

High-energy logarithms and precision at the LHC

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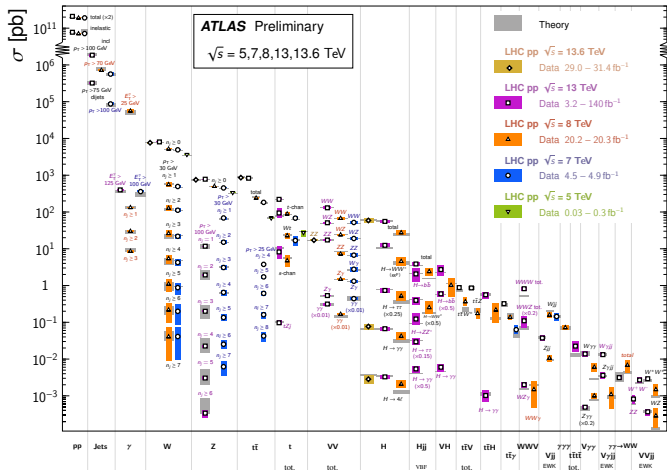
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Lessons on Perturbative QCD at High Energies

Standard Model Production Cross Section Measurements

Status: June 2024



Incredible agreement between fixed order calculations and total cross sections.

Will discuss situations where perturbative corrections **beyond fixed order** are required to obtain good agreement for **distributions**.

This would be old news in the case of p_t -hierarchies (requiring p_t -resummation, parton showers etc.)

New situation: considerations of logarithmic corrections of **High Energy-origin** are necessary for relevant distributions

Lessons from this study

Seemingly unrelated issues at fixed order are caused by the high energy logarithm

- ① **Highly asymmetric scale variations** also in observables unrelated to $m_{j_1 j_2}$
- ② The custom of **choosing a special renormalisation scale** $m_{j_1 j_2}$ for $m_{j_1 j_2}$ studies whereas all other observables are studied with a p_t -based scale

We will show that the special scale choice $\mu_R = m_{j_1 j_2}$ has been hiding a very clear breakdown of the perturbative series.

This breakdown is unrelated to the UV running of the coupling and any numerical effects of this scale choice is an accidental effect mixing UV and unrelated High Energy evolution. The breakdown in fixed-order predictions is repaired by **properly addressing the logarithmic corrections to all orders.**

Outline of talk:

1. Effects of the special scale choice $\mu_R = m_{j_1 j_2}$ in fixed order
2. Amplitudes in the High Energy Limit
3. Predictions for $pp \rightarrow ZJJ$ NLO vs. HEJ

High Energy Jets (HEJ):

- **Factorisation of matrix elements** using **currents** retains analytic properties such as **crossing symmetries**
- systematic **power expansion of QCD amplitudes** for real emissions
- all-order **leading and sub-leading logarithmic corrections**
- Recent experimental results showcasing the importance of the high-energy corrections

Scale choice for one-scale problems

Standard argument goes (for e.g. $e^+e^- \rightarrow$ hadrons R -ratio)

$$\alpha_s(\mu_R^2) = \frac{\alpha_s(Q_0^2)}{1 + b_0 \alpha_s(Q_0^2) \log(\mu_R^2/Q_0^2) - \frac{b_1}{b_0} \alpha_s(Q_0^2) \log \frac{\alpha_s(\mu_R^2)(b_0 + b_1 \alpha_s(Q_0^2))}{\alpha_s(Q_0^2)(b_0 + b_1 \alpha_s(\mu_R^2))} + \dots}$$

$$b_0 = \frac{11C_A - 2n_f}{12\pi}, \quad b_1 = \frac{34C_A^2 - 8C_F - 40n_f}{48\pi^2},$$

$$R(Q) = R_{EW}(Q)(1 + \delta_{QCD}(Q)), \quad \delta_{QCD}(Q) = \sum_{n=1}^{\infty} \bar{c}_n(\mu_R^2/Q^2) \cdot \left(\frac{\alpha_s(\mu_R^2)}{\pi} \right)^n,$$

$$\bar{c}_1(\mu_R^2/Q^2) = c_1, \quad \bar{c}_2(\mu_R^2/Q^2) = c_2 + \pi b_0 c_1 \log(\mu_R^2/Q^2), \dots$$

$$c_1 = 1, \quad c_2 = 1.9857 - 0.1152n_f, \dots$$

One scale problem: Only one logarithm arises. Depends on unphysical parameter μ_R .
Choose $\mu_R^2 = Q^2$ to remove logarithmic corrections.

