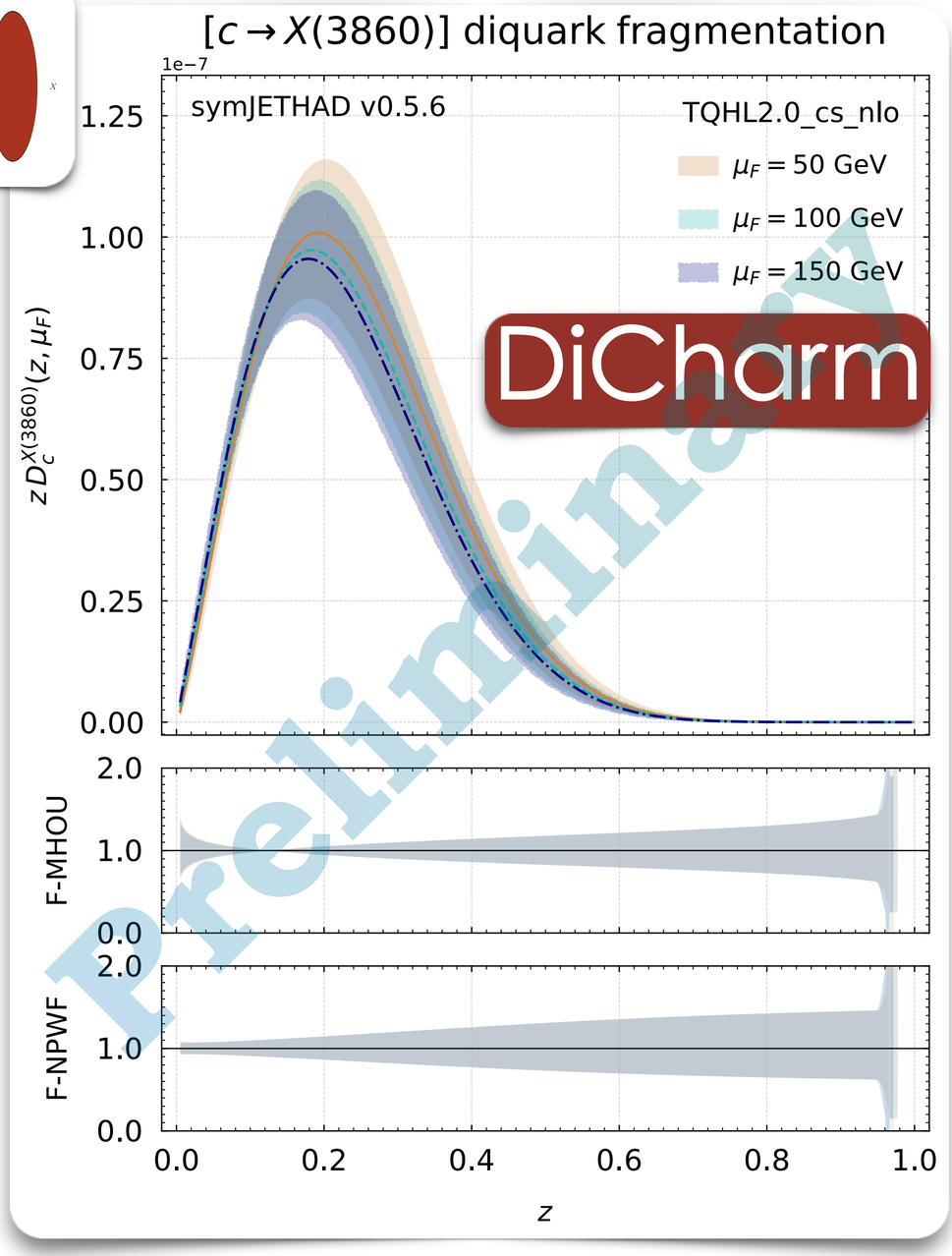

TQHL2.0_cs_nlo
JETHAD
v0.5.5

TQHL2.0: Charm Fragmentation to X(3860)

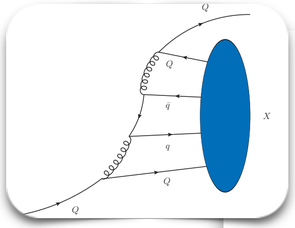


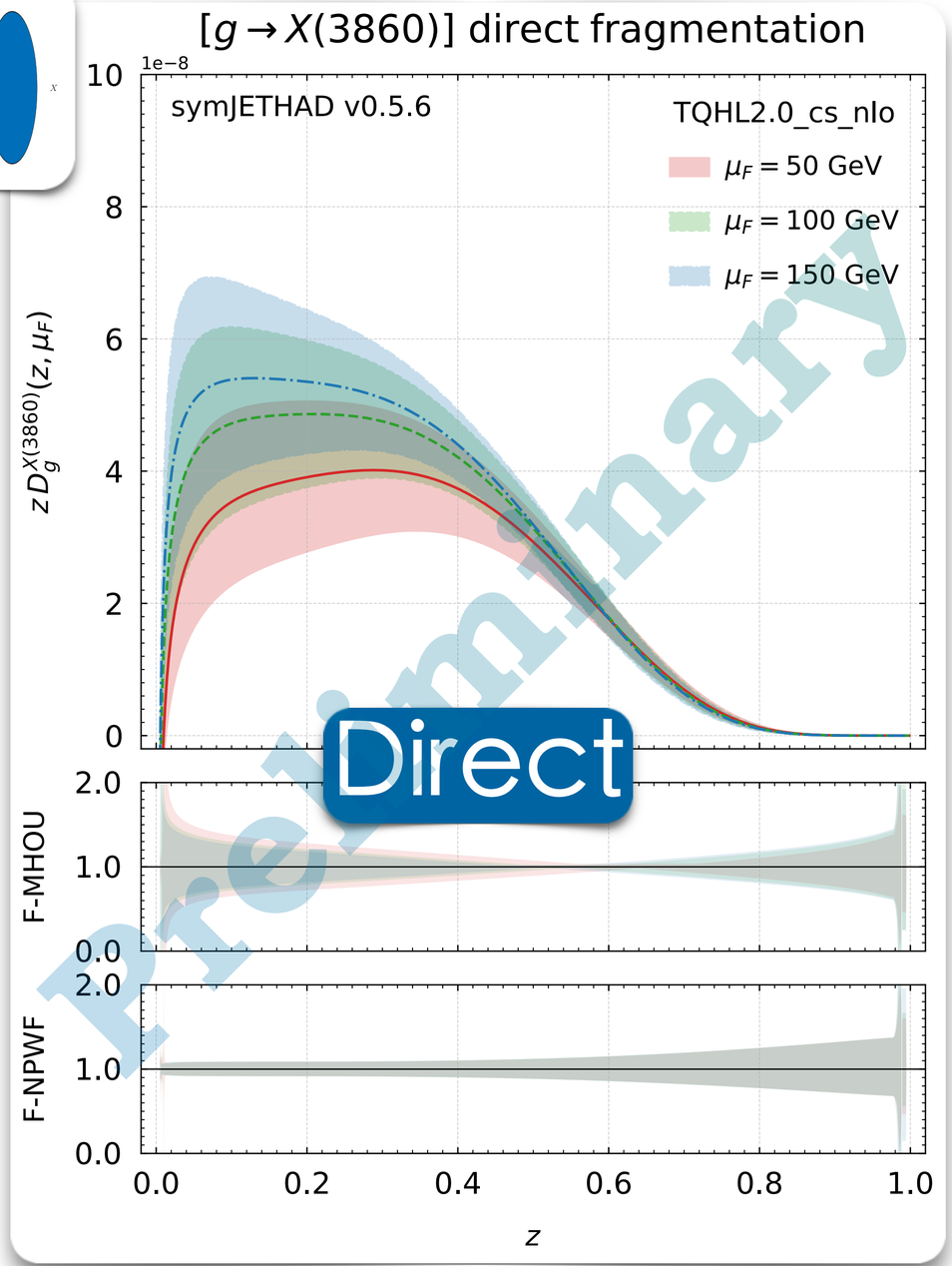


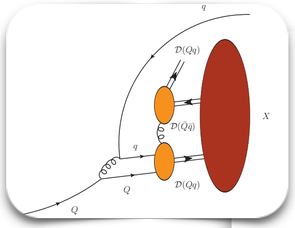


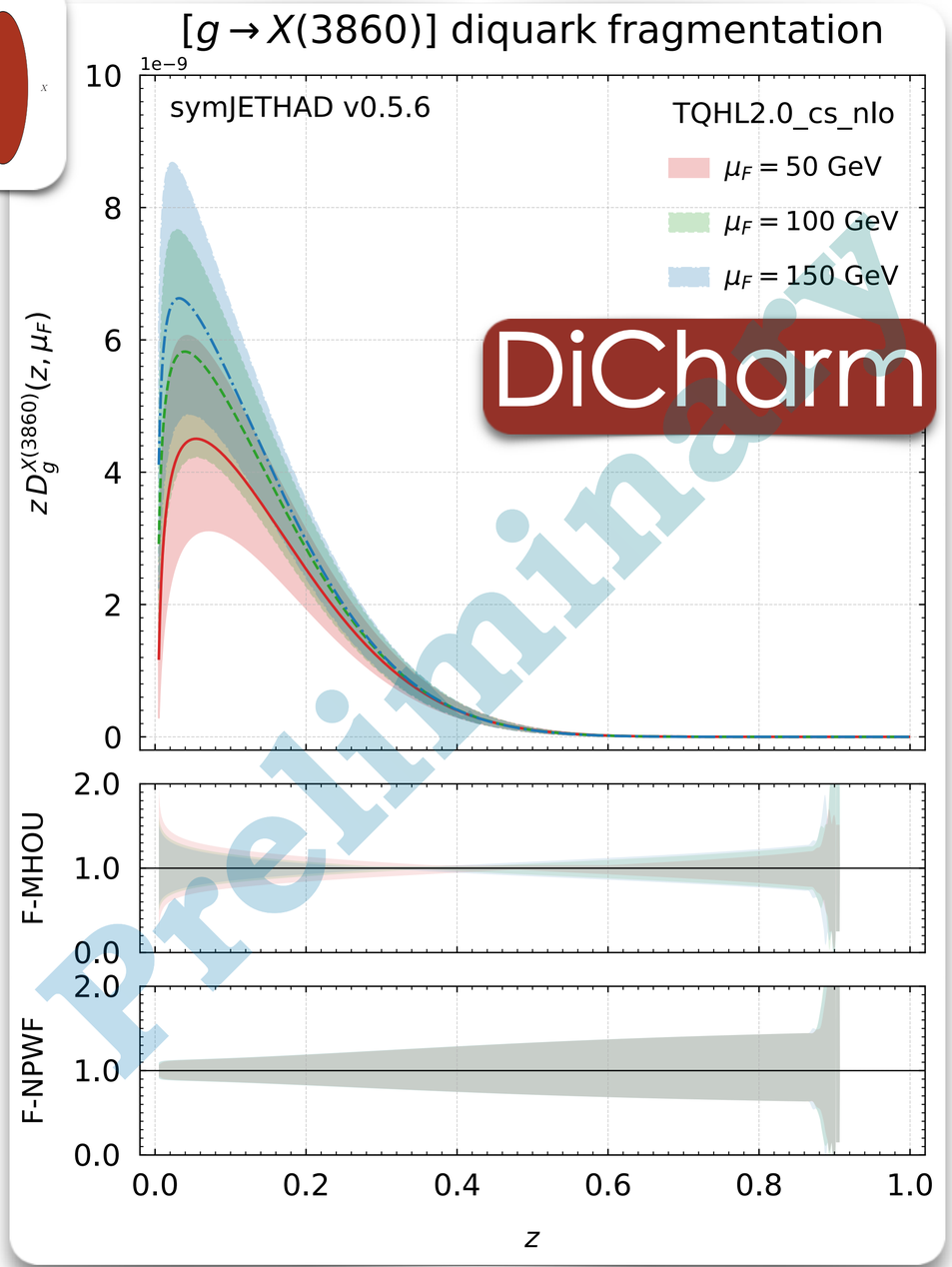


TQHL2.0: Gluon Fragmentation to X(3860)









ALL-HEAVY PENTAQUARK FRAGMENTATION

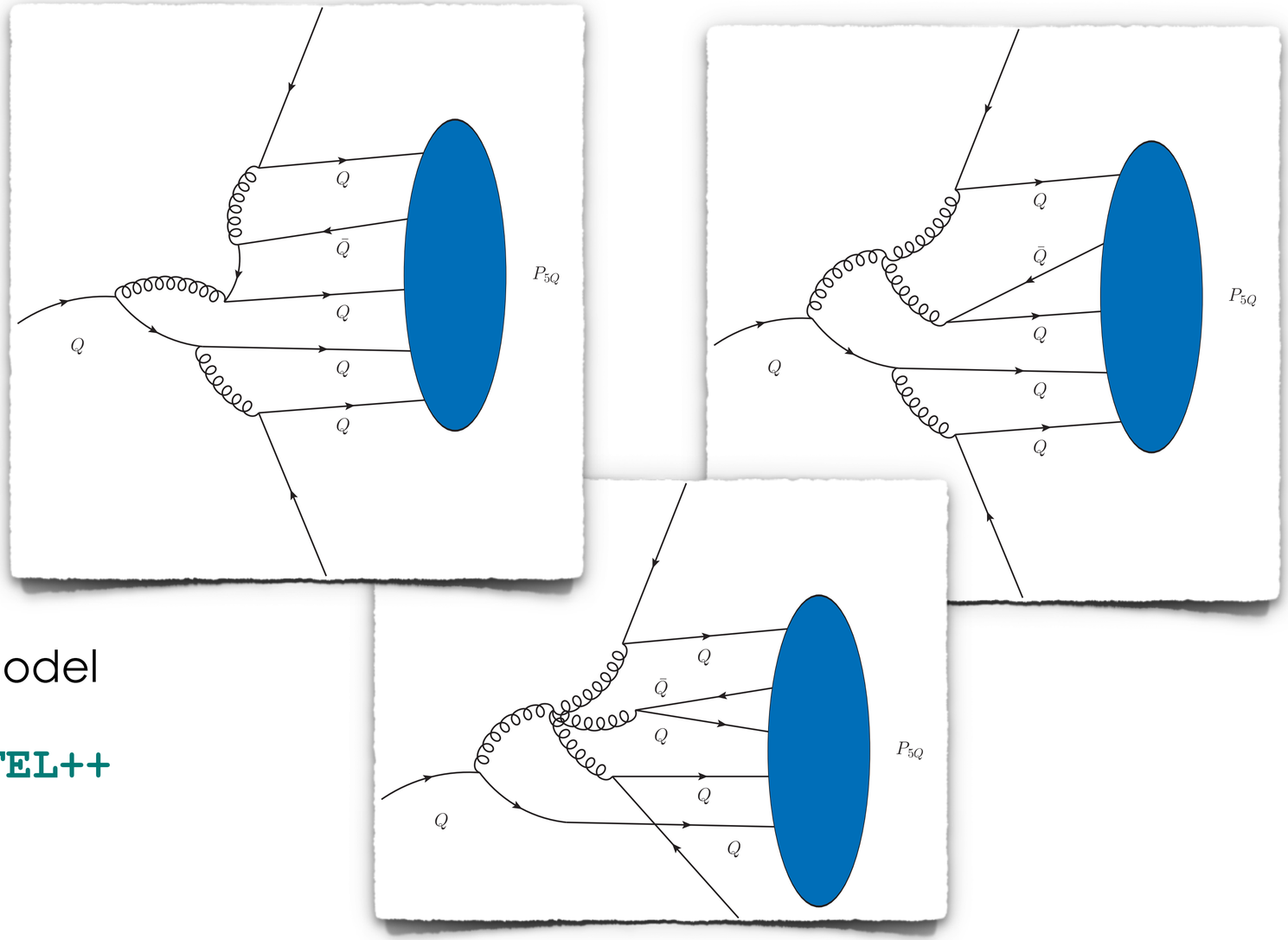
MultiModal Fragmentation to PentaQuarks

Mode 1
Direct MultiQuark
Scalar S-wave P_{5c}

🔗 [F. G. C., Eur. Phys. J. C 85 (2025) 12, 1395]; 🔗 [F. G. C., Phys. Rev. D. (to appear), arXiv:2605.01539]

MultiModal Fragmentation to PentaQuarks

Mode 1
Direct MultiQuark
Scalar S-wave P_{5c}



Initial-scale input: 🔗🔗 Suzuki model

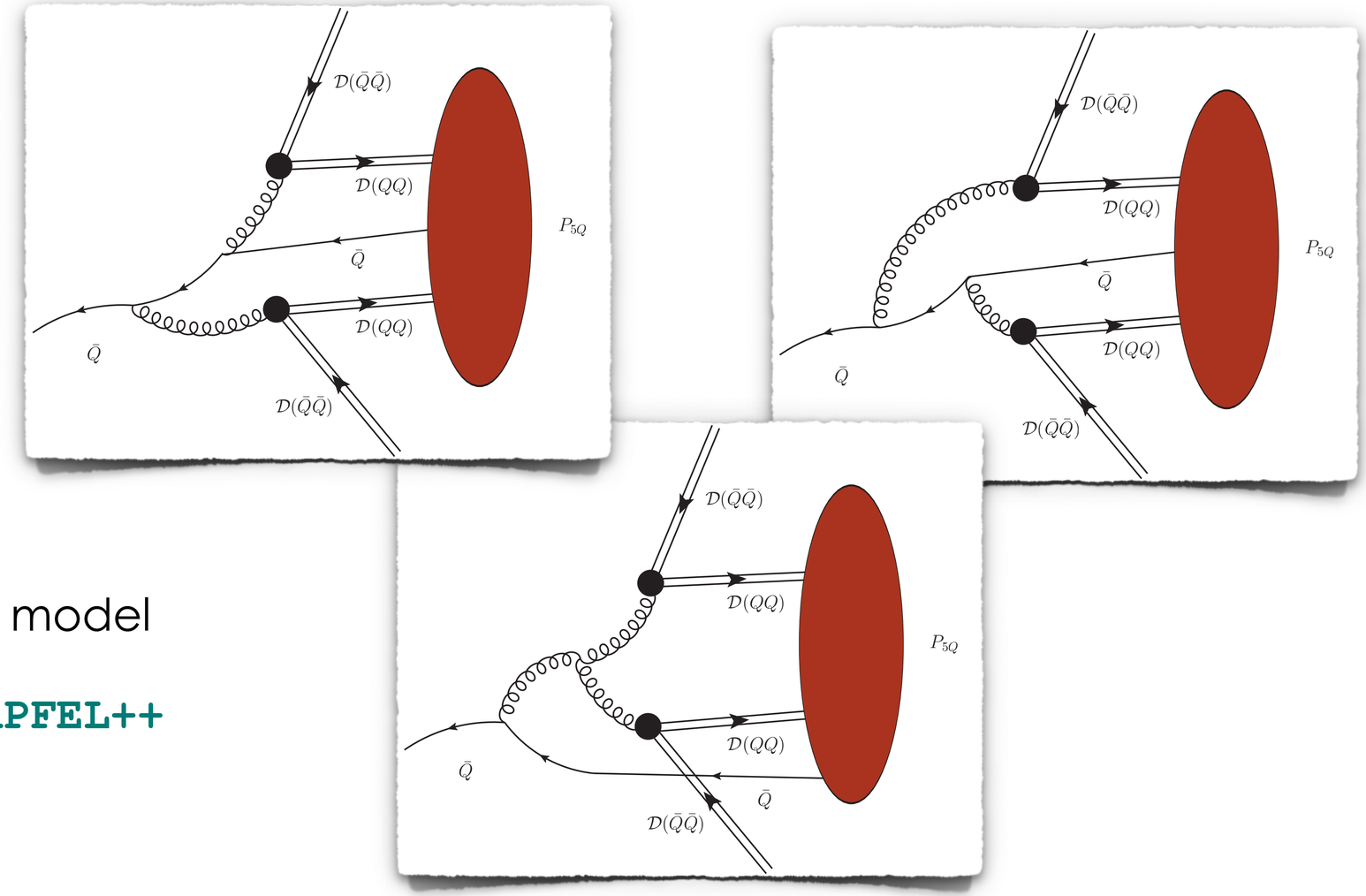
PQ5Q1.y FFs: $(Q \to P_{c\bar{c}ccc}) \otimes$ **APFEL++**

