
Recent progress on logarithmic accuracy in Panscales

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PanScales

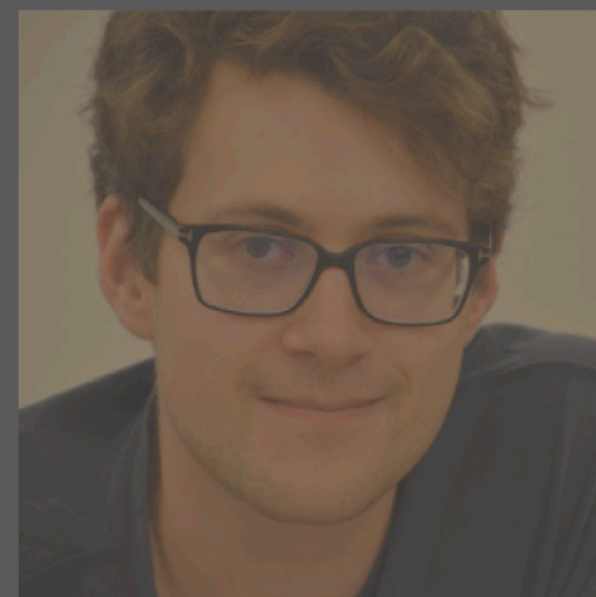
A project to bring
logarithmic
understanding and
accuracy to parton
showers



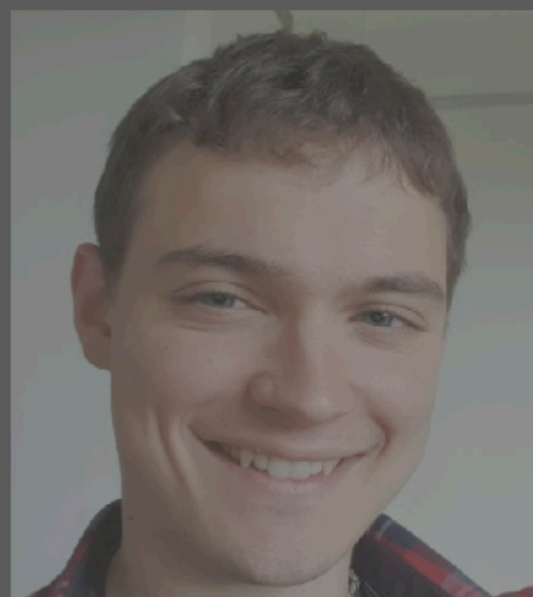
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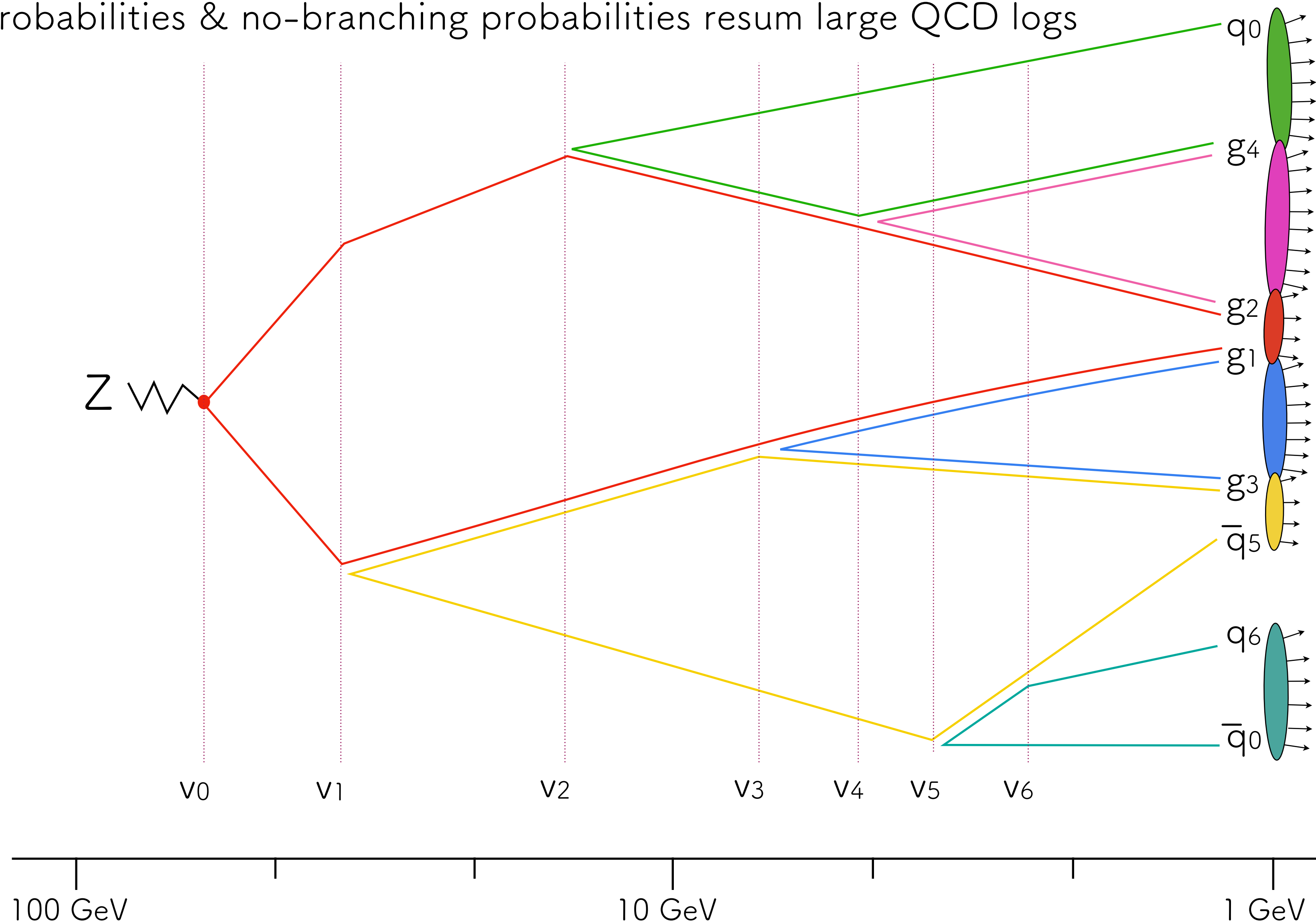


**Scarlett
Woolnough**

Former members

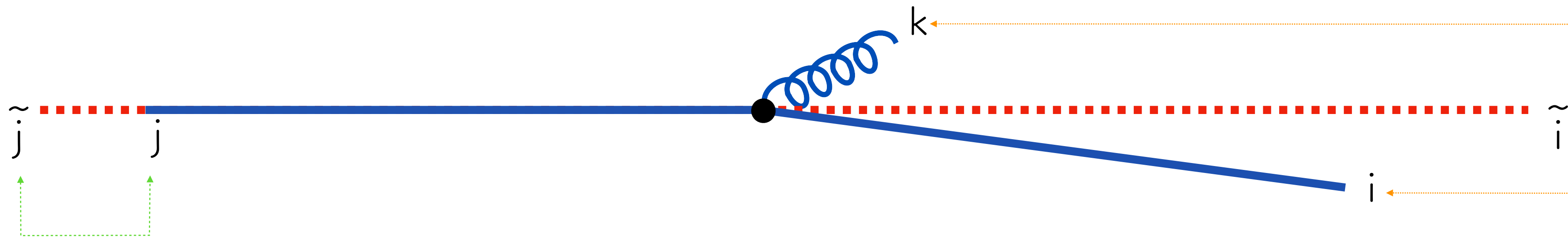
Parton showers

- Showers simulate cascades of QCD rad^n as a series of branchings ordered by a decreasing kinematic scale
- Branching probabilities & no-branching probabilities resum large QCD logs



Momentum conservation and recoil

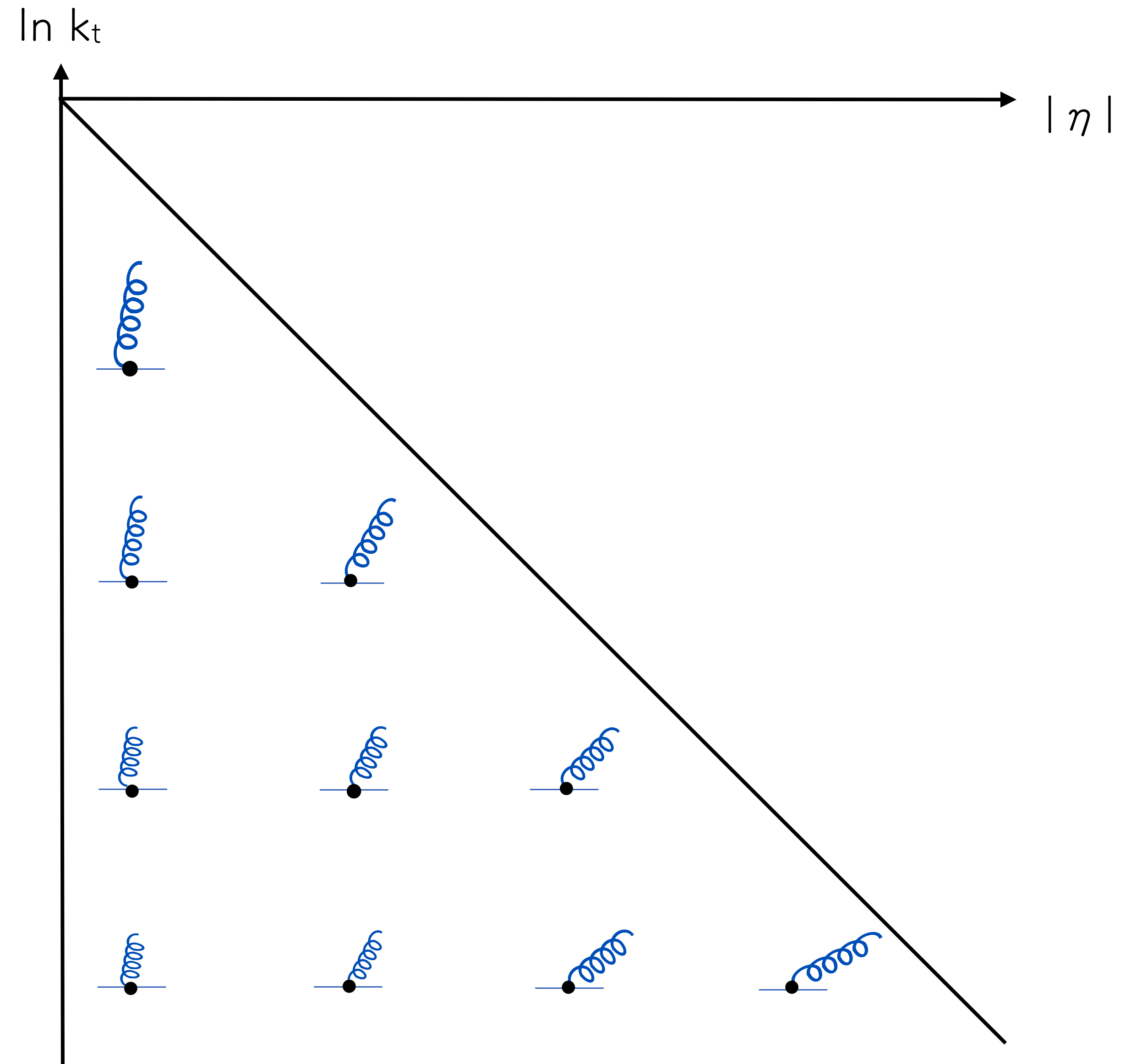
- The total momentum in the event is conserved before and after every branching
- Other particles need to recoil to balance each new emission
- E.g. in Pythia & PanLocal the emitter, i , takes the transverse recoil of k in the $\tilde{i}\tilde{j}$ C.o.M frame



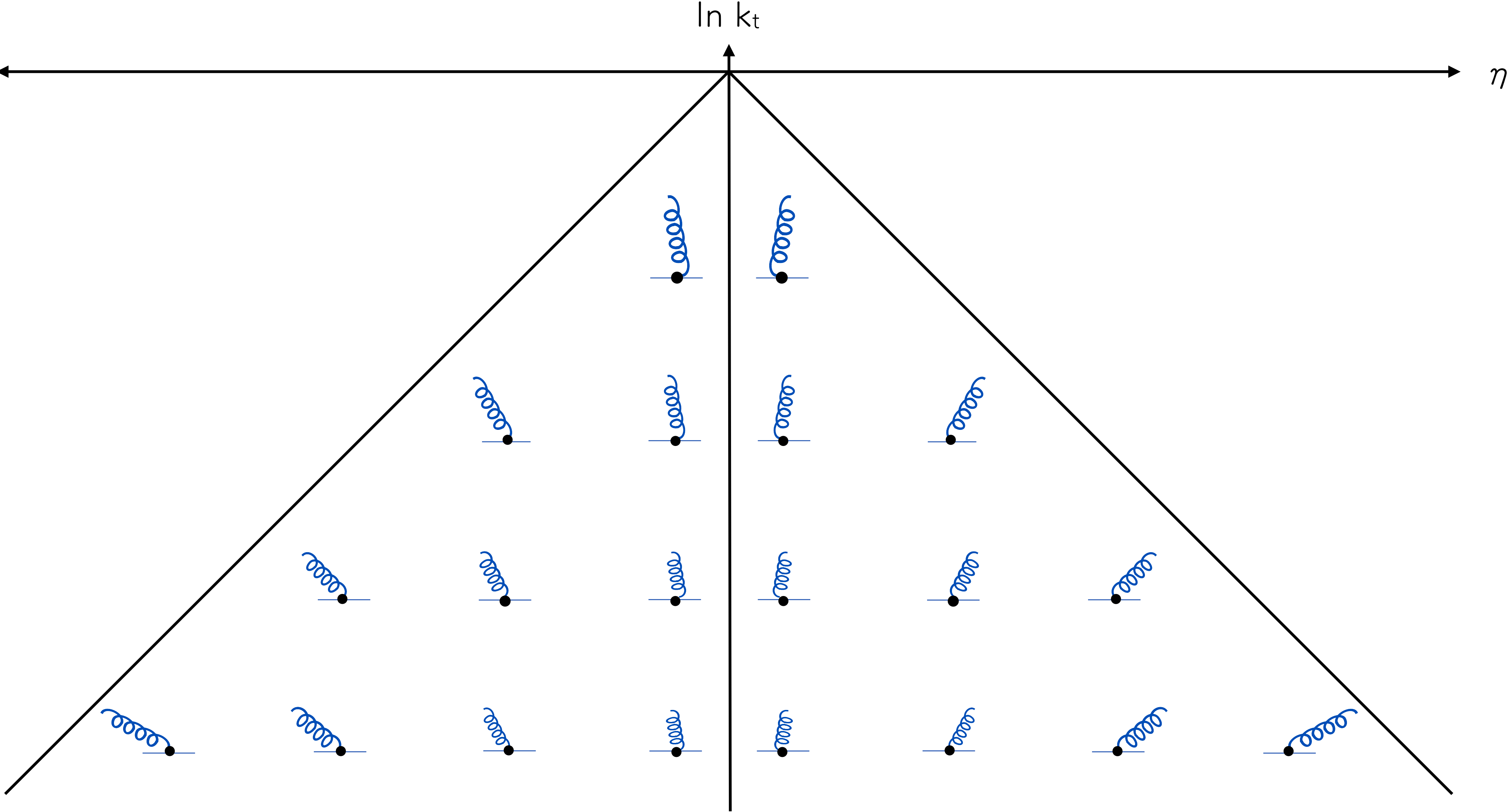
- Residual longitudinal momentum imbalance absorbed by rescaling $\tilde{j} \rightarrow j$ and the +ve light-cone component of i
[spectator]
- Not so good recoil prescriptions can do horrible things, e.g. new emissions recoiling & mangling older ones

Lund Plane

- Shower configurations can be visualised on the Lund plane
- Vertical axis : log of the transverse momentum of each emission relative to the jet axis
- Horizontal axis : each emission's rapidity or absolute rapidity
- ϕ is a 3rd coordinate : it can be averaged **or** a region in ϕ specified
- Each emission corresponds to a point