Scale choice for $2 \rightarrow 2 + n$ in QCD processes

A certain set of processes (see later) develops another set of logarithms

$$\hat{\sigma}_{2j}^{\text{LO}} \sim C \alpha_s^2(\mu_R^2)$$
$$\hat{\sigma}_{2j}^{\text{NLO}} \sim \hat{\sigma}_{2j}^{\text{LO}} \left(1 - \alpha_s(\mu_R^2) (C_2 C_A \log m_{j_1 j_2}^2 / p_t^2 - 2b_0 \log(\mu_R^2 / p_t^2)) \right).$$

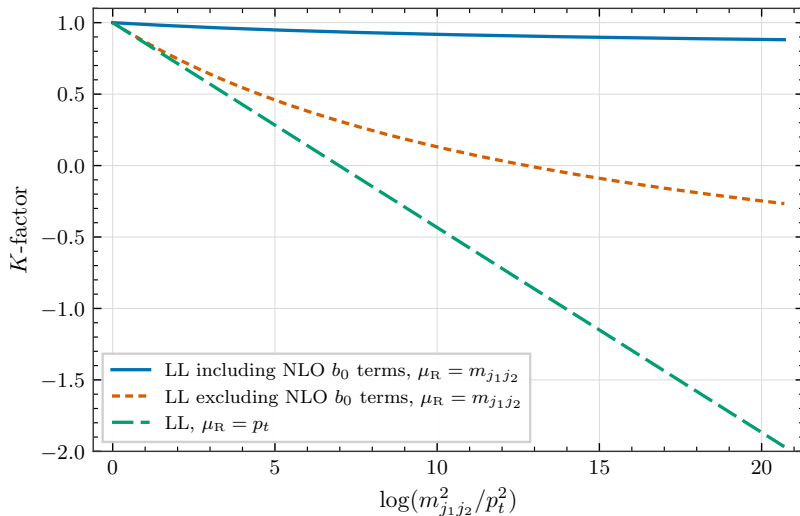
(dropping all terms non-logarithmic in $m_{j_1 j_2}^2 / p_t^2$ but keeping the UV logarithm which can be chosen to be leading by setting $\mu_R = m_{j_1 j_2}$). Consider the region $m_{j_1 j_2} \gg p_t$.

Two separate problems arise with $\mu_R = m_{j_1 j_2}$ in studies of $m_{j_1 j_2} \gg p_t$:

- 1 The UV logarithm is **maximised** by hand and allowed to modify the coefficient of the high energy logarithm (which is independent of the running of the coupling)
- 2 In the large $m_{j_1 j_2}$ limit the perturbative correction becomes **unperturbative**

Dramatic change in coefficient of HE logarithm when setting

$$\mu_R = m_{j_1 j_2}$$



Unperturbative Correction with $\mu_R = m_{j_1 j_2}$

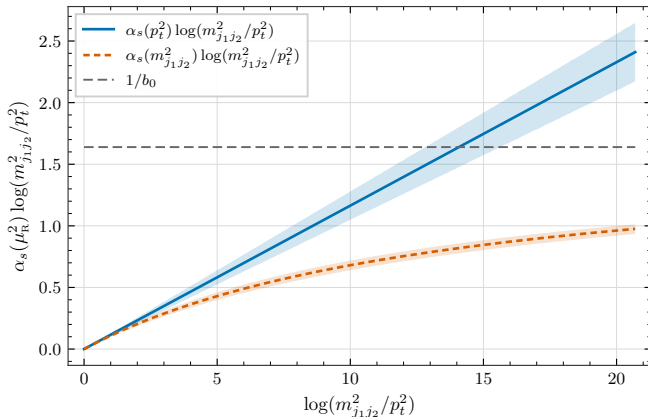
$$\alpha_s(m_{j_1 j_2}^2) \ln(m_{j_1 j_2}^2/p_t^2) \rightarrow \frac{1}{b_0} \quad \text{for} \quad m_{j_1 j_2} \rightarrow \infty$$

The perturbative correction $\alpha_s(m_{j_1 j_2}^2) \log(m_{j_1 j_2}^2/p_t^2)$ becomes **independent of** α_s in the large $m_{j_1 j_2}$ limit.