F-MHOUs [P] + F-NPWF [NP]

🔗 [F. G. C., Eur. Phys. J. C 85 (2025) 12, 1395]; 🔗 [F. G. C., Phys. Rev. D. (to appear), arXiv:2605.01539]

MultiModal Fragmentation to PentaQuarks

Mode 2
Scalar DiQuark
Scalar S-wave P_{5c}

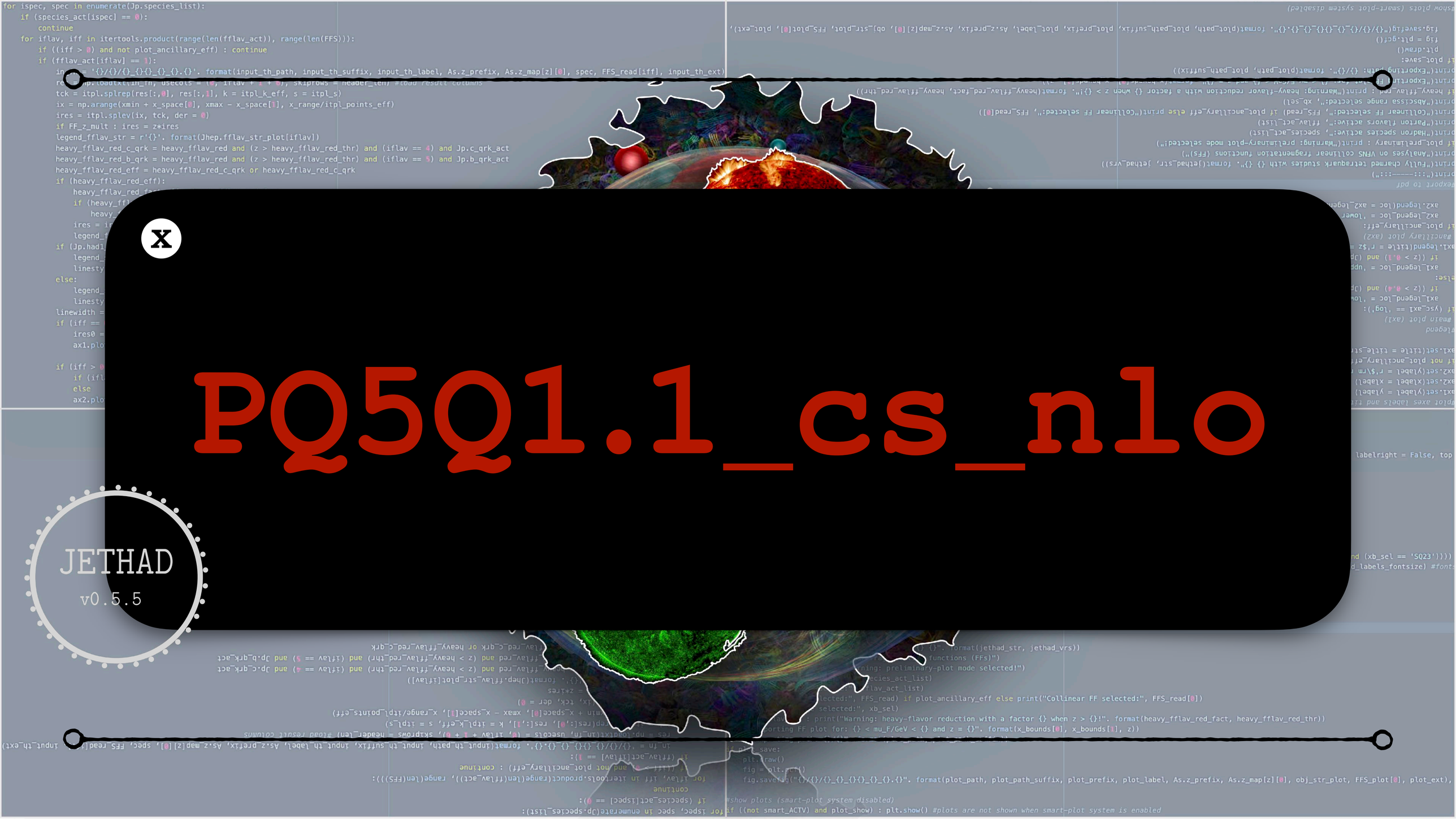


Initial-scale input: 🔗🔗 Suzuki model

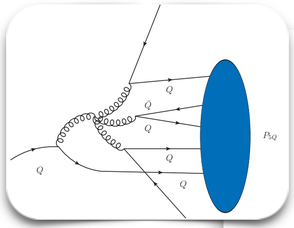
PQ5Q1.y FFs: $(Q \to P_{c\bar{c}ccc}) \otimes$ **APFEL++**

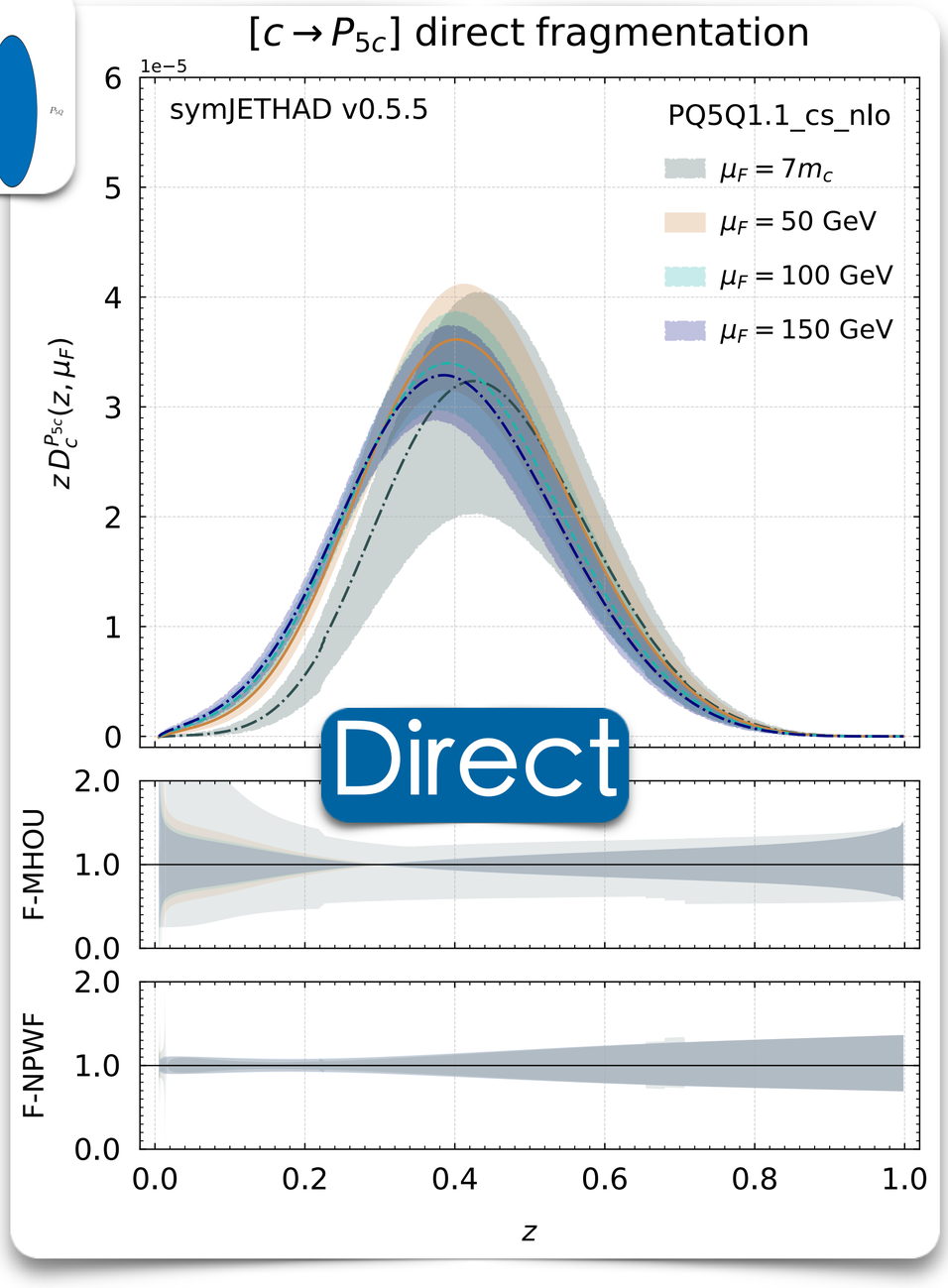
F-MHOUs [P] + F-NPWF [NP]

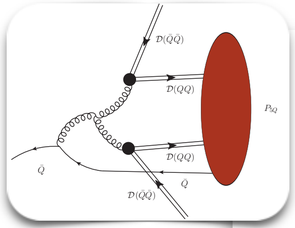
🔗 [F. G. C., Eur. Phys. J. C 85 (2025) 12, 1395]; 🔗 [F. G. C., Phys. Rev. D. (to appear), arXiv:2605.01539]