Lund Plane

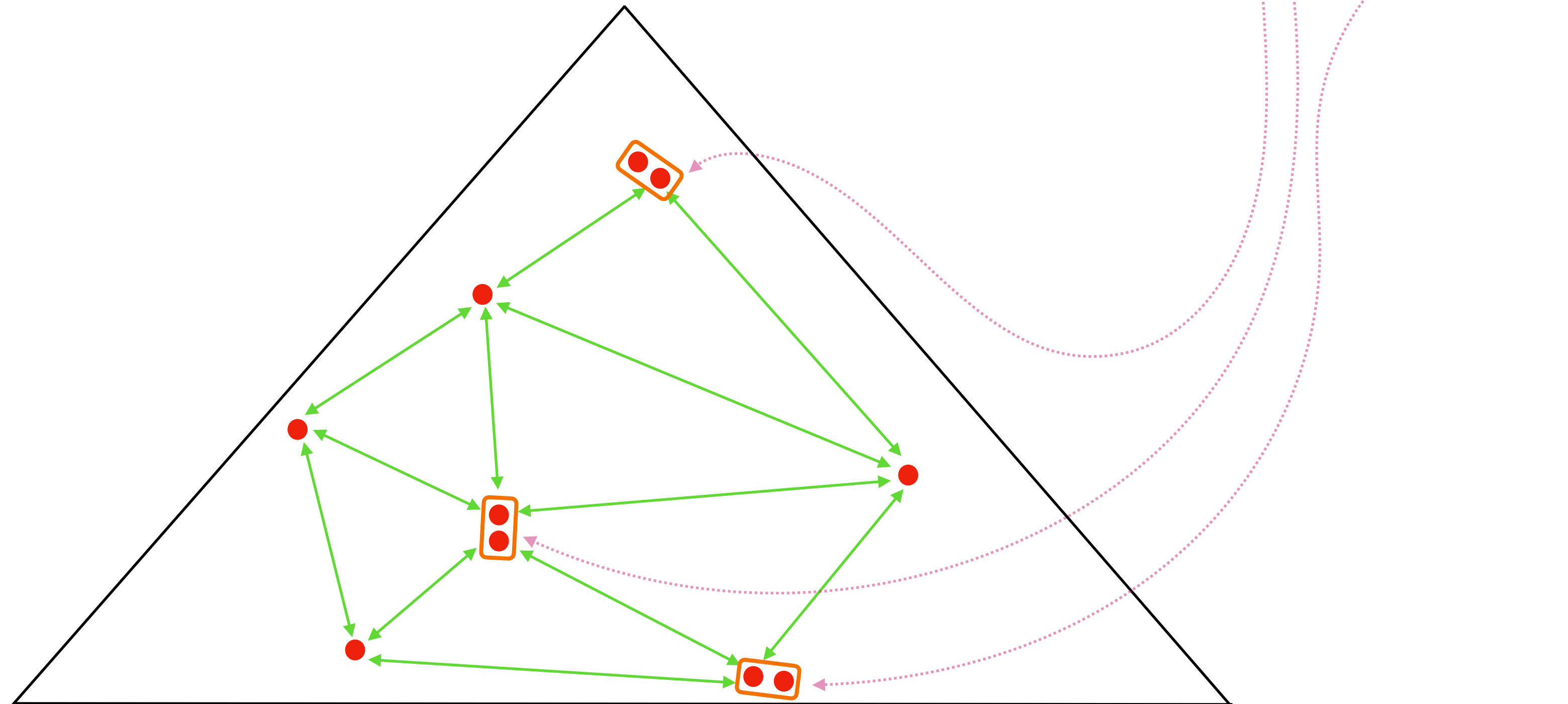


Getting from NLL to NNLL accuracy for event shapes with Panscales

- Include inclusive NLO hard-collinear corr^n and NNLO soft corr^n to single emission kernel :

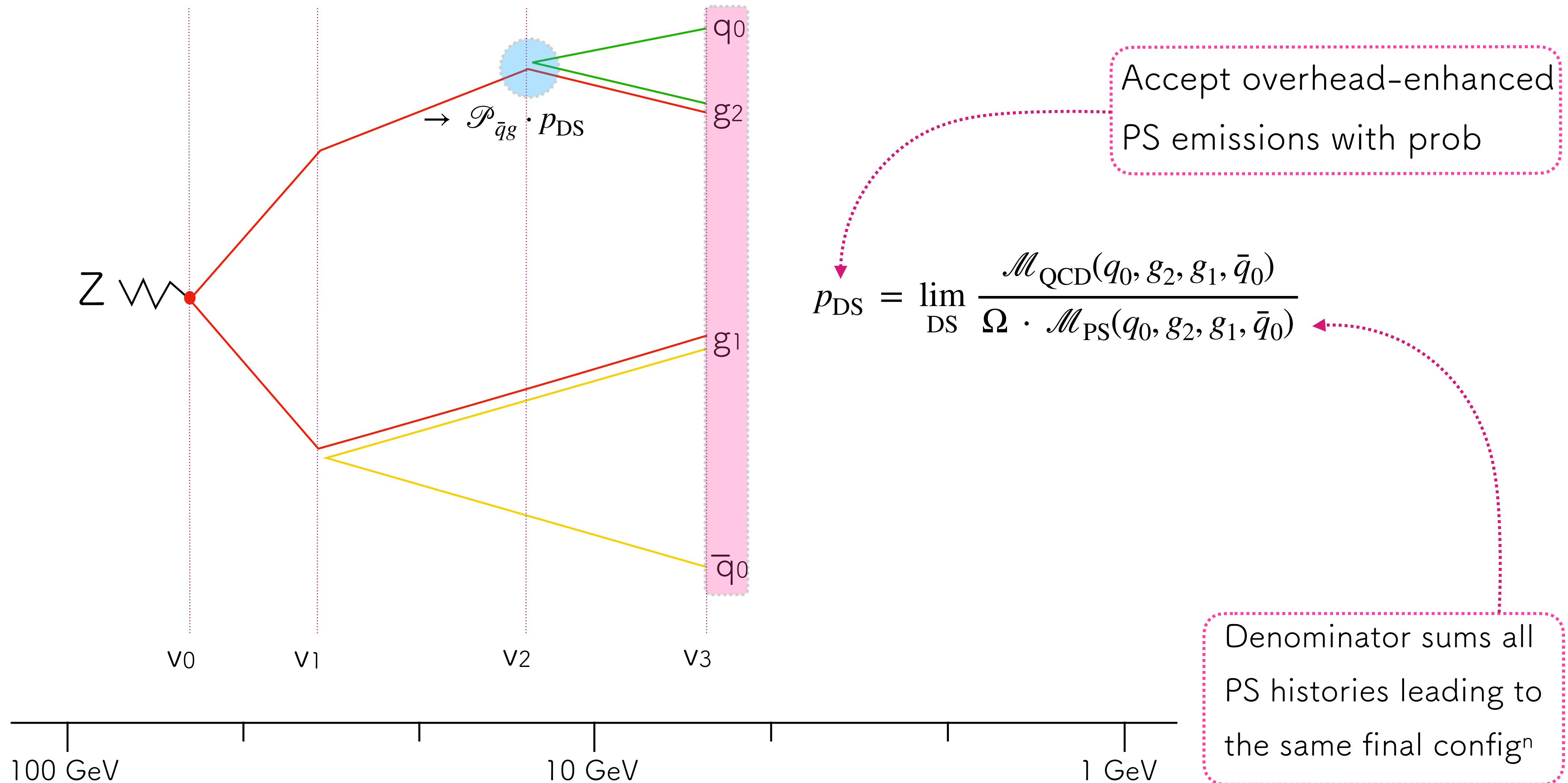
- $\alpha_s(k_t) \rightarrow \alpha_{s,\text{eff}} = \alpha_s \left[1 + \frac{\alpha_s}{2\pi} (K_1 + \Delta K_1(\bar{\eta}) + B_2(z) + \Delta B_2(z)) + \left(\frac{\alpha_s}{2\pi} \right)^2 (K_2 + \Delta K_2) \right]$

- Include NLO fixed order matching
- PS must reproduce tree-level QCD MEs for configurations with arbitrarily many double-soft **pairs** of emissions all widely separated from one another in the Lund plane, e.g.,



Correcting double-soft pairs

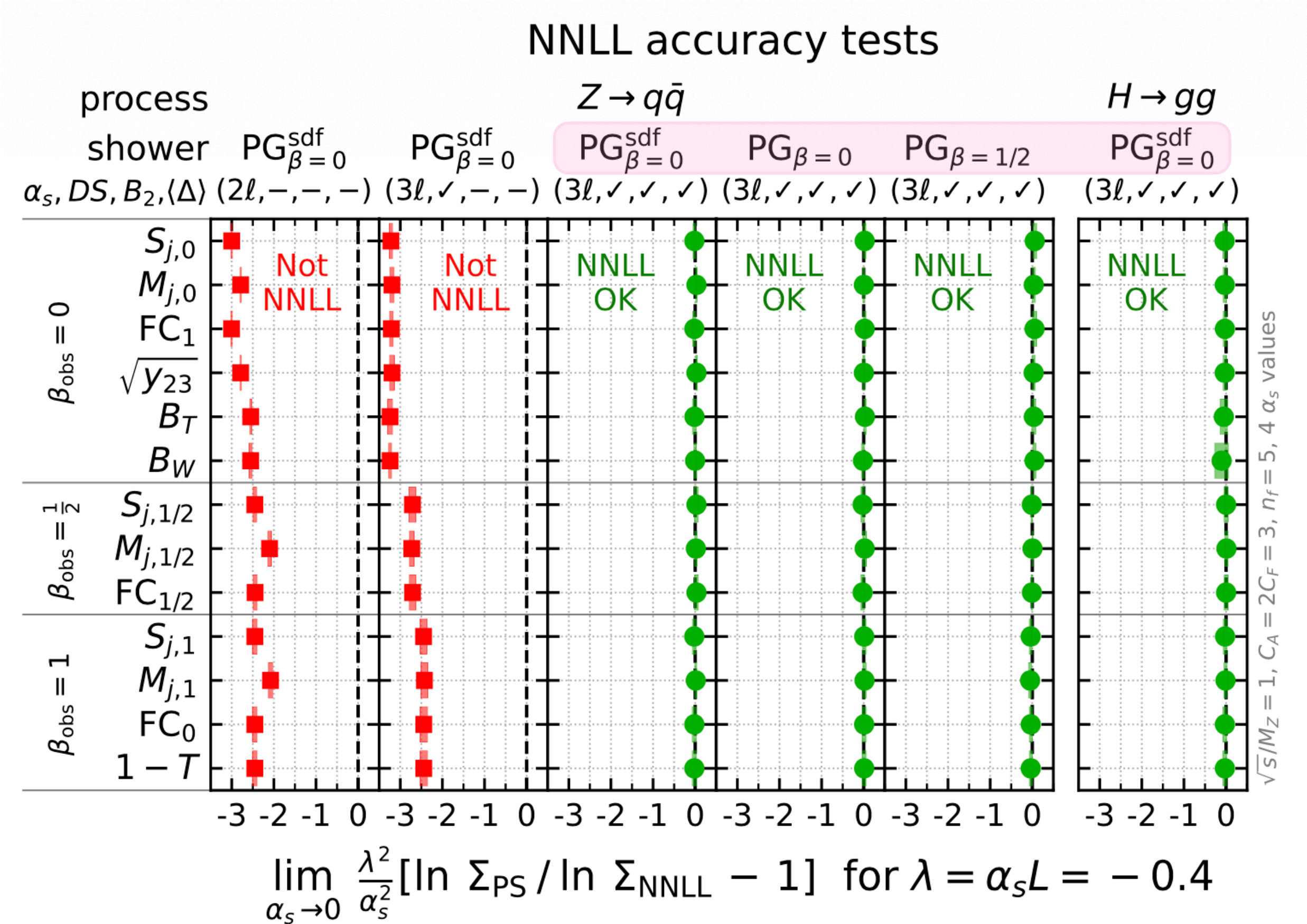
- Identify each parton shower emission as part of a would-be double-soft configuration
- Apply more-or-less a parton shower matrix-element corrⁿ / powheg-type corrⁿ to get desired distⁿ



NNLL event shapes with Panscales

- Panscales developed **one class** of **NNLL PS** following the latter scheme:
 - PanLocal : local recoil dipole PS
 - PanGlobal : global recoil antenna PS
- **NNLL** event shape accuracy demonstrated with **PanGlobal shower class** in 2024

Global observables

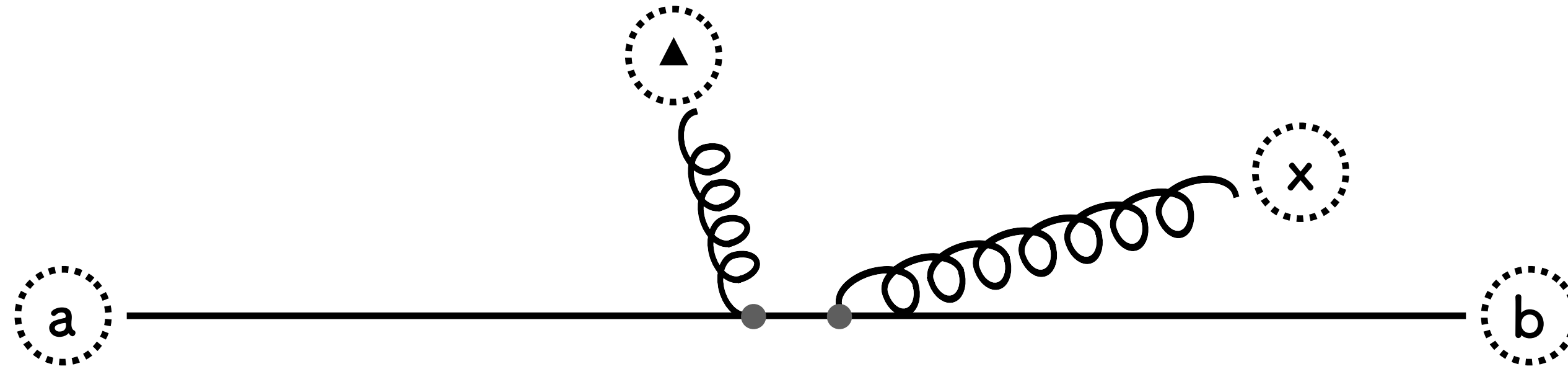


- Why only PanGlobal, why not also PanLocal ?

- Is PanScales NNLL methodology limited to that one class of shower ?

