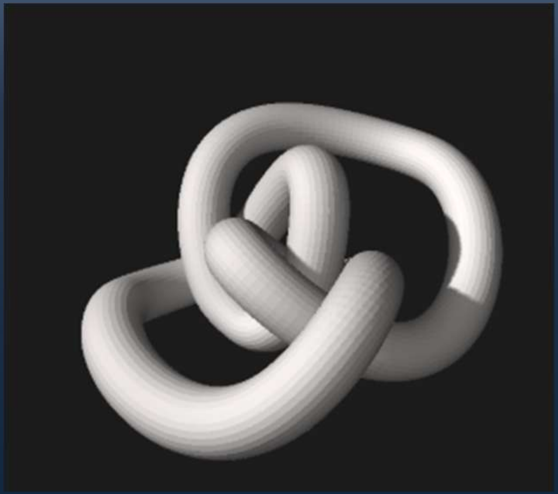
Topological states across platforms

Tami Pereg-Barnea

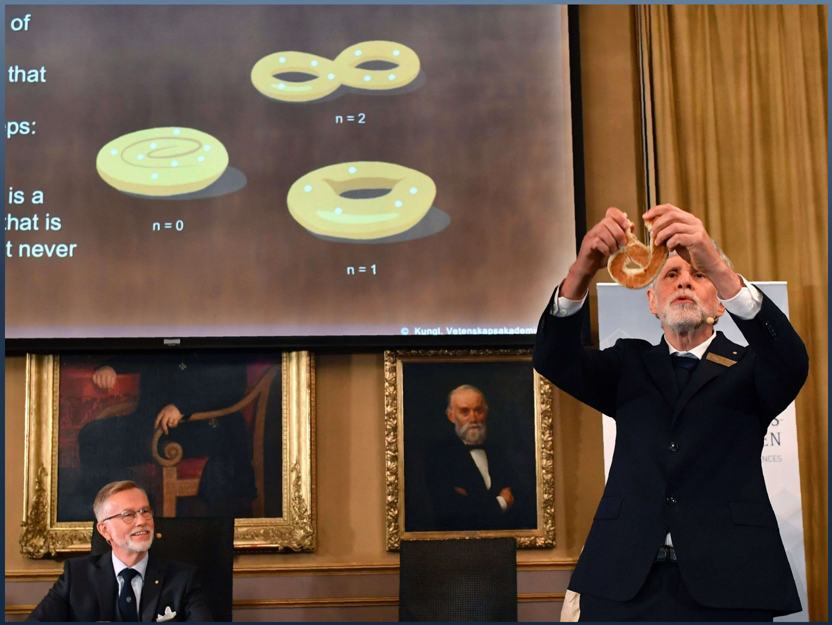


McGill

Theory Canada 18,
Universite de Montreal, 20.6.2026

A word about topology

Topology (from the Greek words τόπος, 'place, location', and λόγος, 'study') is the branch of mathematics concerned with the properties of a geometric object that are preserved under continuous deformations, such as stretching, twisting, crumpling, and bending; that is, without closing holes, opening holes, tearing, gluing, or passing through itself.



- Topology deals with classifies the possible map between spaces
- In condensed matter we are interested in real space or momentum space (the Brillouin zone) and mapping it to a Hilbert space

2016 Nobel prize in Physics --- topology in condensed matter

David J. Thouless, F. Duncan M. Haldane and , J. Michael Kosterlitz,

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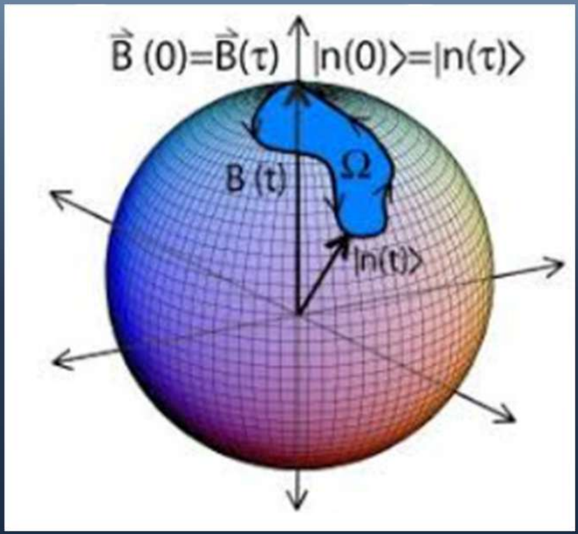
Two level system and the Berry phase

- Imagine a spin in a magnetic field B whose direction changes slowly and forms a closed loop, will follow the field direction.
- Its initial and final wavefunction will defer only by a phase.
- The phase has two contributions – the dynamical phase and Berry's phase:

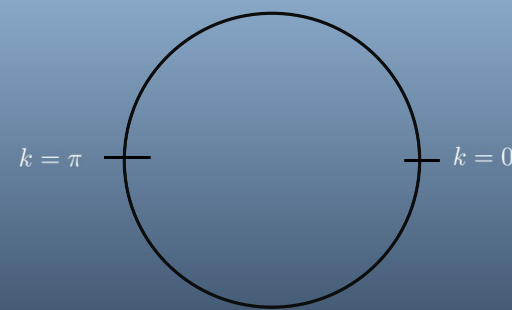
$$\gamma_n = i \int_0^t < n(B(t'))| \frac{d}{dt} |n(B(t') > dt'$$

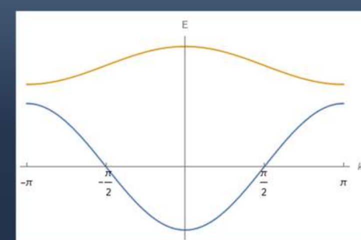
- A trip around the equator gives a Berry phase of $\pm\pi$

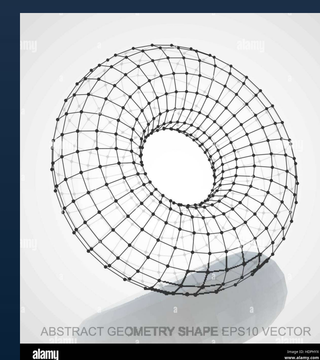


Energy Bands

- The lattice of ions provides a periodic potential for electrons.
- Momentum is replaced by lattice momentum – confined to the Brillouin zone (loop in 1D, torus in 2D or 3D)
- Imagine scanning momentum and following the wavefunction
  - The total accumulated phase is the Chern number (loosely defined)

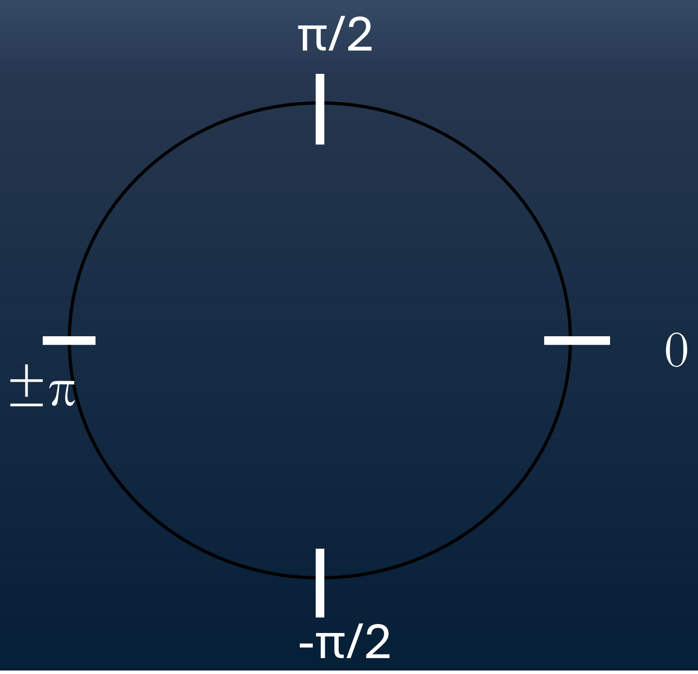


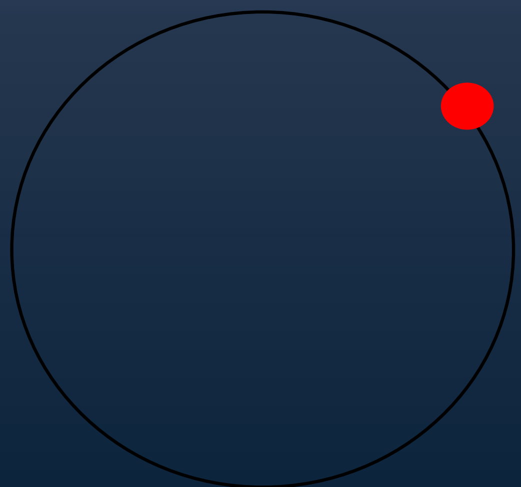




Simplest example in 1D

$$\mathcal{H} = \sum_k \begin{pmatrix} \psi_\uparrow^\dagger & \psi_\downarrow^\dagger \end{pmatrix} \begin{pmatrix} h_z(k) & h_x(k) - ih_y(k) \\ h_x(k) + ih_y(k) & -h_z(k) \end{pmatrix} \begin{pmatrix} \psi_\uparrow \\ \psi_\downarrow \end{pmatrix}$$





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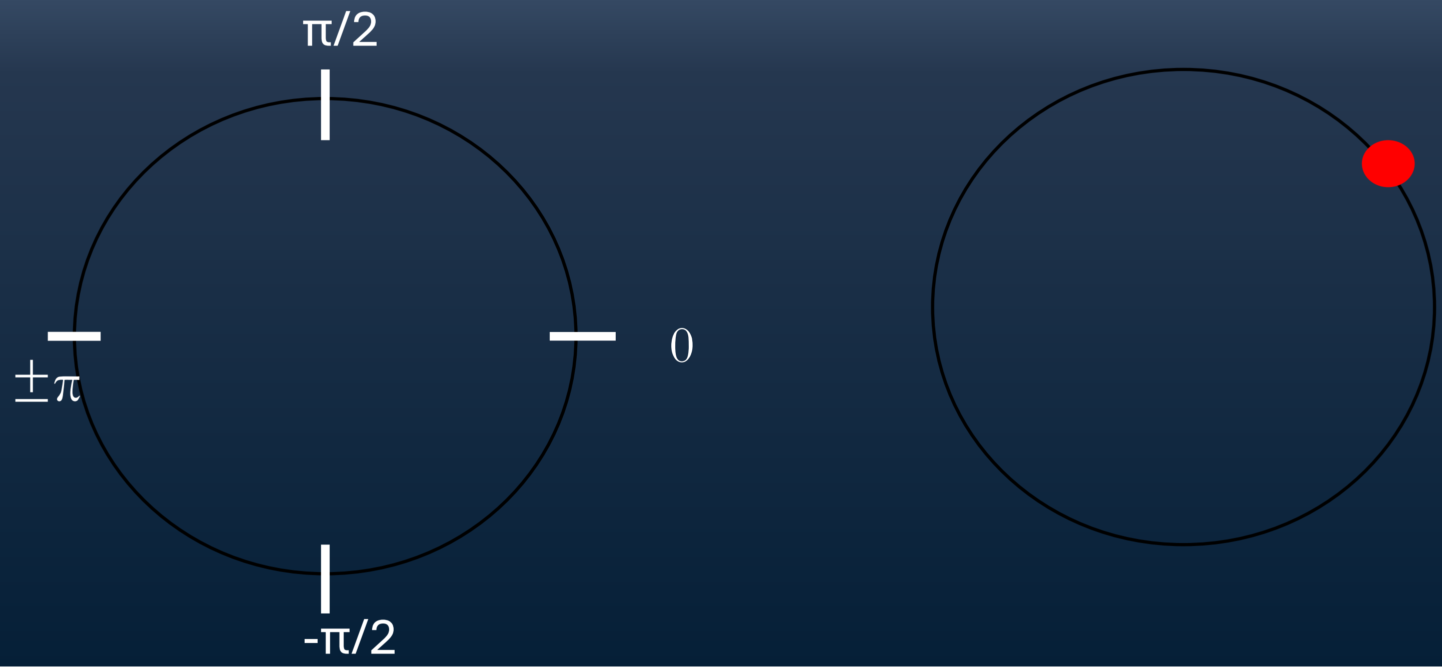

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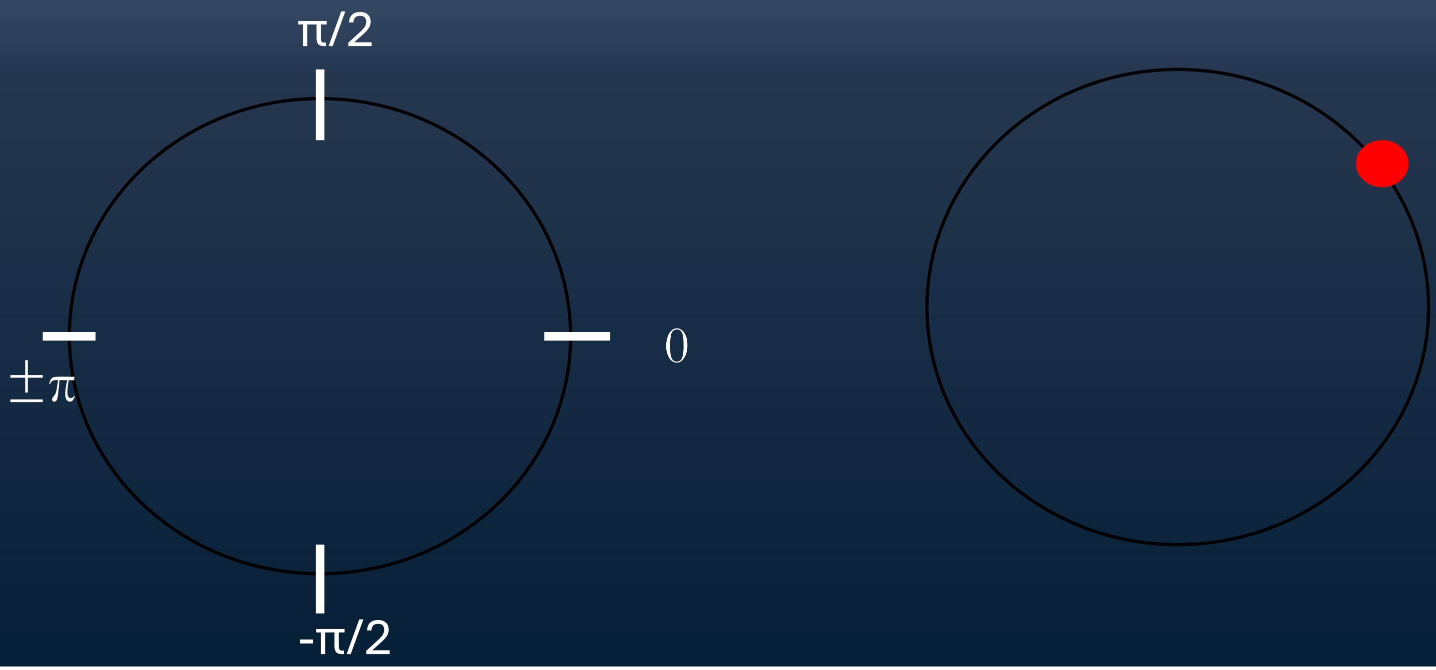
$$\mathcal{H} = \sum_k \psi^\dagger \left(\vec{h}(k) \cdot \vec{\sigma} \right) \psi$$