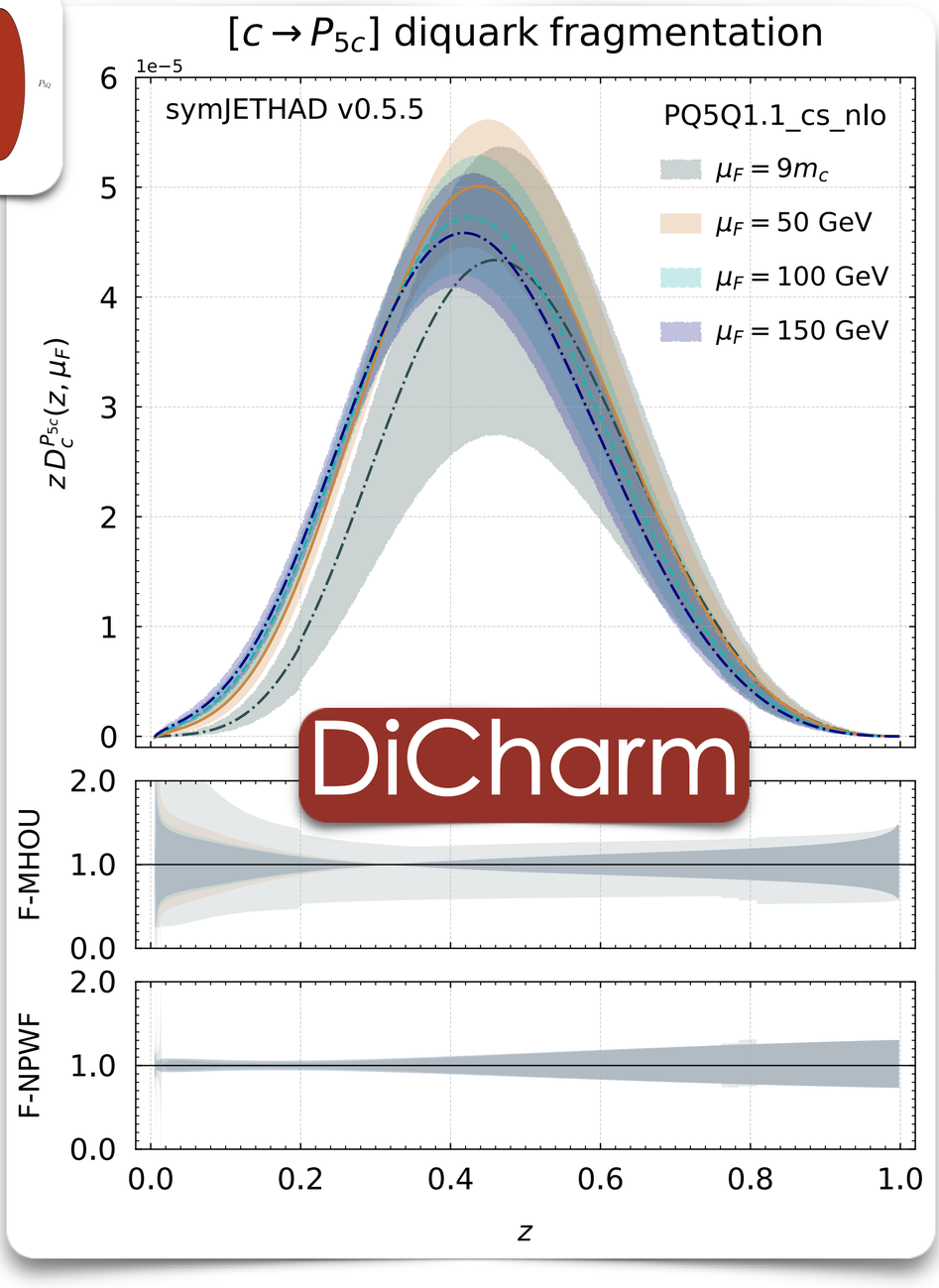

PQ5Q1.1_cs_nlo
JETHAD
v0.5.5

PQ5Q1.1: Charm Fragmentation to PentaCharm

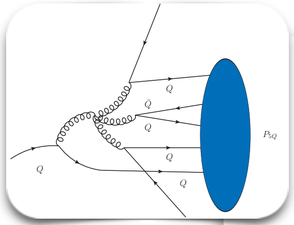


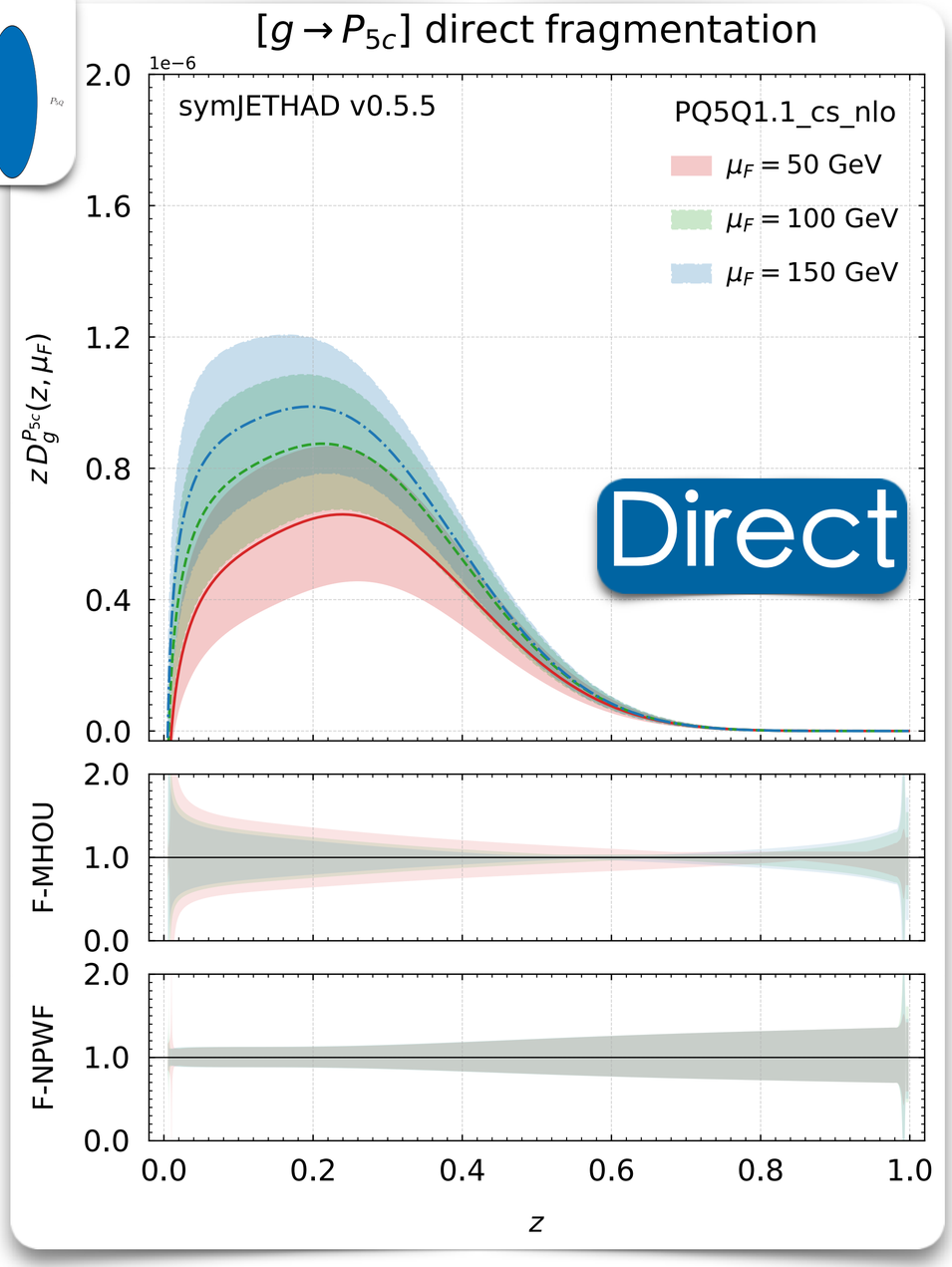


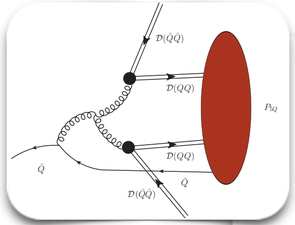


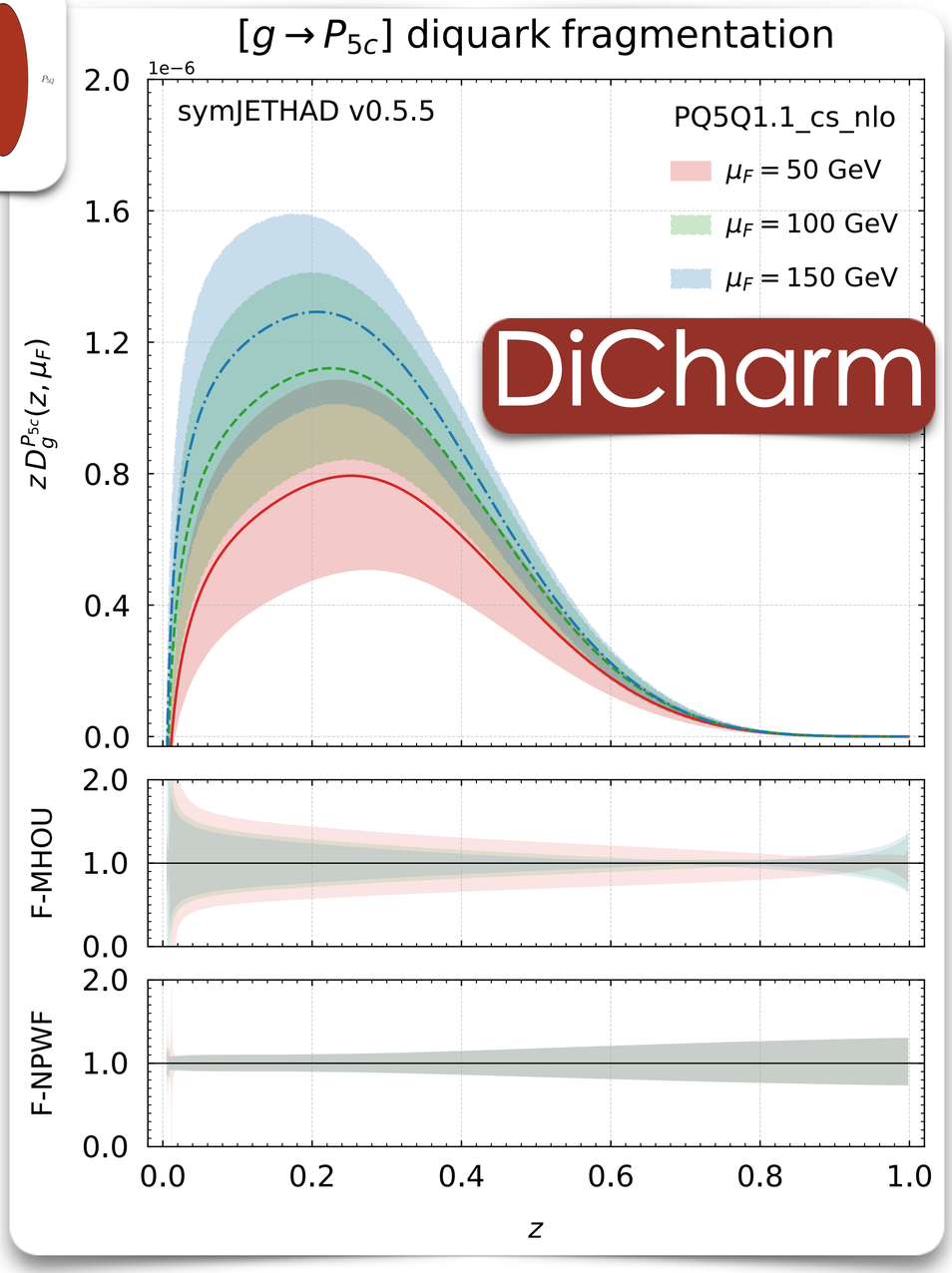


PQ5Q1.1: Gluon Fragmentation to PentaCharm

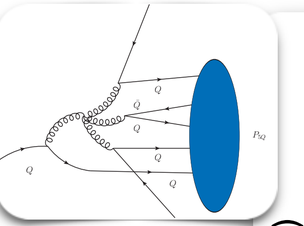


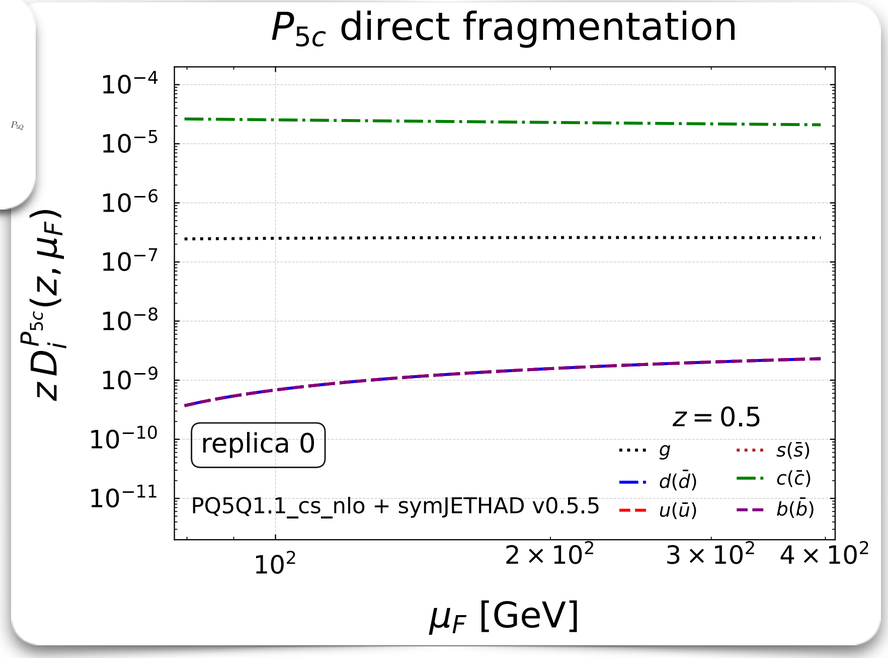


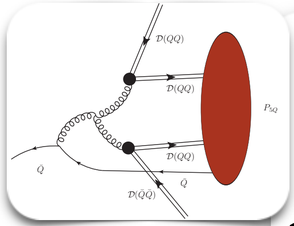


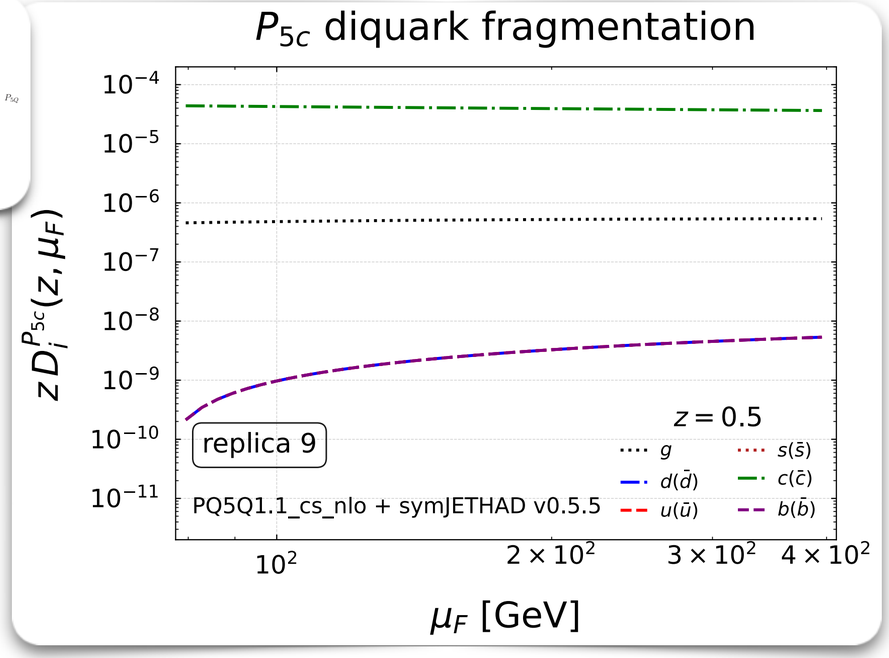


PQ5Q1.1: Energy Spectrum





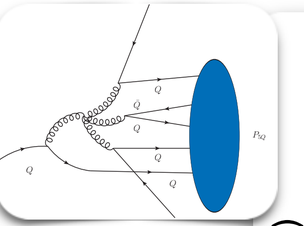


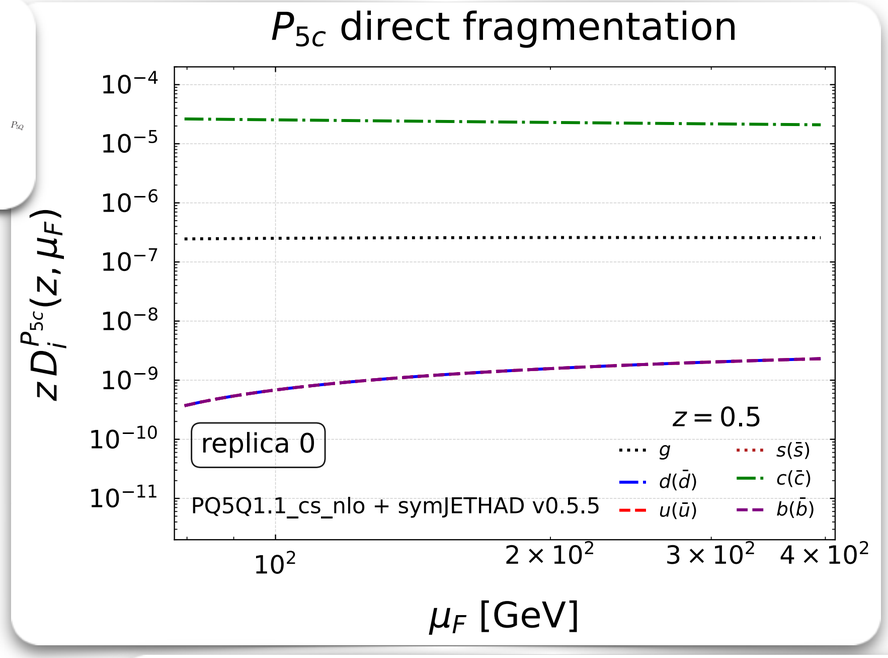


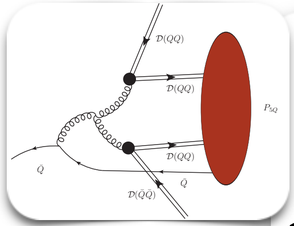
DiCharm

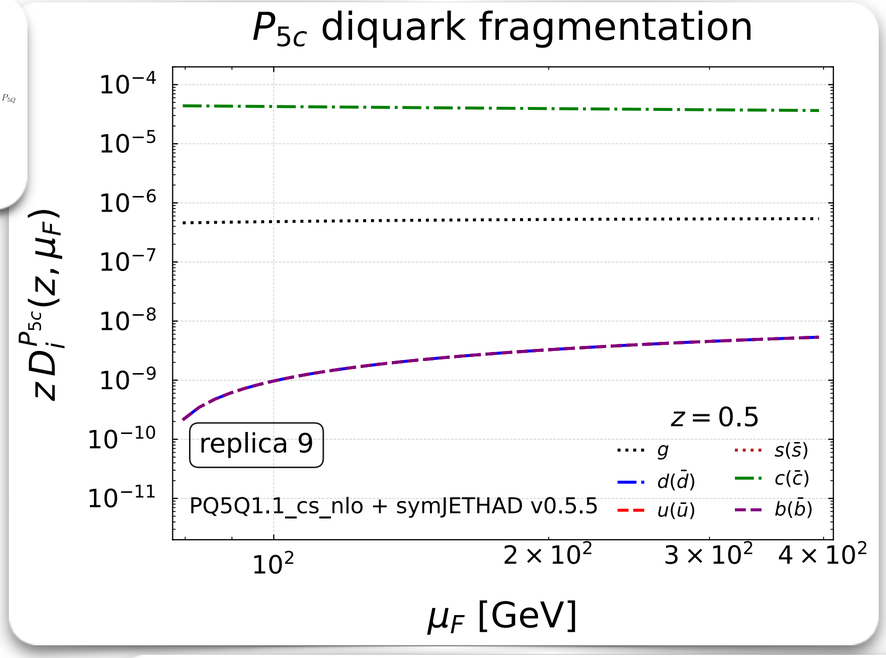
Direct

PQ5Q1.1: Energy Spectrum



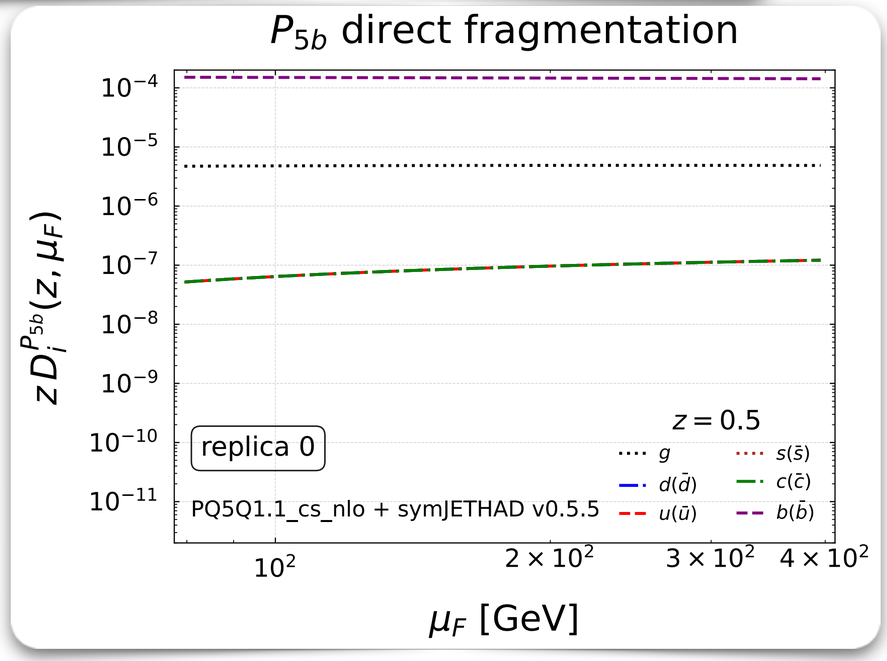


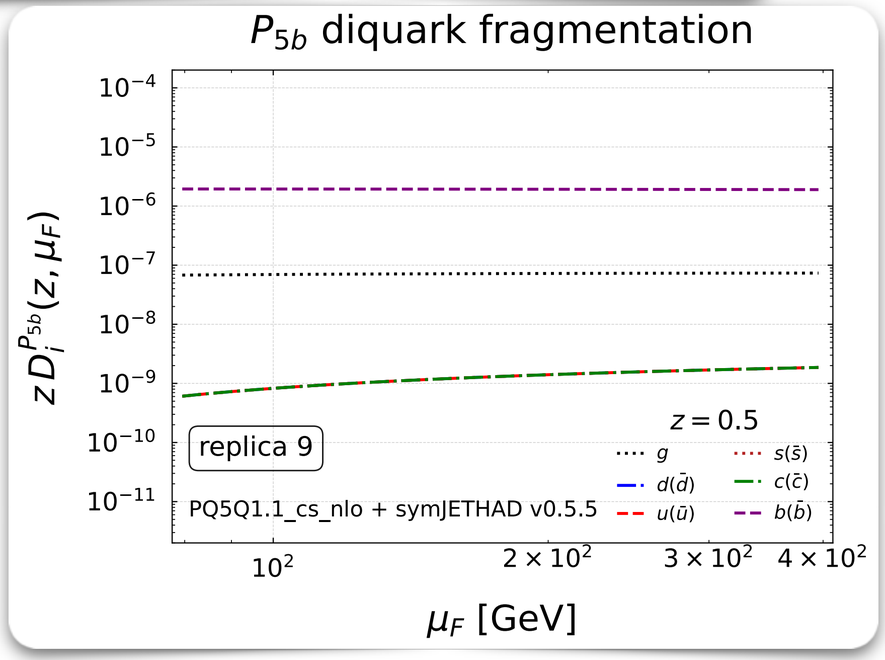




DiCharm

DiRect



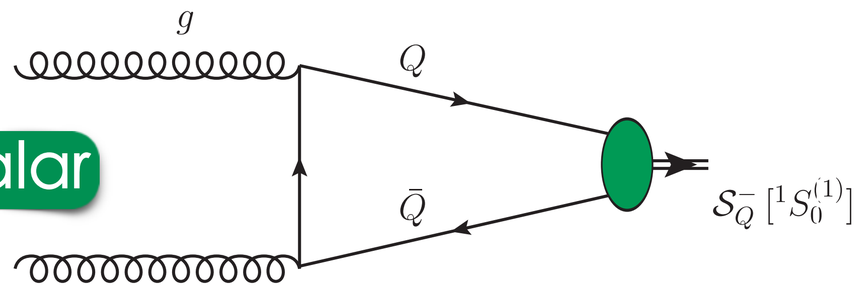


QUARKONIUM FRAGMENTATION

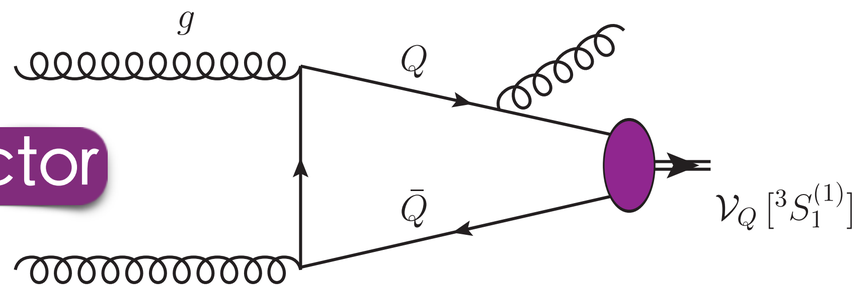
From low energies to high energies

Low p_T $\Rightarrow$ two-parton FFs
Genuine higher-twist FFNS FFs

scalar

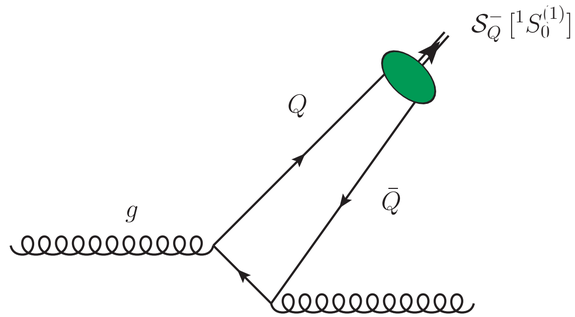


vector

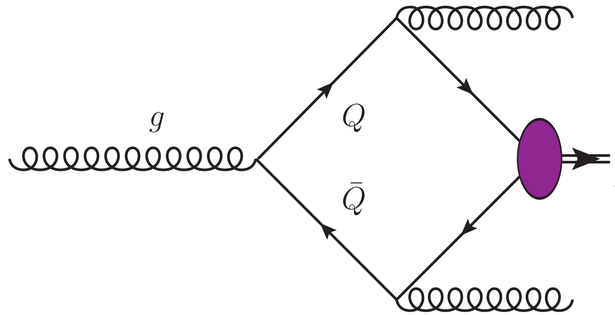


High p_T $\Rightarrow$ single-parton FFs
Twist-2 VFNS FFs + power corrections

scalar



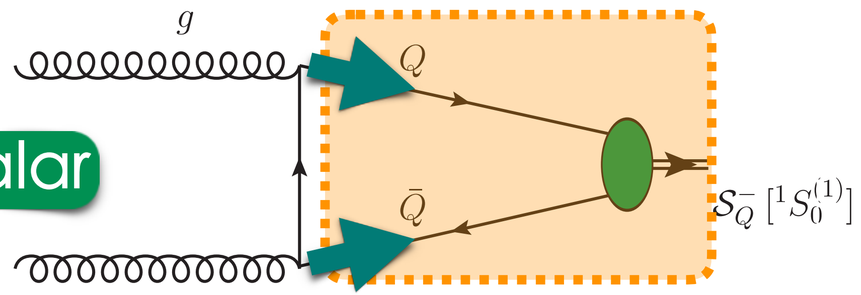
vector



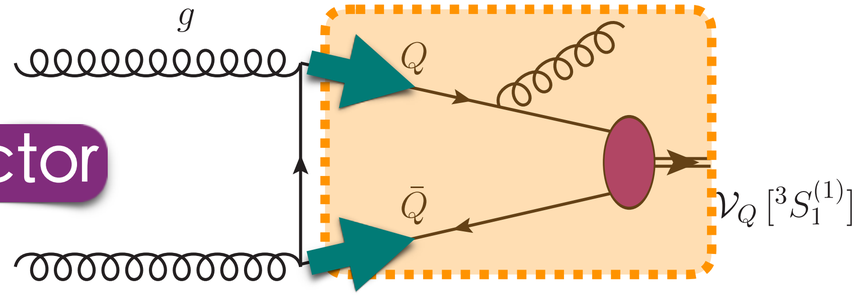
From low energies to high energies

Low p_T ⇒ two-parton FFs
Genuine higher-twist FFNS FFs

scalar

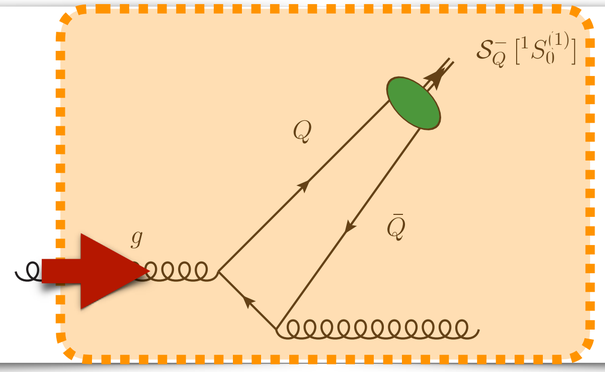


vector

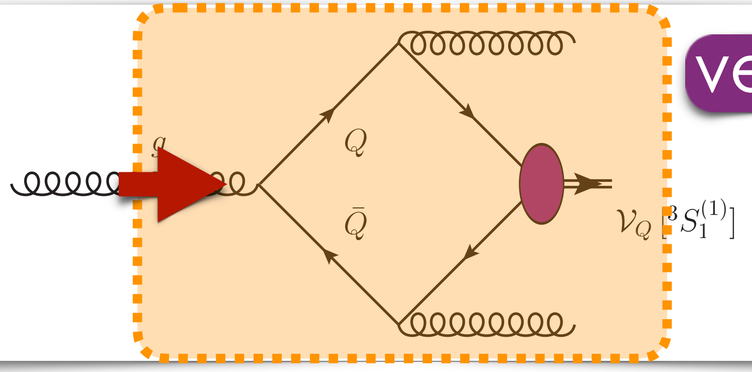


High p_T ⇒ single-parton FFs
Twist-2 VFNS FFs + power corrections

scalar



vector



Collinear Fragmentation Correlator (CFC)

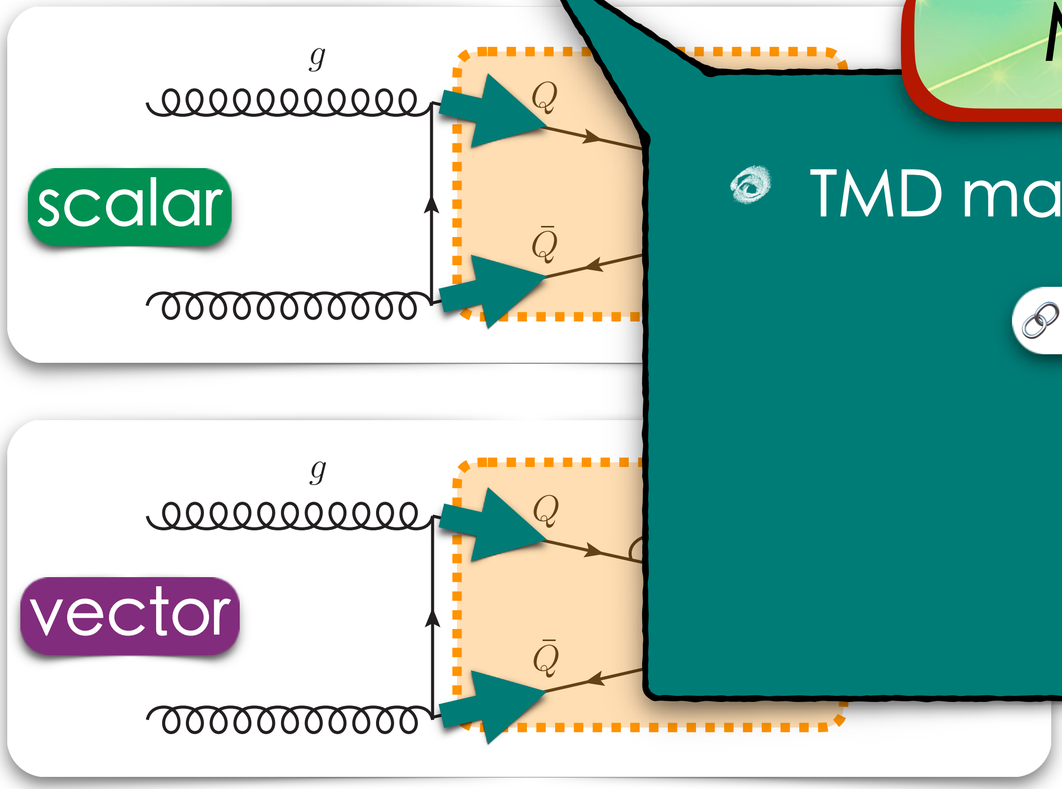
From low energies to high energies

Low p_T $\Rightarrow$ two-parton FFs

Genuine higher-twist FFNS FFs

High p_T $\Rightarrow$ single-parton FFs