Examining the double-soft radiation pattern

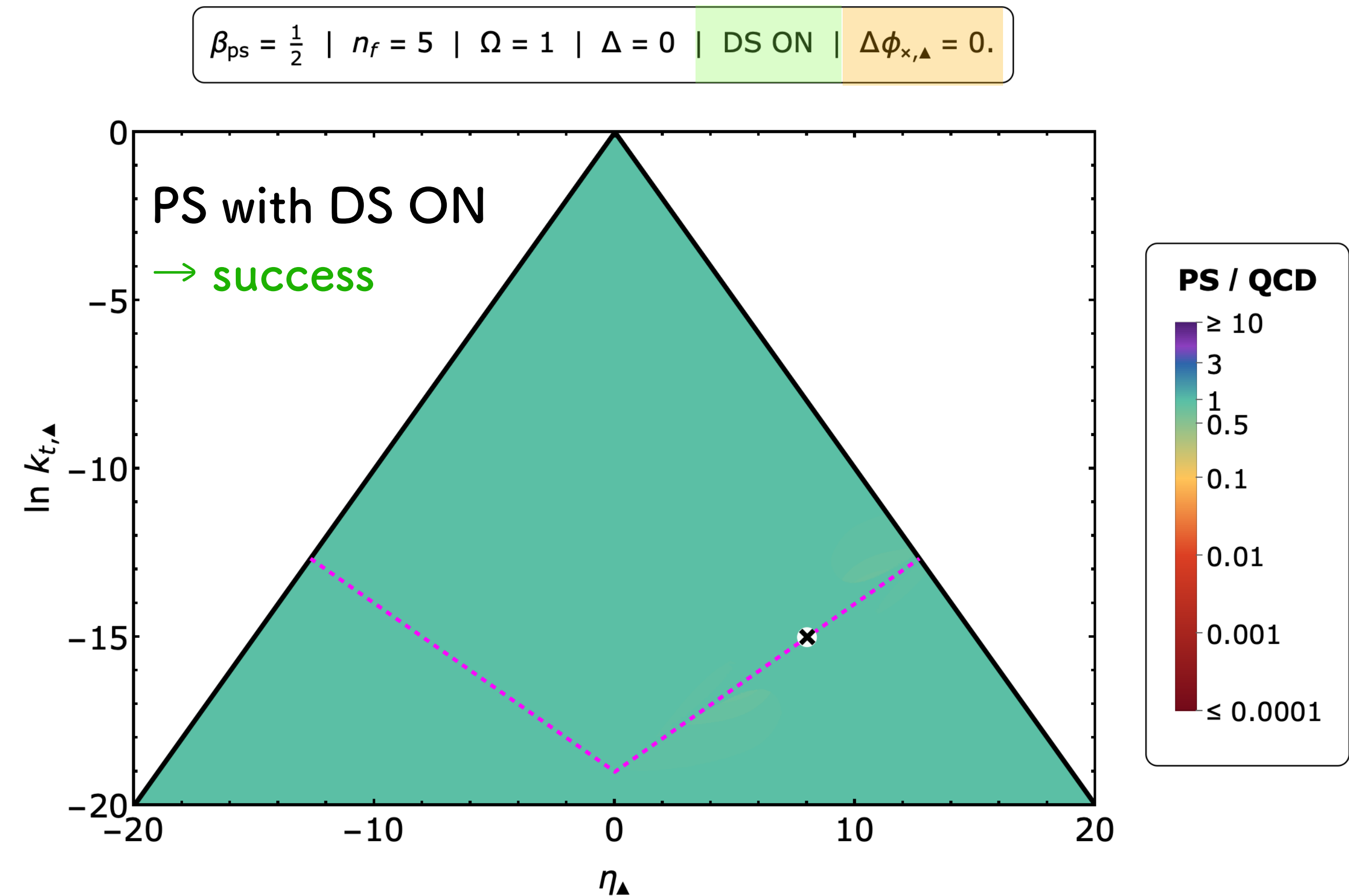
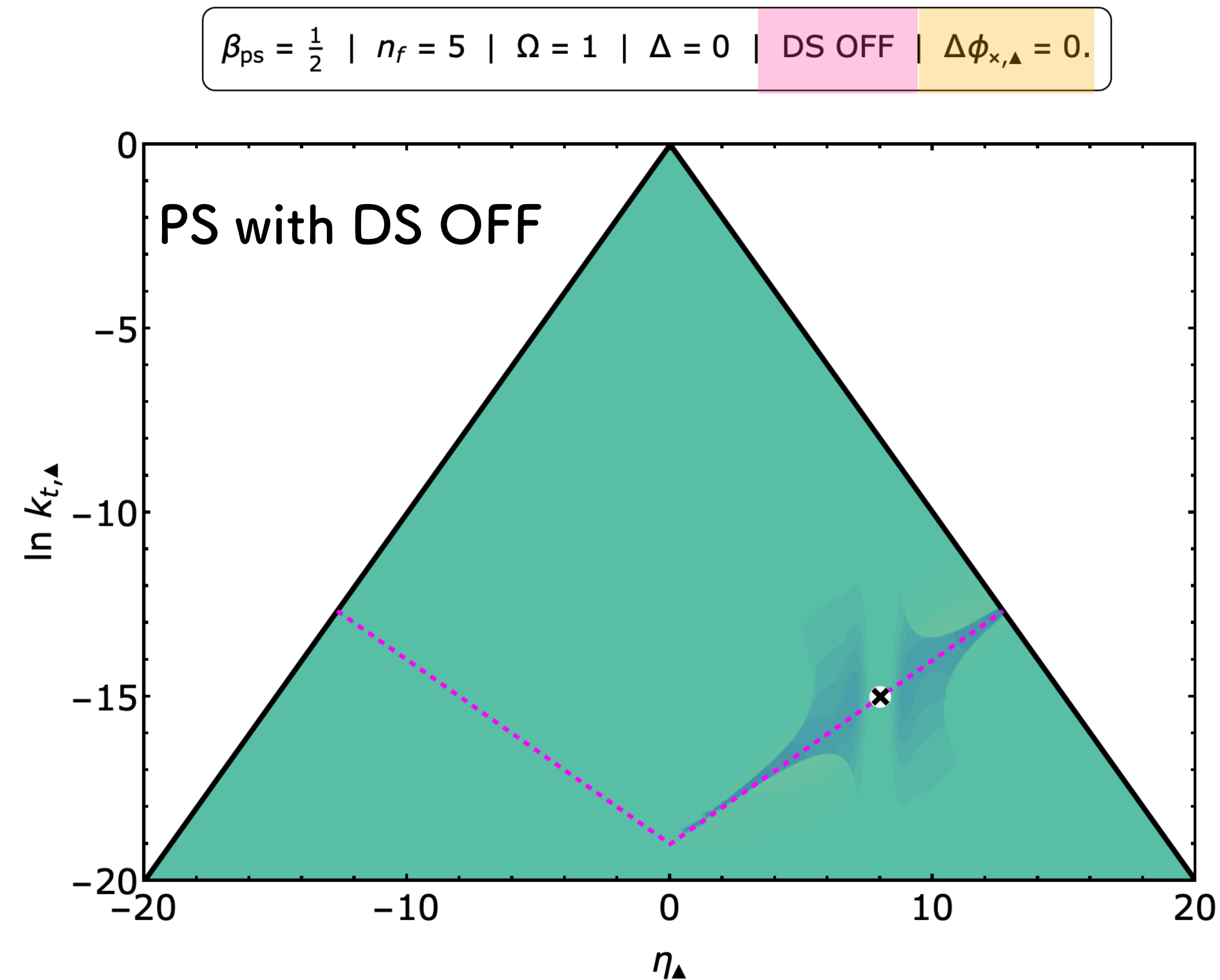
- Consider a dipole comprised of particles **a** and **b** that emits two soft particles **x** and **▲** :



- Work in the rest frame of **a + b** : $p_a = (Q/2) \{ 1; 0, 0, 1 \}$, $p_b = (Q/2) \{ 1; 0, 0, -1 \}$
 - Fix **x** somewhere in the soft-collinear (bulk) region of the Lund plane : $\{ \ln k_{t,x}, \eta_x, \phi_x \} = \{ -15, 8, 0 \}$
 - a**, **b**, **x** are now totally nailed down
 - PS emission rate $\div$ QCD emission rate, $\frac{\Omega \cdot \mathcal{M}_{\text{PS}}}{\mathcal{M}_{\text{QCD}}}$, is now just a fⁿ of $\{ \ln k_{t,\blacktriangle}, \eta_{\blacktriangle}, \phi_{\blacktriangle} \}$
-

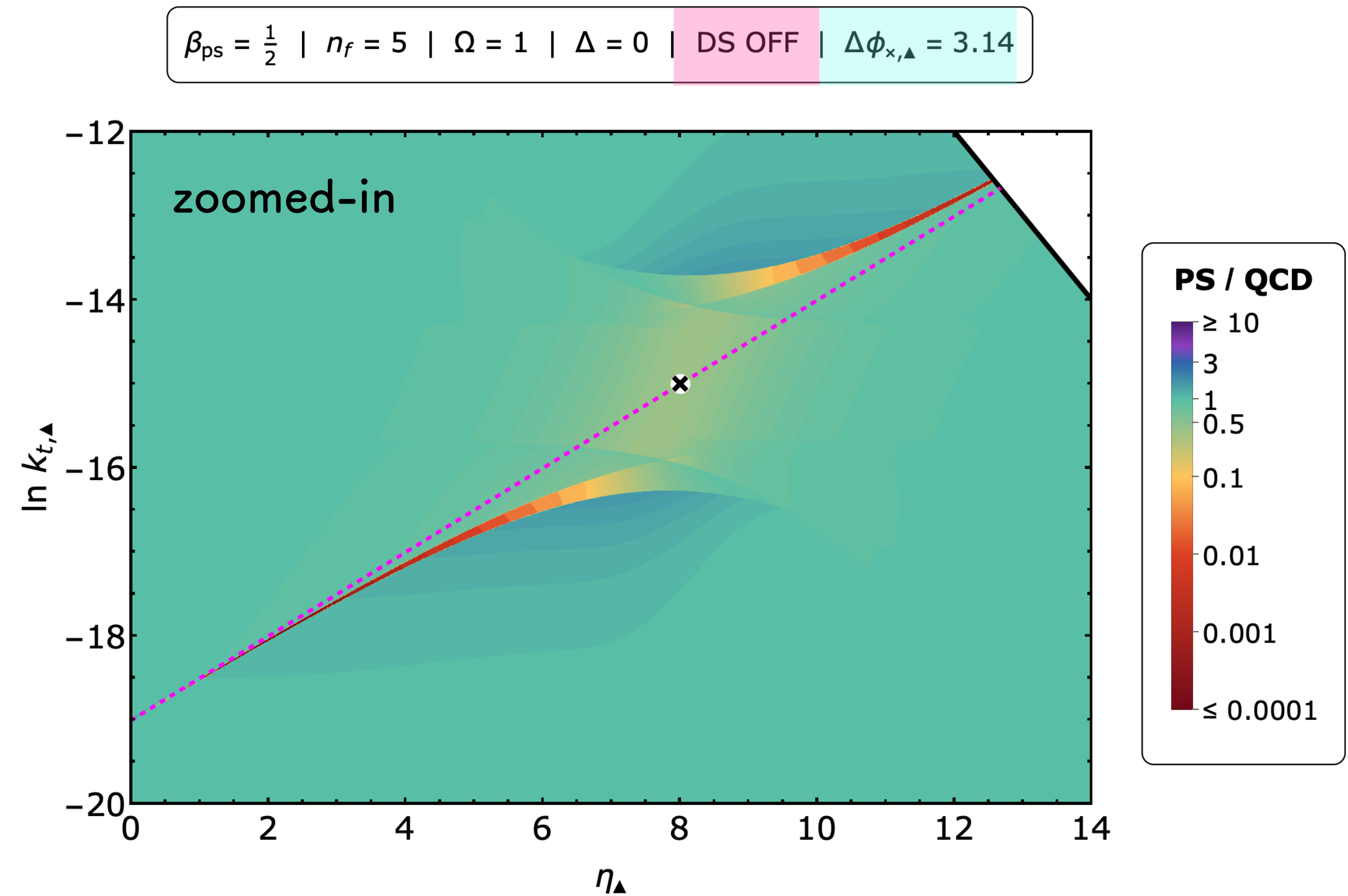
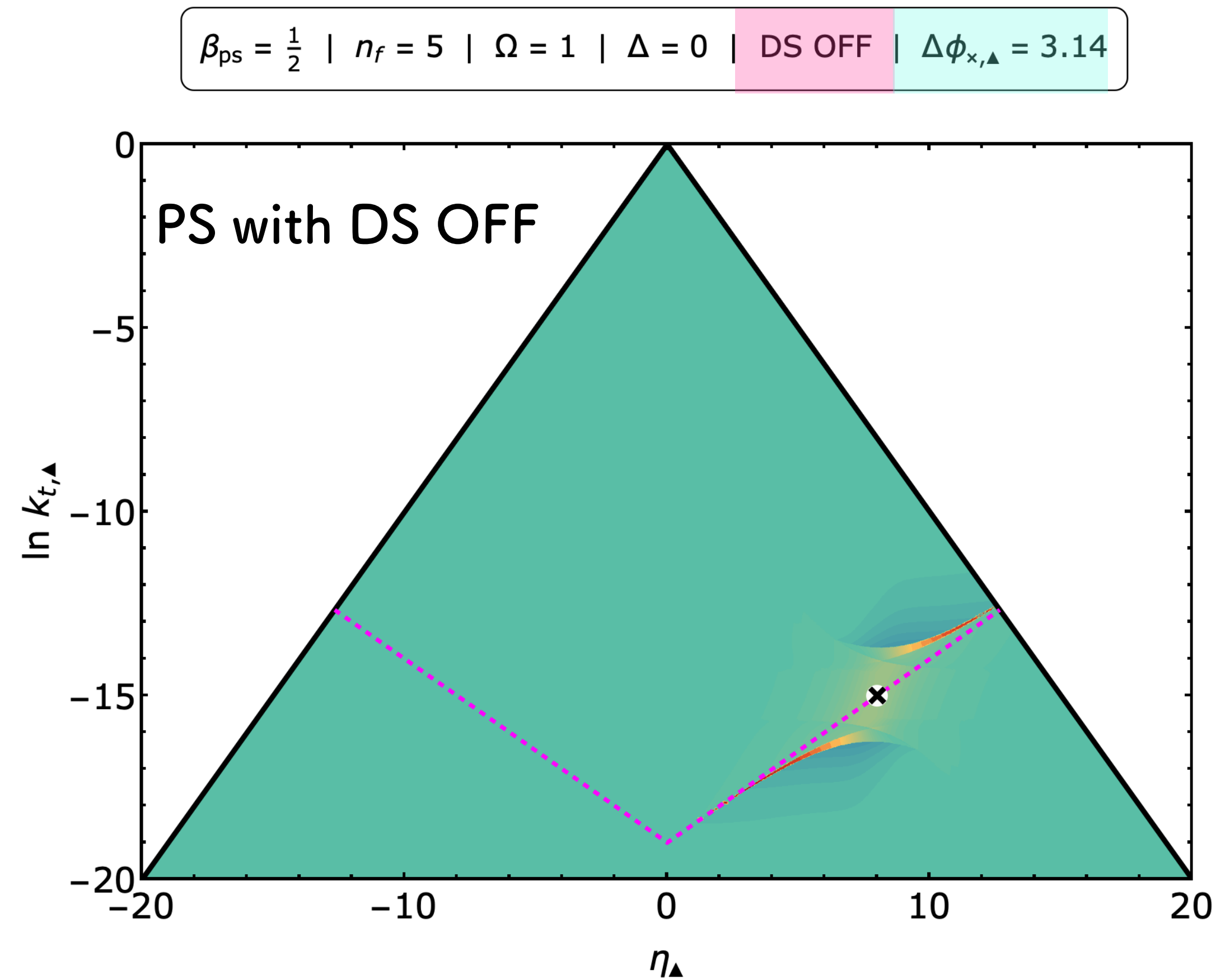
PS $\div$ QCD emission rate : at equal azimuth $\Delta\phi_{x,\blacktriangle} = 0$ the DS algorithm works

- LHS : **default** PS $\div$ QCD emission rate
- RHS : **corrected** PS $\div$ QCD emission rate — equals one — **success**



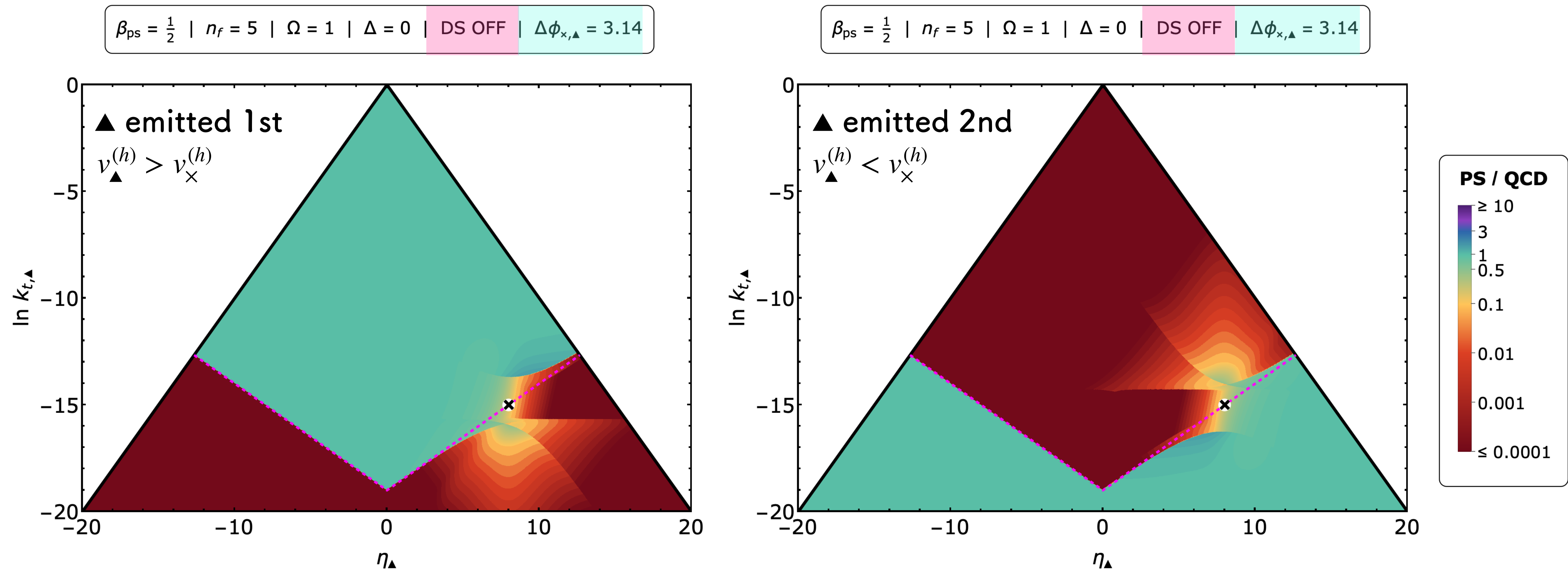
PS $\div$ QCD emission rate : opposite azimuth $\Delta\phi_{x,\blacktriangle} = \pi$ reveals deep fissures

- Keep everything the same as before and but change $\phi_{\blacktriangle} = 0$ to $\phi_{\blacktriangle} = \pi$
- Previously PS > QCD rate everywhere, now there are **deep fissures** in the PS rate that run almost to zero