The perturbative correction becomes **unperturbative**!

The sensitivity to α_s is much reduced already far from that limit

Unreasonable scale choice



The Emergence and Universality of the High Energy Logarithm

The elements necessary to the High Energy Logarithm

$$\mathcal{M}_{ij \rightarrow ij}^{(1LL)aa'bb'} = \mathcal{M}_{ij \rightarrow ij}^{(0LL)aa'bb'} \alpha(t) \log \left(\frac{s}{-t} \right).$$

$$\alpha(t) = g_s^2 C_A \frac{2}{\varepsilon} \left(\frac{\mu^2}{-t} \right)^\varepsilon c_\Gamma$$
$$c_\Gamma = \frac{1}{(4\pi)^{2-\varepsilon}} \frac{\Gamma(1+\varepsilon) \Gamma^2(1-\varepsilon)}{\Gamma(1-2\varepsilon)}.$$

And a real emission contribution

$$\frac{|\mathcal{M}_{n+1}|^2}{|\mathcal{M}_n|^2} \rightarrow \frac{4g_s^2 C_A}{p_{i\perp}^2}$$

The Emergence and Universality of the High Energy Logarithm

Loop factors from Kunszt, Signer, Trocsanyi $qQ \rightarrow qQ$.

$$F_{a;1}^{--}(\varepsilon, s_{12}, s_{13}, s_{14}) = \left(-\frac{\mu^2}{s_{14}}\right)^\varepsilon \left\{ N_C \left[-\frac{2}{\varepsilon^2} - \frac{3}{\varepsilon} + \frac{11}{3\varepsilon} - \frac{2}{\varepsilon} \log \frac{s_{14}}{s_{12}} + \frac{13}{9} + \pi^2 \right] + n_f \left[-\frac{2}{3\varepsilon} - \frac{10}{9} \right] - \frac{1}{N_C} \left[-\frac{2}{\varepsilon^2} - \frac{3}{\varepsilon} - \frac{2}{\varepsilon} \log \frac{s_{12}}{s_{13}} - 8 + \frac{1}{2} \frac{s_{14}}{s_{12}} \left(1 - \frac{s_{13}}{s_{12}} \right) \left(\log^2 \frac{s_{14}}{s_{12}} + \pi^2 \right) + \frac{s_{14}}{s_{12}} \log \frac{s_{14}}{s_{13}} \right] \right\} - \frac{1}{\varepsilon} \beta_0,$$

$$F_{a;2}^{--}(\varepsilon, s_{12}, s_{13}, s_{14}) = \left(-\frac{\mu^2}{s_{14}}\right)^\varepsilon \frac{N_C^2 - 1}{N_C} \left[-\frac{2}{\varepsilon} \log \frac{s_{12}}{s_{13}} + \frac{1}{2} \frac{s_{14}}{s_{12}} \left(1 - \frac{s_{13}}{s_{12}} \right) \left(\log^2 \frac{s_{14}}{s_{13}} + \pi^2 \right) + \frac{s_{14}}{s_{12}} \log \frac{s_{14}}{s_{13}} \right].$$

In the high energy limit $s_{12} \gg s_{14}$ all kinematic contributions vanish except the high energy logarithm

One-loop colour-stripped primitive amplitudes by Bern, Dixon, Kosower

$$A_6^{(1)} = g_s^2 g \left(M^{(0)} V + iF \right),$$

$$\begin{aligned} V^{cc} &= -\frac{1}{\varepsilon^2} \left(\left(\frac{\mu^2}{-s_{12}} \right)^\varepsilon + \left(\frac{\mu^2}{-s_{23}} \right)^\varepsilon + \left(\frac{\mu^2}{-s_{34}} \right)^\varepsilon \right) - \frac{2}{\varepsilon} \left(\frac{\mu^2}{-s_{56}} \right)^\varepsilon - 4. \\ &= -\left(\frac{\mu^2}{-s_{23}} \right)^\varepsilon \left[\frac{1}{\varepsilon^2} \left(\left(\frac{s_{23}}{s_{12}} \right)^\varepsilon + 1 + \left(\frac{s_{23}}{s_{34}} \right)^\varepsilon \right) \right] - \frac{2}{\varepsilon} \left(\frac{\mu^2}{-s_{56}} \right)^\varepsilon - 4. \end{aligned}$$