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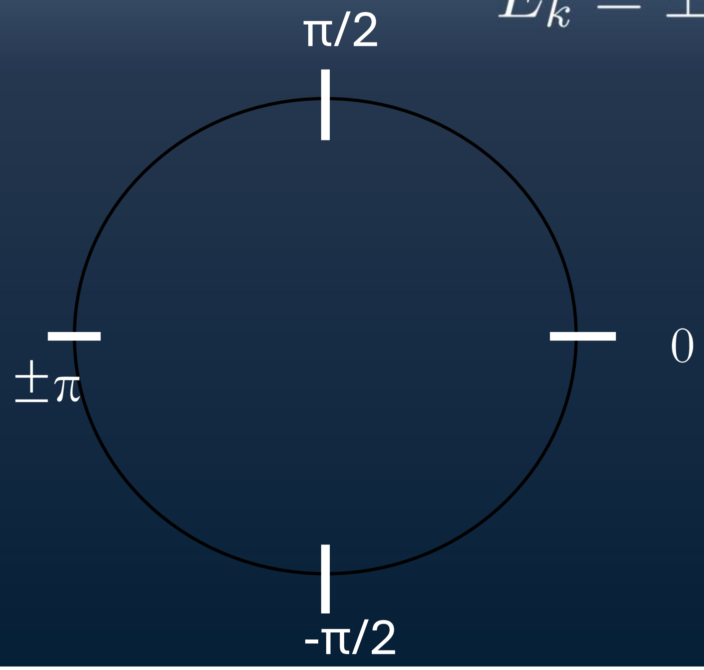
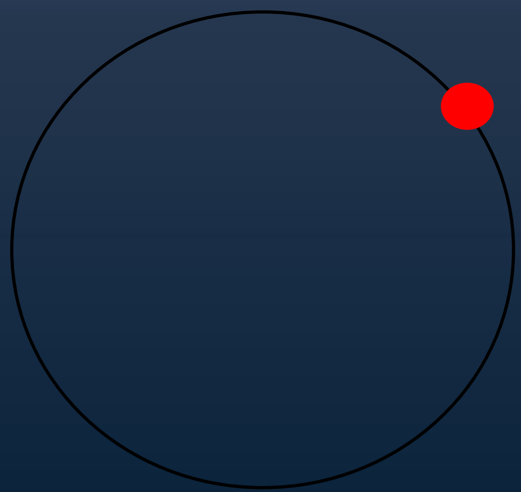
$$E_k = \pm|\vec{h}(k)|$$





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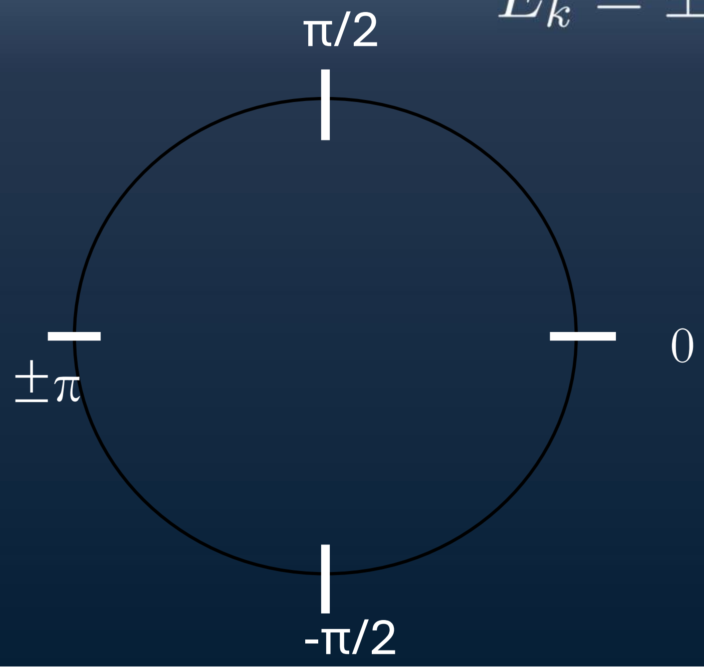

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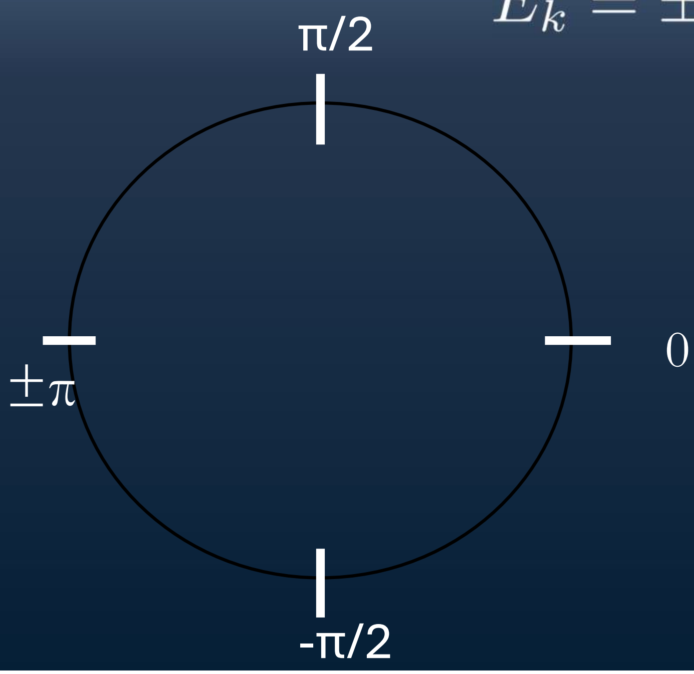

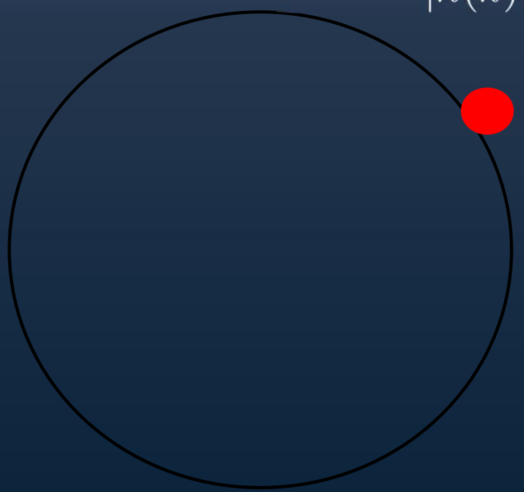

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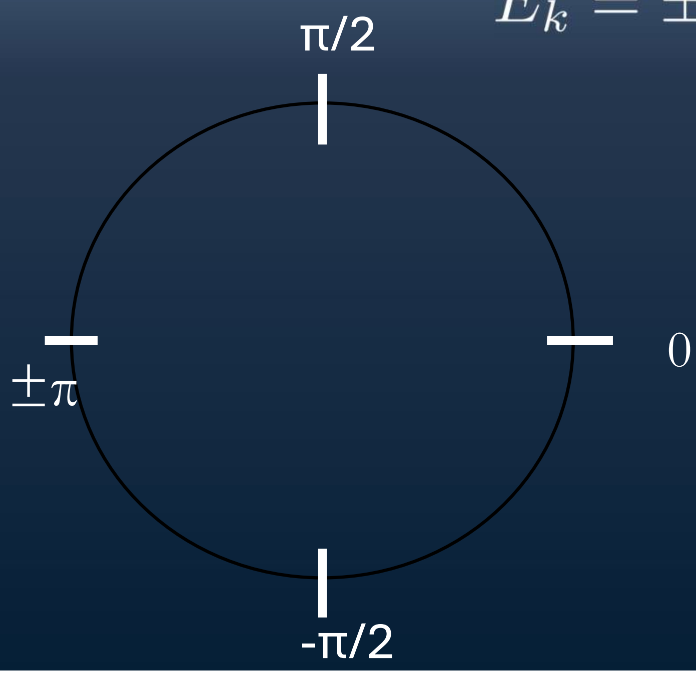

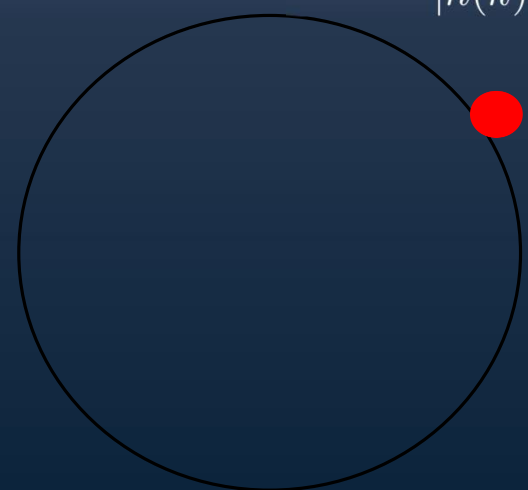

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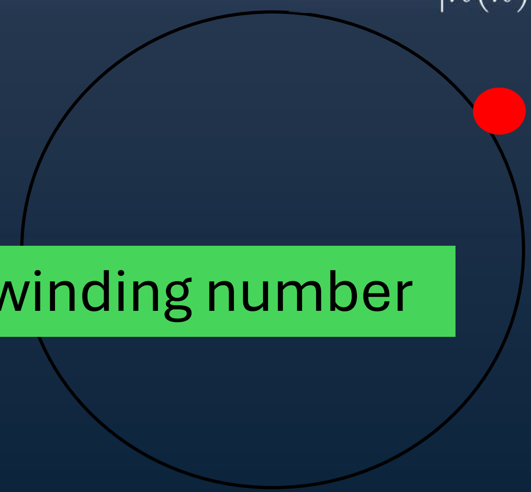

Topological invariant = winding number

1D - SSH model

Solitons in Polyacetylene
Su, Schrieffer, and Heeger
Phys. Rev. Lett. 42, 1698 (1979)

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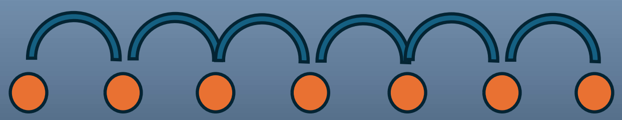

$$\mathcal{H} = t \sum_j c_j^\dagger c_{j+1} + h.c$$

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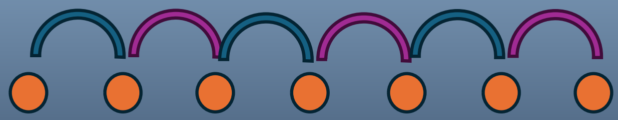

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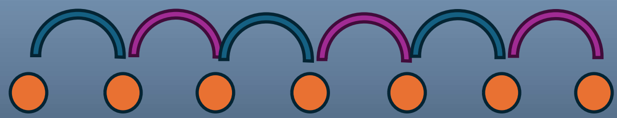

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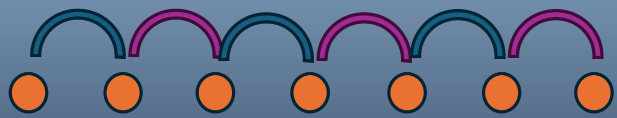

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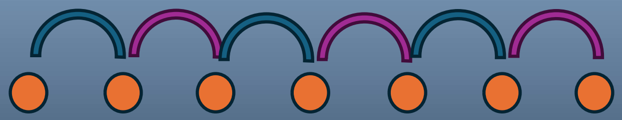

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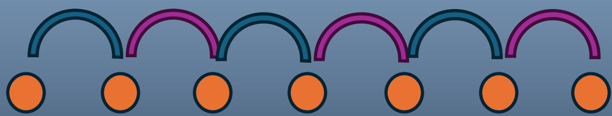

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Chiral symmetry –
no σ_z

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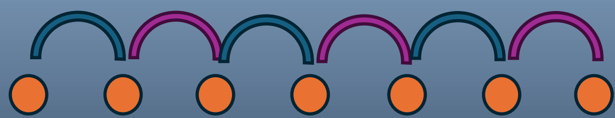

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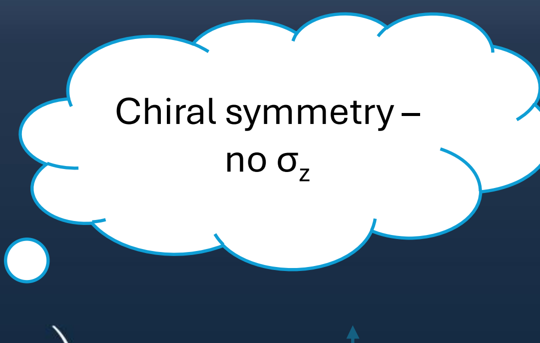

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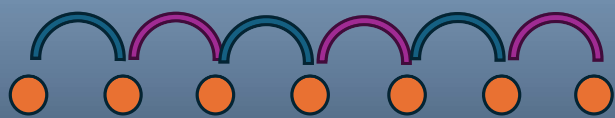

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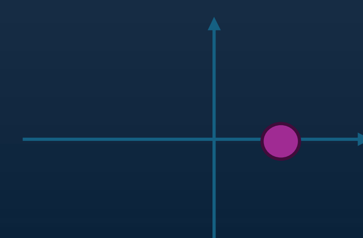

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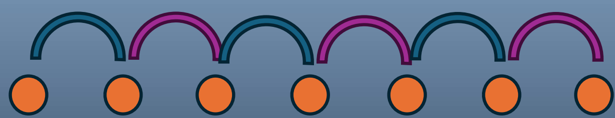

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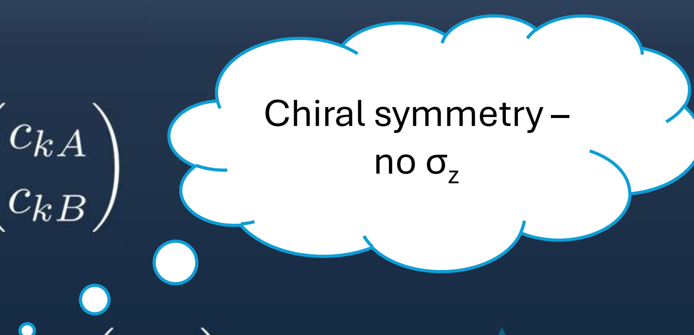
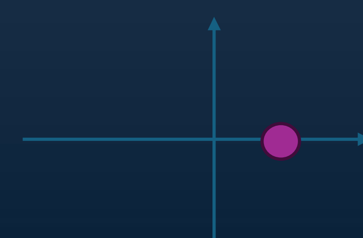
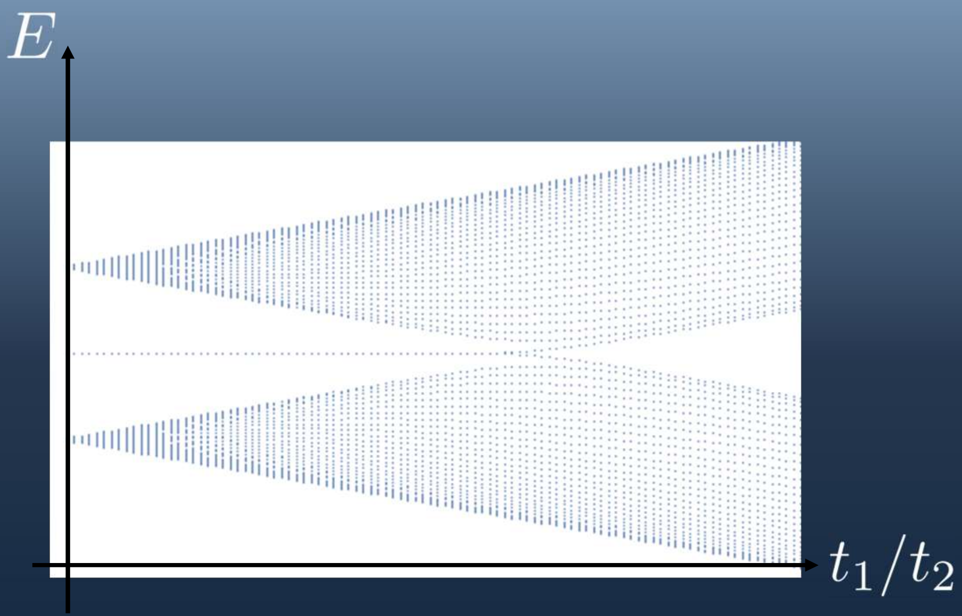