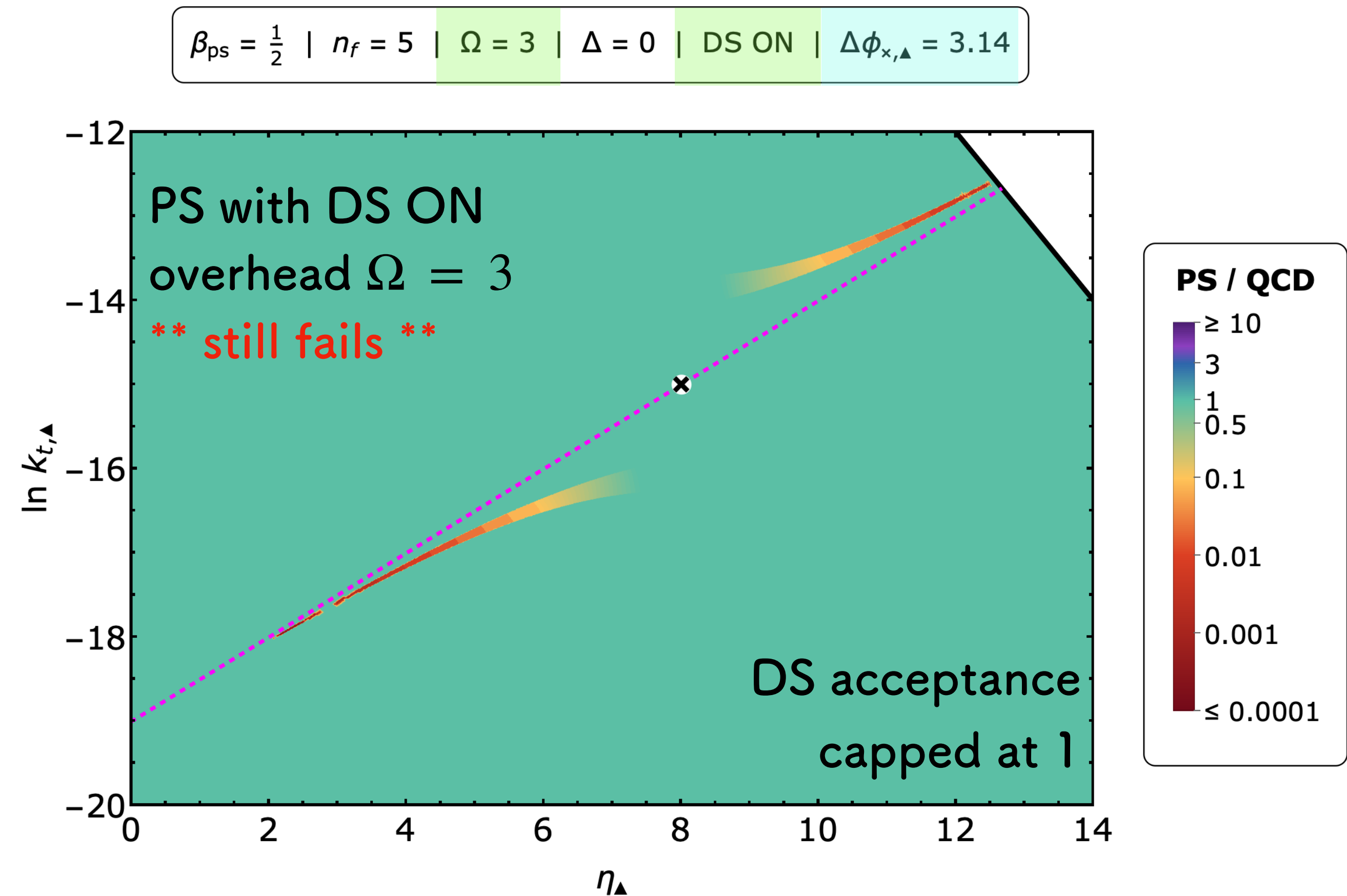
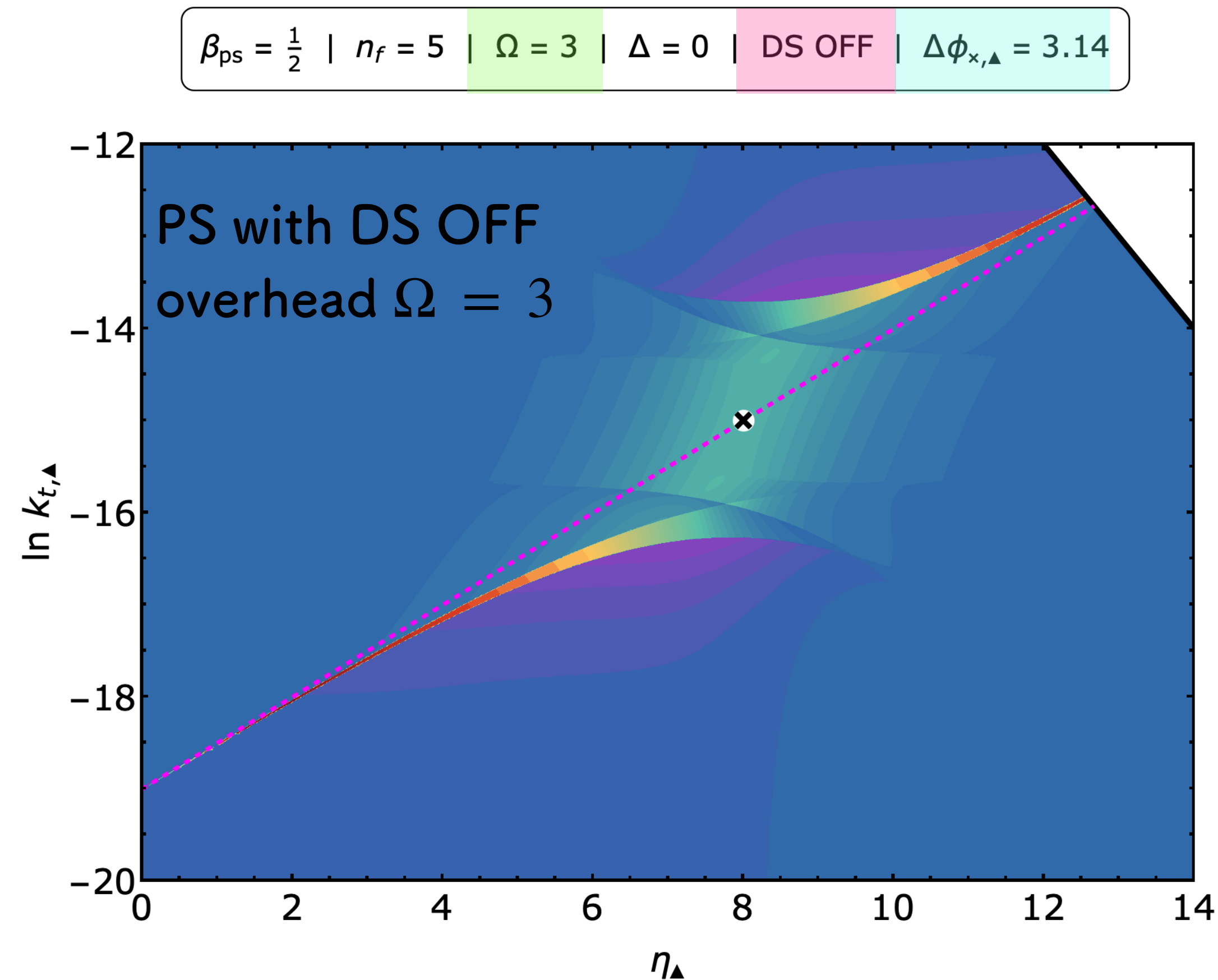
Recoil + emission ordering generates a gap between the dominant contribⁿs

- Look at how the PS builds that radiation pattern from histories **▲ emitted 1st** vs **▲ emitted 2nd**
- PS **▲ emitted 1st** & **▲ emitted 2nd** rates no longer touch/overlap at their edges everywhere → **fissures**



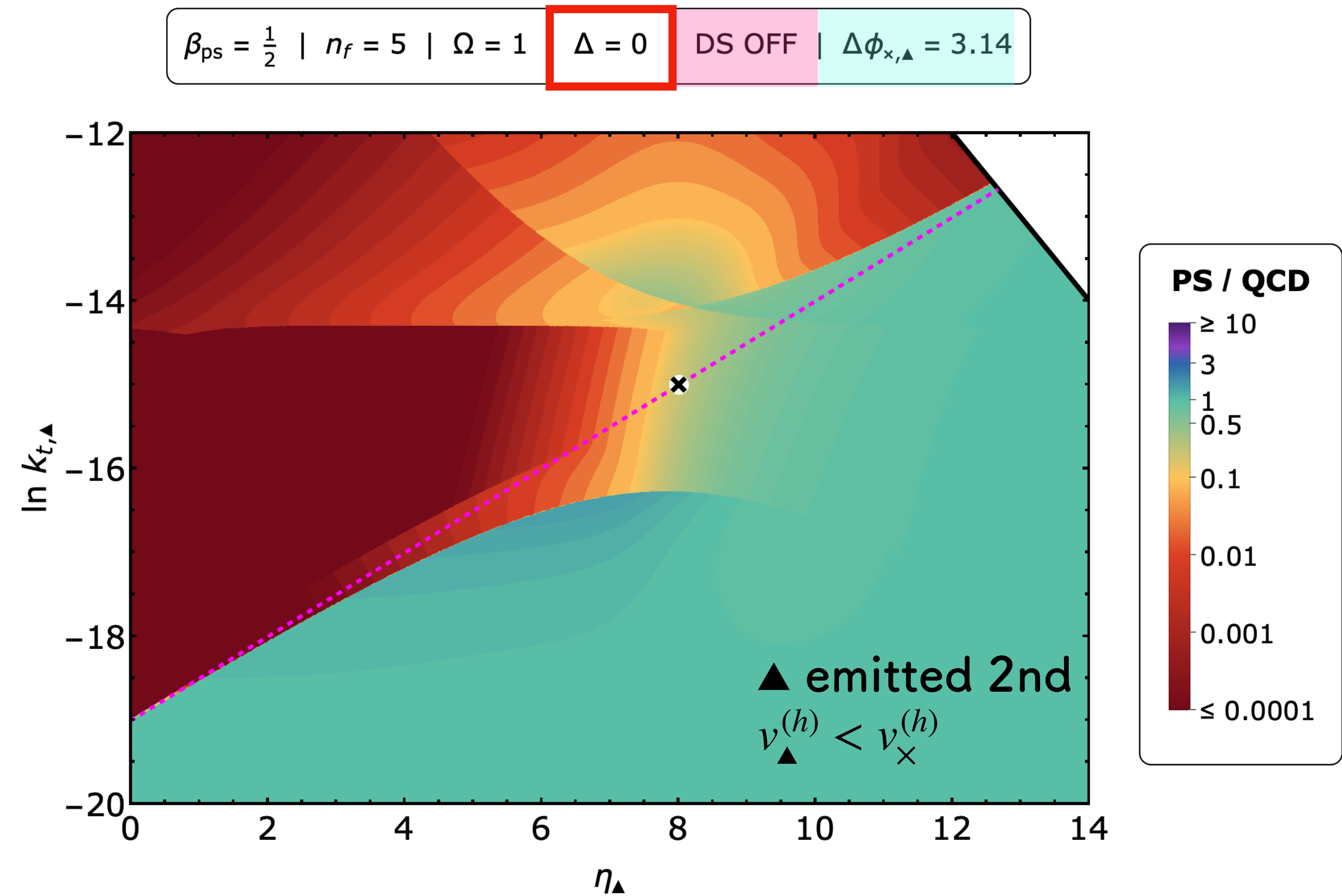
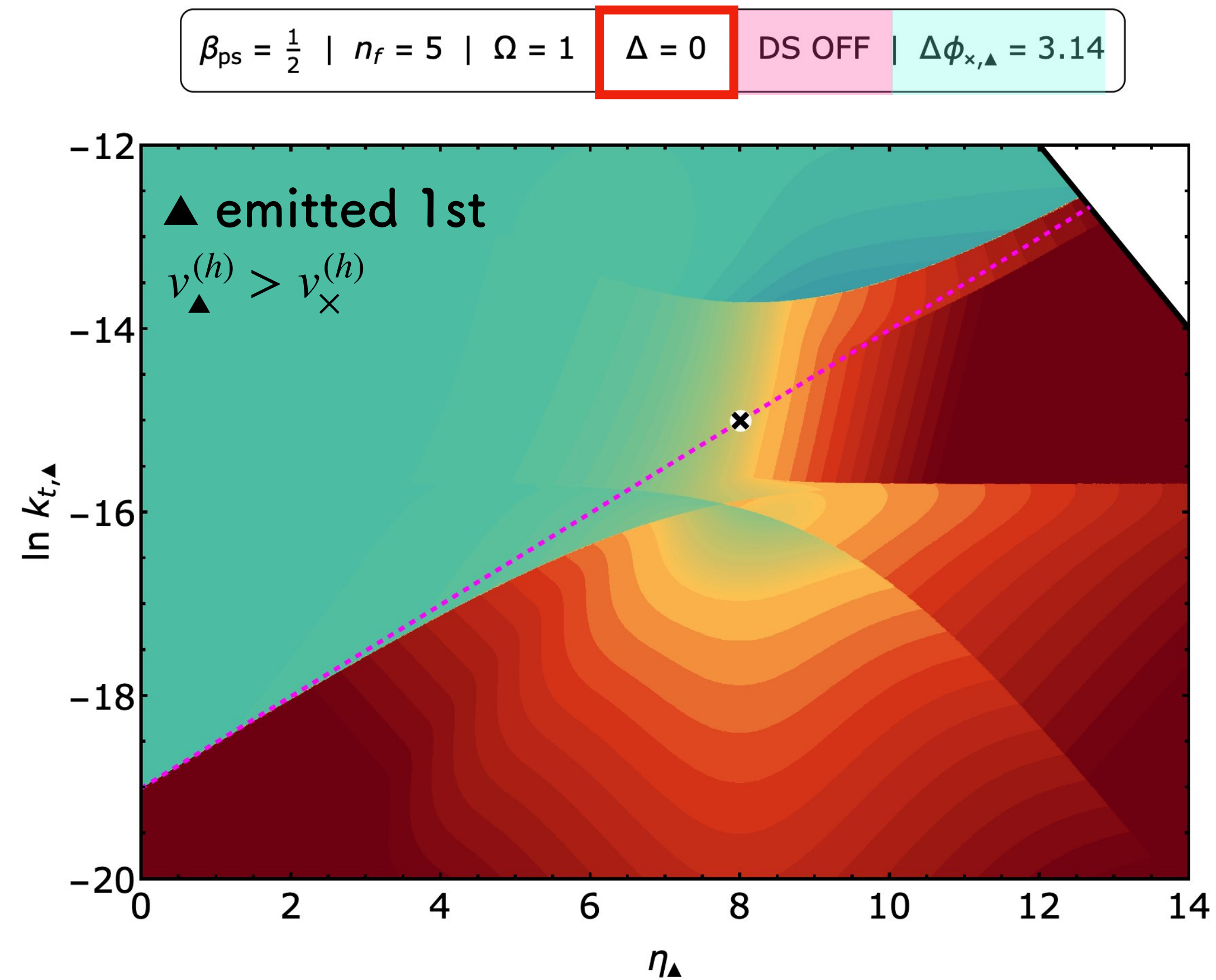
Deficit at proposal level guarantees a deficit after acceptance correction

- Enhancing PS with an overhead $\Omega = 3$ helps a tiny bit : things go slightly more teal on RH plot below
- A global overhead big enough to cover the deepest deficits would make the PS completely unusable