$$V^{cc} \supset \left(\frac{\mu^2}{-s_{23}} \right)^\varepsilon \left[-\frac{1}{\varepsilon} (\log(s_{23}/s_{12}) + \log(s_{23}/s_{34})) \right].$$

This process contains the virtual component of the high energy logarithm

The general case

QCD soft singular term for loop helicity amplitudes (Kunszt, Signer, Trocsanyi)

$$\begin{aligned} \mathcal{A}_{\text{soft}}^{1-\text{loop}}(2 \rightarrow n-2)_{c_1 c_2 \dots c_n} \\ = - \sum_{i < j} \left(\frac{g}{4\pi} \right)^2 g \frac{1}{\epsilon^2} \left(-\frac{\mu^2}{s_{ij}} \right)^\epsilon \sum_{a, c'_i, c'_j} t_{c_i c'_i}^a t_{c_j c'_j}^a \mathcal{A}^{\text{tree}}(2 \rightarrow n-2)_{c_1 \dots c'_i \dots c'_j \dots c_n}, \end{aligned}$$

The high energy logarithm generalises to all processes with a colour octet exchange and generates a correction $C_A \log(s/t)$.

Manifestations of the high energy logarithm

Lepton rapidity

$$|y| \leq 1.37 \text{ or } 1.52 \leq |y| \leq 2.47$$

Leading lepton transverse momentum

$$p_T > 80 \text{ GeV}$$

Sub-leading lepton transverse momentum

$$p_T > 7 \text{ GeV}$$

Dilepton mass m_{ll}

$$m_{ll} \in (66, 116) \text{ GeV}$$

p_T^{recoil}

$$p_T^{\text{recoil}} > 200 \text{ GeV}$$

Leading jet transverse momentum

$$p_{T,j_1} > 80 \text{ GeV}$$

Sub-leading jet transverse momentum

$$p_{T,j_2} > 50 \text{ GeV}$$

Jet rapidity

$$|y_j| < 4.4$$

Dijet invariant mass $m_{j_1 j_2}$

$$m_{j_1 j_2} > 200 \text{ GeV}$$

Rapidity difference between two hardest jets

$$|\Delta y_{j_1 j_2}| > 1$$

Jets with rapidity in-between two hardest jets

$$\text{None with } p_T > 30 \text{ GeV}$$

Jet definition

$$\text{anti-}k_t, R = 0.4$$

ATLAS JHEP 08 (2024) 223 arxiv:2403.02793

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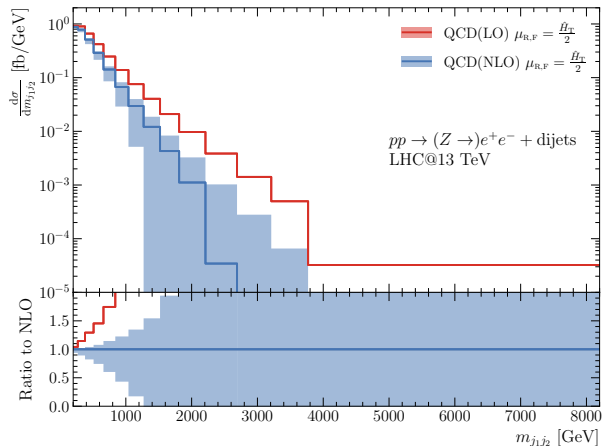
Clear logarithmic correction at NLO

$$\hat{\sigma}_{2j}^{\text{NLO}} \sim \hat{\sigma}_{2j}^{\text{LO}} \left(1 - \alpha_s(\mu_R^2) (C_2 C_A \log m_{j_1 j_2}^2 / p_t^2) \right).$$