Twist-2 VFNS FFs + power corrections



Matching low & high p_T

- TMD matching tails $\Rightarrow$ different singularities

[D. Boer, J. Bor, L. Maxia, C. Pisano, F. Yuan (2023)]

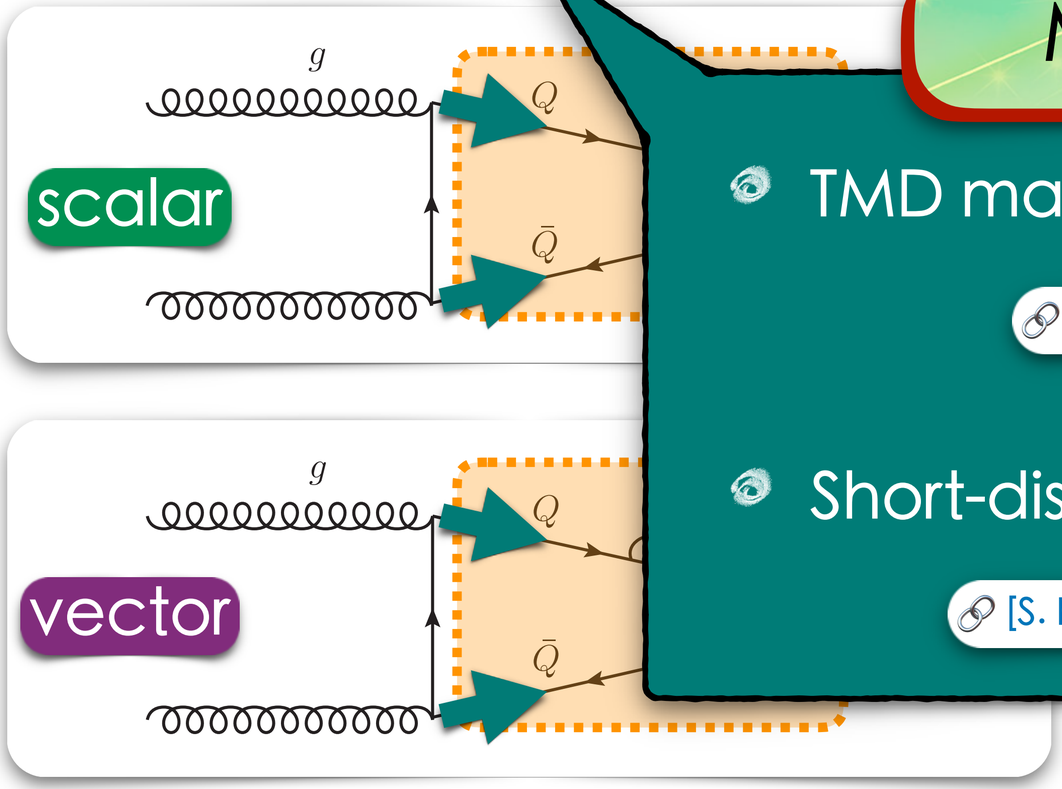
From low energies to high energies

Low p_T $\Rightarrow$ two-parton FFs

Genuine higher-twist FFNS FFs

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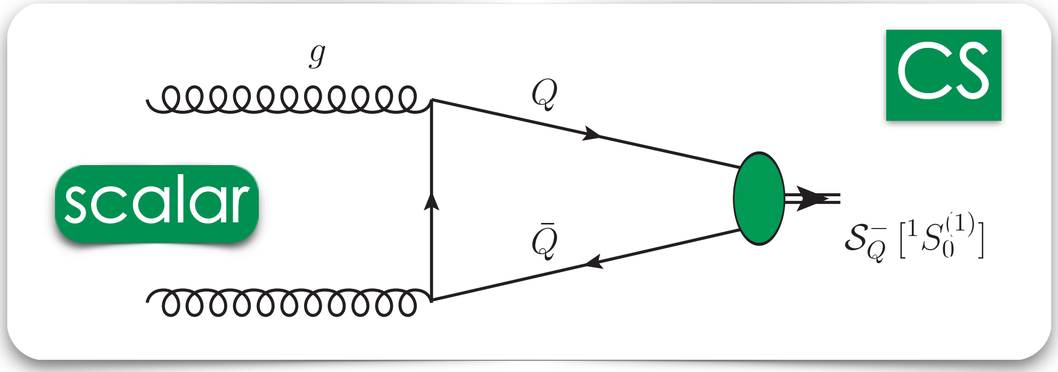
[D. Boer, J. Bor, L. Maxia, C. Pisano, F. Yuan (2023)]

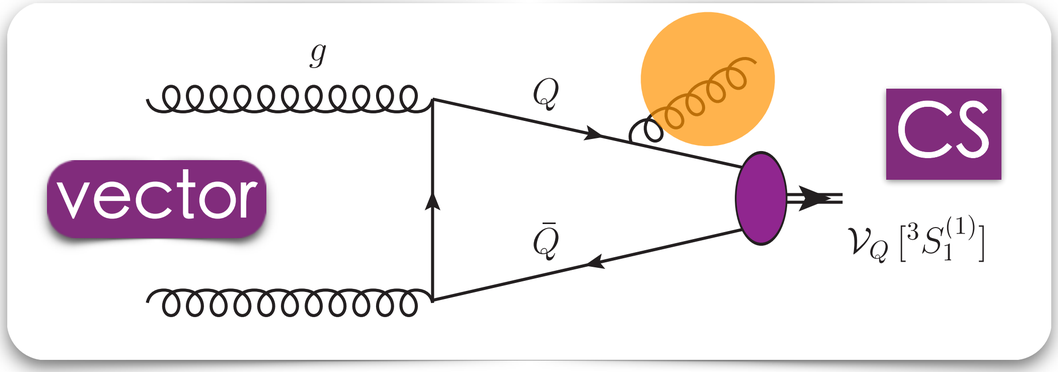
- Short-distance $\Leftrightarrow$ double-parton FFs (SCET)

[S. Fleming, A.K. Leibovich, T. Mehen, I.Z. Rothstein (2012)]

From low energies to high energies

Low energy, low p_T
Short-distance production



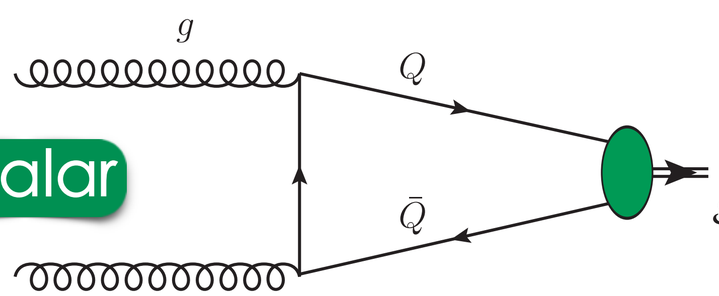


Extra gluon due to Landau-Yang selection rule

From low energies to high energies

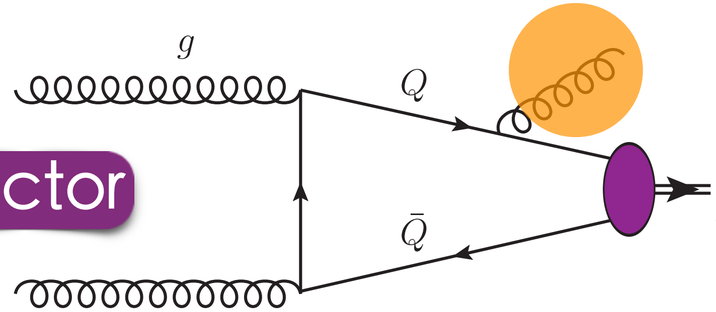
Low energy, low p_T
Short-distance production

scalar



CS

vector

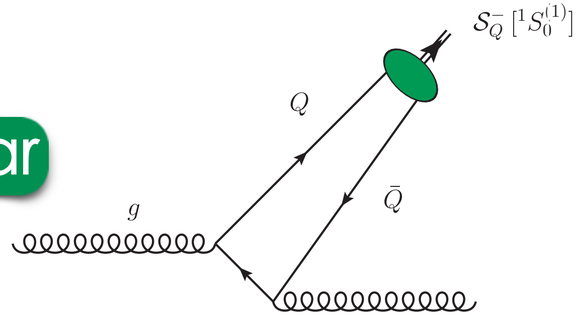


CS

$p_T \gtrsim 10 \div 15$ GeV
(charmonia)

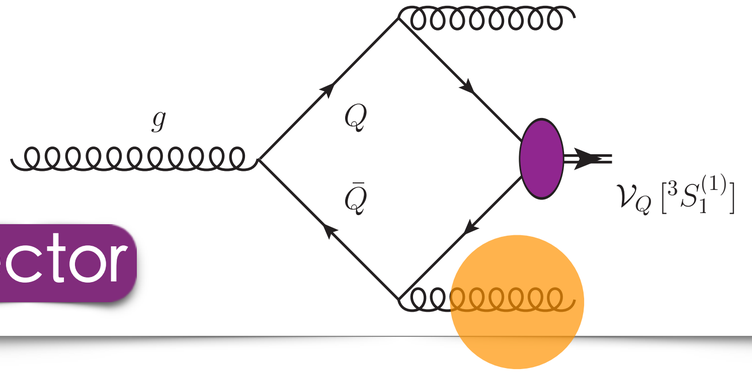
High energy, high p_T
Single-parton fragmentation

scalar



CS

vector



CS

Extra gluon due to Landau-Yang selection rule

HF-NRevo: Initial-scale NRCQD inputs

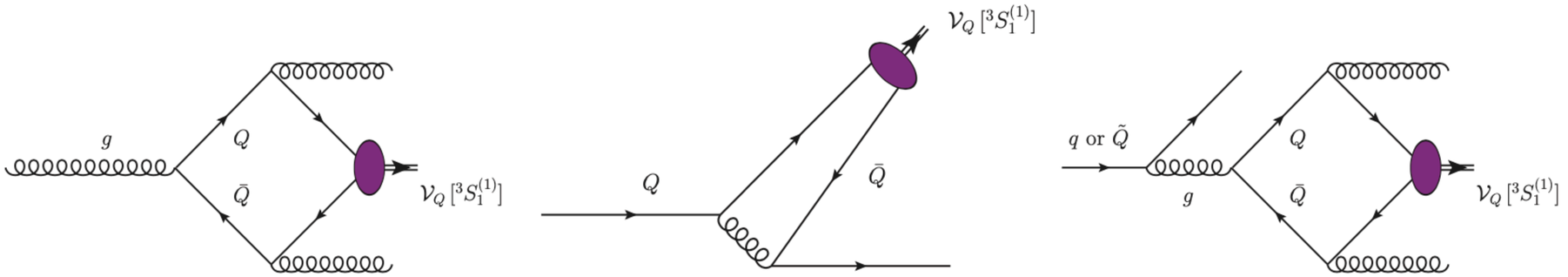


Fig. 1 Left panel: a leading diagram for the fragmentation of a gluon to $\mathcal{V}_Q\,[^3S_1^{(1)}]$ at $\mathcal{O}(\alpha_s^3)$. Central panel: one of the leading diagrams for the fragmentation of a constituent heavy quark (Q) to a S-wave color singlet vector quarkonium $\mathcal{V}_Q\,[^3S_1^{(1)}]$ at $\mathcal{O}(\alpha_s^2)$. Right panel: a leading diagram for the fragmentation of a nonconstituent light (q) or heavy ($\tilde{Q}$) quark to $\mathcal{V}_Q\,[^3S_1^{(1)}]$ at $\mathcal{O}(\alpha_s^4)$. Violet blobs refer to the $\mathcal{V}_Q\,[^3S_1^{(1)}]$ nonperturbative NRQCD LDME.