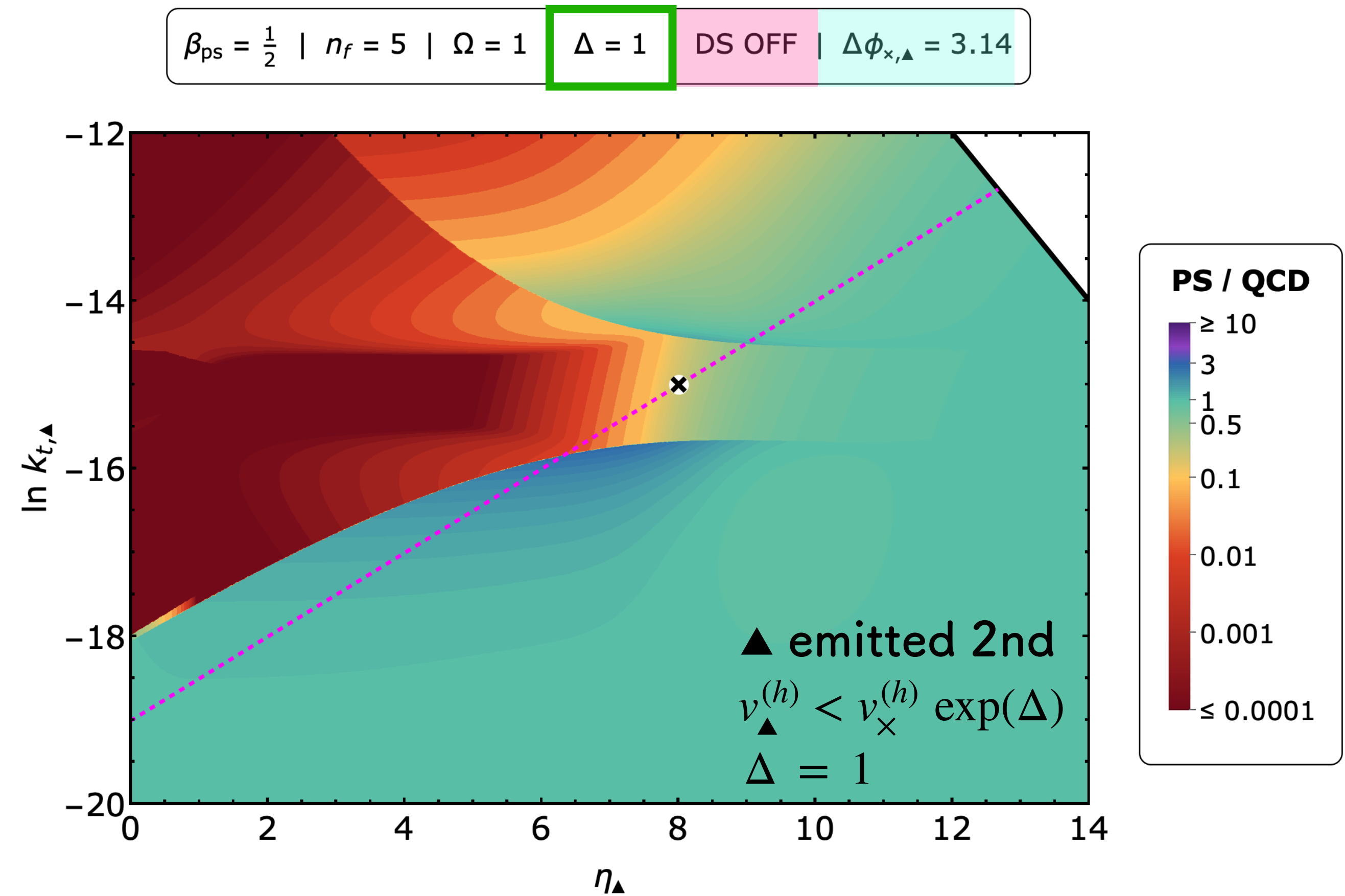
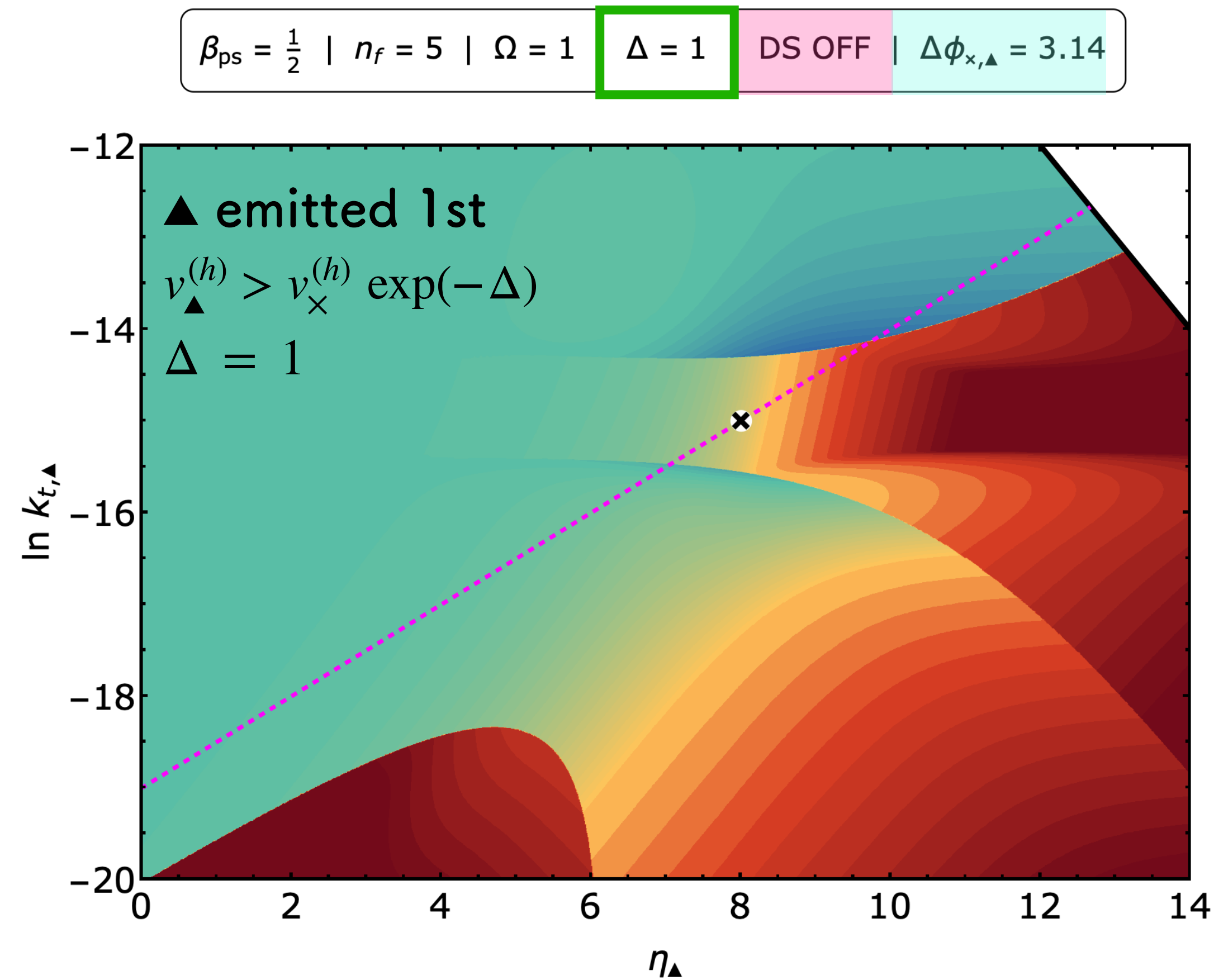
Fixing PanLocal : reminder

- Recall : issue seems to be **deep fissures** between **▲ emitted 1st** region & **▲ emitted 2nd** region for $\Delta\phi_{x,\Delta} = \pi$



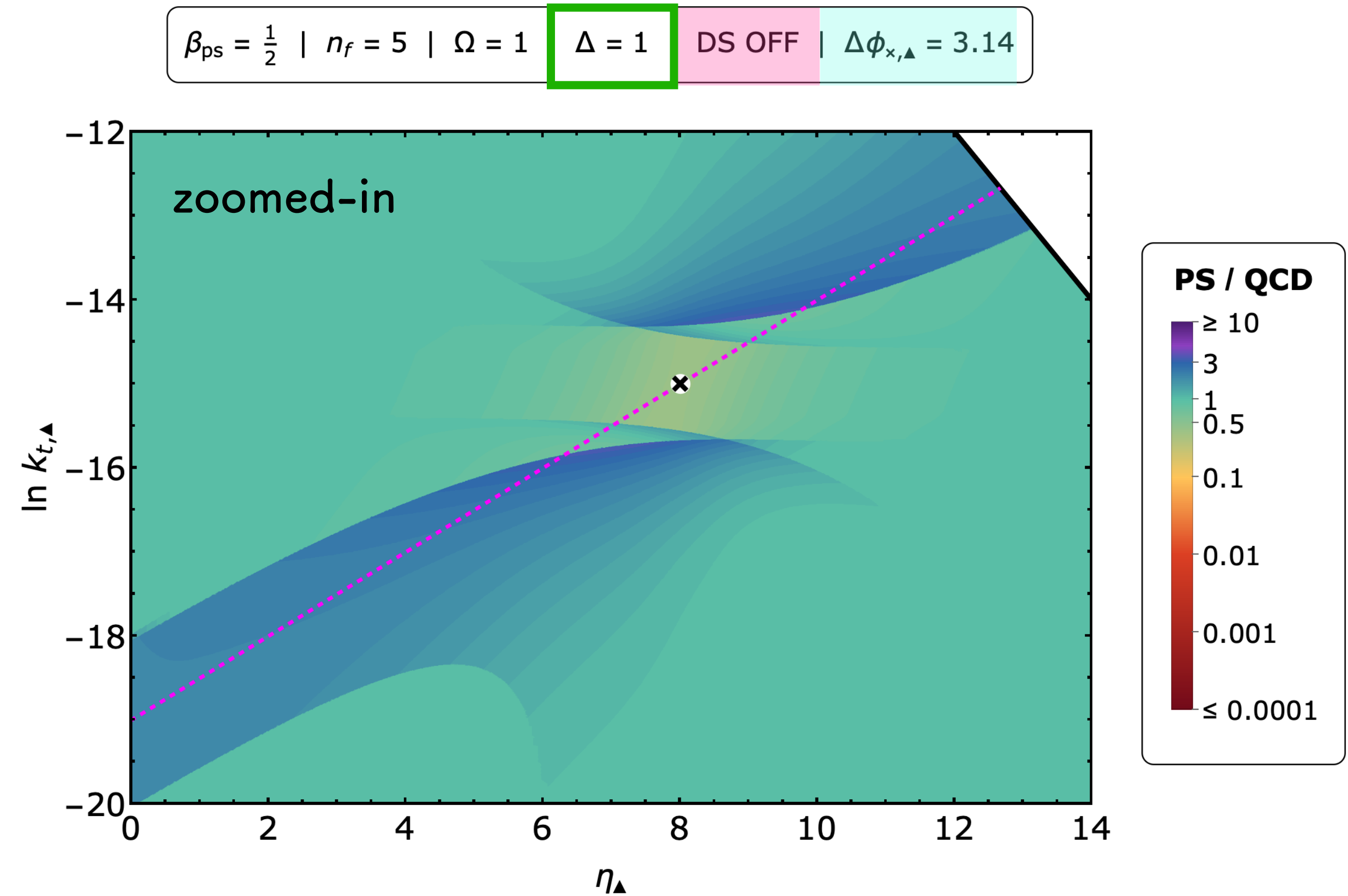
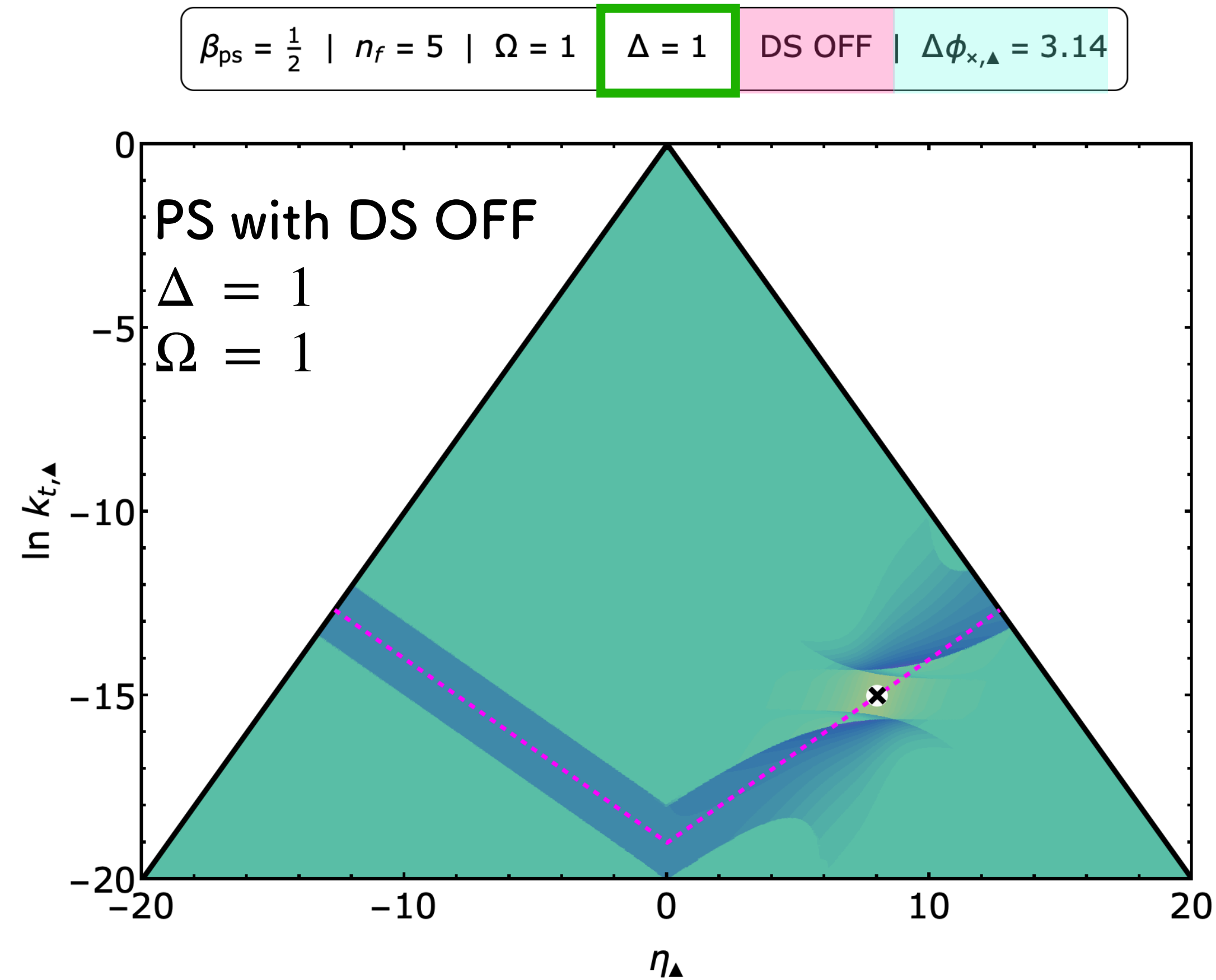
Fixing PanLocal : step 1

- **Relax PS emission ordering** from strict $v_2^{(h)} < v_1^{(h)}$ ordering to $v_2^{(h)} < v_1^{(h)} \exp(\Delta)$ with $\Delta = 1$
- **▲ emitted 1st** region extends down as $v_{\Delta}^{(h)} > v_x^{(h)} \exp(-\Delta)$; **▲ emitted 2nd** region extends up as $v_{\Delta}^{(h)} < v_x^{(h)} \exp(\Delta)$



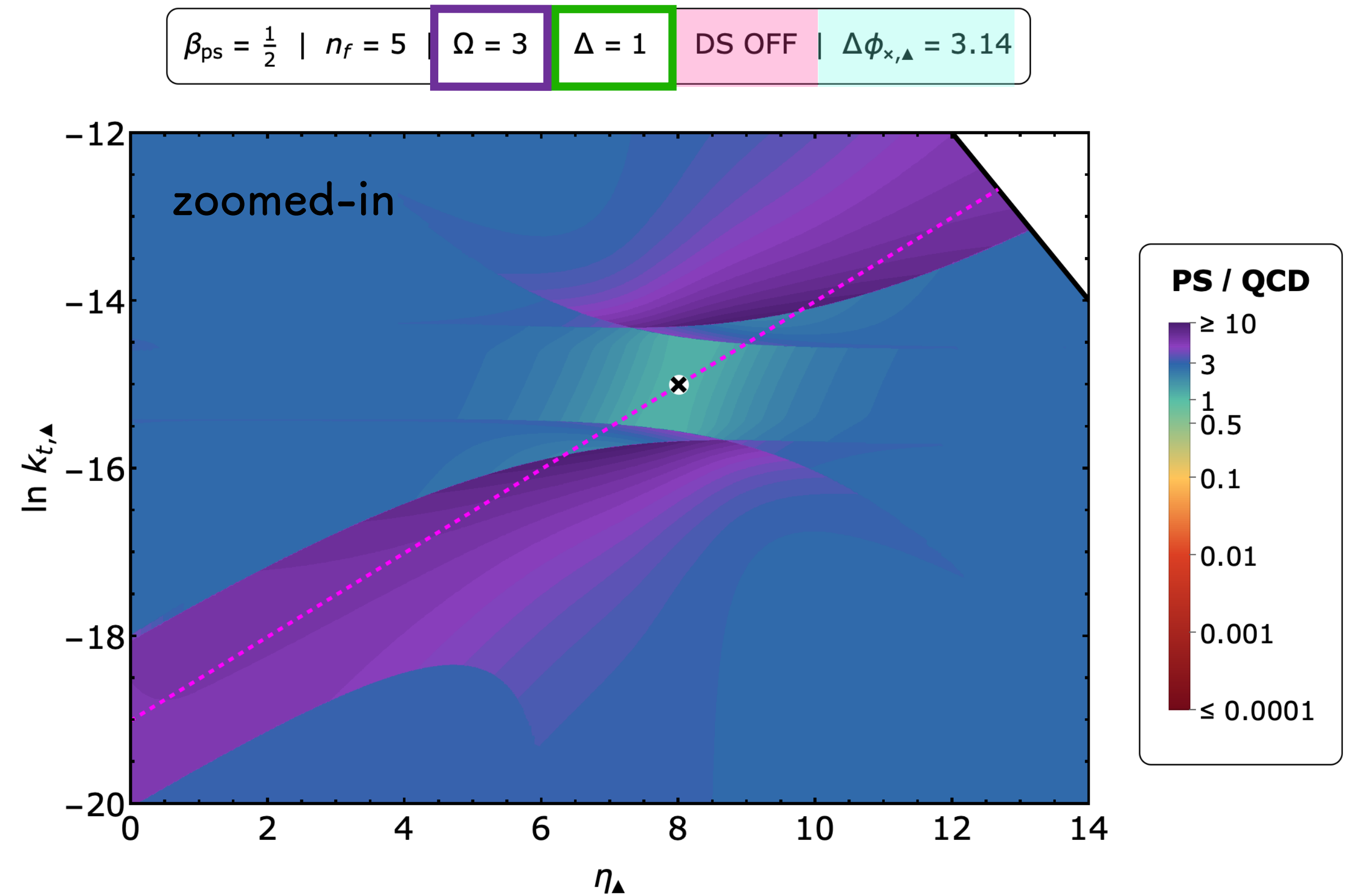
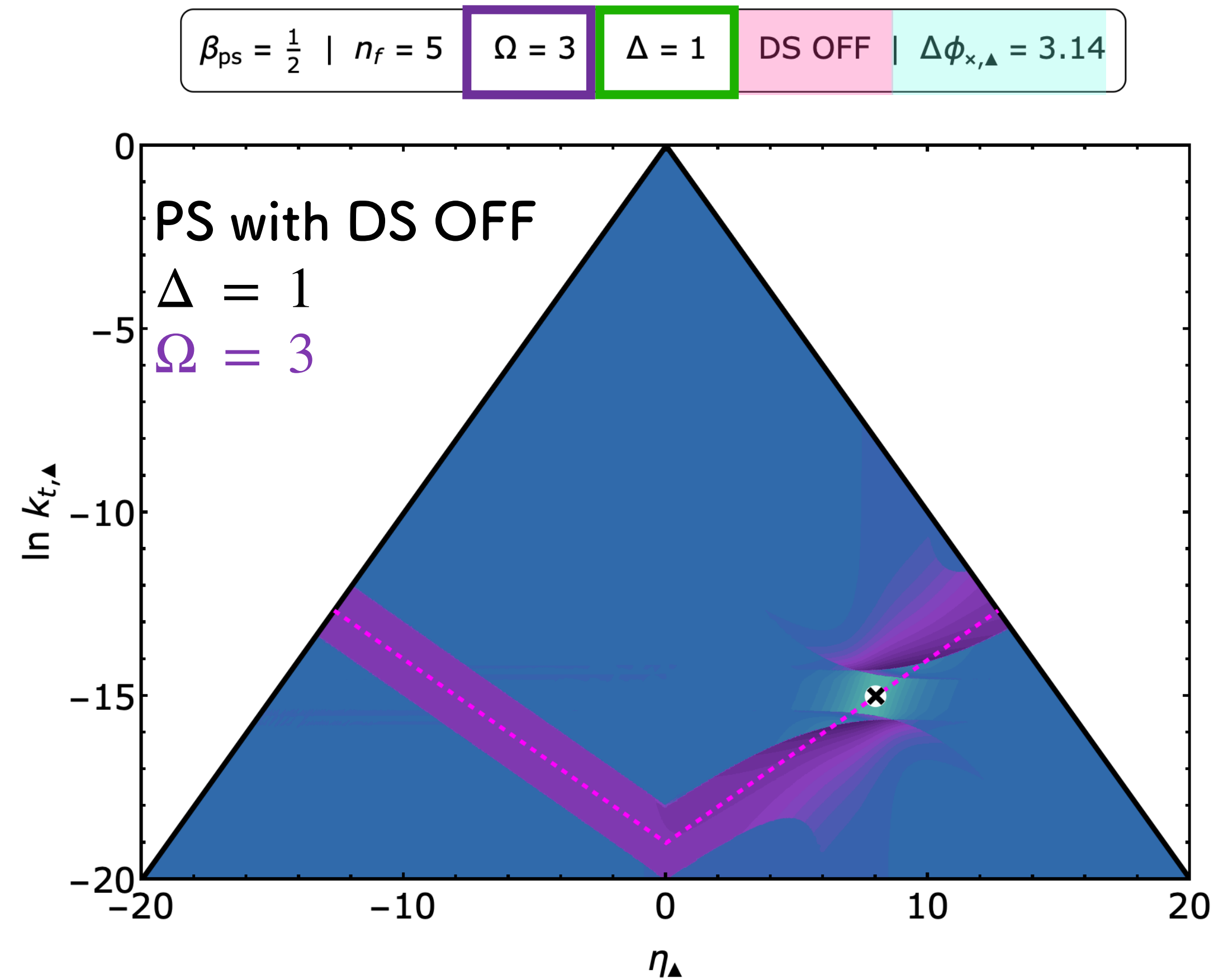
Fixing PanLocal : step 1 [ctd]