NLO cross section even goes negative well within the explored phase space

Add resummation with HEJ

Need to add electroweak process $pp \rightarrow ZJJ$ before comparison to data. Large corrections at large $m_{j_1 j_2}$. Will compare just up to 1.5TeV



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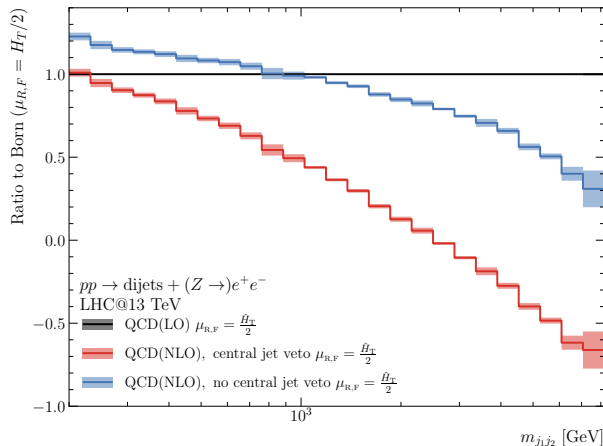
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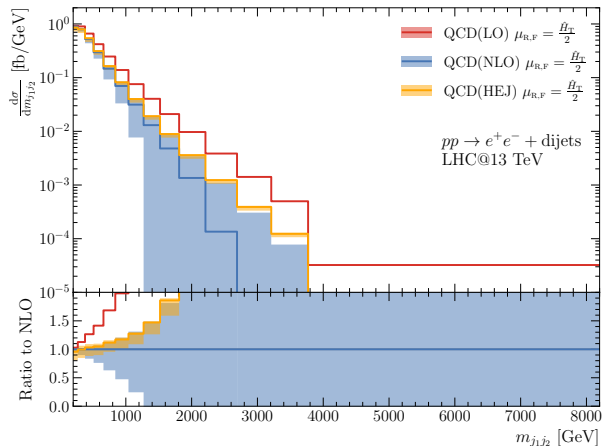
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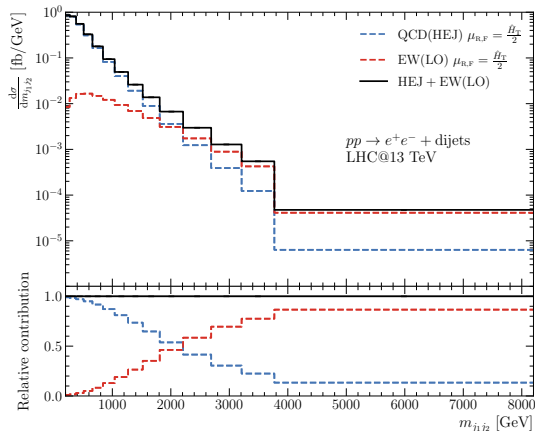
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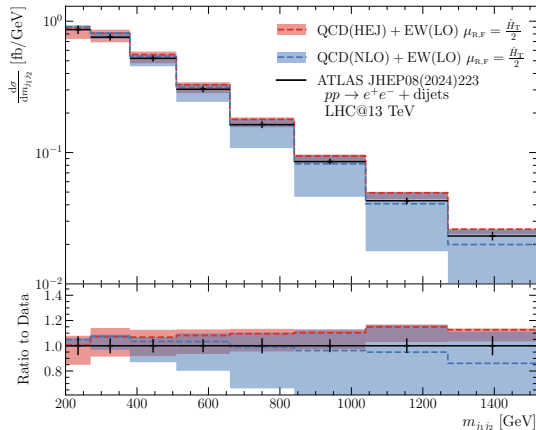
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- Consequence of choosing $\mu_R = m_{j_1 j_2}$ for studies at large $m_{j_1 j_2}$:
 - 1 UV logarithm is maximised and the coefficient of the leading logarithm $\log(m_{j_1 j_2}^2/p_t^2)$ is modified
 - 2 What should be a perturbative correction $\alpha_s(m_{j_1 j_2}) \log(m_{j_1 j_2}^2/p_t^2) \rightarrow 1/b_0$.
- Instead, address the large perturbative corrections by a systematic resummation.