Bulk boundary correspondence



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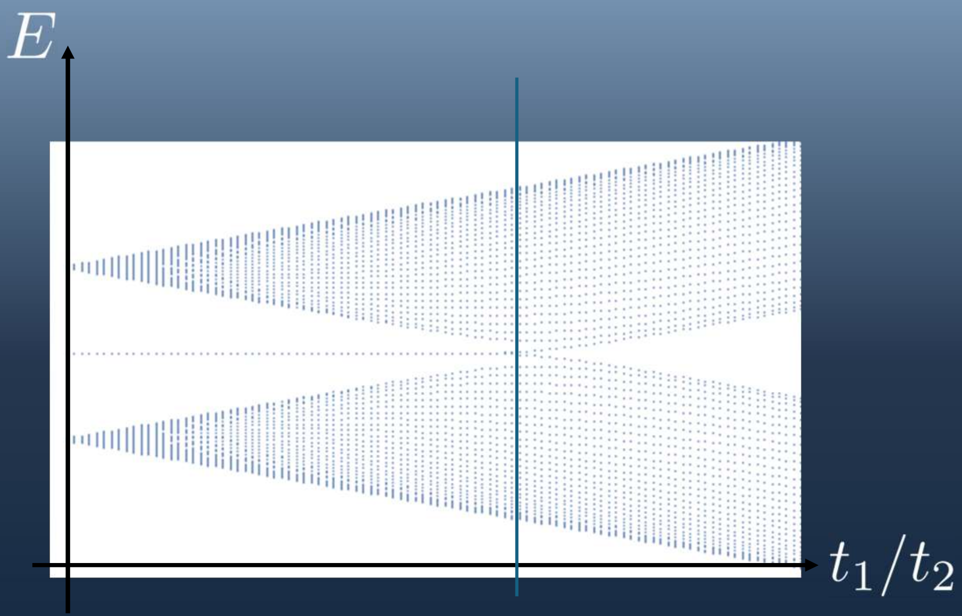

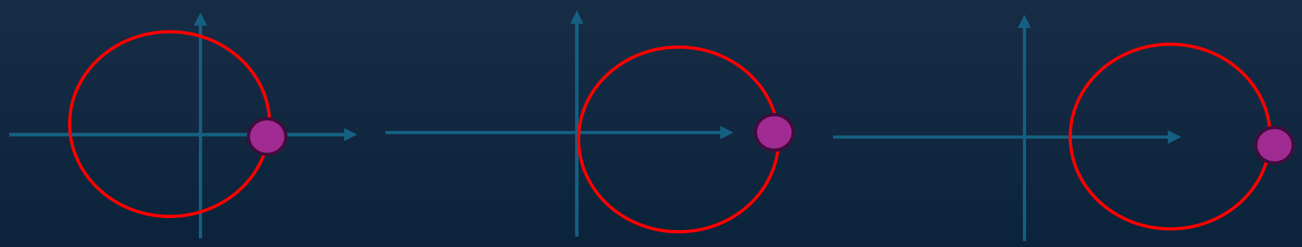

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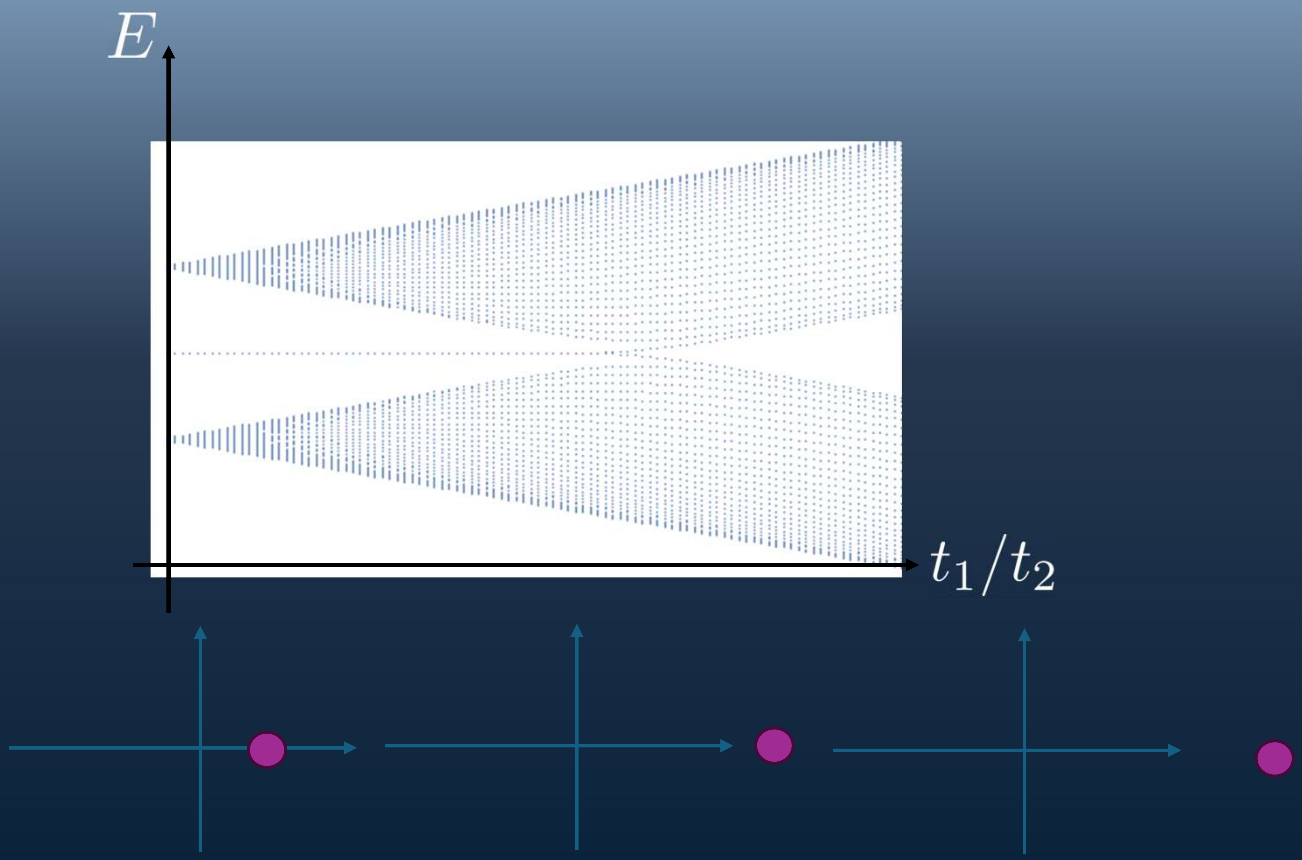

1D topological superconductor – Kitaev model

Particle hole symmetry replaces Chiral symmetry!

1D topological superconductor – Kitaev model

$$\mathcal{H} = \sum_i \left[\frac{t}{2} \hat{c}_i^\dagger \hat{c}_{i+1} + \frac{\Delta}{2} \hat{c}_i^\dagger \hat{c}_{i+1}^\dagger - \mu c_i^\dagger c_i + h.c. \right]$$

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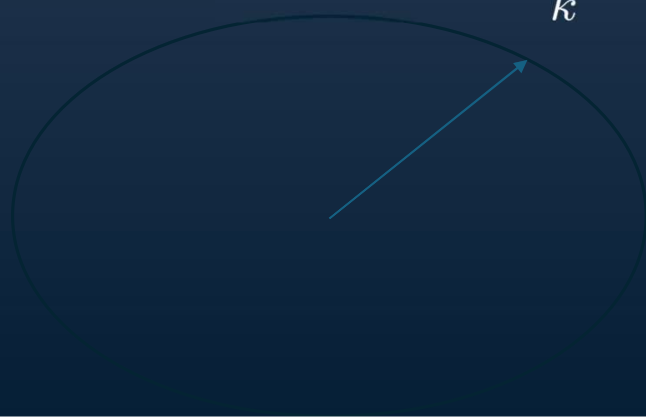

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Fermionic Kitaev chain

Alicea, Rep. Prog. Phys. 2012

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$$c_i^\dagger = \frac{1}{2}(\gamma_{iA} + i\gamma_{iB})$$

$$\begin{aligned} \{\gamma_\alpha, \gamma_\beta\} &= \delta_{\alpha,\beta} \\ \gamma_\alpha^2 &= 1 \end{aligned}$$

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$$\mathcal{H} = \sum_i [i(t+\Delta)\gamma_{ib}\gamma_{i+1a} + i(t-\Delta)\gamma_{ia}\gamma_{i+1b} - i\mu\gamma_{ia}\gamma_{ib}]$$

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$$\{\gamma_\alpha, \gamma_\beta\} = \delta_{\alpha,\beta}$$

$$\gamma_\alpha^2 = 1$$

Alicea, Rep. Prog. Phys. 2012

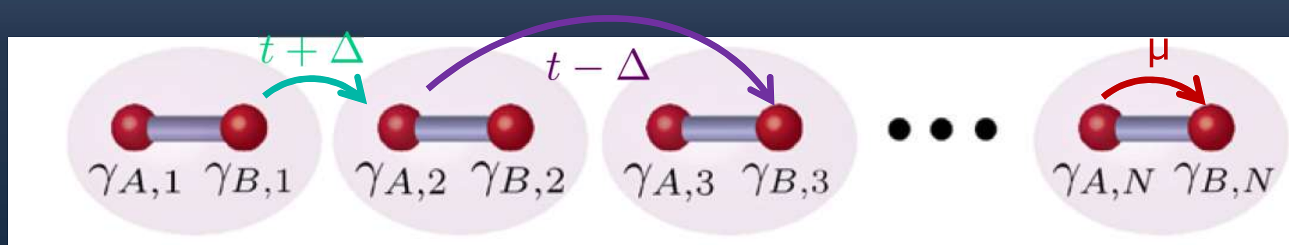
Fermionic Kitaev chain

$$\mathcal{H} = \sum_i \left[\frac{t}{2} \hat{c}_i^\dagger \hat{c}_{i+1} + \frac{\Delta}{2} \hat{c}_i^\dagger \hat{c}_{i+1}^\dagger - \mu c_i^\dagger c_i + h.c. \right]$$

$$\mathcal{H} = \sum_i \left[i(t+\Delta)\gamma_{ib}\gamma_{i+1a} + i(t-\Delta)\gamma_{ia}\gamma_{i+1b} - i\mu\gamma_{ia}\gamma_{ib} \right]$$

$$c_i^\dagger = \frac{1}{2}(\gamma_{iA} + i\gamma_{iB})$$

$$\begin{aligned} \{\gamma_\alpha, \gamma_\beta\} &= \delta_{\alpha,\beta} \\ \gamma_\alpha^2 &= 1 \end{aligned}$$



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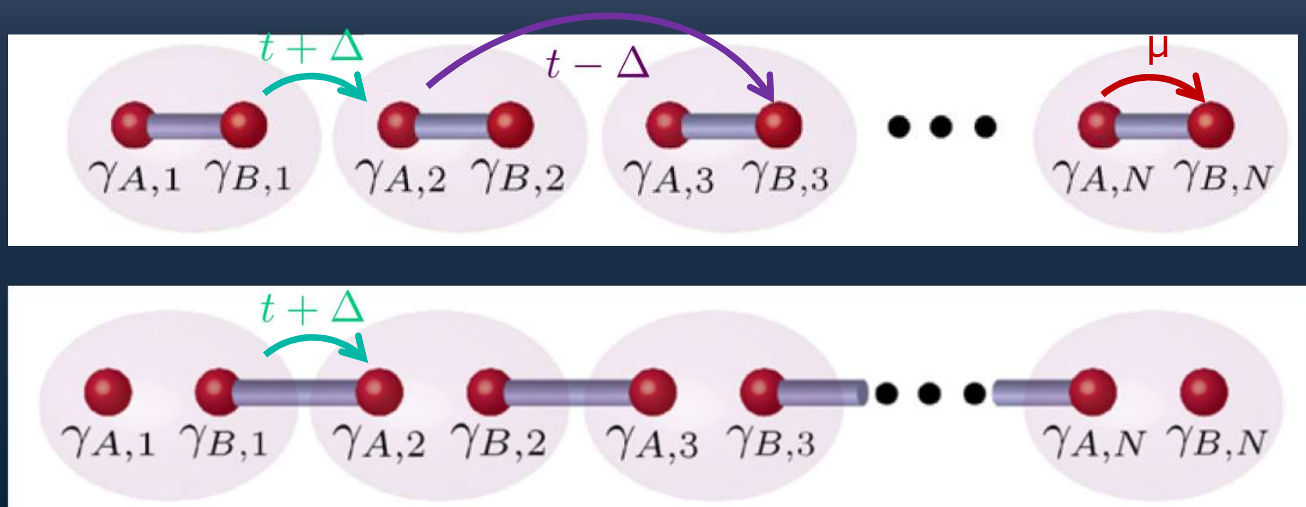

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Majorana Fermions

$$\gamma = c + c^\dagger = \gamma^\dagger$$

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1D Bosonic Kitaev Chain

- Similar model, with bosons, very different physics

$$\mathcal{H} = \sum_j (it\hat{a}^\dagger_{j+1}\hat{a}_j + i\Delta\hat{a}^\dagger_{j+1}\hat{a}^\dagger_j + h.c)$$

McDonald, TPB and Clerk
Phys. Rev. X **8**, 041031 (2018)

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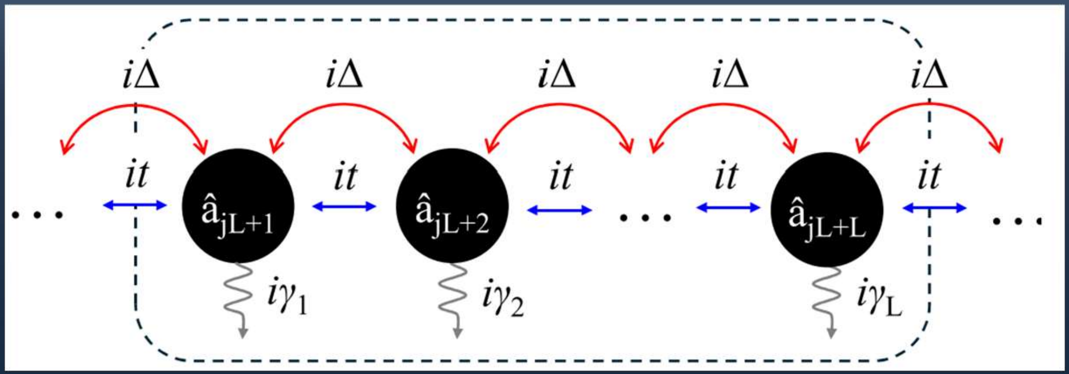