- With ordering relaxed to $v_2^{(h)} < v_1^{(h)} \exp(\Delta)$ **▲ emitted 1st** region overlaps **▲ emitted 2nd** : fissures all filled
- Not so fast ... sizeable deficit remains around **×** [yellow] so still need modest overhead for rejection sampling to work



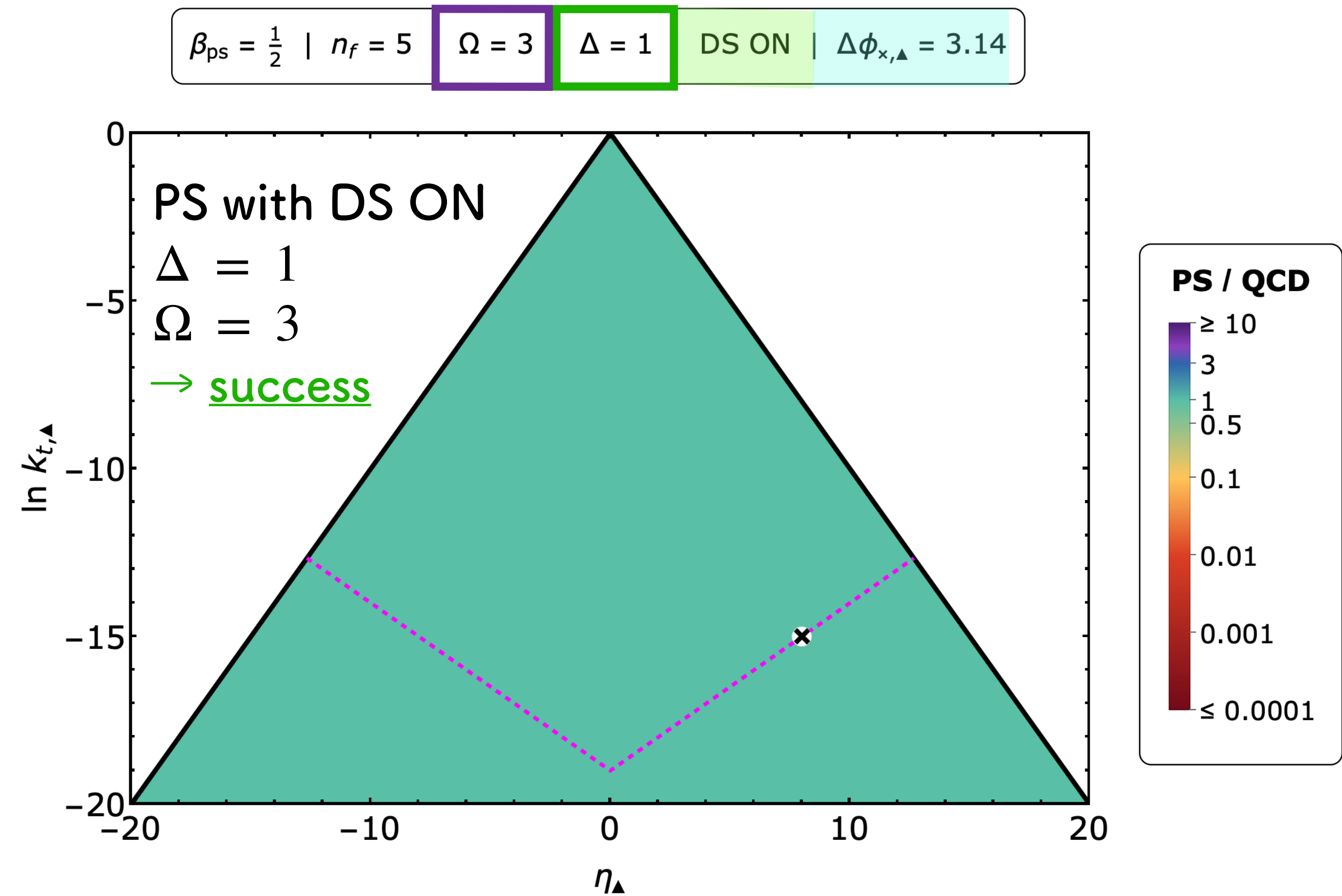
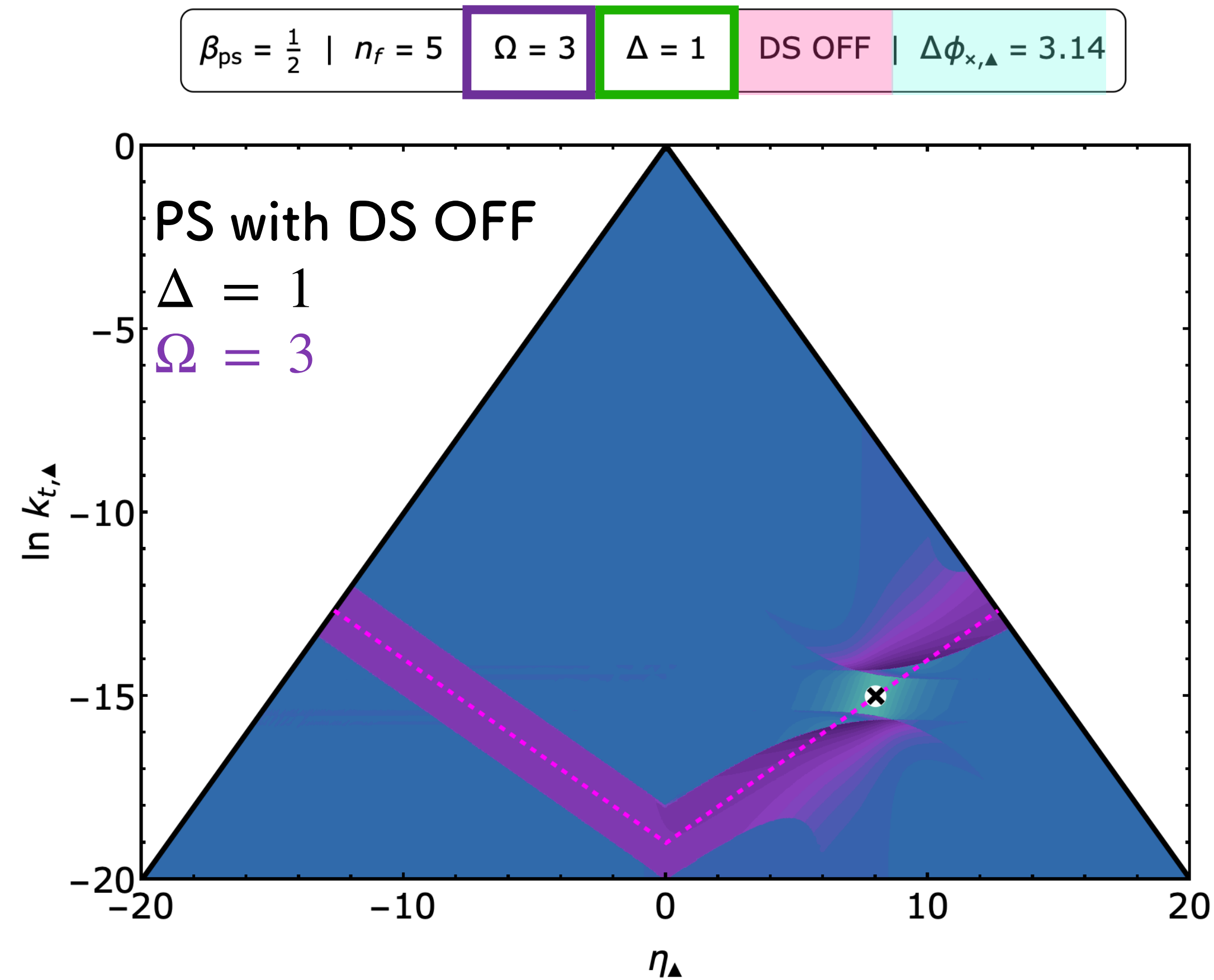
Fixing PanLocal : step 2

- Supplement **relaxed ordering condition** with **PS overhead $\Omega = 3$** : no more deficit
- Everywhere the PS rate is $>$ QCD : unweighted shower implementation now feasible [rejection sampling]



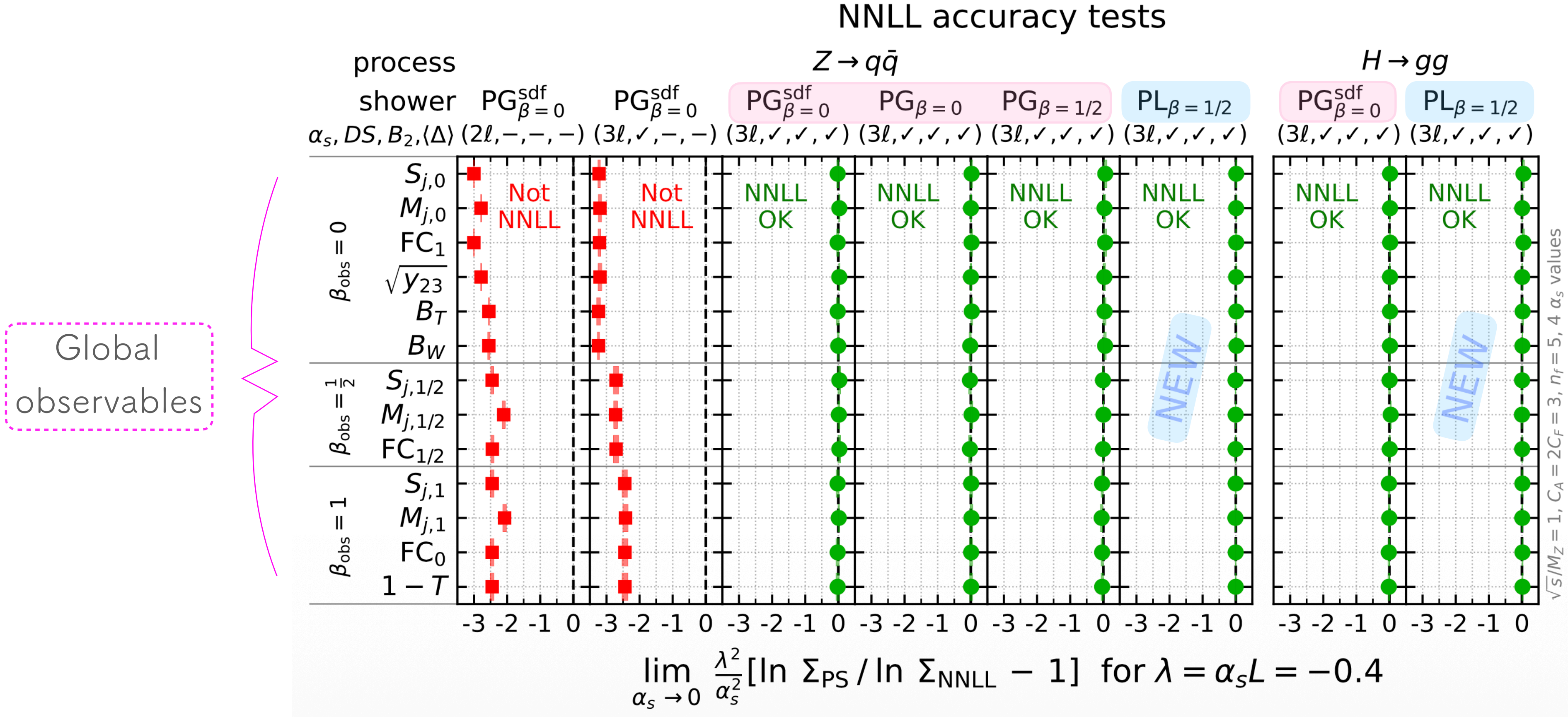
Fixing PanLocal : step 3

- Ordered histories supply as much of the QCD rate as possible
- Disordered emissions **strictly limited to filling deficits** between [$\Omega = 3$ enhanced] ordered emissions vs QCD



NNLL event shapes with Panscales

- Panscales developed **two classes** of NNLL PS following the latter scheme:
 - PanLocal : local recoil dipole PS
 - PanGlobal : global recoil antenna PS
- Demonstrated NNLL accuracy with PanGlobal shower class in 2024 and PanLocal shower class in 2026



- Why only PanGlobal, why not also PanLocal ?

- Recoil and strict ordering depleted parts of the double-soft radiation pattern, requiring large correction factors that would render the shower completely unusable.

- Is PanScales NNLL methodology limited to that one class of shower ?