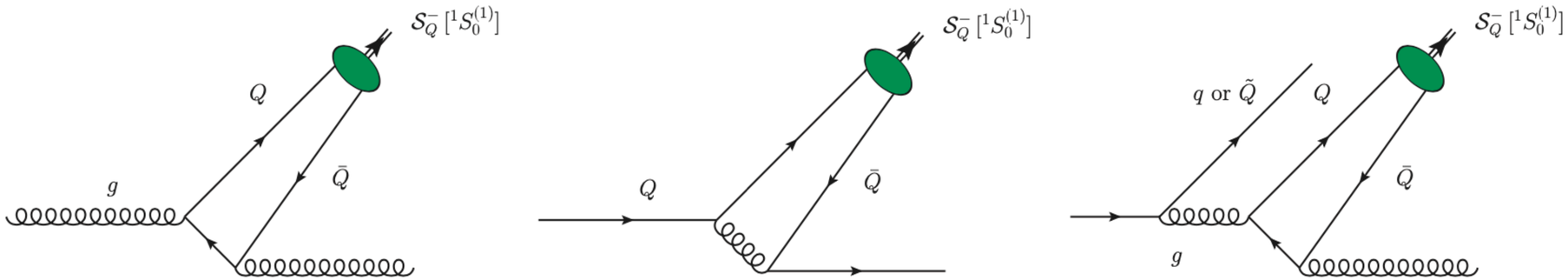
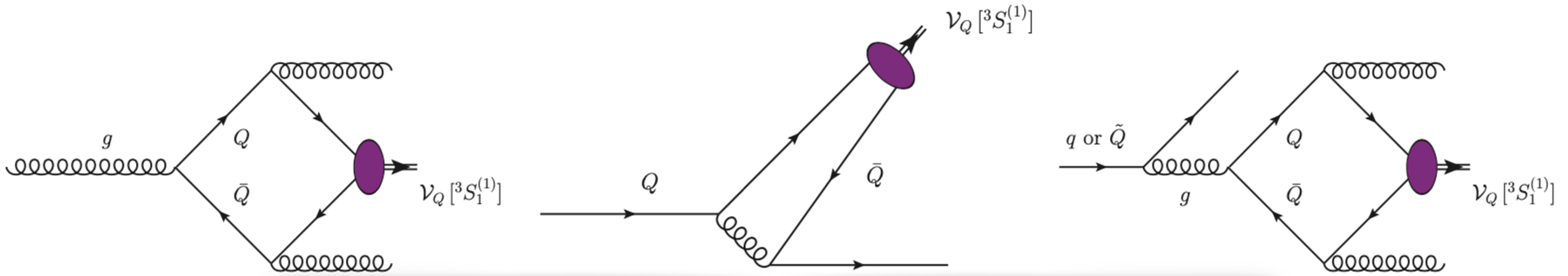


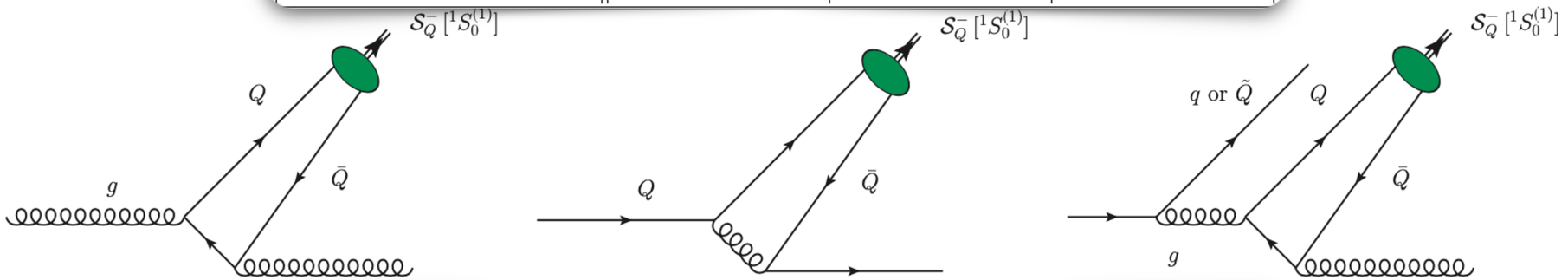
Fig. 2 Left panel: a leading diagram for the fragmentation of a gluon to $\mathcal{S}_Q^-\,[^1S_0^{(1)}]$ at $\mathcal{O}(\alpha_s^2)$. Central panel: one of the leading diagrams for the fragmentation of a constituent heavy quark (Q) to a S-wave color singlet pseudoscalar quarkonium $\mathcal{S}_Q^-\,[^1S_0^{(1)}]$ at $\mathcal{O}(\alpha_s^2)$. Right panel: a leading diagram for the fragmentation of a nonconstituent light (q) or heavy ($\tilde{Q}$) quark to $\mathcal{S}_Q^-\,[^1S_0^{(1)}]$ at $\mathcal{O}(\alpha_s^3)$. Green blobs refer to the $\mathcal{S}_Q^-\,[^1S_0^{(1)}]$ nonperturbative NRQCD LDME.

[F. G. C., in preparation]

HF-NRevo: Initial-scale NRCQD inputs



Quarkonium ($\mathcal{Q}$)	$g \to \mathcal{Q}$	$Q \to \mathcal{Q}$	q or $\tilde{Q} \to \mathcal{Q}$
$\mathcal{Q} \equiv \mathcal{V}_Q\,[^1S_0^{(1)}] \equiv J/\psi, \Upsilon$	–	$\mathcal{O}(\alpha_s^2) \equiv$ LO	–
	$\mathcal{O}(\alpha_s^3) \equiv$ NLO	$\mathcal{O}(\alpha_s^3) \equiv$ NLO	–
	$\mathcal{O}(\alpha_s^4) \equiv$ NNLO	$\mathcal{O}(\alpha_s^4) \equiv$ NNLO	$\mathcal{O}(\alpha_s^4) \equiv$ NNLO
	⋮	⋮	⋮

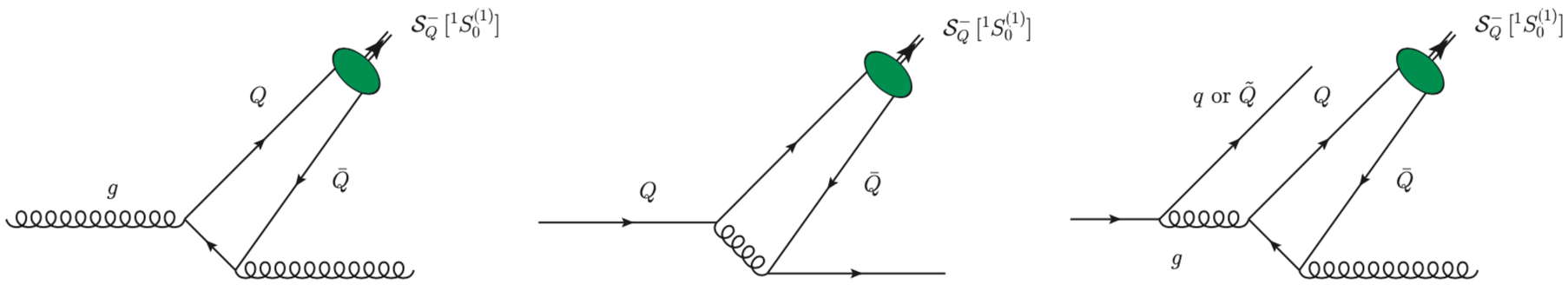


$\mathcal{Q} \equiv \mathcal{S}_Q^-\,[^3S_1^{(1)}] \equiv \eta_c, \eta_b$	$\mathcal{O}(\alpha_s^2) \equiv$ LO	$\mathcal{O}(\alpha_s^2) \equiv$ LO	–
	$\mathcal{O}(\alpha_s^3) \equiv$ NLO	$\mathcal{O}(\alpha_s^3) \equiv$ NLO	$\mathcal{O}(\alpha_s^3) \equiv$ NLO
	$\mathcal{O}(\alpha_s^4) \equiv$ NNLO	$\mathcal{O}(\alpha_s^4) \equiv$ NNLO	$\mathcal{O}(\alpha_s^4) \equiv$ NNLO
	⋮	⋮	⋮

[F. G. C., in preparation]

HF-NRevo: Initial-scale NRCQD inputs

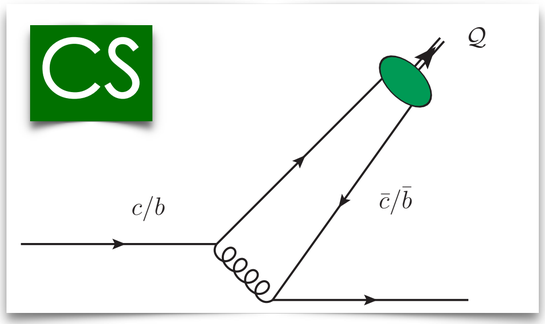
$\mathcal{S}_Q^-\,[^1S_0^{(1)}]$ quarkonium	$\mu_{F,0}(g \to \mathcal{S}_Q^-)$	$\mu_{F,0}(d,u,s \to \mathcal{S}_Q^-)$	$\mu_{F,0}(Q \to \mathcal{S}_Q^-)$	$\mu_{F,0}(\tilde{Q} \to \mathcal{S}_Q^-)$	$Q_0 \equiv \max(\{\mu_{F,0}\})$
η_c	$2m_c$	$m_{d,u,s} + 2m_c$	$3m_c$	$m_b + 2m_c$	$m_b + 2m_c$
η_b	$2m_b$	$m_{d,u,s} + 2m_b$	$3m_b$	$m_c + 2m_b$	$3m_b$



[F. G. C., in preparation]

Vector quarkonium from single-parton fragmentation

¡ *Let us consider* J/ψ *and* Υ *at large* p_T $\rightarrow$ *initial-scale* ***heavy-quark*** + ***gluon*** *from* ***NRQCD***!



$$D_{\mathcal{Q}}^{\mathcal{Q}}(z,\mu_F \equiv \mu_0) = D_{\mathcal{Q}}^{\mathcal{Q},\mathrm{LO}}(z) + \frac{\alpha_s^3(\mu_R)}{m_Q^3}\,|\mathcal{R}_{\mathcal{Q}}(0)|^2\,\Gamma^{\mathcal{Q},\mathrm{NLO}}(z)$$

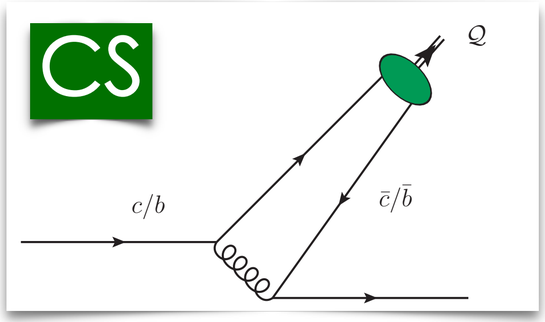
$(\mathcal{Q} \to \mathcal{Q}\,Q)$ at $\mu_0 = 3m_Q$

(LO) [E. Braaten et al., Phys. Rev. D 48 (1993) 4230-4235]
(NLO) [X. Zheng et al., Phys. Rev. D 100 (2019) 1, 014005]

[My talk at the Higgs Centre, Edinburgh (2023)]
[F. G. C., M. Fucilla, Eur. Phys. J. C 82 (2022) 10, 929]

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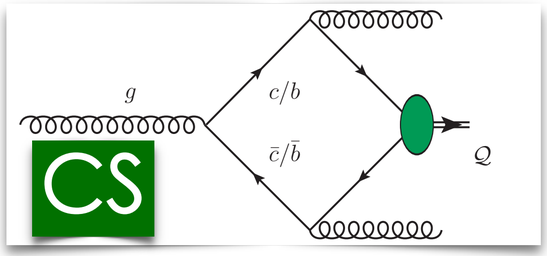
$$D_{\mathcal{Q}}^{\mathcal{Q}}(z,\mu_F \equiv \mu_0) = D_{\mathcal{Q}}^{\mathcal{Q},\mathrm{LO}}(z) + \frac{\alpha_s^3(\mu_R)}{m_Q^3}\,|\mathcal{R}_{\mathcal{Q}}(0)|^2\,\Gamma^{\mathcal{Q},\mathrm{NLO}}(z)$$

$(Q \rightarrow \mathcal{Q}\,Q)$ at $\mu_0 = 3m_Q$

(LO) 🔗 [E. Braaten et al., Phys. Rev. D 48 (1993) 4230-4235]

(NLO) 🔗 [X. Zheng et al., Phys. Rev. D 100 (2019) 1, 014005]

+



$$D_g^{\mathcal{Q}}(z,2m_Q) = \frac{5}{36(2\pi)^2}\alpha_s^3(2m_Q)\frac{|\mathcal{R}_{\mathcal{Q}}(0)|^2}{m_{\mathcal{Q}}^3}\int_0^z \mathrm{d}\xi \int_{(\xi+z^2)/2z}^{(1+\xi)/2} \mathrm{d}\tau \frac{1}{(1-\tau)^2(\tau-\xi)^2(\tau^2-\xi)^2}$$

$$\sum_{i=1}^{2} z^i \left[f_i^{(g)}(\xi,\tau) + g_i^{(g)}(\xi,\tau)\frac{1+\xi-2\tau}{2(\tau-\xi)\sqrt{\tau^2-\xi}} \ln\left(\frac{\tau-\xi+\sqrt{\tau^2-\xi}}{\tau-\xi-\sqrt{\tau^2-\xi}}\right)\right],$$

$(g \rightarrow \mathcal{Q}\,gg)$ at $\mu_0 = 2m_Q$

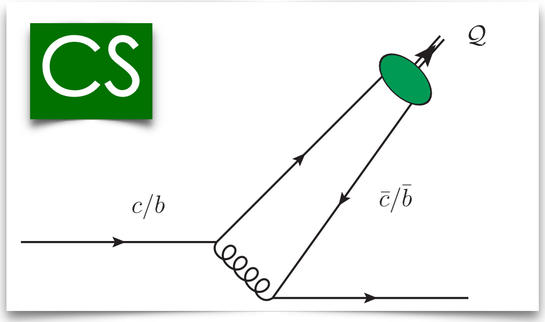
(LO) 🔗 [A. Braaten, T.C Yuan, Phys. Rev. Lett. 71 (1993), 1673]

🔗 [My talk at the Higgs Centre, Edinburgh (2023)]

🔗 [F. G. C., M. Fucilla, Eur. Phys. J. C 82 (2022) 10, 929]

Vector quarkonium from single-parton fragmentation

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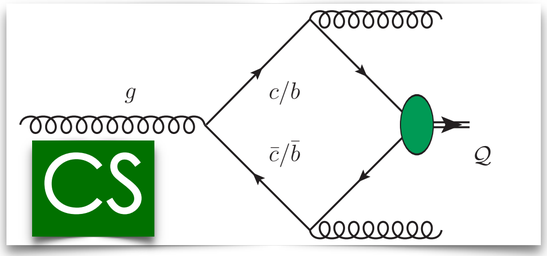


$$D_{\mathcal{Q}}^{\mathcal{Q}}(z,\mu_F \equiv \mu_0) = D_{\mathcal{Q}}^{\mathcal{Q},\mathrm{LO}}(z) + \frac{\alpha_s^3(\mu_R)}{m_Q^3}\,|\mathcal{R}_{\mathcal{Q}}(0)|^2\,\Gamma^{\mathcal{Q},\mathrm{NLO}}(z)$$

$(Q \to \mathcal{Q}\,Q)$ at $\mu_0 = 3m_Q$

(LO) 🔗 [E. Braaten et al., Phys. Rev. D 48 (1993) 4230-4235]
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+



$$D_g^{\mathcal{Q}}(z,2m_Q) = \frac{5}{36(2\pi)^2}\alpha_s^3(2m_Q)\frac{|\mathcal{R}_{\mathcal{Q}}(0)|^2}{m_{\mathcal{Q}}^3}\int_0^z \mathrm{d}\xi \int_{(\xi+z^2)/2z}^{(1+\xi)/2} \mathrm{d}\tau \frac{1}{(1-\tau)^2(\tau-\xi)^2(\tau^2-\xi)^2}$$

$$\sum_{i=1}^{2} z^i \left[f_i^{(g)}(\xi,\tau) + g_i^{(g)}(\xi,\tau)\frac{1+\xi-2\tau}{2(\tau-\xi)\sqrt{\tau^2-\xi}} \ln\left(\frac{\tau-\xi+\sqrt{\tau^2-\xi}}{\tau-\xi-\sqrt{\tau^2-\xi}}\right)\right],$$

$(g \to \mathcal{Q}\,gg)$ at $\mu_0 = 2m_Q$

$\otimes$

(LO) 🔗 [A. Braaten, T.C Yuan, Phys. Rev. Lett. 71 (1993), 1673]

🔗 [My talk at the Higgs Centre, Edinburgh (2023)]
🔗 [F. G. C., M. Fucilla, Eur. Phys. J. C 82 (2022) 10, 929]

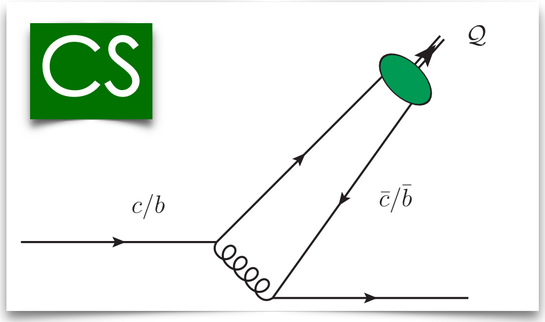
NRFF1.0
quarkonium FFs

=

APFEL++/EKO

Vector quarkonium from single-parton fragmentation

¡ *Let us consider* J/ψ *and* Υ *at large* p_T → *initial-scale* ***heavy-quark*** + ***gluon*** *from* ***NRQCD***!