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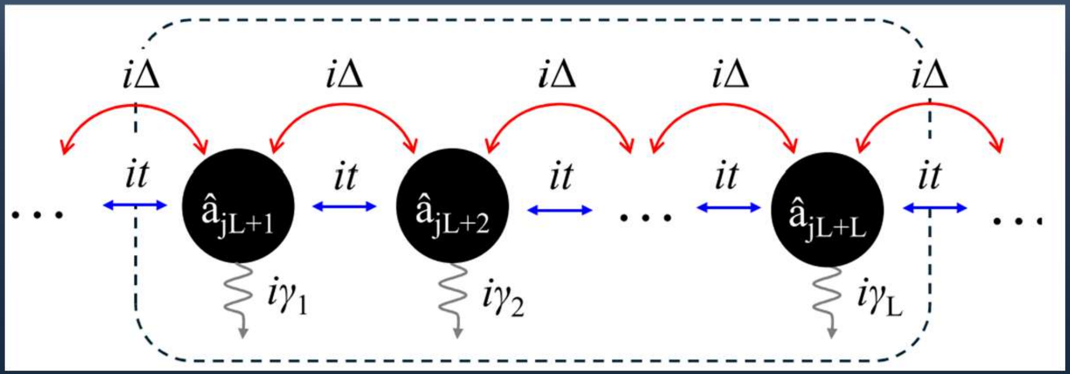

$$\mathcal{H} = \sum_k [t\sin k a^\dagger_k a_k + i\frac{\Delta}{2}\cos k\left(a^\dagger_k a^\dagger_{-k} - h.c.\right)]$$

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Bosonic Kitaev Chain

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Bosonic Kitaev Chain

- Quadratures decouple

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- Quadratures decouple

$$
\begin{aligned}
\hat{a}_j^\dagger &= \frac{1}{\sqrt{2}}(\hat{x}_j + i\hat{p}_j) \\
\mathcal{H} &= \sum_j ((t+\Delta)\hat{x}_j\hat{p}_{j+1} - (t-\Delta)\hat{x}_{j+1}\hat{p}_j)
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$$

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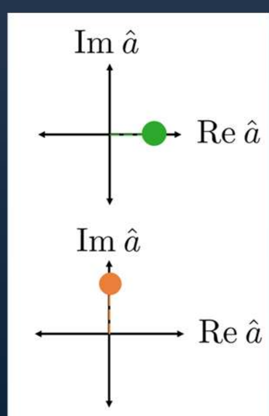

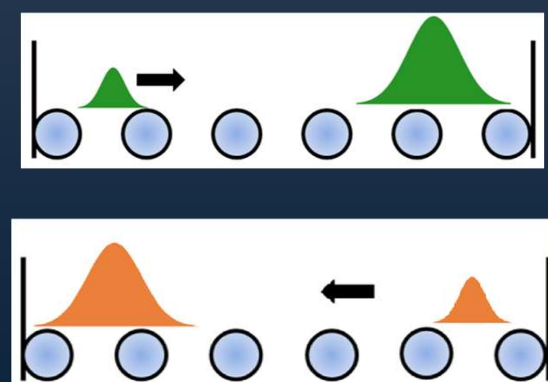

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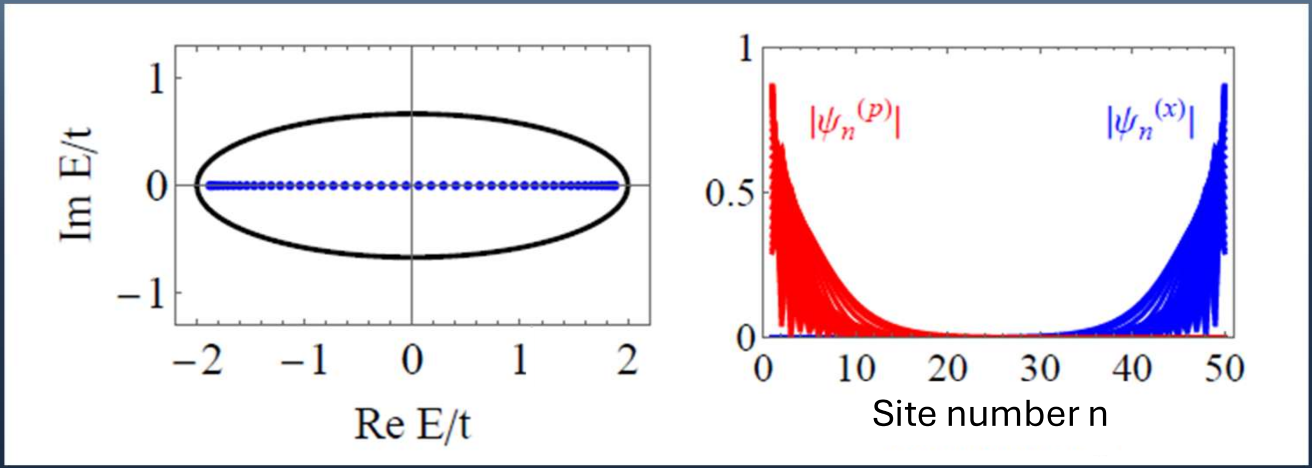
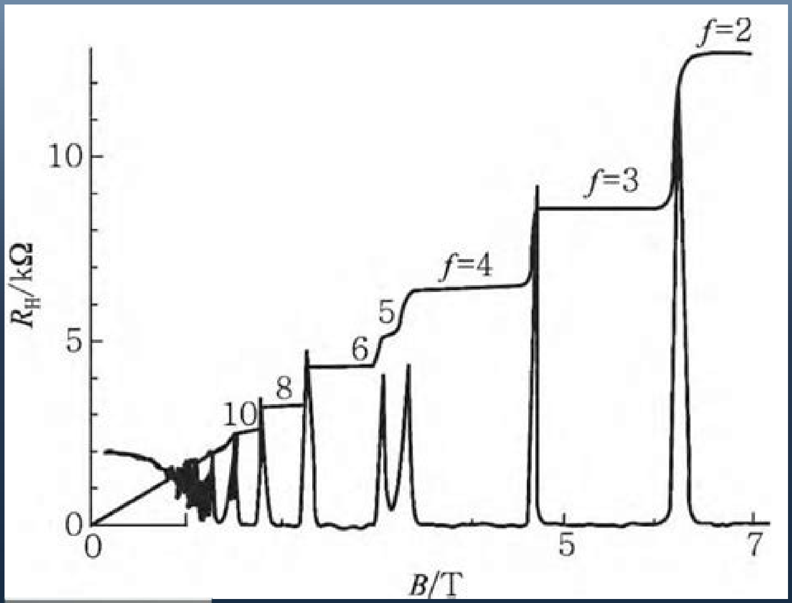
Bosonic Kitaev Chain

- Non-Hermitian skin effect



Fortin, Wang and TPB, Phys. Rev. B **112**, 064208 (2025)
Fortin, Wang and TPB, Phys. Rev. B **112**, 174208 (2025)
Fortin, Wang and TPB, arXiv:2606.17881

2D: Quantum Hall Effect - Chern number



$$\sigma_{xy} = n\frac{e^2}{h}$$

Klaus von Klitzing
1985 Nobel Prize

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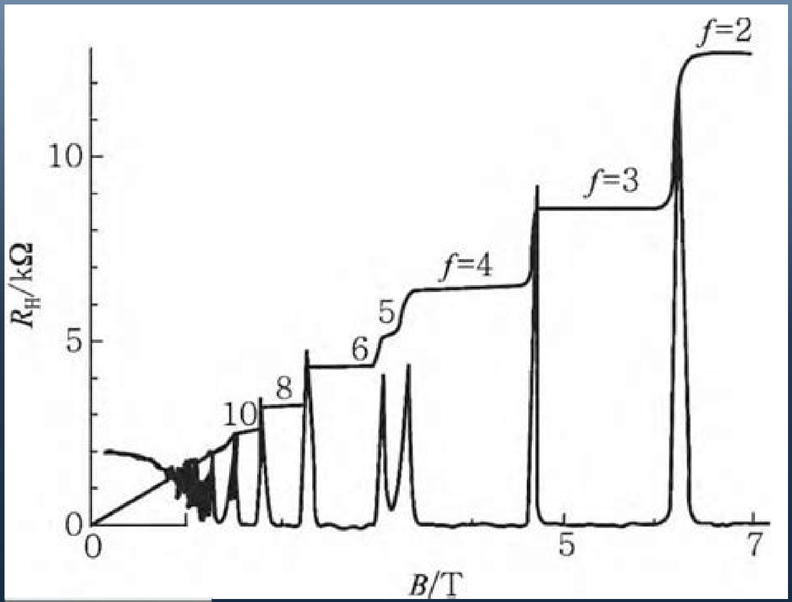

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Topological connection:

$$\sigma_{xy} = n\frac{e^2}{h}$$

n is an integer
= #of filled Landau levels
= # of edge modes
= sum of Chern numbers C_n of the filled Landau levels (each LL contributes 1)

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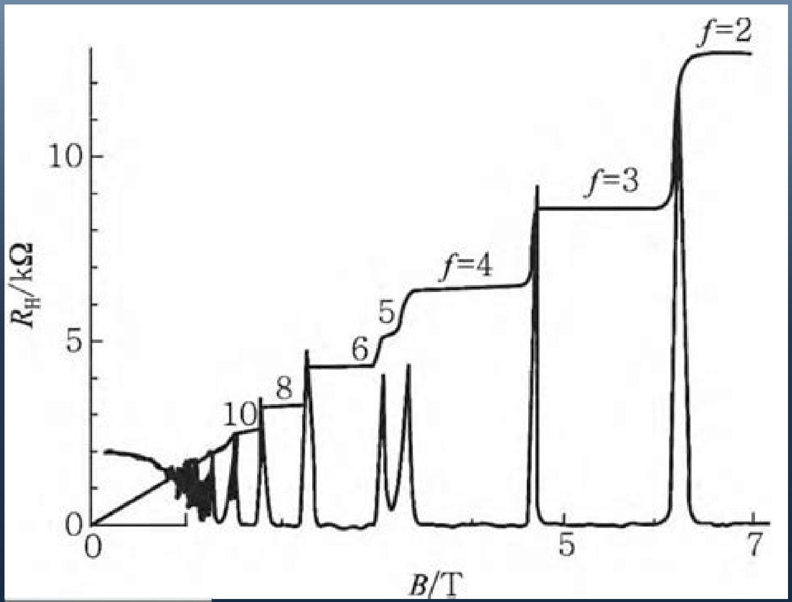

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2D. Topological insulator - Z_2 invariant

König *et al.* Quantum Spin Hall Insulator State in HgTe Quantum Wells. *Science* **318**,766-770(2007)

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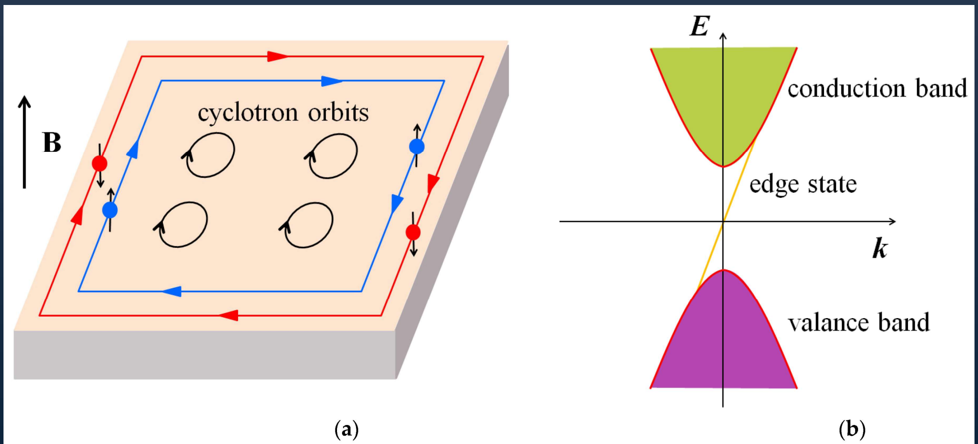

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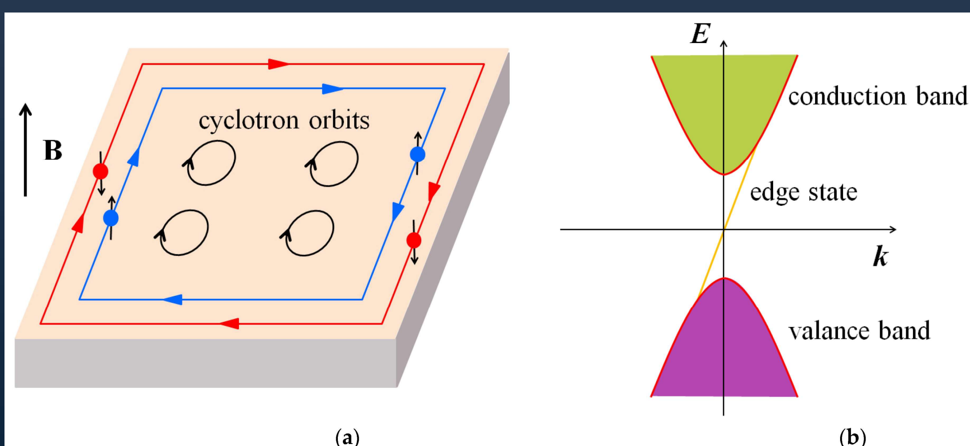
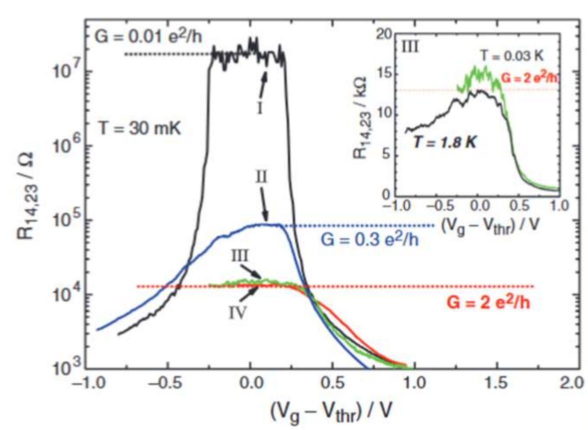


Fig. 4. The longitudinal four-terminal resistance, $R_{14,23}$, of various normal (d = 5.5 nm) (I) and inverted (d = 7.3 nm) (II, III, and IV) QW structures as a function of the gate voltage measured for B = 0 T at T = 30 mK. The device sizes are (20.0 × 13.3) μm^2 for devices I and II, (1.0 × 1.0) μm^2 for device III, and (1.0 × 0.5) μm^2 for device IV. The inset shows $R_{14,23}(V_g)$ of two samples from the same wafer, having the same device size (III) at 30 mK (green) and 1.8 K (black) on a linear scale.