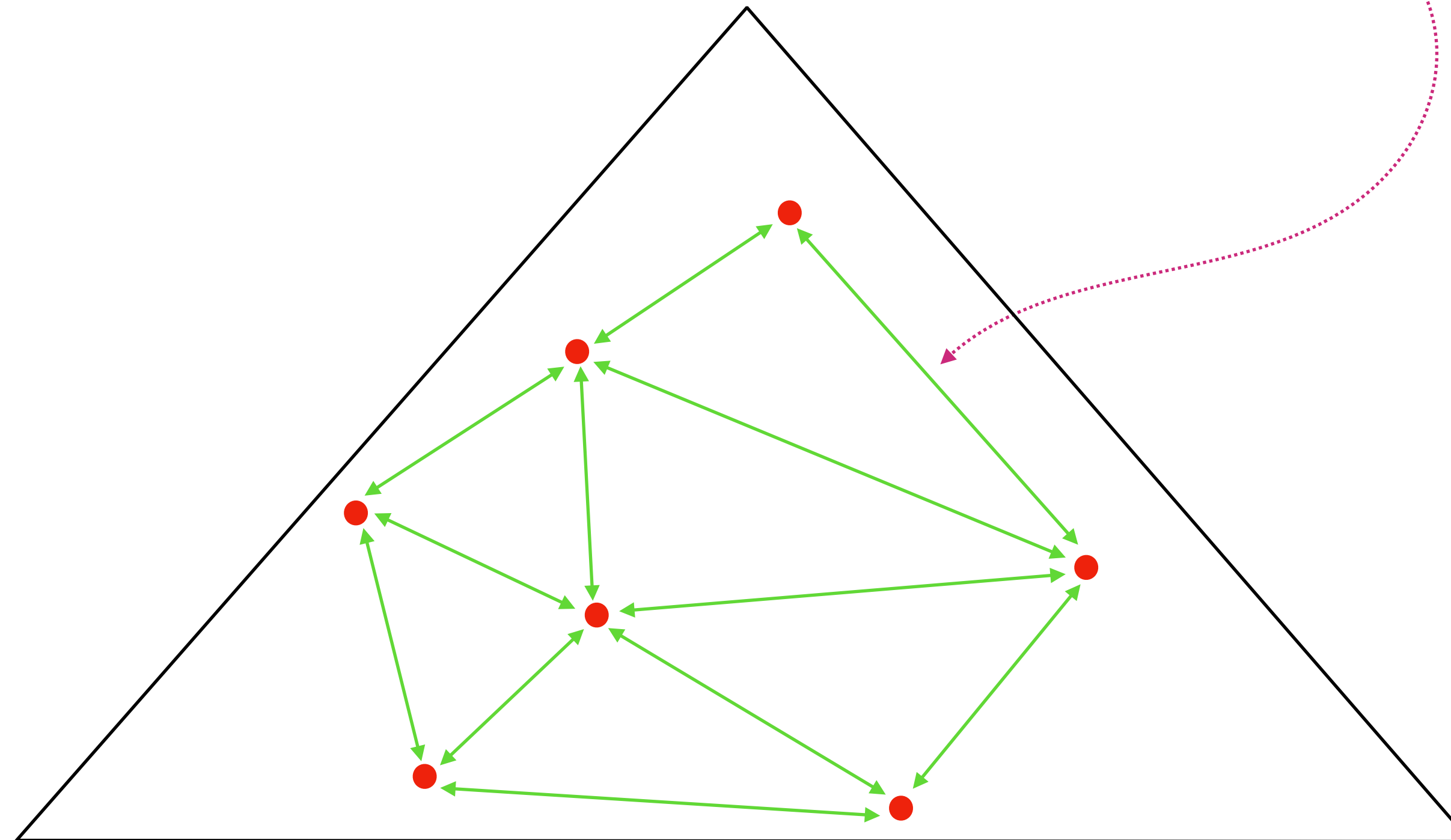
- No. Controlled disordered emissions restore coverage, enabling NNLL event-shape accuracy with PanLocal as well as PanGlobal.

Extra

- Include inclusive NLO soft corrⁿ to single emission kernel :

- $\alpha_s(k_t) \rightarrow \alpha_s^{\text{eff}}(k_t) = \alpha_s(k_t) \left[1 + \frac{\alpha_s(k_t)}{2\pi} K_1 \right]$ with 2-loop running

- No need for NLO fixed order matching
- PS must reproduce tree-level QCD MEs for configurations with arbitrarily many emissions all widely separated from one another in the Lund plane, e.g.,



Log accuracy : LL / NLL / NNLL

- For jet rates & event shapes frac^n of events where the value of observable v_{obs} is below a cut v has the form

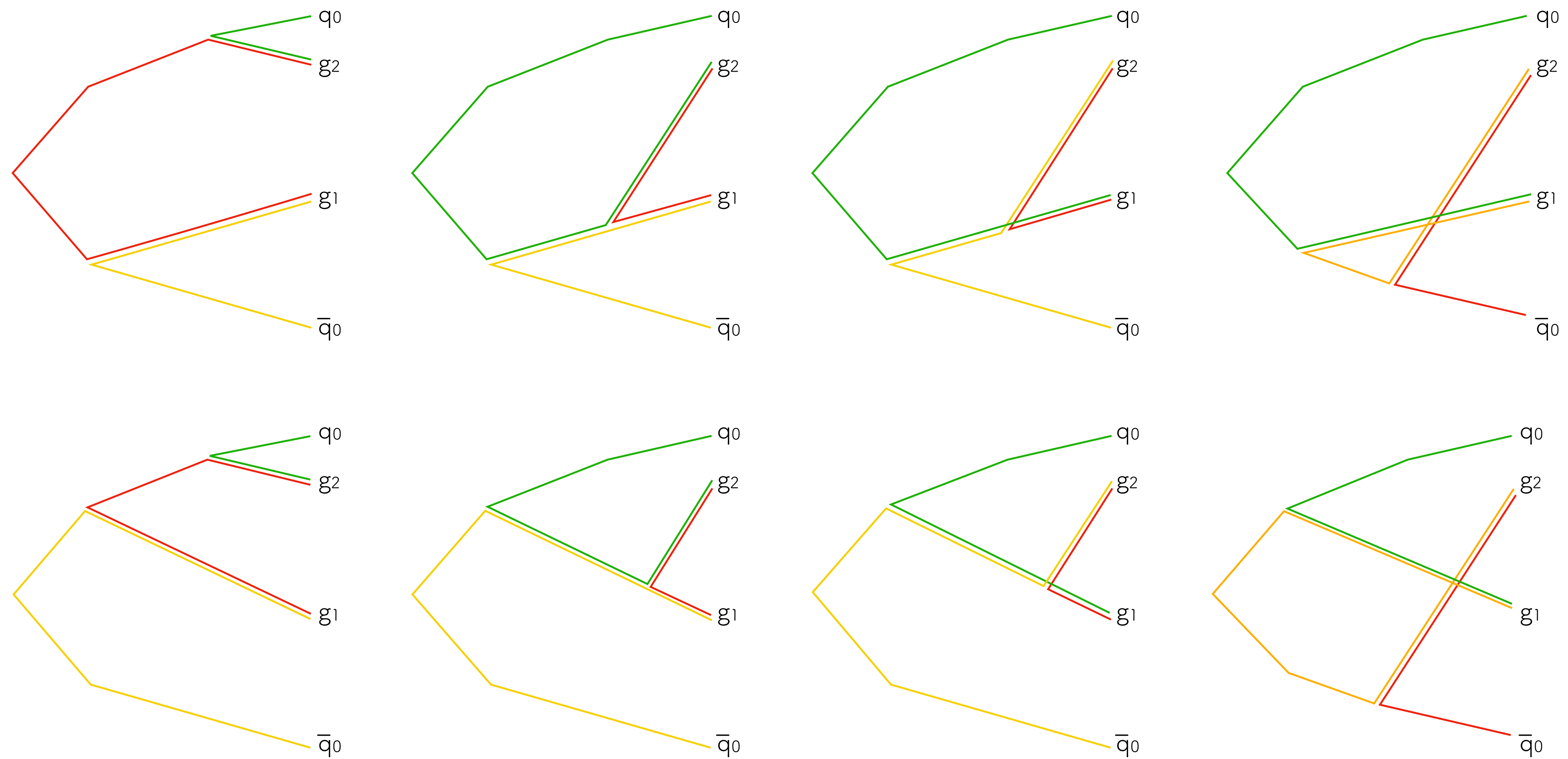
$$\Sigma(\alpha_s, \alpha_s L) = (1 + \underbrace{\alpha_s C_1}_{\text{NNLL} \sim \alpha_s^n L^{n-1} \sim O(\alpha_s)}) \exp \left[\underbrace{\alpha_s^{-1} g_1(\alpha_s L)}_{\text{LL} \sim \alpha_s^n L^{n+1} \sim O(\alpha_s^{-1})} + \underbrace{g_2(\alpha_s L)}_{\text{NLL} \sim \alpha_s^n L^n \sim O(1)} + \underbrace{\alpha_s g_3(\alpha_s L)}_{\text{NNLL} \sim \alpha_s^n L^{n-1} \sim O(\alpha_s)} \right]$$

frac^n of events where observable value $< e^L$

- This is a reorganisation of perturbation theory valid for describing the region $\alpha_s L \sim O(1)$
 - To test PS vs QCD examine ratios $\Sigma_{\text{PS}} / \Sigma_{\text{QCD}}$ or $\ln \Sigma_{\text{PS}} / \ln \Sigma_{\text{QCD}}$ and extrapolate to $\alpha_s \rightarrow 0$ at **fixed** $\alpha_s L \sim O(1)$
 - If PS is **LL** the following probes for residual **NLL** difference : $\lim_{\alpha_s \rightarrow 0} \frac{1}{\alpha_s} \left[\frac{\ln \Sigma_{\text{PS}}}{\ln \Sigma_{\text{QCD}}} - 1 \right] = \frac{\Delta g_2}{g_1}$
 - If PS is **NLL** the following probes for residual **NNLL** difference : $\lim_{\alpha_s \rightarrow 0} \frac{1}{\alpha_s^2} \left[\frac{\ln \Sigma_{\text{PS}}}{\ln \Sigma_{\text{QCD}}} - 1 \right] = \frac{\Delta C_1 + \Delta g_3}{g_1}$
-

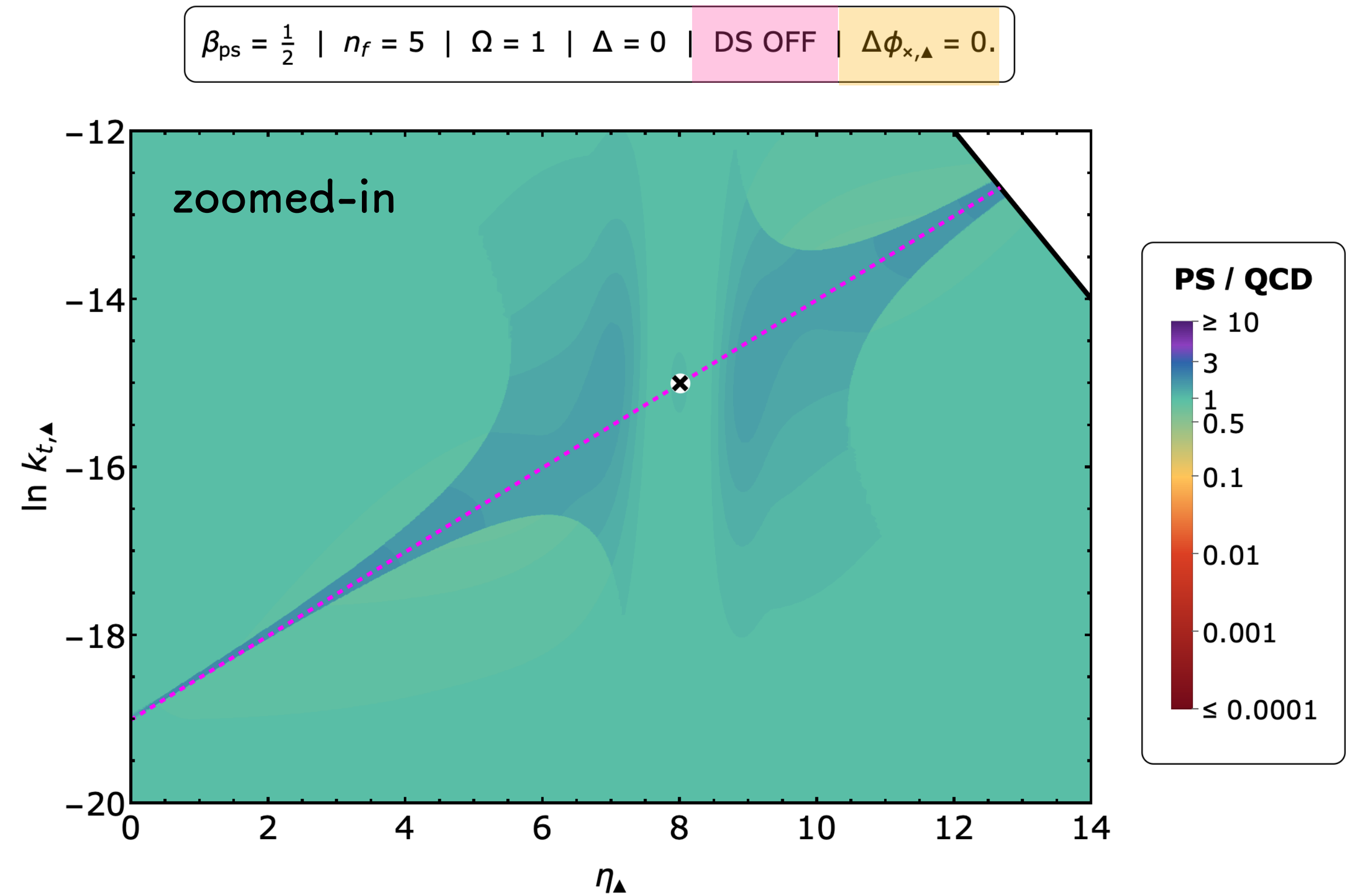
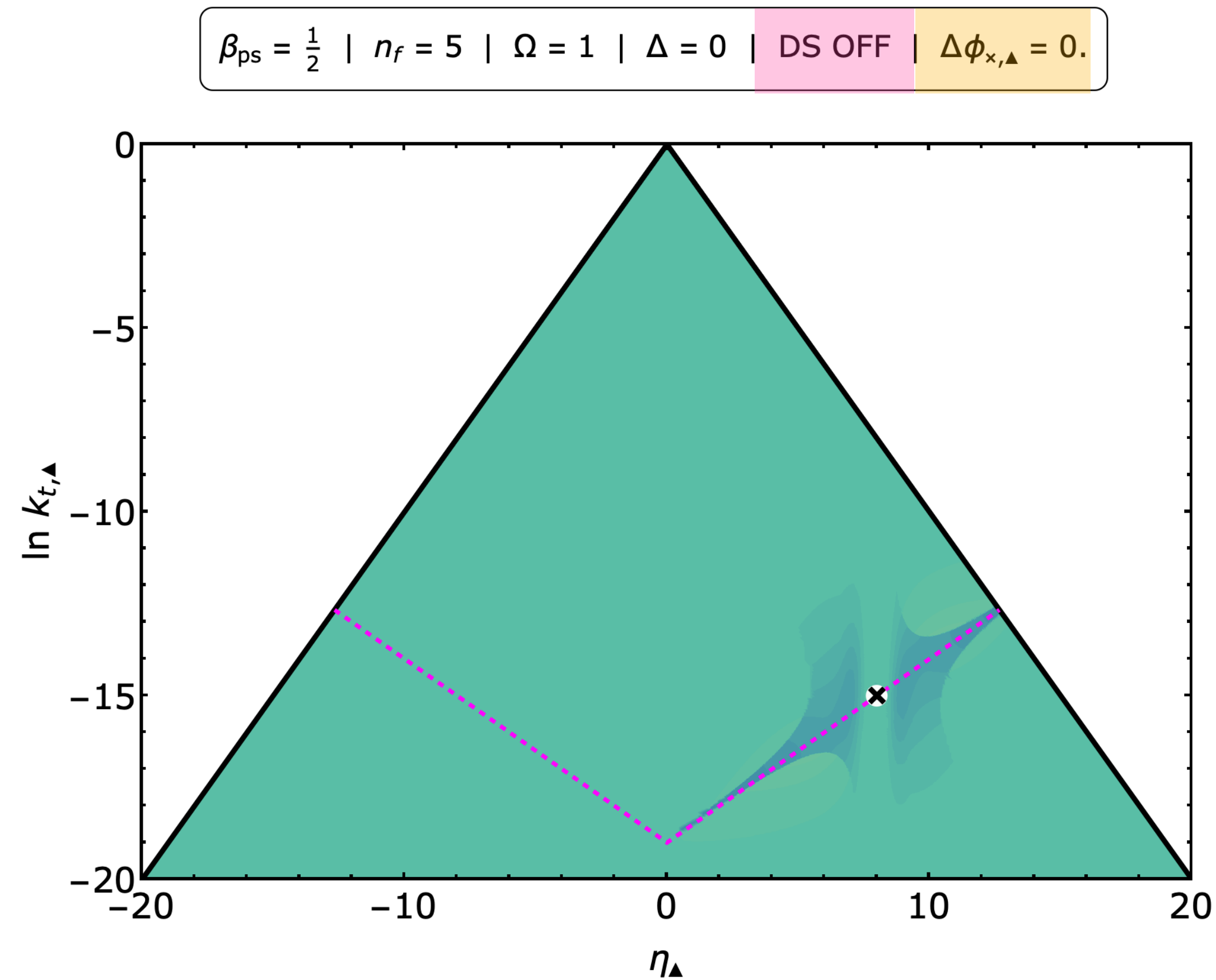
Correcting double-soft pairs

- To get $\mathcal{M}_{\text{PS}}(q_0, g_2, g_1, \bar{q}_0)$ we sum emission rates of all possible routes to that configuration, 8 out of 16 are :