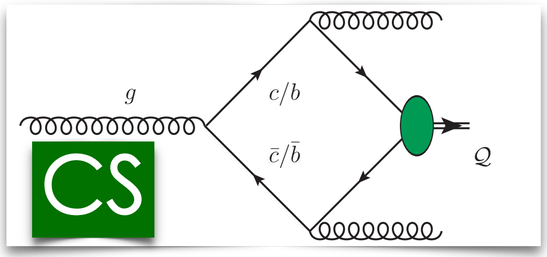


$$D_Q^{\mathcal{Q}}(z,\mu_F \equiv \mu_0) = D_Q^{\mathcal{Q},\mathrm{LO}}(z) + \frac{\alpha_s^3(\mu_R)}{m_Q^3}\,|\mathcal{R}_{\mathcal{Q}}(0)|^2$$

(LO) 🔗 [E. Braaten et al., Phys. Rev. D 48 (1993) 4230-42
(NLO) 🔗 [X. Zheng et al., Phys. Rev. D 100 (2019) 1, 014005]

$(Q \to \mathcal{Q}\,Q)$ at $\mu_0 = 3m_Q$

+



$$D_g^{\mathcal{Q}}(z,2m_Q) = \frac{5}{36(2\pi)^2}\alpha_s^3(2m_Q)\frac{|\mathcal{R}_{\mathcal{Q}}(0)|^2}{m_{\mathcal{Q}}^3}\int_0^z \mathrm{d}\xi \int_{(\xi+z^2)/2z}^{(1+\xi)/2} \mathrm{d}\tau \frac{1}{(1-\tau)^2(z}$$

$$\sum_{i=1}^{2} z^i \left[f_i^{(g)}(\xi,\tau) + g_i^{(g)}(\xi,\tau)\frac{1+\xi-2\tau}{2(\tau-\xi)\sqrt{\tau^2-\xi}} \ln\left(\frac{\tau-\xi+\sqrt{\tau^2-\xi}}{\tau-\xi-\sqrt{\tau^2-\xi}}\right)\right]$$

(LO) 🔗 [A. Braaten, T.C Yuan, Phys. Rev. Lett. 71 (1993), 1673]

$(g \to \mathcal{Q}\,gg)$ at $\mu_0 = 2m_Q$

⊗

🔗 [My talk at the Higgs Centre, Edinburgh (2023)]
🔗 [F. G. C., M. Fucilla, Eur. Phys. J. C 82 (2022) 10, 929]

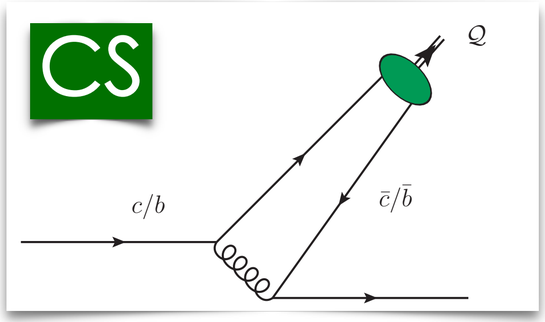
NRFF1.0
quarkonium FFs

=

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Vector quarkonium from single-parton fragmentation

¡ *Let us consider* J/ψ *and* Υ *at large* p_T → *initial-scale* ***heavy-quark*** + ***gluon*** *from* ***NRQCD***!

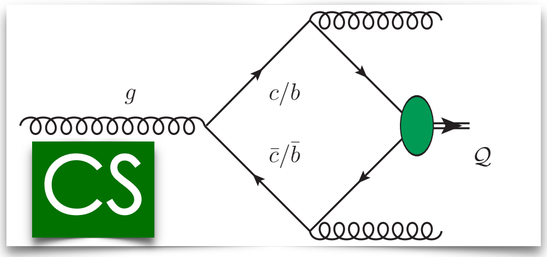


$$D_Q^{\mathcal{Q}}(z,\mu_F\equiv\mu_0)=D_Q^{\mathcal{Q},\mathrm{LO}}(z)+\frac{\alpha_s^3(\mu_R)}{m_Q^3}\,|\mathcal{R}_{\mathcal{Q}}(0)|^2$$

(LO) 🔗 [E. Braaten et al., Phys. Rev. D 48 (1993) 4230-42
(NLO) 🔗 [X. Zheng et al., Phys. Rev. D 100 (2019) 1, 014005]

$(Q \to \mathcal{Q}\,Q)$ at $\mu_0 = 3m_Q$

+



$$D_g^{\mathcal{Q}}(z,2m_Q)=\frac{5}{36(2\pi)^2}\alpha_s^3(2m_Q)\frac{|\mathcal{R}_{\mathcal{Q}}(0)|^2}{m_{\mathcal{Q}}^3}\int_0^z \mathrm{d}\xi\int_{(\xi+z^2)/2z}^{(1+\xi)/2}\mathrm{d}\tau\,\frac{1}{(1-\tau)^2(z}$$

$$\sum_{i=1}^{2} z^i\left[f_i^{(g)}(\xi,\tau)+g_i^{(g)}(\xi,\tau)\frac{1+\xi-2\tau}{2(\tau-\xi)\sqrt{\tau^2-\xi}}\ln\left(\frac{\tau-\xi+\sqrt{\tau^2-\xi}}{\tau-\xi-\sqrt{\tau^2-\xi}}\right)\right]$$

(LO) 🔗 [A. Braaten, T.C Yuan, Phys. Rev. Lett. 71 (1993), 1673]

$(g \to \mathcal{Q}\,gg)$ at $\mu_0 = 2m_Q$

🔗 [My talk at the Higgs Centre, Edinburgh (2023)]
🔗 [F. G. C., M. Fucilla, Eur. Phys. J. C 82 (2022) 10, 929]

NRFF1.0

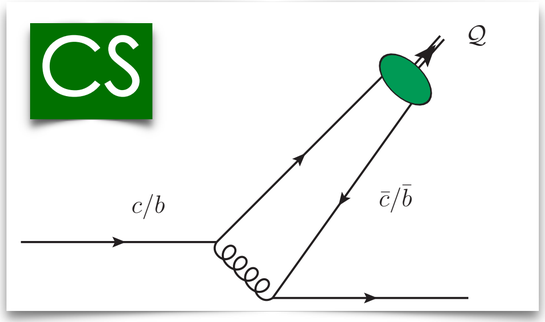
quarkonium FFs

=

APFEL++/EKO

Vector quarkonium from single-parton fragmentation

¡ *Let us consider* J/ψ *and* Υ *at large* p_T → *initial-scale* ***heavy-quark*** + ***gluon*** *from* ***NRQCD***!

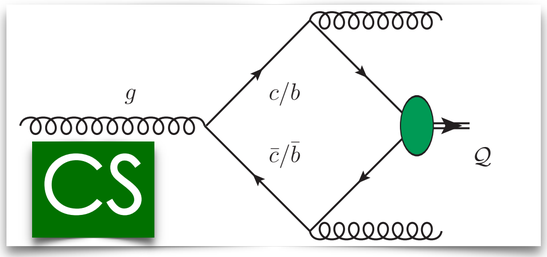


$$D_Q^{\mathcal{Q}}(z,\mu_F\equiv\mu_0)=D_Q^{\mathcal{Q},\mathrm{LO}}(z)+\frac{\alpha_s^3(\mu_R)}{m_Q^3}\,|\mathcal{R}_{\mathcal{Q}}(0)|^2$$

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(LO) 🔗 [A. Braaten, T.C Yuan, Phys. Rev. Lett. 71 (1993), 1673]

$(g \to \mathcal{Q}\,gg)$ at $\mu_0 = 2m_Q$

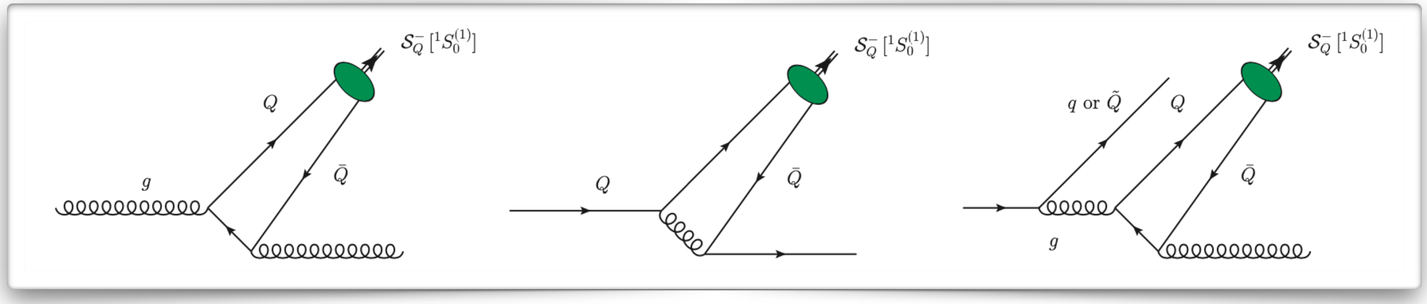
🔗 [My talk at the Higgs Centre, Edinburgh (2023)]
🔗 [F. G. C., M. Fucilla, Eur. Phys. J. C 82 (2022) 10, 929]

NRFF1.0
quarkonium FFs

=

APFEL++/EKO

HF-NRevo: Initial-scale NRCQD inputs



Scalar η_c

HF-NRevo for $\eta_c[^1S_0^{(1)}]$ collinear fragmentation

Partonic hard factor

DGLAP AOevo

All-Order evolution (numeric)

DGLAP EDevo

Expanded Decoupled evolution

NRQCD

LDMEs

SDCs

g
$\tilde{q}$
c
b
$2m_c$

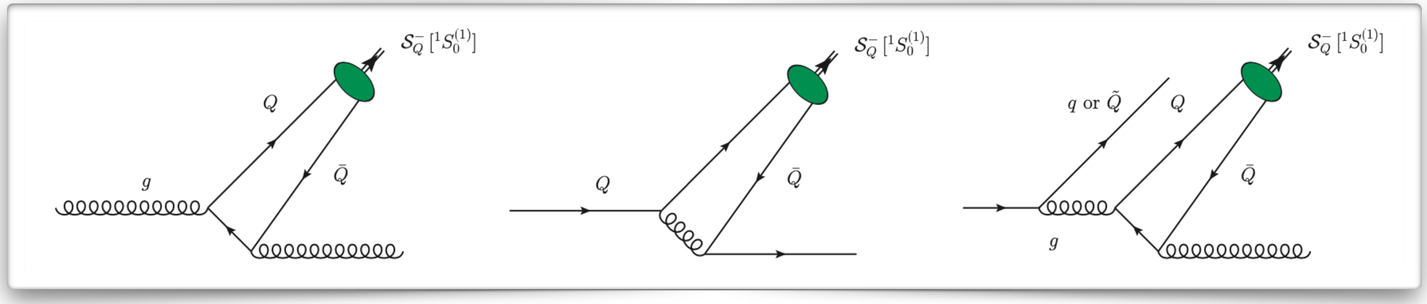
$\mathcal{D}_i^{\mathcal{Q}}(z,[n]) \otimes \langle \mathcal{O}^{\mathcal{Q}}([n]) \rangle$

Energy

Evolution-ready scale $Q_0 \equiv m_b + 2m_c \quad 3m_c \quad m_{\tilde{q}} + 2m_c$

Time

HF-NRevo: Initial-scale NRCQD inputs



Scalar η_c

HF-NRevo for $\eta_c[^1S_0^{(1)}]$ collinear fragmentation

Partonic hard factor

DGLAP AOevo

All-Order evolution (numeric)

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Expanded Decoupled evolution

NRQCD

LDMEs

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g
$\tilde{q}$
c
b
$2m_c$

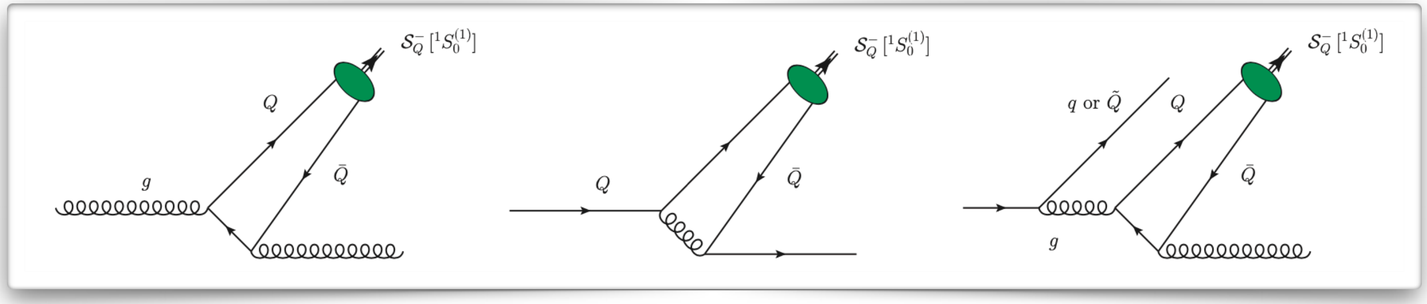
$\mathcal{D}_i^{\mathcal{Q}}(z,[n]) \otimes \langle \mathcal{O}^{\mathcal{Q}}([n]) \rangle$

Energy

Evolution-ready scale $Q_0 \equiv m_b + 2m_c \quad 3m_c \quad m_{\tilde{q}} + 2m_c$

Time

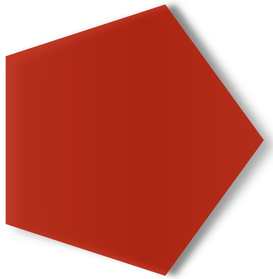
HF-NRevo: Initial-scale NRCQD inputs



Scalar η_c

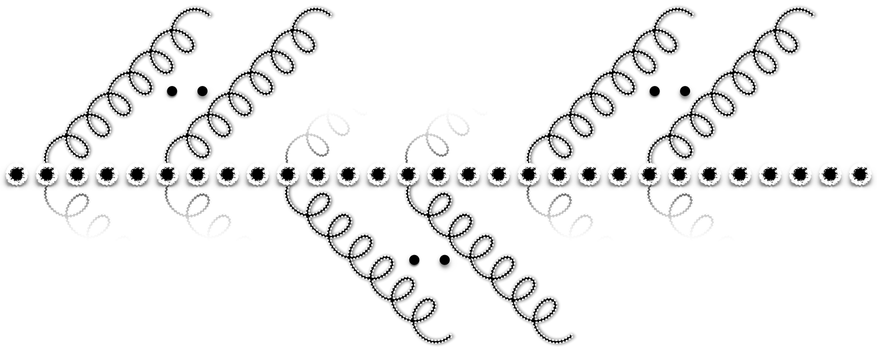
HF-NRevo for $\eta_c[^1S_0^{(1)}]$ collinear fragmentation

Partonic hard factor



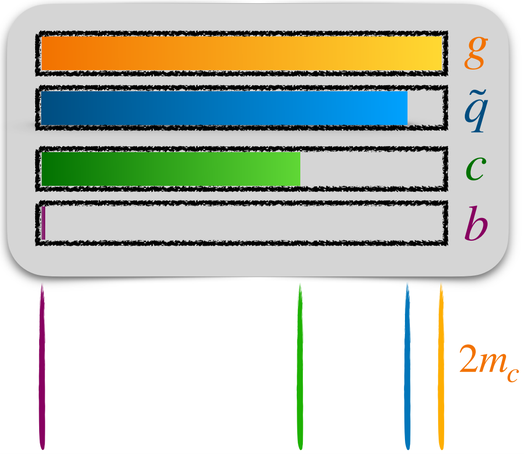
DGLAP AOevo

All-Order evolution (numeric)



DGLAP EDevo

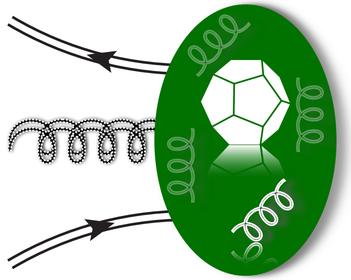
Expanded Decoupled evolution



NRQCD

LDMEs

SDCs



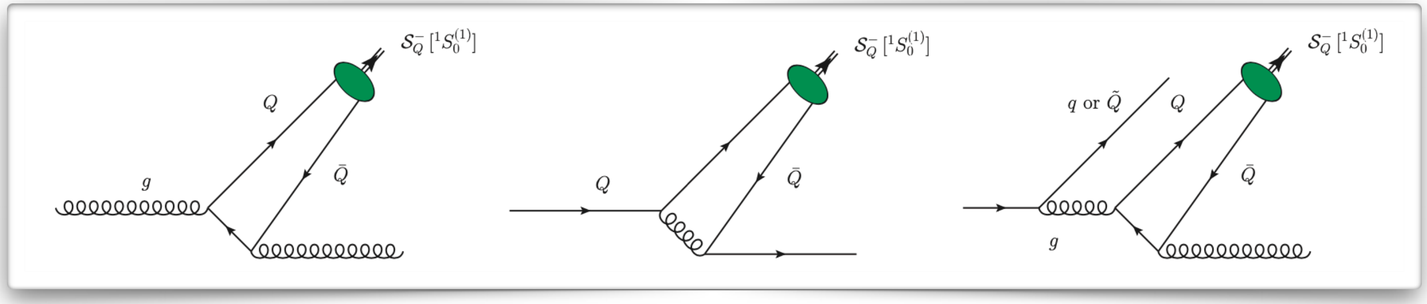
$\mathcal{D}_i^{\mathcal{Q}}(z,[n]) \otimes \langle \mathcal{O}^{\mathcal{Q}}([n]) \rangle$

Energy

Evolution-ready scale $Q_0 \equiv m_b + 2m_c$ $3m_c$ $m_{\tilde{q}} + 2m_c$

Time

HF-NRevo: Initial-scale NRCQD inputs



Scalar η_b

HF-NRevo for $\eta_b[^1S_0^{(1)}]$ collinear fragmentation

Partonic hard factor

DGLAP AOevo

All-Order evolution (numeric)

DGLAP EDevo

Expanded Decoupled evolution

g
$\tilde{q}$
c
b
$2m_b$

NRQCD

SDCs

LDMEs

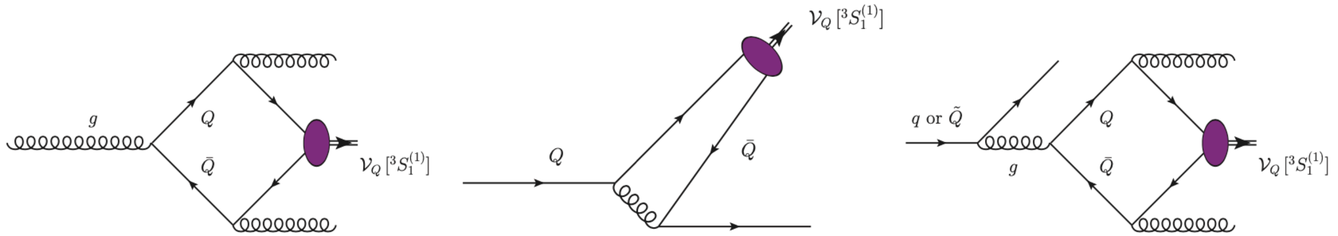
$\mathcal{D}_i^{\mathcal{Q}}(z,[n]) \otimes \langle \mathcal{O}^{\mathcal{Q}}([n]) \rangle$