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Classification

Classification of topological insulators and superconductors in three spatial dimensions

Andreas P. Schnyder,[1] Shinsei Ryu,[1] Akira Furusaki,[2] and Andreas W. W. Ludwig[3]

		TRS	PHS	SLS	$d=1$	$d=2$	$d=3$
Standard	A (unitary)	0	0	0	-	$\mathbb{Z}$	-
(Wigner-Dyson)	AI (orthogonal)	+1	0	0	-	-	-
	AII (symplectic)	−1	0	0	-	$\mathbb{Z}_2$	$\mathbb{Z}_2$
Chiral	AIII (chiral unitary)	0	0	1	$\mathbb{Z}$	-	$\mathbb{Z}$
(sublattice)	BDI (chiral orthogonal)	+1	+1	1	$\mathbb{Z}$	-	-
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BdG	D	0	+1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	-
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QSHE

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SC, Majoranas

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d+1 – Floquet topological insulators/superconductors

- Topology requires strong spin-orbit coupling and/or band inversion
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Quasi-energy

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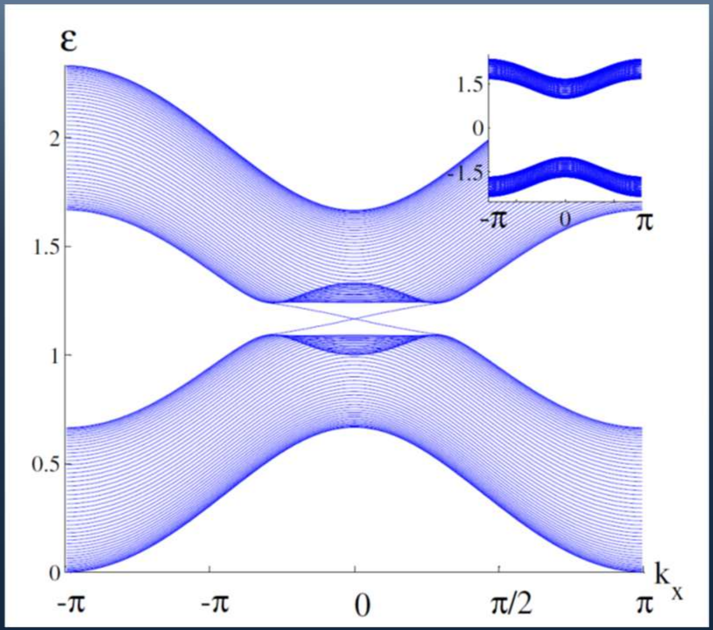
$$U(T) = \tau e^{-i\int_0^T H(t)dt} \equiv e^{-iH_F T}$$

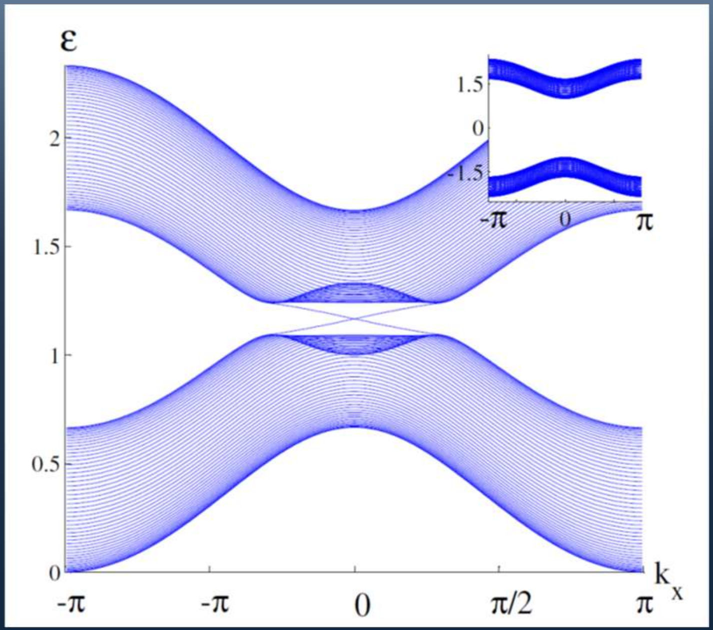
$$H_F \neq \frac{1}{T}\int_0^T H(t)dt$$

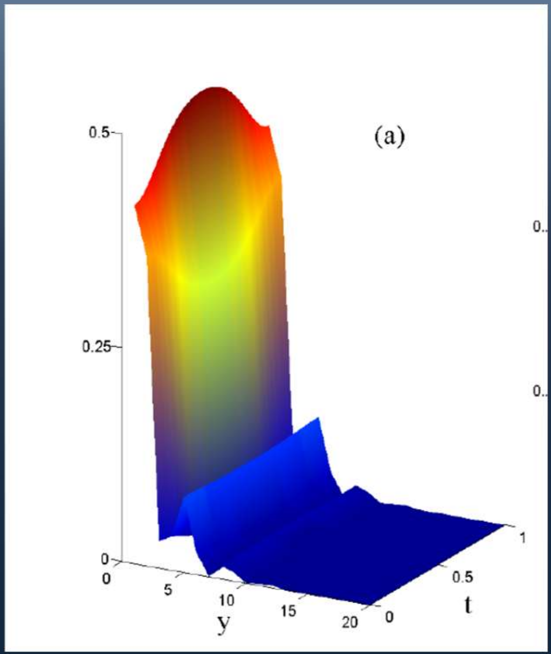
Floquet Topological Insulator



Lindner *et al*, Nature Physics (2011)

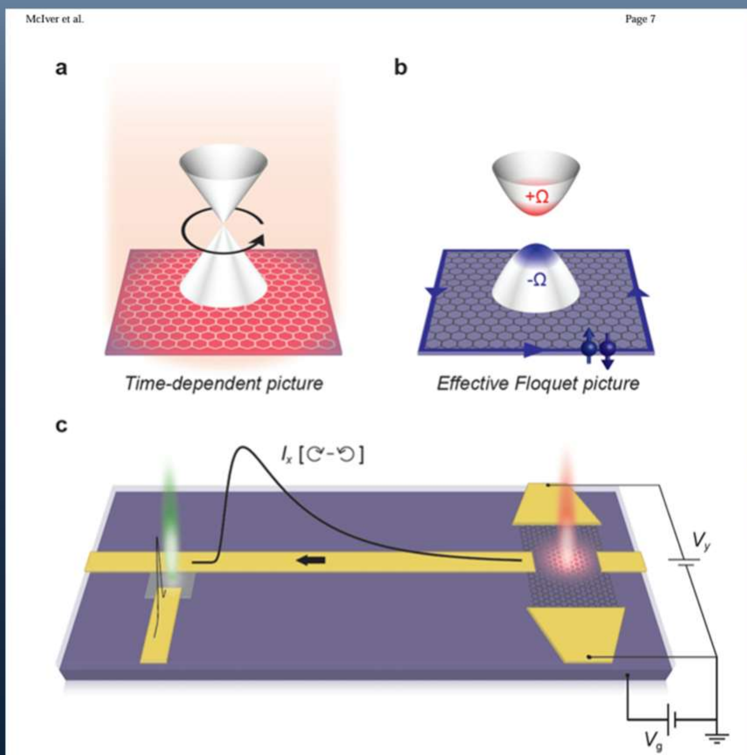
Floquet Topological Insulator

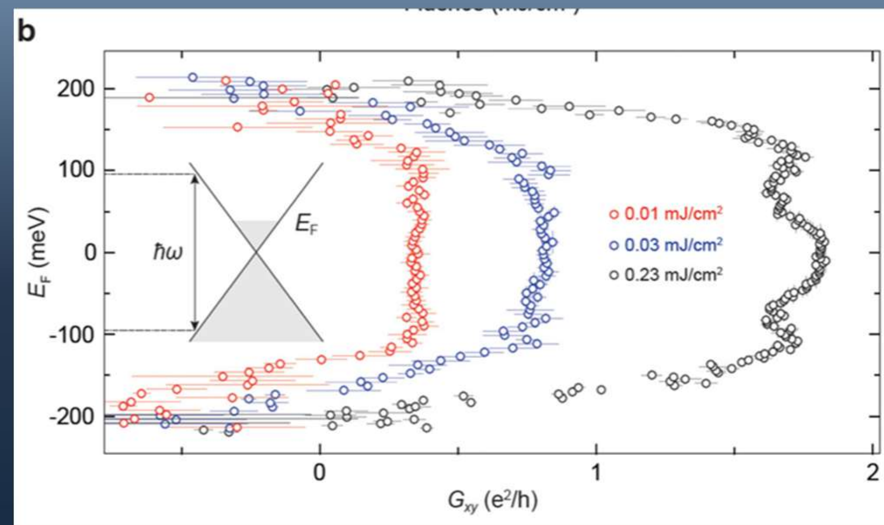




Lindner *et al*, Nature Physics (2011)

Irradiated Graphene





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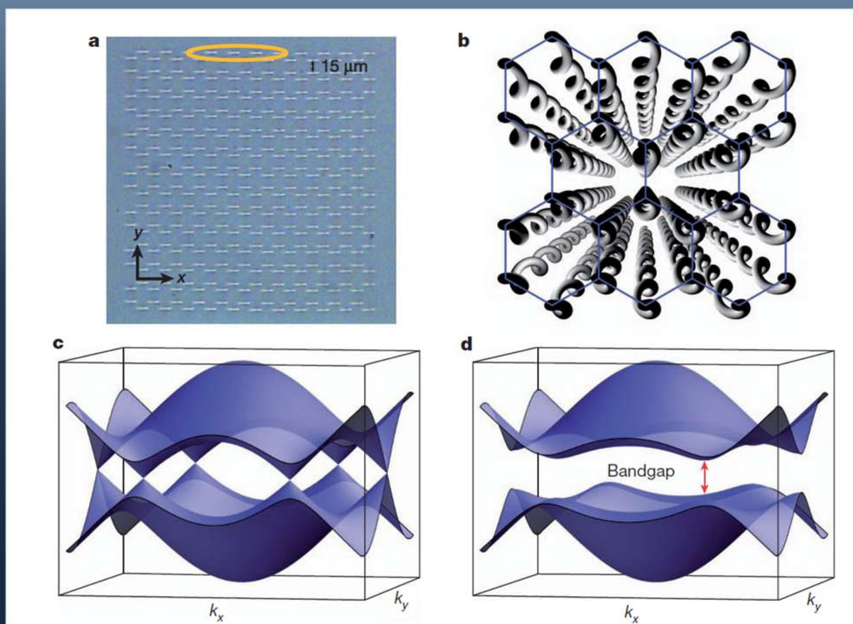
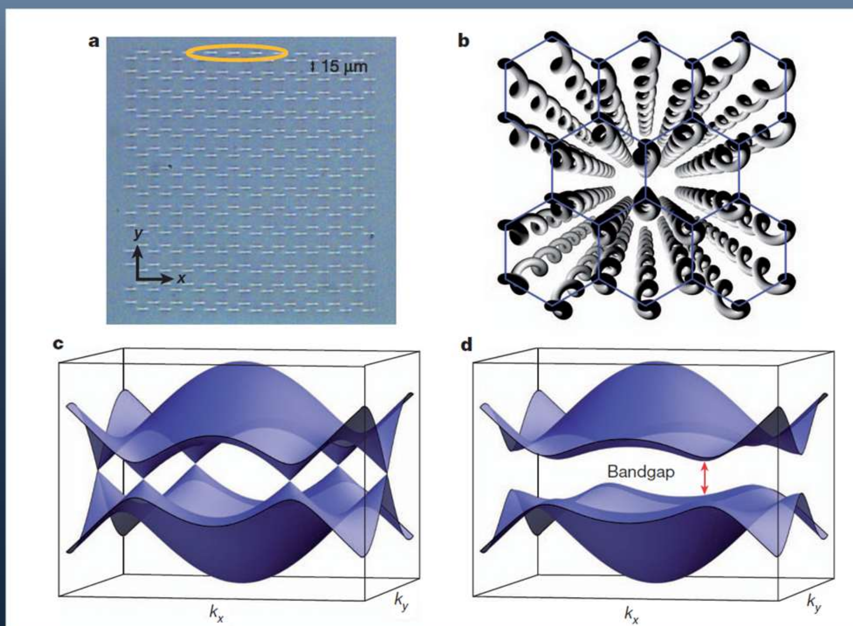
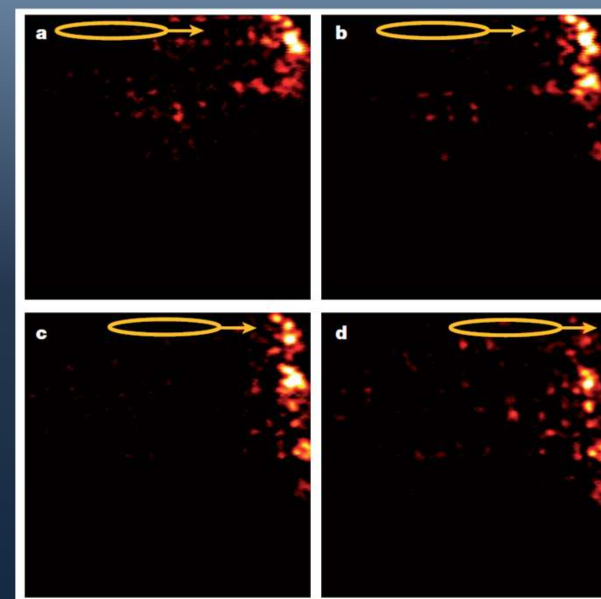
Photonic Floquet topological insulators



Photonic Floquet topological insulators
Rechtsman, M., Zeuner, J., Plotnik, Y. *et al. Nature* **496**, 196–200 (2013)

Photonic Floquet topological insulators

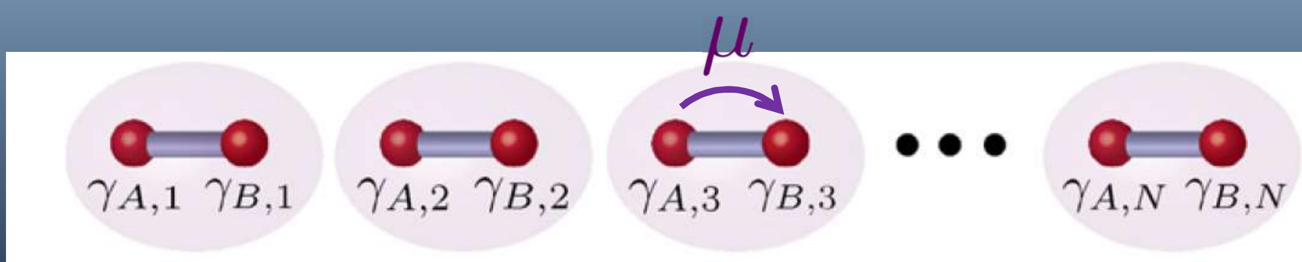




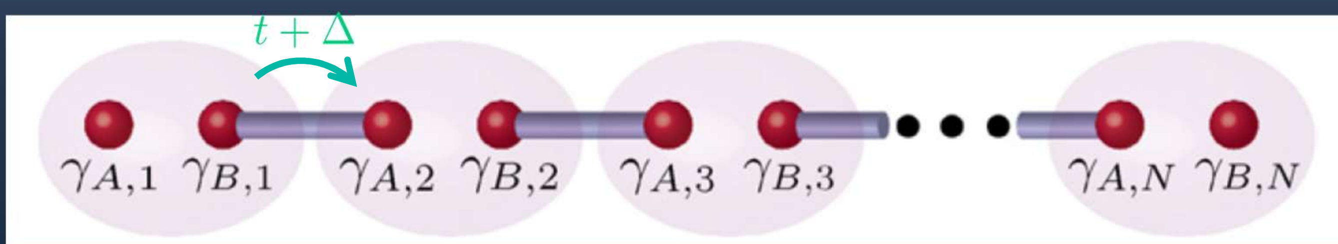
Photonic Floquet topological insulators
Rechtsman, M., Zeuner, J., Plotnik, Y. *et al. Nature* **496**, 196–200 (2013)