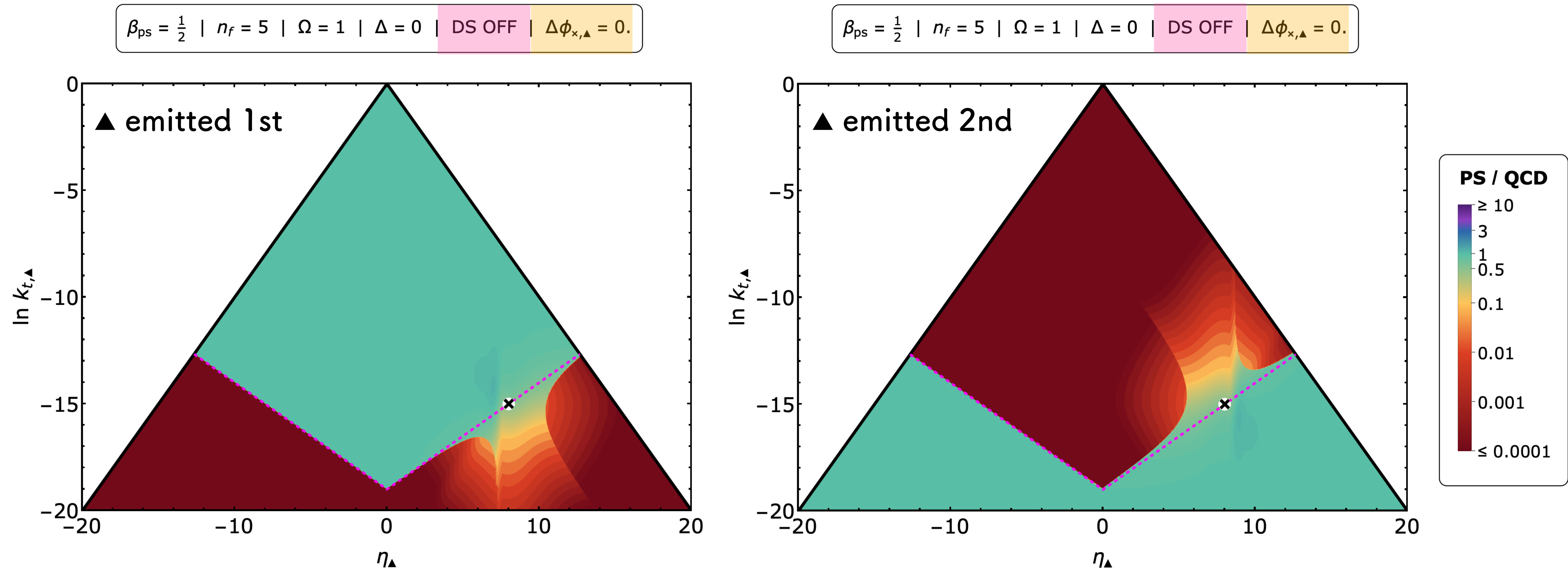
PS $\div$ QCD emission rate : equal azimuth $\Delta\phi_{\mathbf{x},\blacktriangle} = 0$

- Dipole $\mathbf{a}\mathbf{b}$ emits soft particles $\mathbf{x}$ and $\blacktriangle$ with $\mathbf{x}$ at $\{ \ln k_{t,\mathbf{x}}, \eta_{\mathbf{x}}, \phi_{\mathbf{x}} \} = \{ -15, 8, 0 \}$
- Plot ratio of PS rate $\div$ QCD rate for $\phi_{\blacktriangle} = 0$ which is just a fⁿ of $\{ \ln k_{t,\blacktriangle}, \eta_{\blacktriangle} \}$



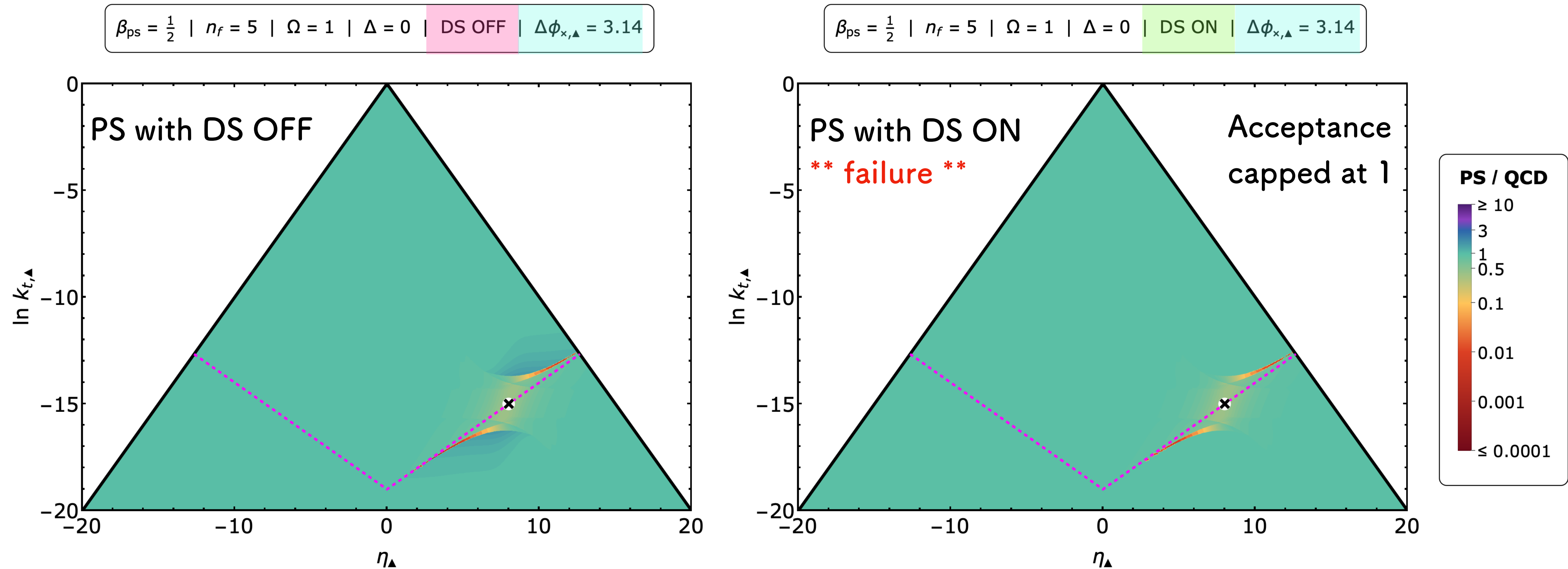
PS $\div$ QCD emission rate : equal azimuth $\Delta\phi_{x,\blacktriangle} = 0$

- Look at how the PS builds that radiation pattern from histories **x emitted last** vs **▲ emitted last**
- PS rates for **▲ emitted 1st** & **▲ emitted 2nd** coincide or overlap each other all along dashed magenta line



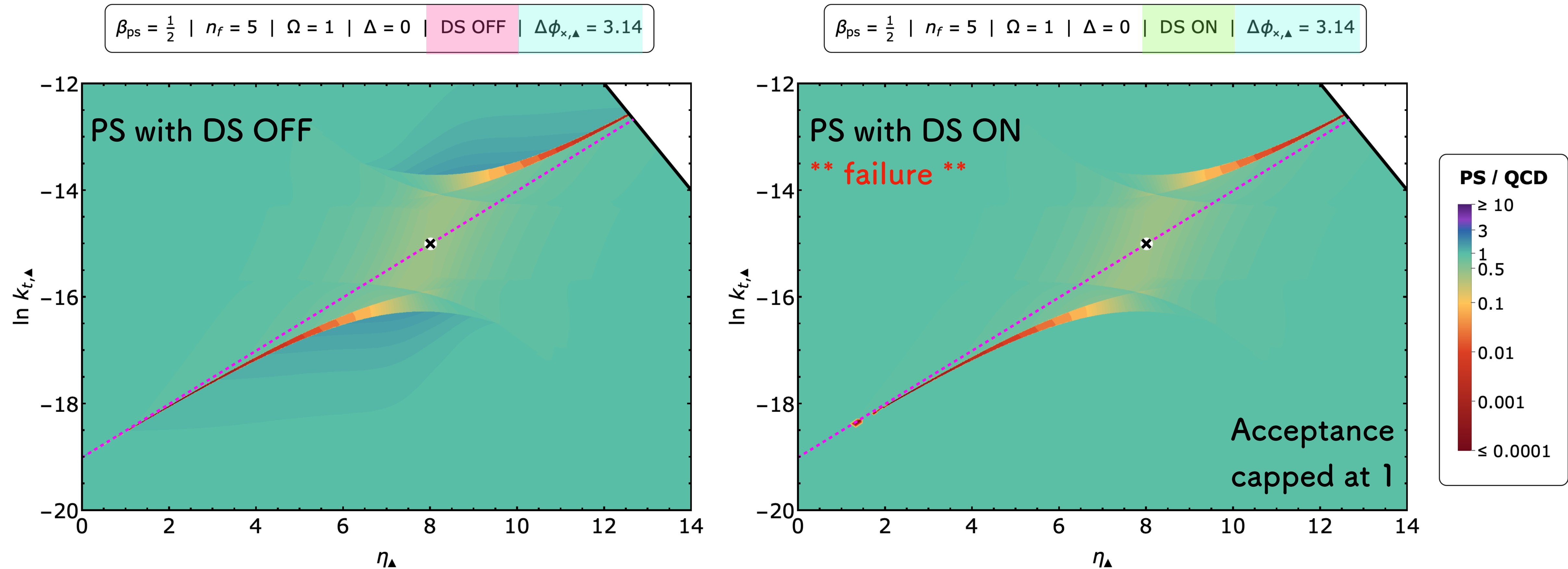
Deficit at proposal level guarantees deficit after acceptance correction

- LHS : **default** PS $\div$ QCD emission rate
- RHS : **corrected** PS $\div$ QCD emission rate — shows huge deficits in **fissures** — **failure**



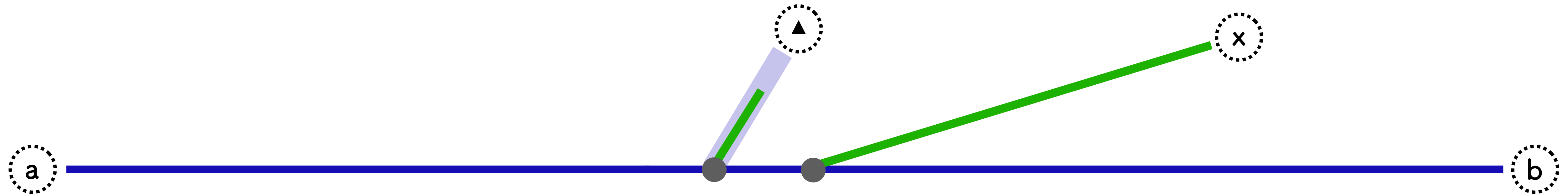
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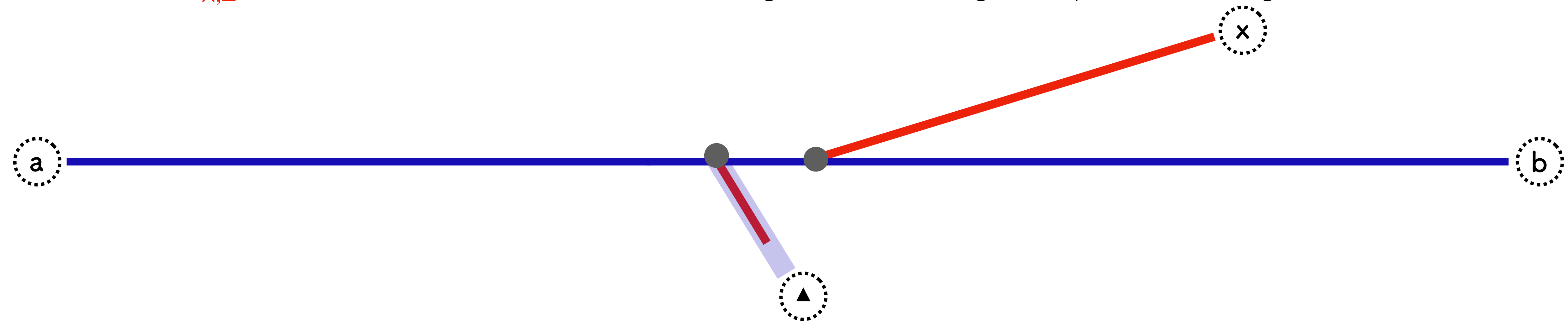


Dominant history **above** lower left fissure : **b** emits $\blacktriangle$ then **b** emits $\mathbf{x}$

- Hard particle **b** takes k_T recoil due to emission of $\mathbf{x}$, pre-branching $\blacktriangle$ momentum just gets overall rescaling
- For $\Delta\phi_{\mathbf{x},\blacktriangle} = 0$ before $\mathbf{x}$ was emitted $\blacktriangle$ [blue] is just a rescaling of its post-branching self [green]



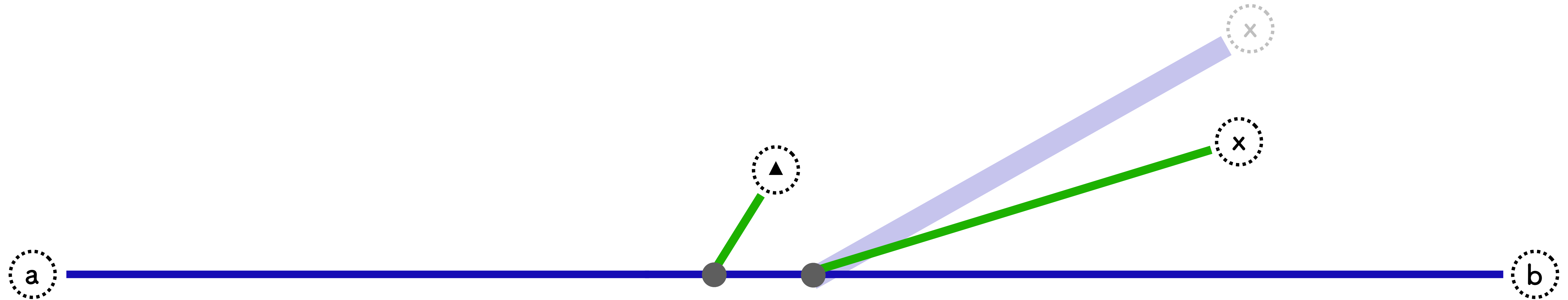
- For $\Delta\phi_{\mathbf{x},\blacktriangle} = \pi$ before $\mathbf{x}$ was emitted $\blacktriangle$ [blue] is [again!] a rescaling of its post-branching self [red]



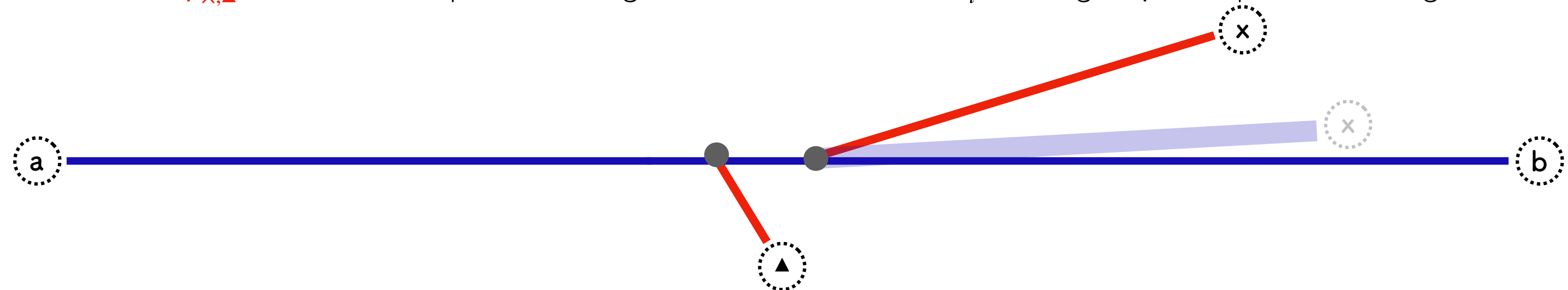
- In both cases pre-branching $\blacktriangle$ is basically the same : hence $v_{\mathbf{x}} < v_{\blacktriangle}$ boundary unchanged by $\Delta\phi_{\mathbf{x},\blacktriangle} = 0 \rightarrow \pi$

Dominant history **below** lower left fissure : **b** emits **x** then **x** emits **▲**

- Soft **x** takes, to excellent approx, entire recoil of **▲** [**a** takes -ve light cone fracⁿ, but that's fractionally ~ 0]
- Hence for $\Delta\phi_{x,\blacktriangle} = 0$ pre-branching **x** [blue] was at higher k_t and smaller $|\eta|$ than post-branching **x** [green]



- For $\Delta\phi_{x,\blacktriangle} = \pi$ this means pre-branching **x** [blue] was at smaller k_t and larger $|\eta|$ than post-branching **x** [red]



- Compared to $\Delta\phi_{x,\blacktriangle} = 0$, for $\Delta\phi_{x,\blacktriangle} = \pi$ the initial **x** is more soft-collinear so the $v_{\blacktriangle} < v_x$ boundary gets lower