Energy

Evolution-ready scale $Q_0 \equiv 3m_b$ $m_c + 2m_b$ $m_{\tilde{q}} + 2m_b$

Time

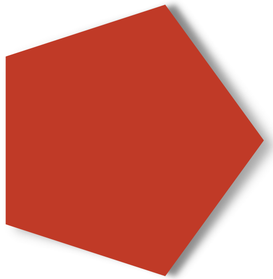
HF-NRevo: Initial-scale NRCQD inputs



Vector J/ψ

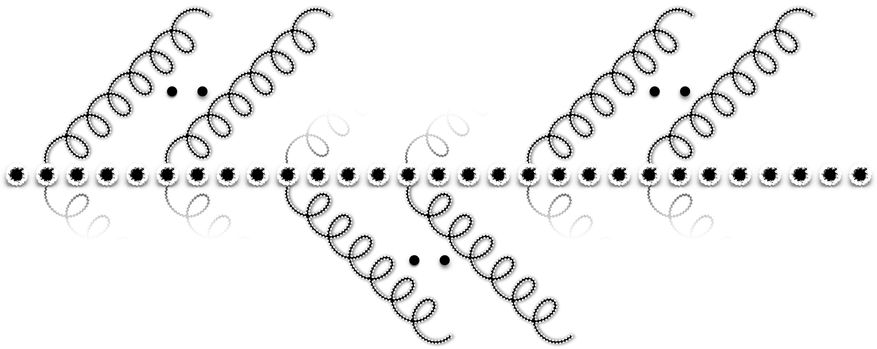
HF-NRevo for $J/\psi[^3S_1^{(1)}]$ collinear fragmentation

Partonic hard factor



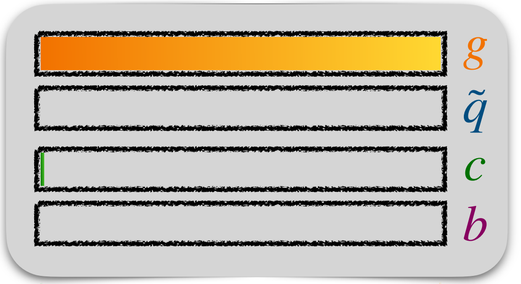
DGLAP AOevo

All-Order evolution (numeric)



DGLAP EDevo

Expanded Decoupled evolution

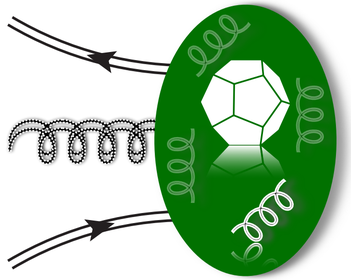


$2m_c$

NRQCD

LDMEs

SDCs



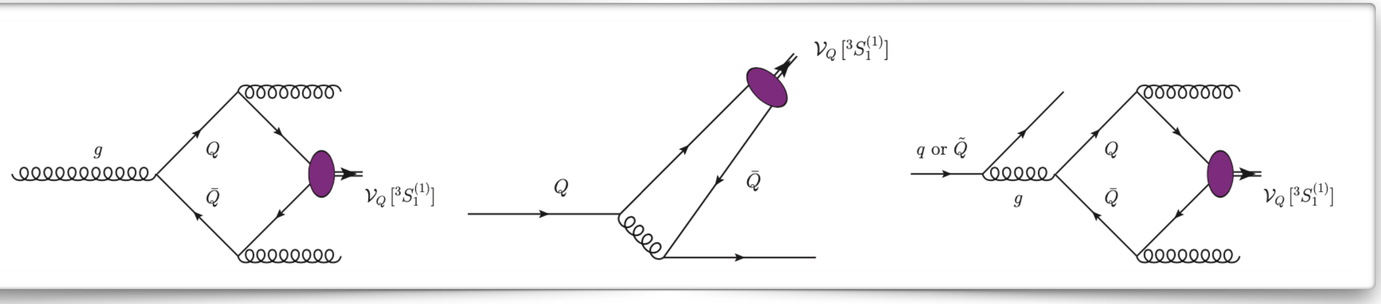
$\mathcal{D}_i^{\mathcal{Q}}(z,[n]) \otimes \langle \mathcal{O}^{\mathcal{Q}}([n]) \rangle$

Energy

Evolution-ready scale $Q_0 \equiv 3m_c$

Time

HF-NRevo: Initial-scale NRCQD inputs



Vector Υ

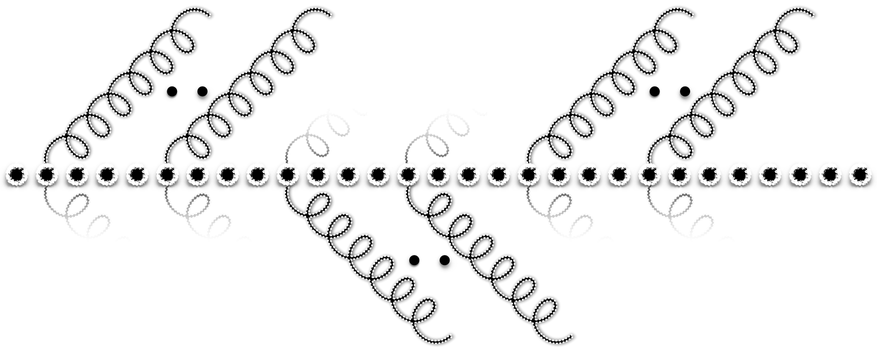
HF-NRevo for $\Upsilon[^3S_1^{(1)}]$ collinear fragmentation

Partonic hard factor



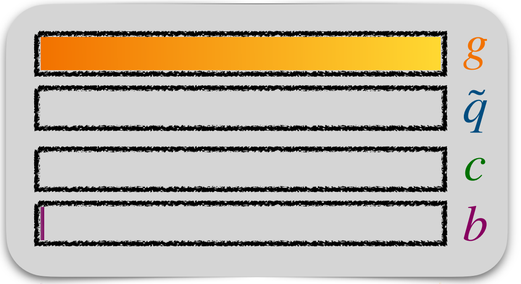
DGLAP AOevo

All-Order evolution (numeric)



DGLAP EDevo

Expanded Decoupled evolution

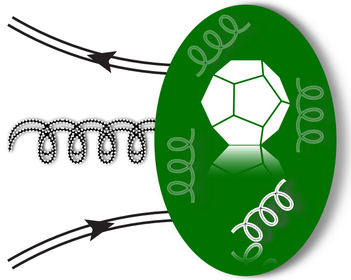


$2m_c$

NRQCD

LDMEs

SDCs



$\mathcal{D}_i^{\mathcal{Q}}(z,[n]) \otimes \langle \mathcal{O}^{\mathcal{Q}}([n]) \rangle$

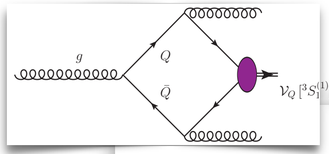
Energy

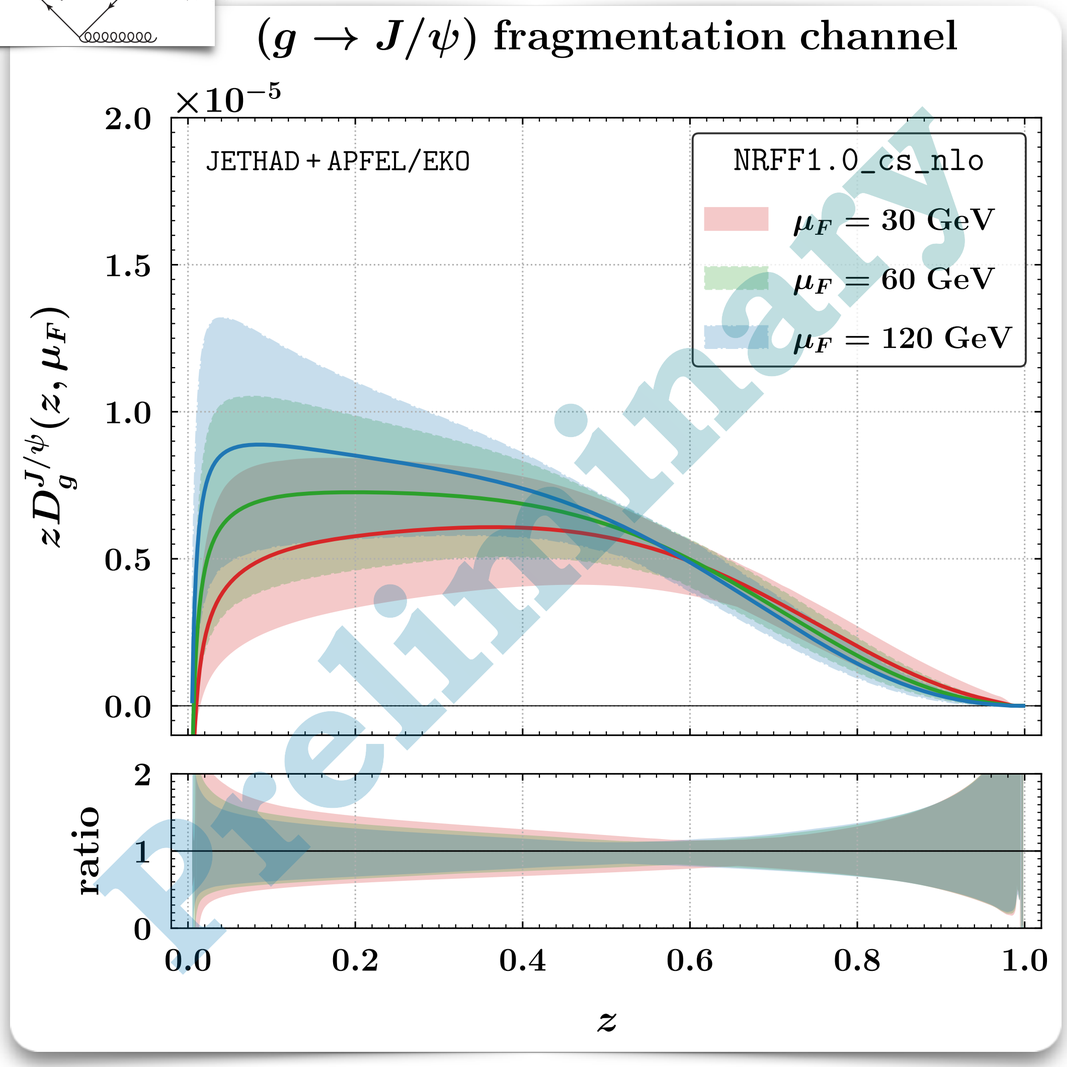
Evolution-ready scale $Q_0 \equiv 3m_b$

Time

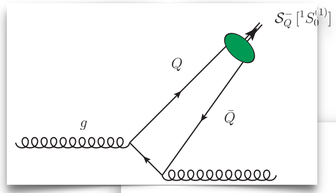
NRFF1.0: Gluon fragmentation to charmonia

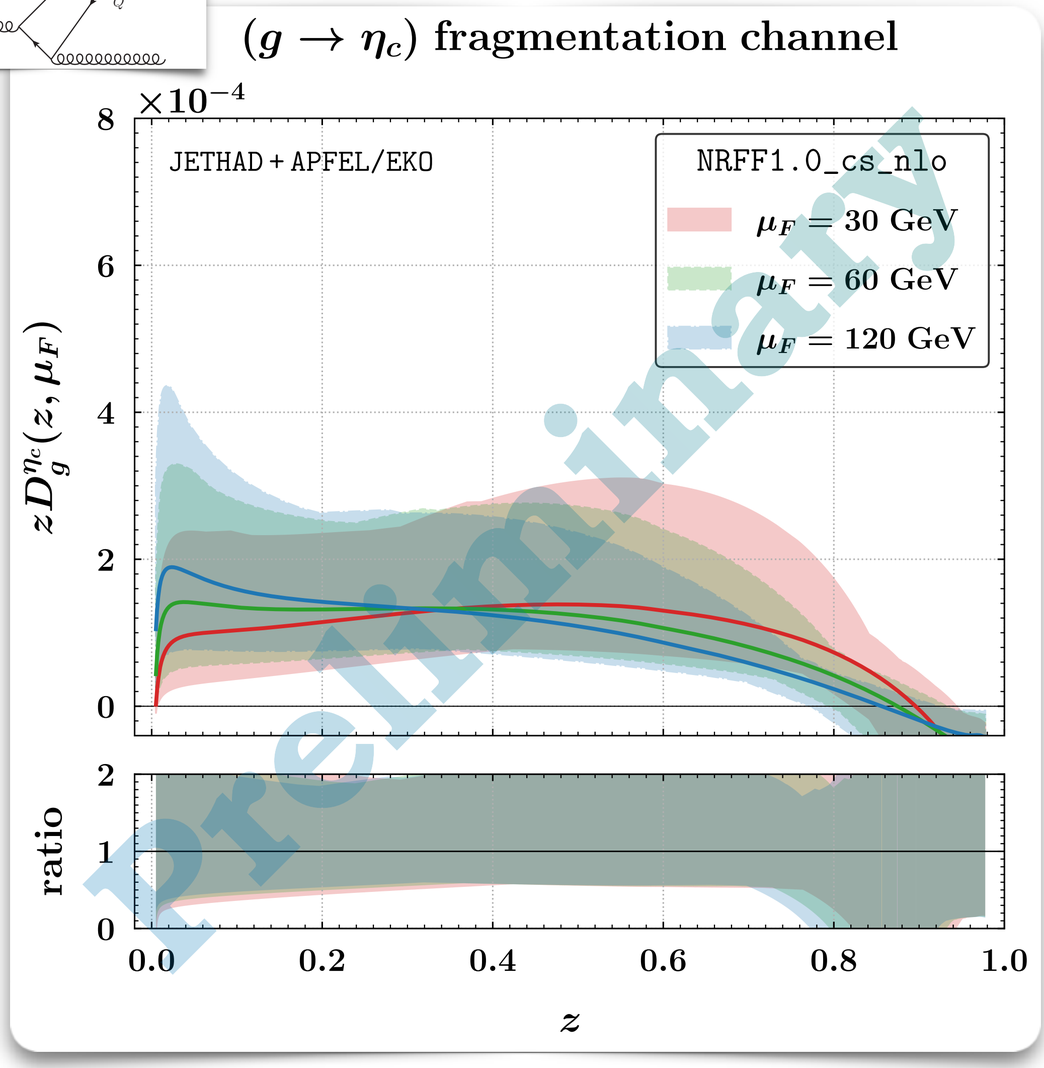
Vector J/ψ





Scalar η_c

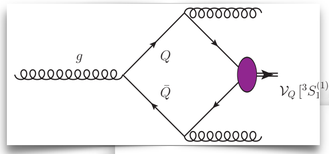


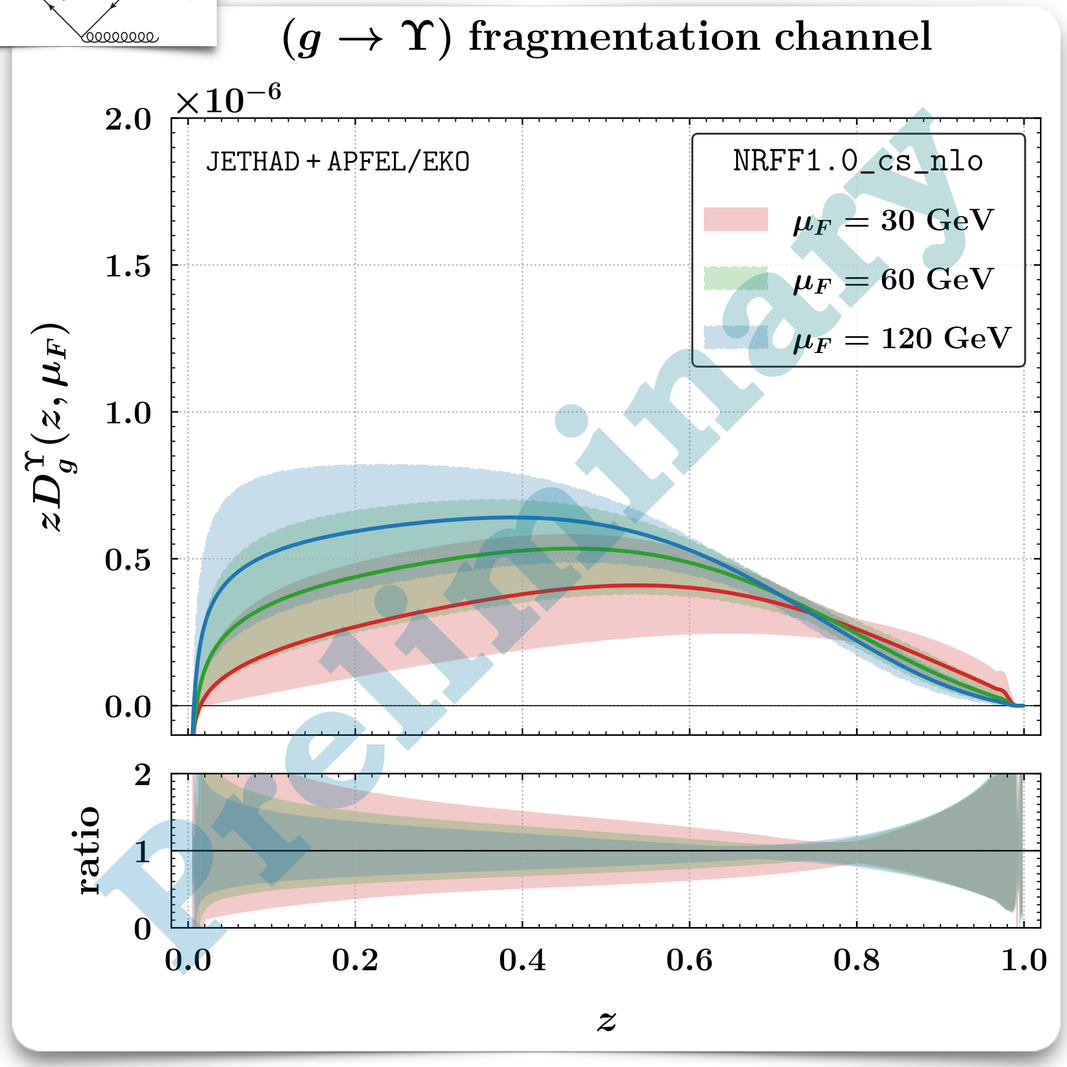


[F. G. C., in preparation]

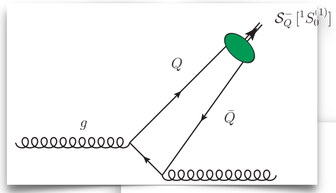
NRFF1.0: Gluon fragmentation to bottomonia