Floquet topological Superconductor

H_1

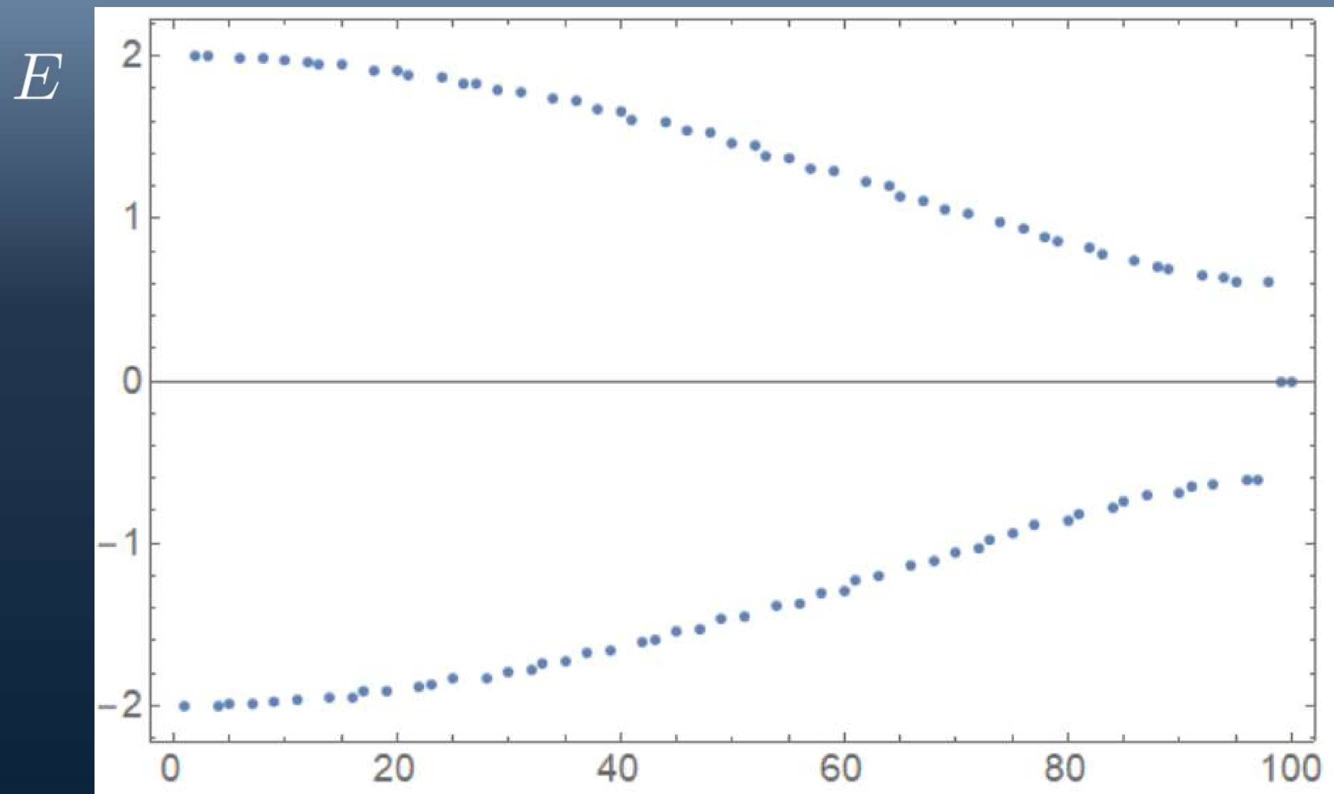


H_2



$$U(T) = e^{-iH_2\frac{T}{2}} e^{-iH_1\frac{T}{2}} \equiv e^{-iH_F T}$$

Topological Superconductor, statis case



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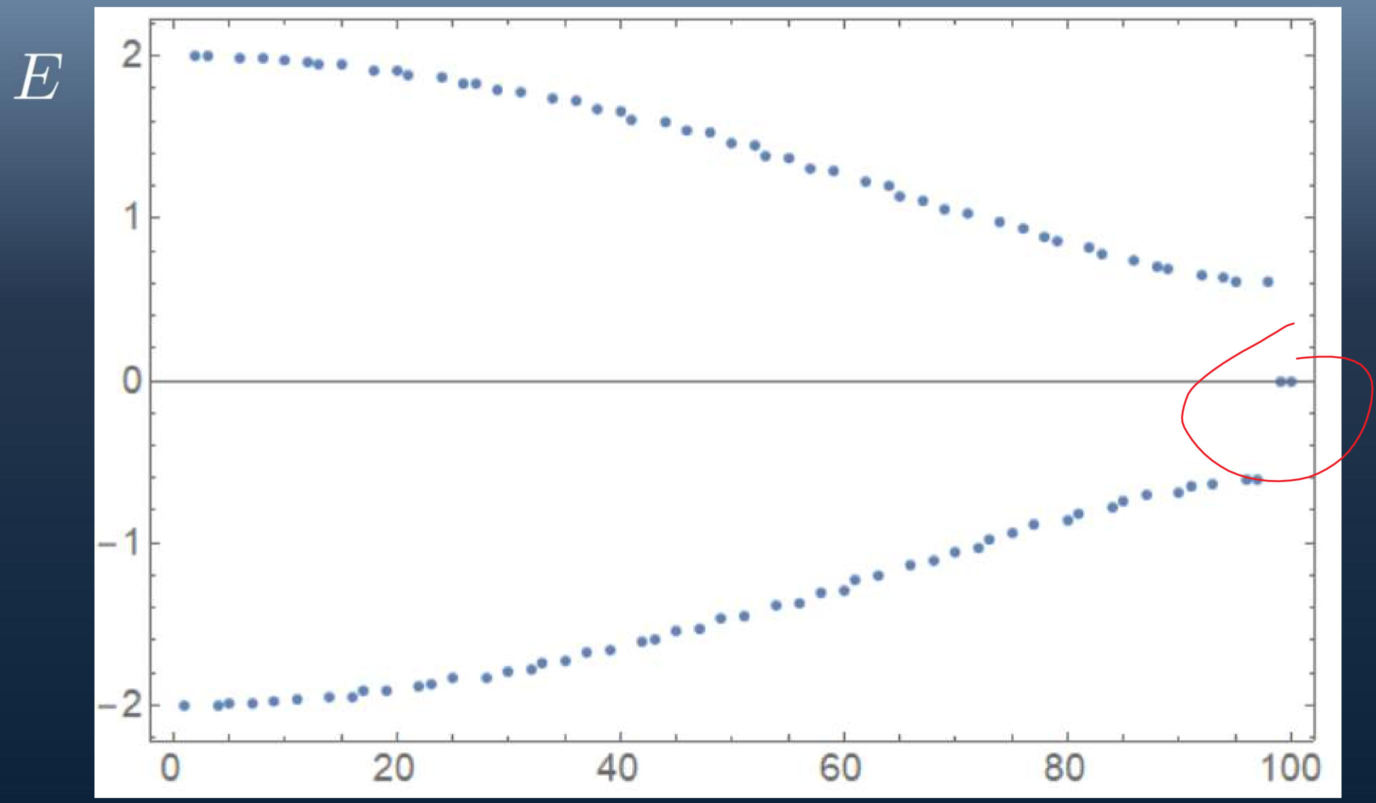


Floquet Topological Superconductor

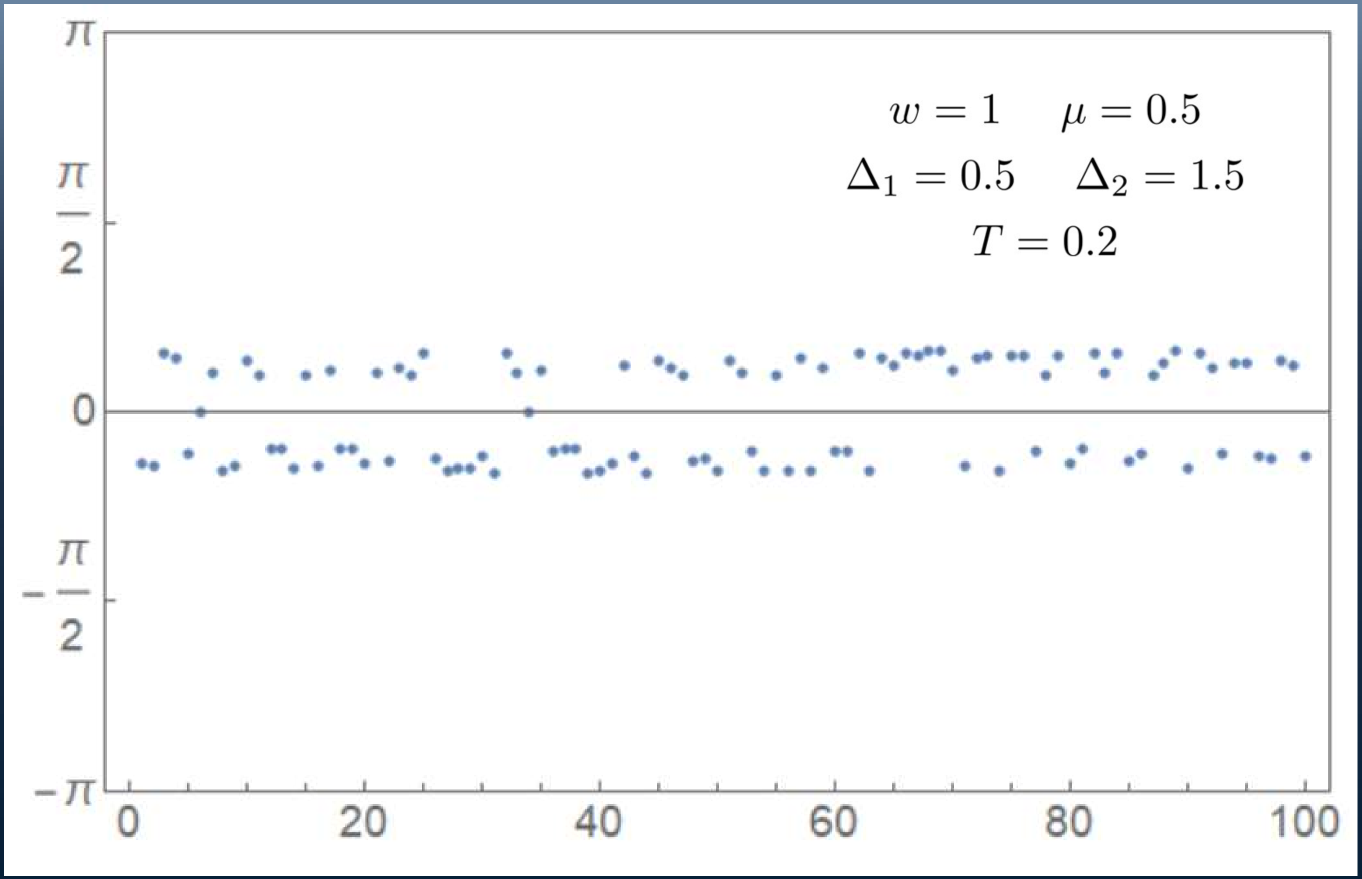
ϵT

$$\mathcal{H} = \sum_i \left(w c^\dagger_{i+1} c_i + i\Delta(t) c_{i+1} c_i - i\Delta(t) c_i c_{i+1} - \mu c^\dagger_i c_i + h.c \right)$$

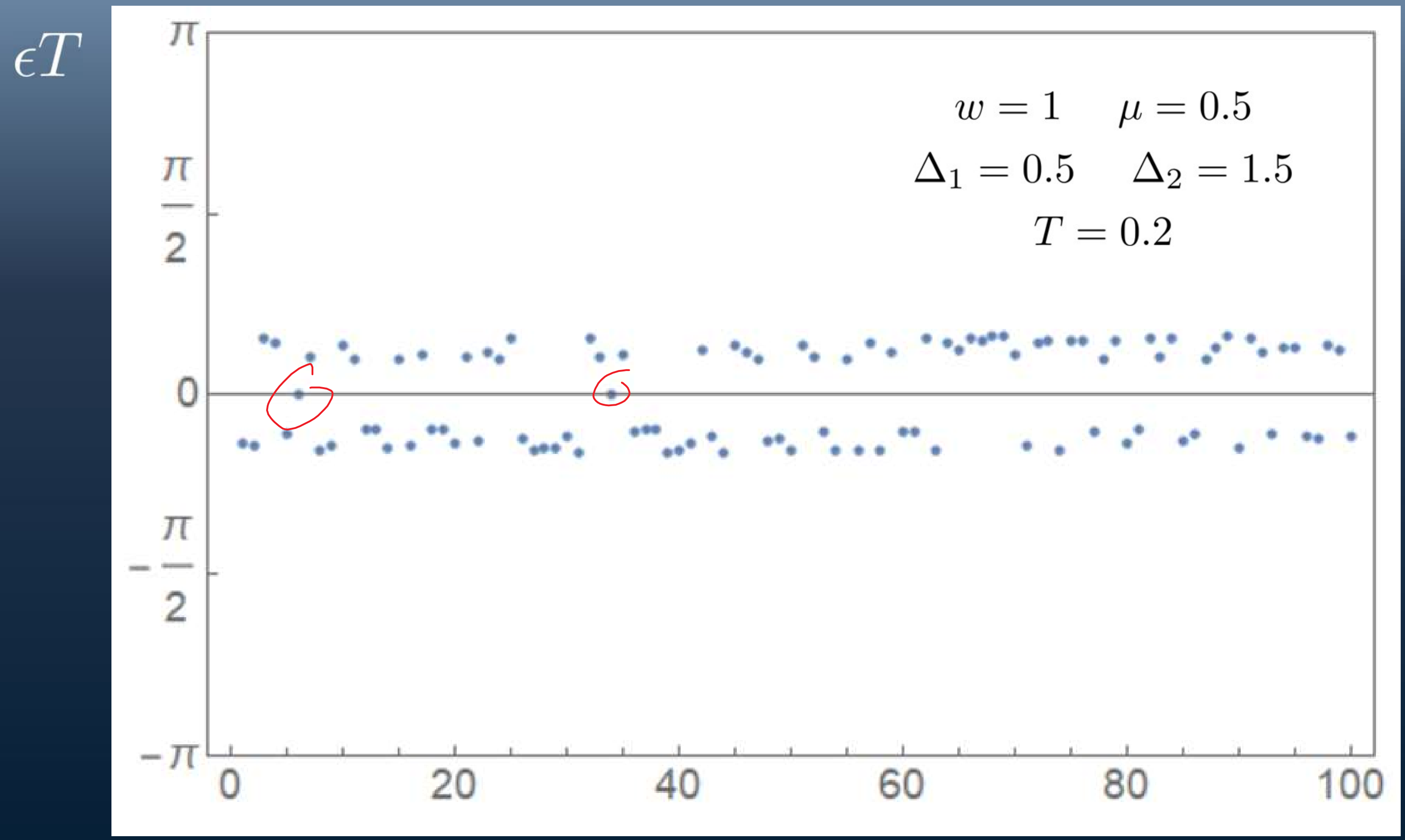
$$\Delta(t) = \begin{cases} \Delta_1 & t \in (nT, n + \frac{1}{2}T) \\ \Delta_2 & t \in (n + \frac{1}{2}T, nT) \end{cases}$$

Floquet Topological Superconductor

ϵT

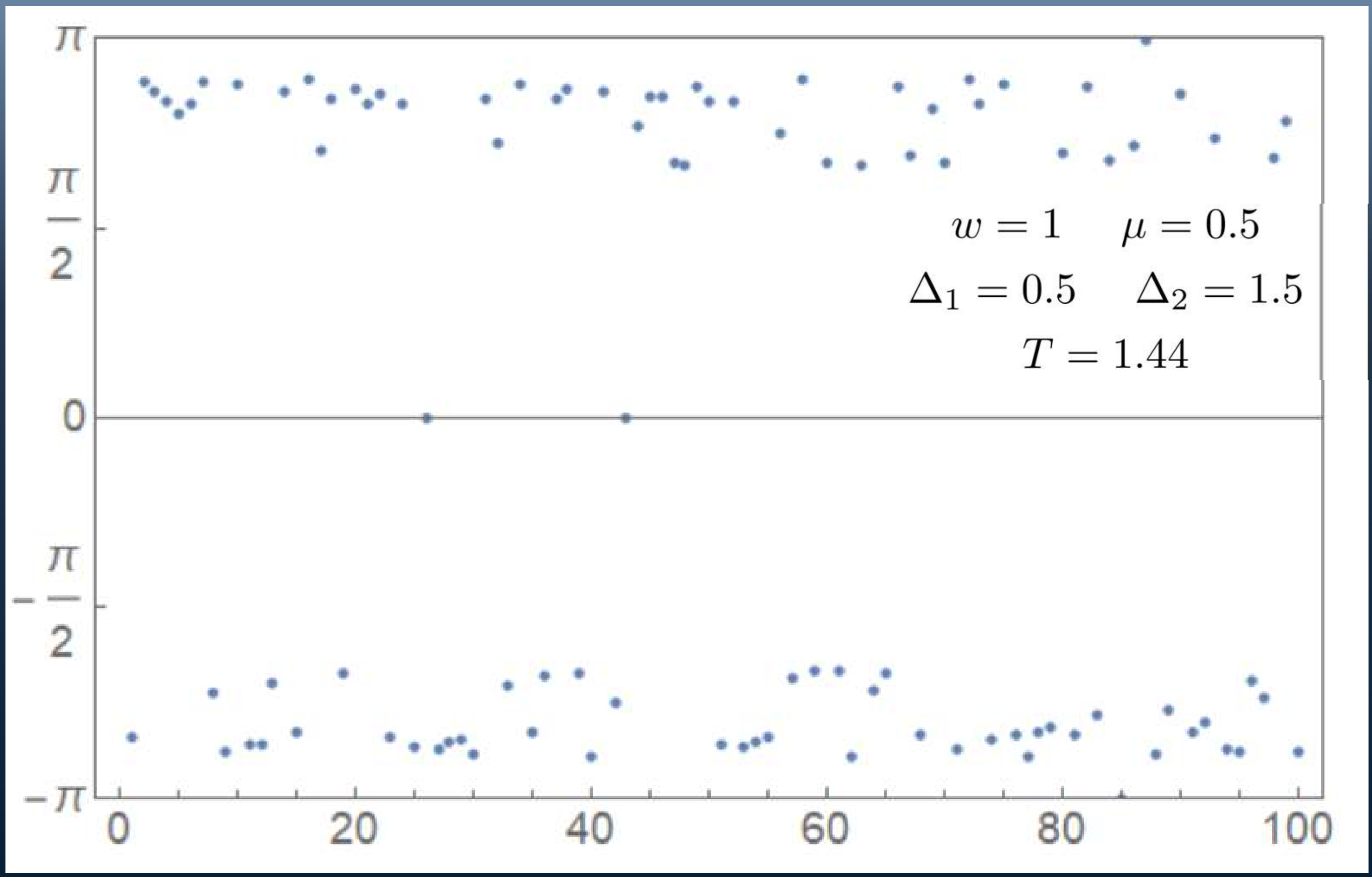


Floquet Topological Superconductor



Floquet Topological Superconductor

ϵT



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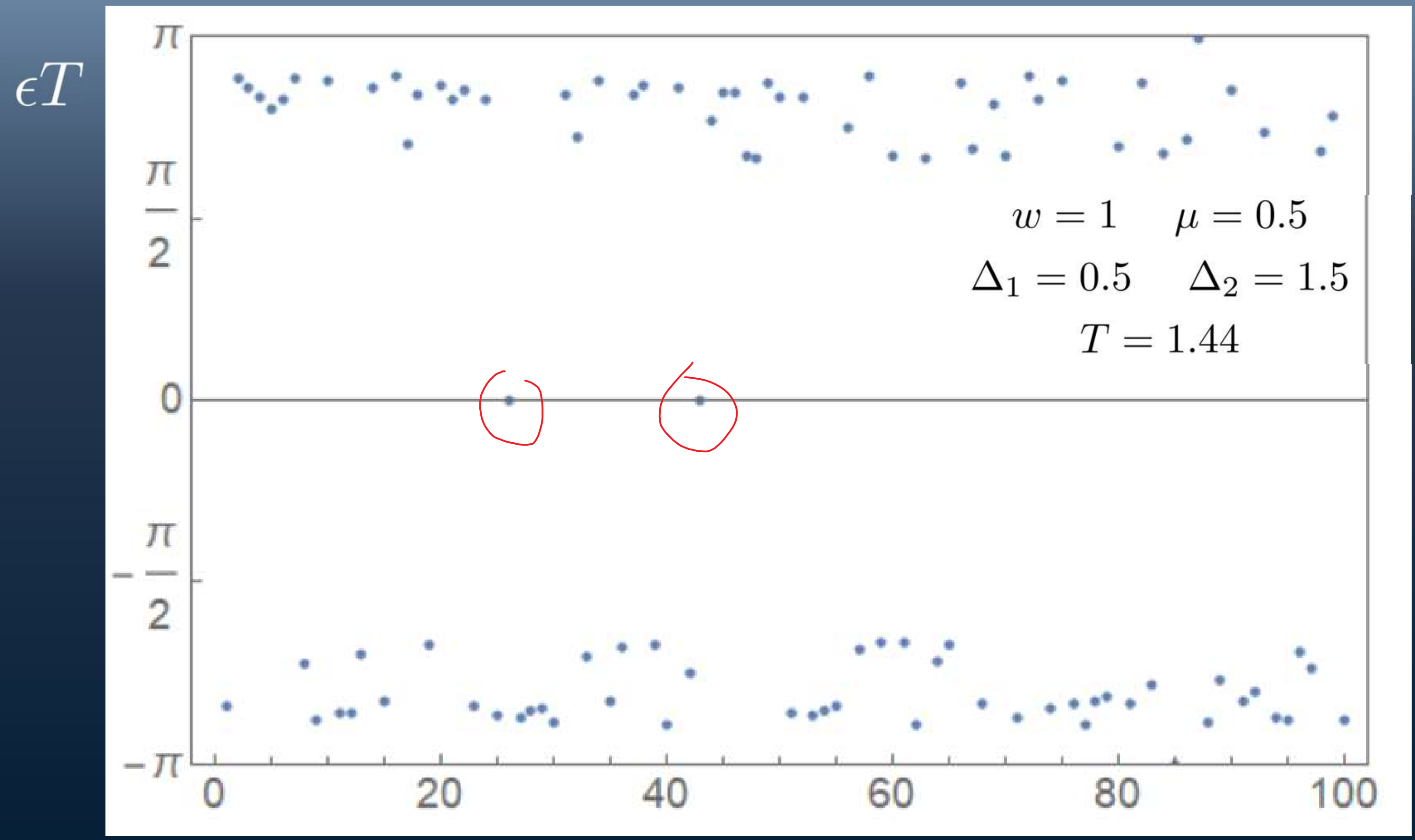

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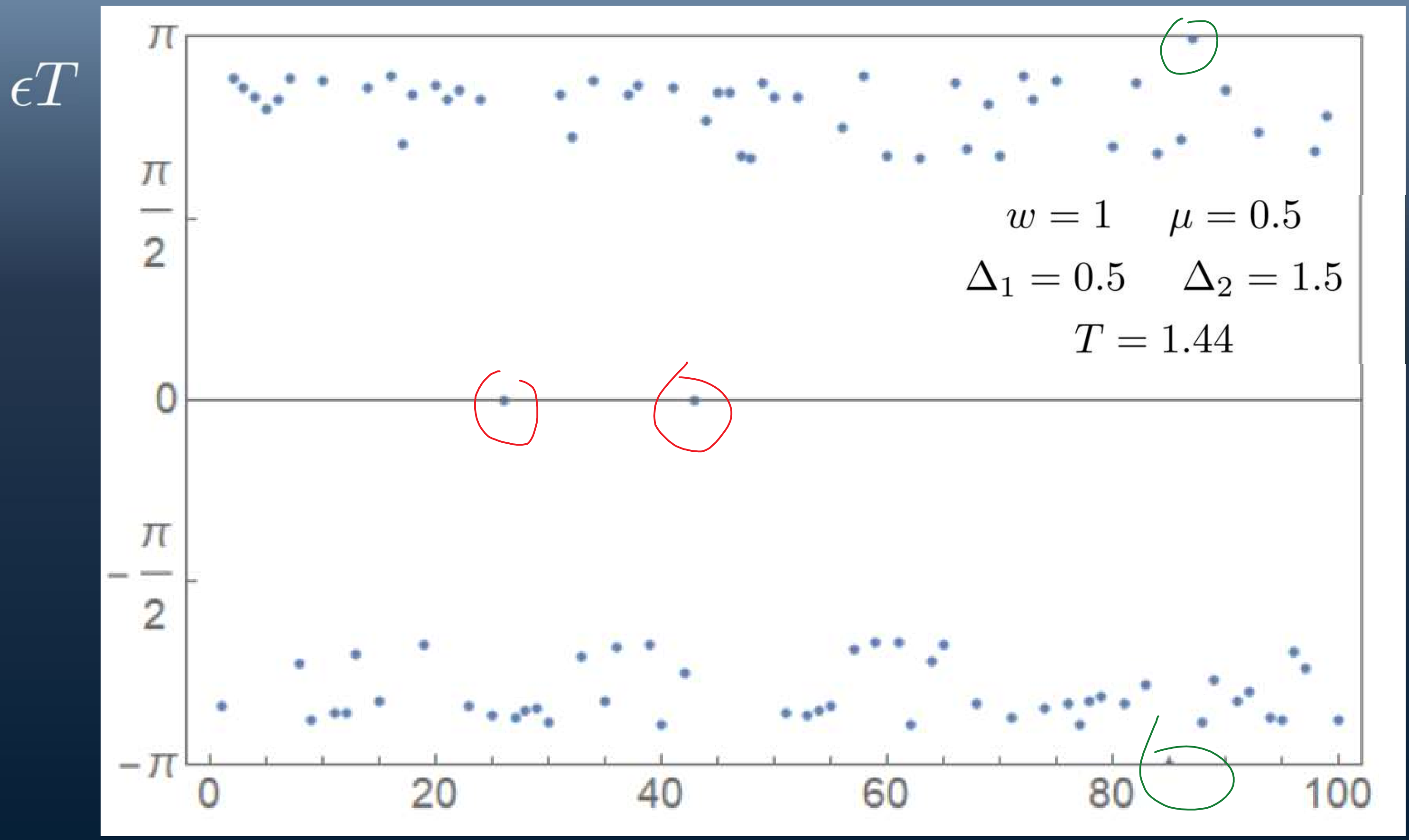
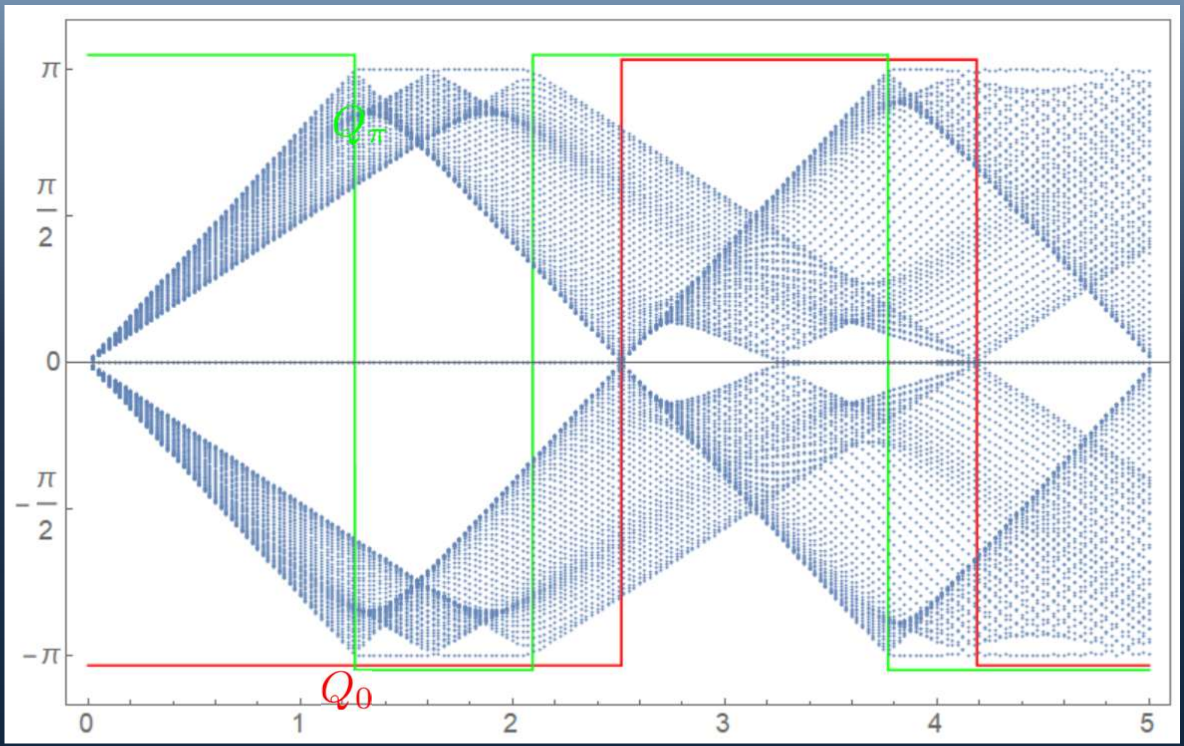
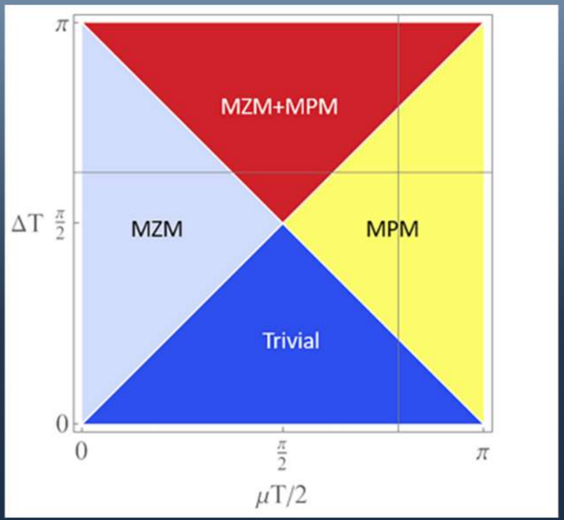


Floquet Topological Superconductor

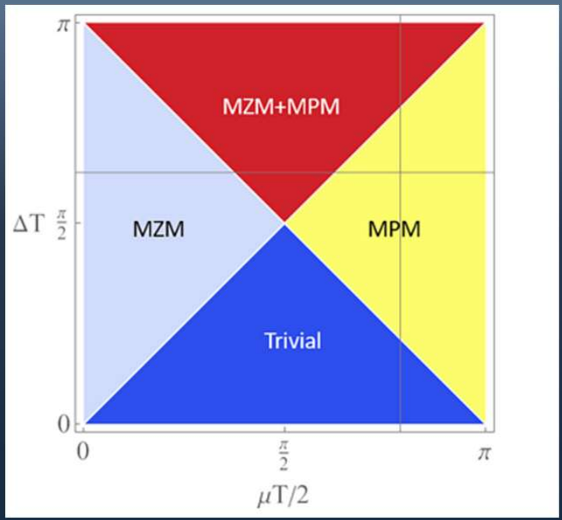


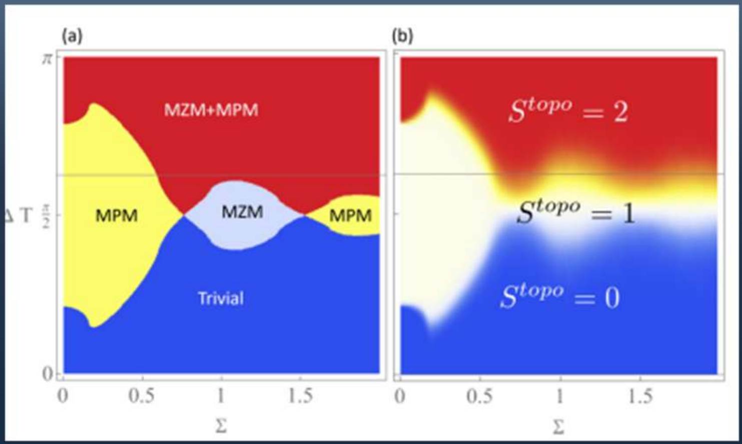
T



Jiang *et al.* PRL (2011)
Kundu and Seradjeh, PRL (2014)