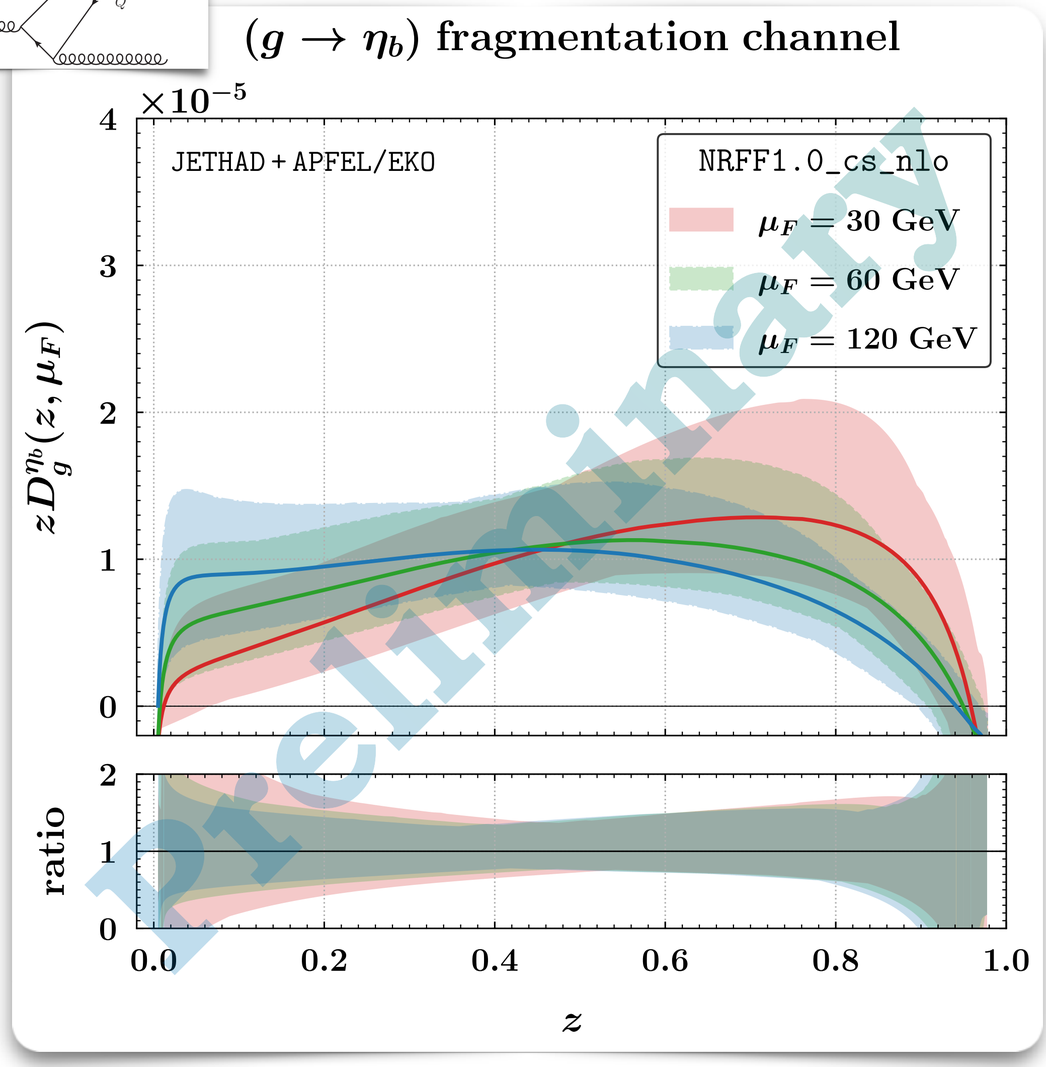
Vector Υ





Scalar η_b

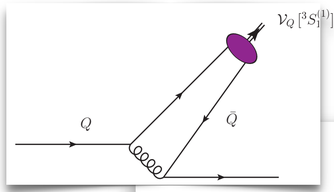


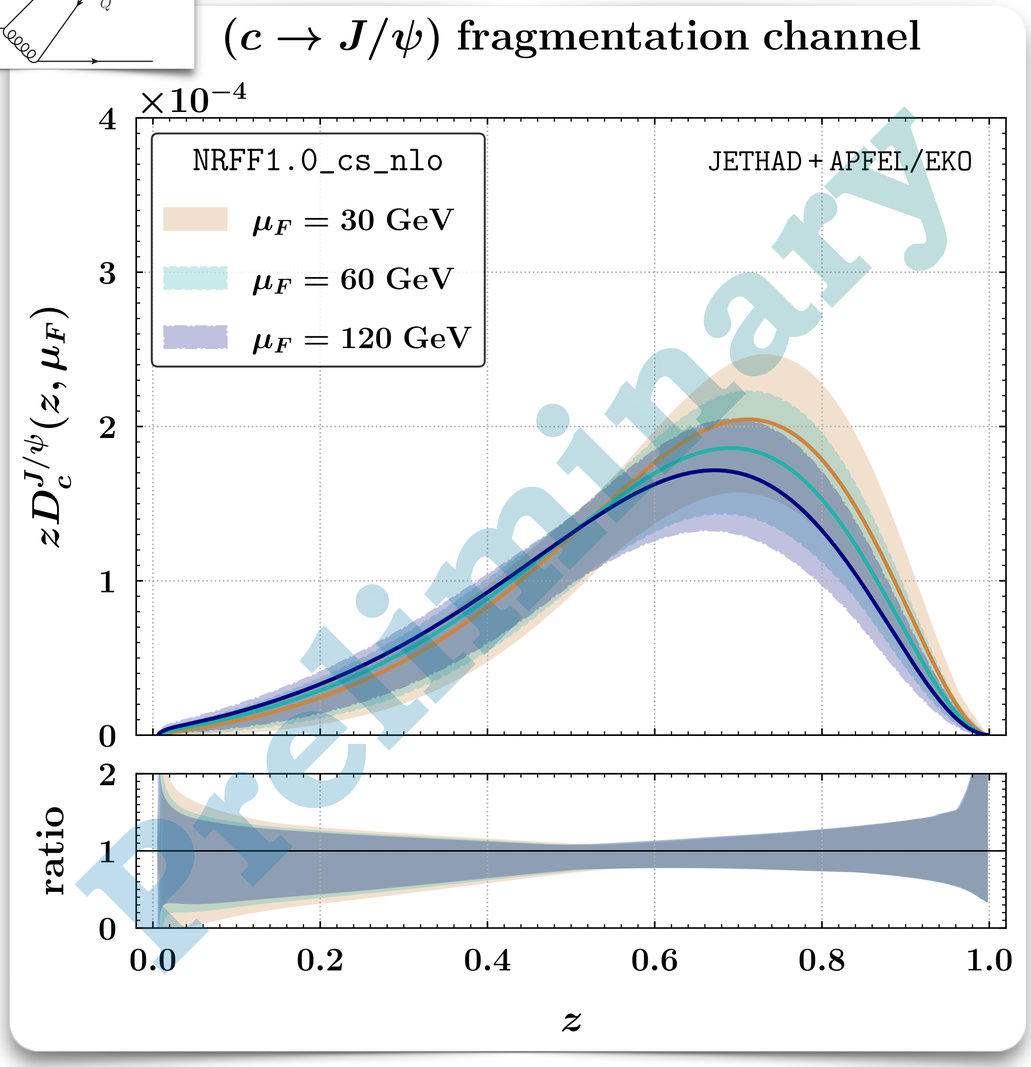


[F. G. C., in preparation]

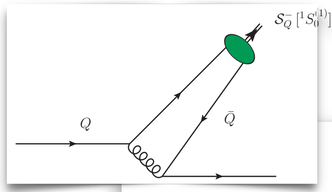
NRFF1.0: Charm fragmentation to charmonia

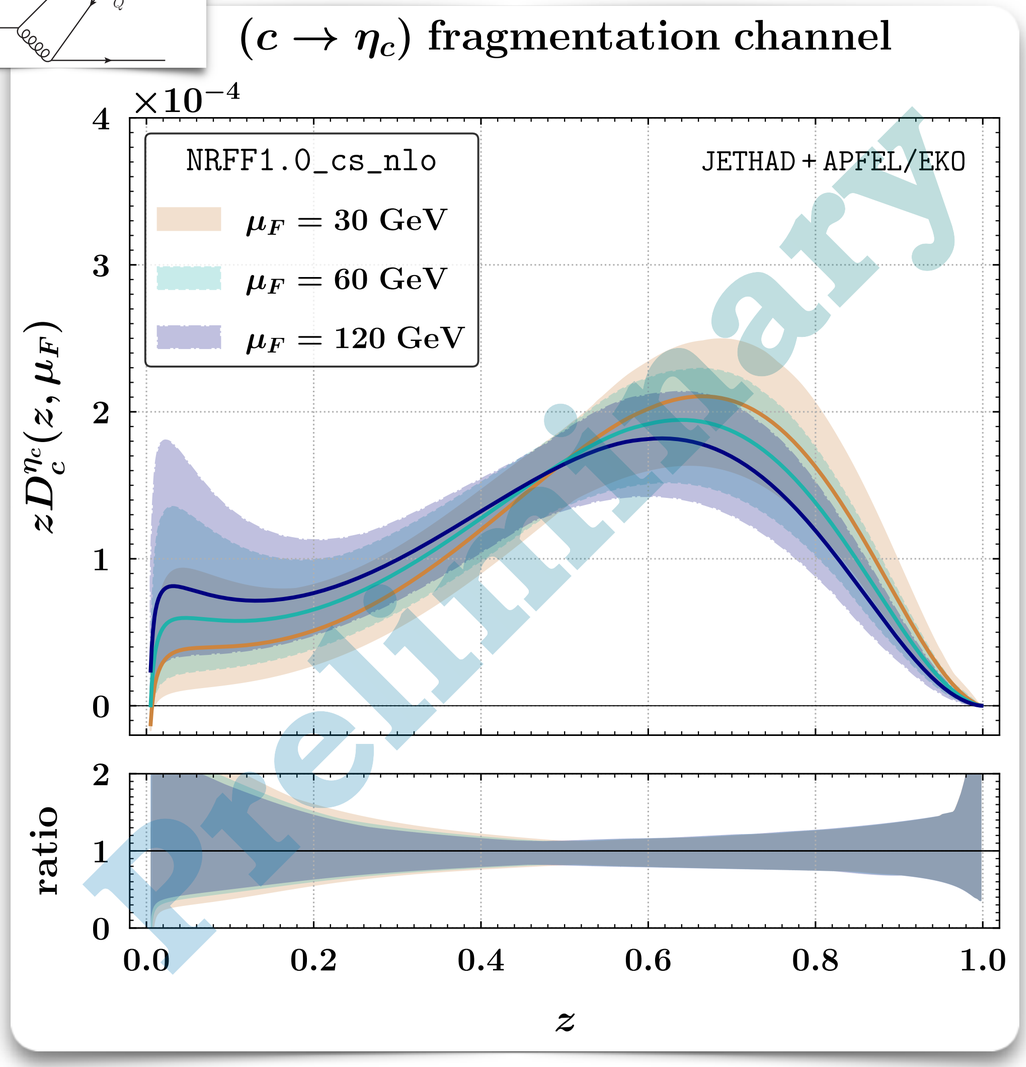
Vector J/ψ





Scalar η_c



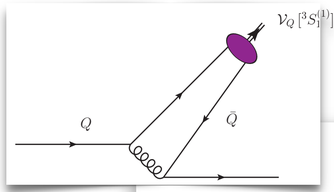


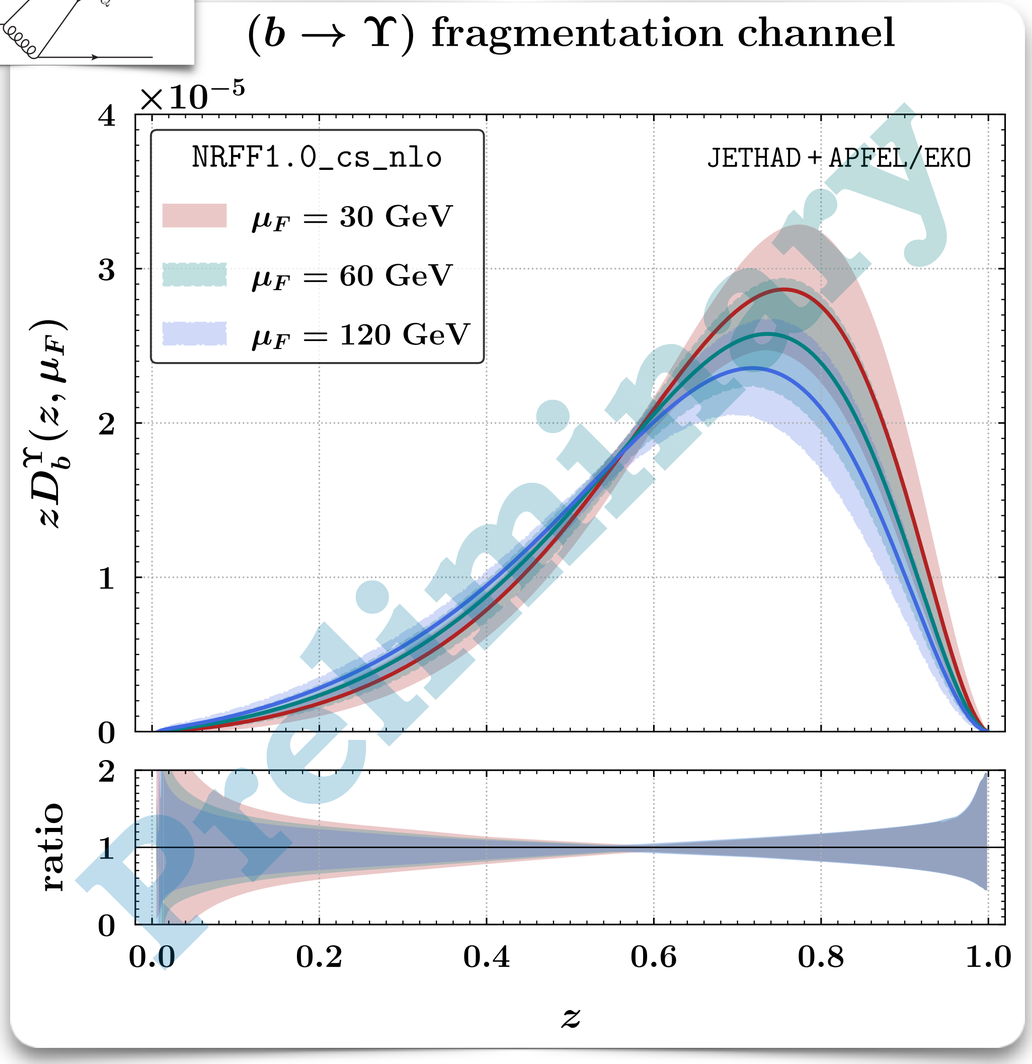
Preliminary

[F. G. C., in preparation]

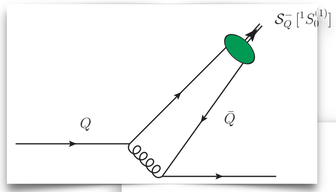
NRFF1.0: Bottom fragmentation to bottomonia

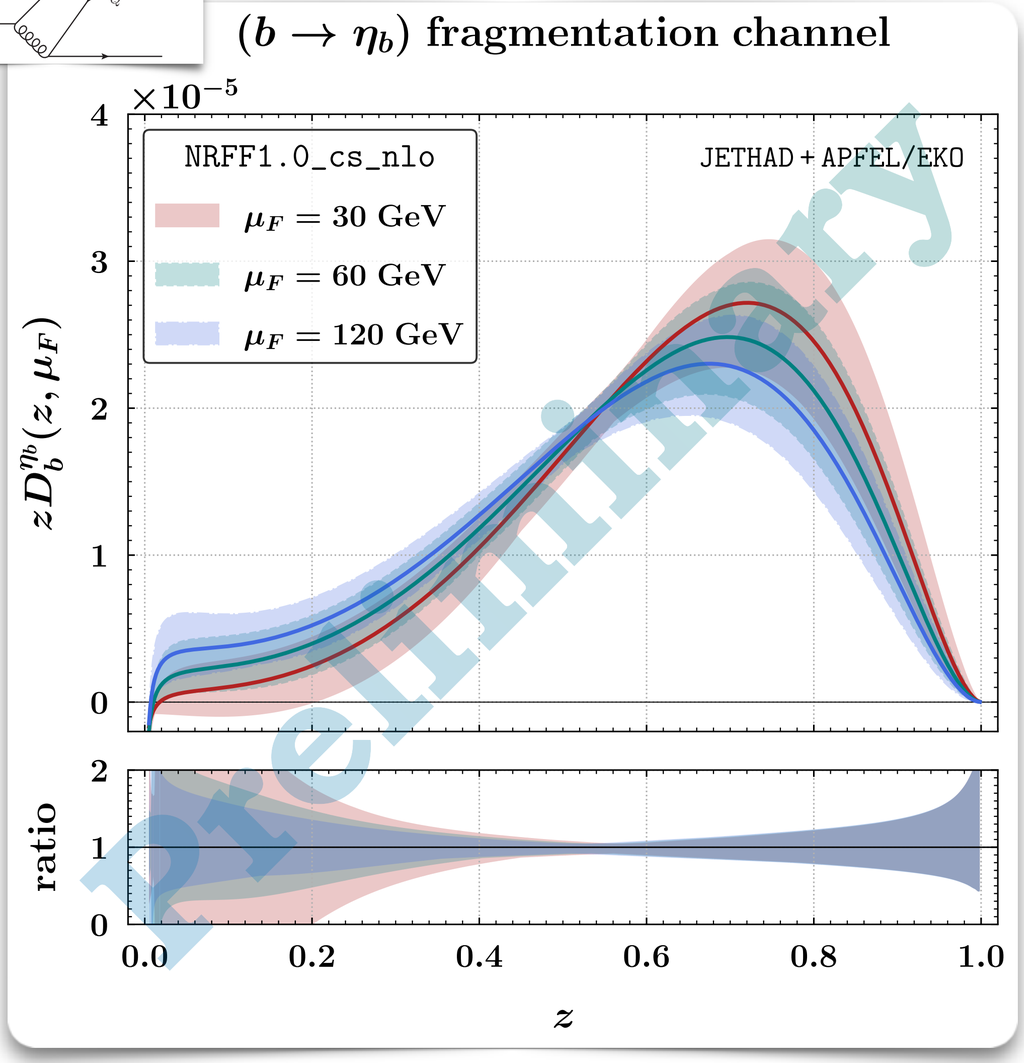
Vector Υ





Scalar η_b





Preliminary

[F. G. C., in preparation]