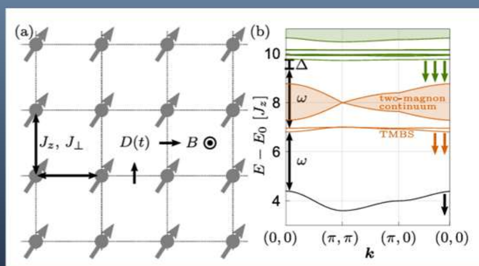
Four possible phases



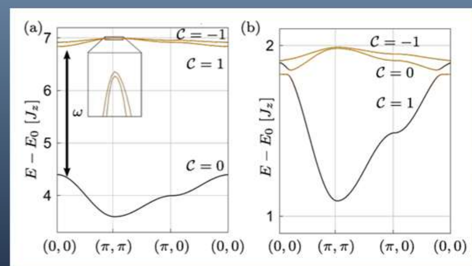


Disorder-induced topological phase transition in a driven Majorana chain
Ling,..., TPB, Phys. Rev. B **109**, 155144 (2024)

Topological magnons



Static DMI Driven



Single-magnon state

$$|\boldsymbol{k}\rangle = \frac{1}{\sqrt{N}} \sum_{\boldsymbol{r}} e^{i\boldsymbol{k}\cdot\boldsymbol{r}} |\boldsymbol{r}\rangle , \quad |\boldsymbol{r}\rangle \equiv S^-_{\boldsymbol{r}} |0\rangle$$

Two-magnon state

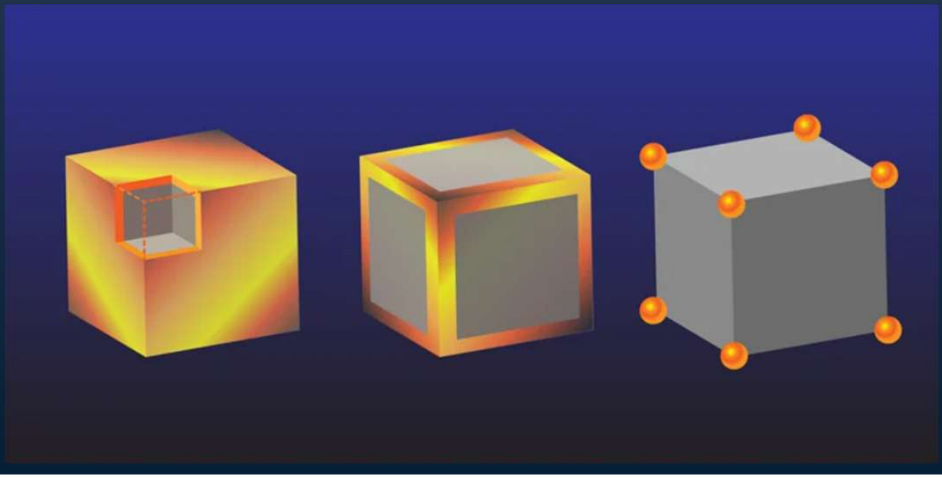
$$|\boldsymbol{k}, \boldsymbol{r}\rangle = \frac{1}{\sqrt{N}} \sum_{\boldsymbol{r}'} e^{i\boldsymbol{k}\cdot(\boldsymbol{r}'+\boldsymbol{r}/2)} |\boldsymbol{r}', \boldsymbol{r}'+\boldsymbol{r}\rangle$$

$$|\boldsymbol{r}, \boldsymbol{r}'\rangle \equiv S^-_{\boldsymbol{r}} S^-_{\boldsymbol{r}'} |0\rangle$$

Tunable multi-magnon Floquet topological edge states
Ivan Martinez-Berumen, TPB and Bill Coish
arXiv:2508.20049

Higher Order Topological Insulator

- Topological insulators in *d* dimensions will have surface states in *d-1* dimensions
- Second order topological insulators have surface states in *d-2* dimensions
- You can have 3D, second order TI with hinge modes,
  2D second order TI with corner states
  3D third order TI with corner states



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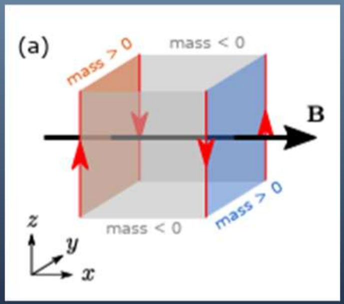

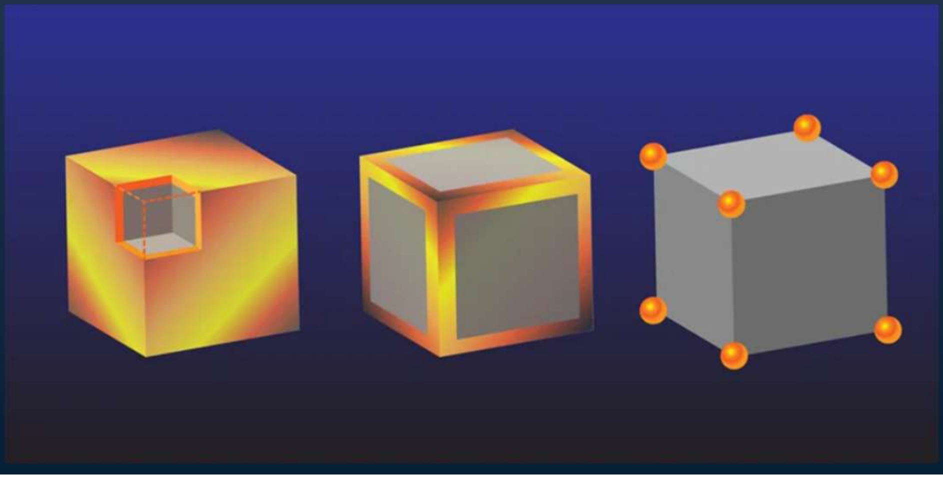

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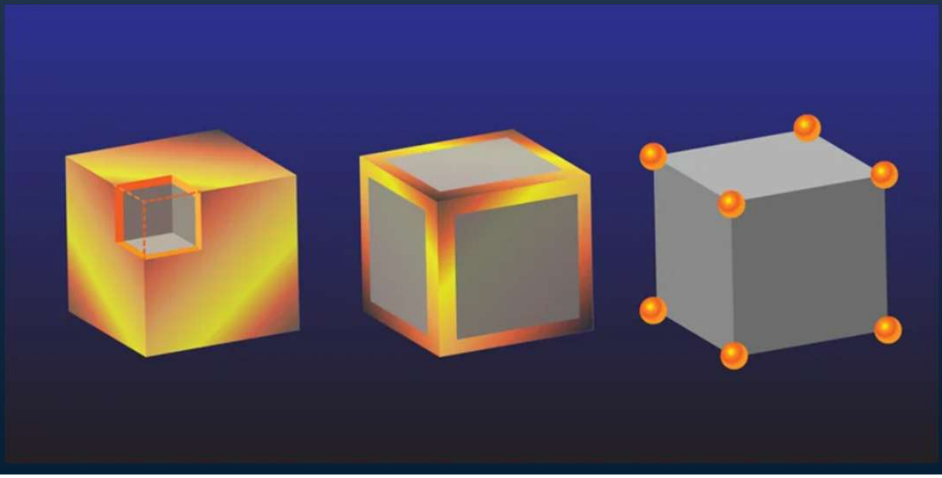
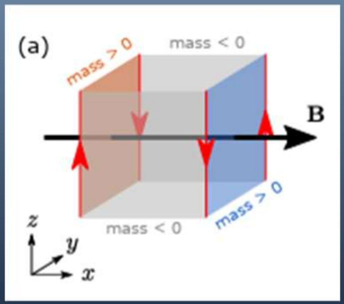
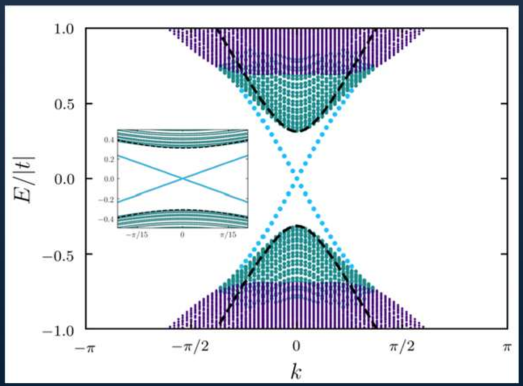
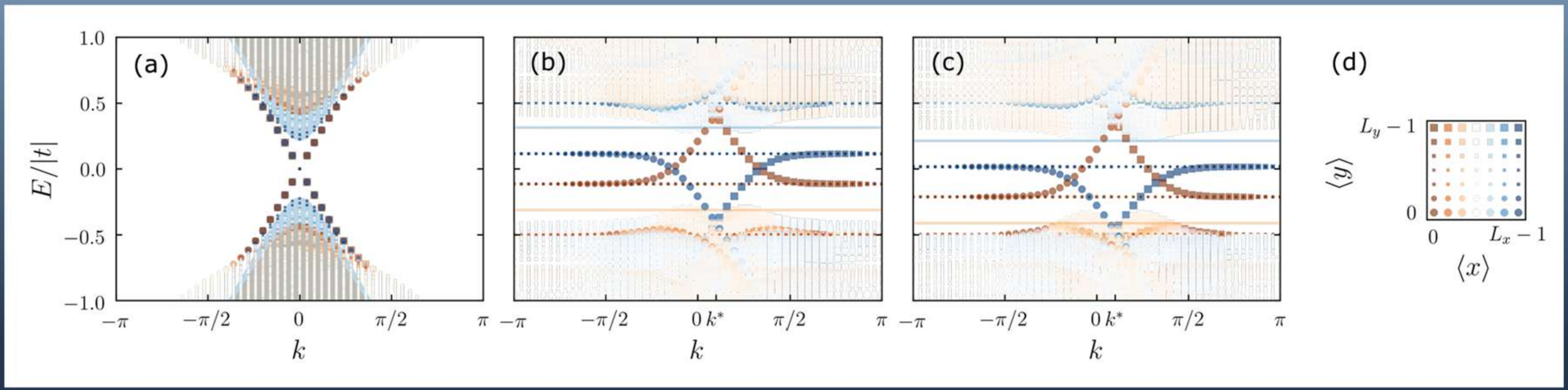
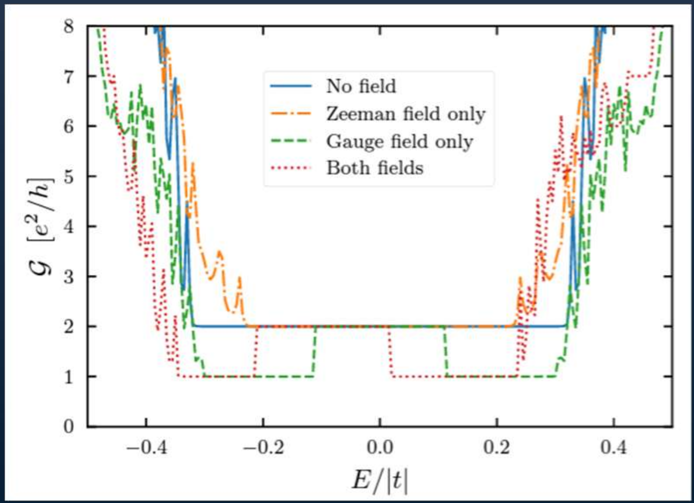






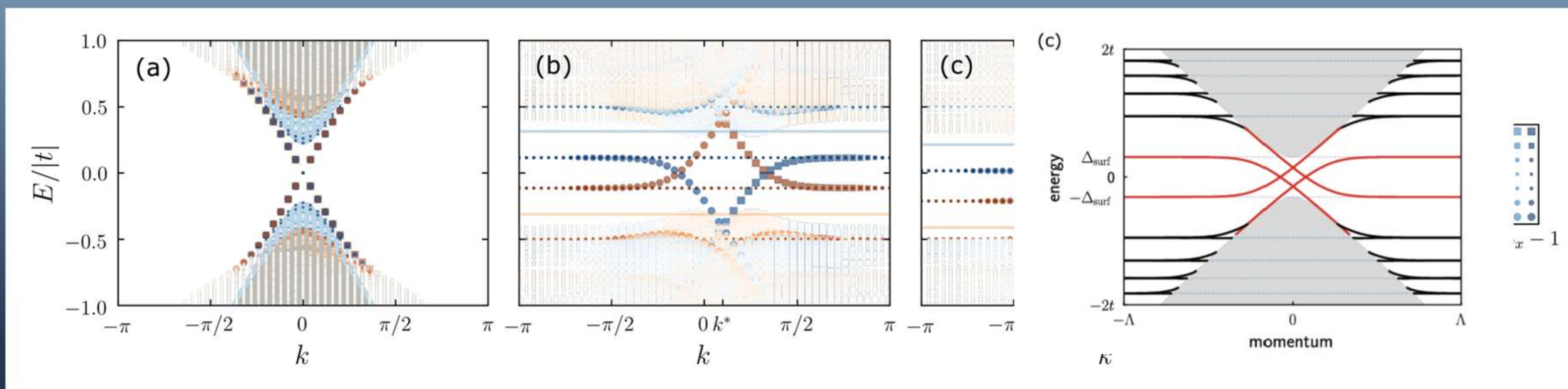
3D SOTI in magnetic field

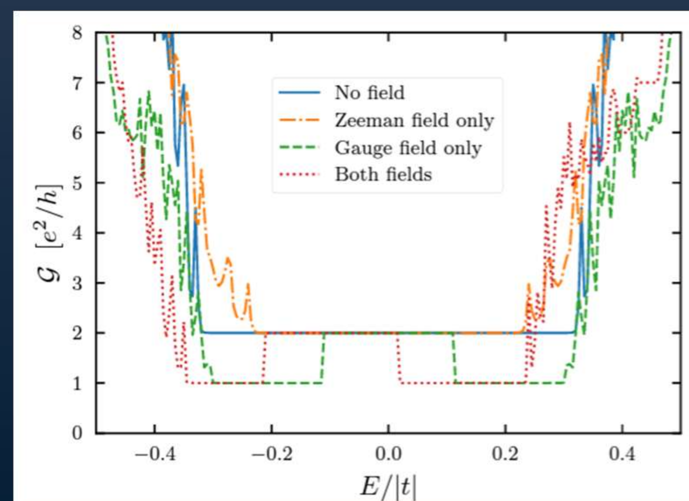




Levitan and TPB,
PRR **2**, 033327 (2020)
Levitan, Goutte and TPB,
PRB **104**, 125105 (2021)

3D SOTI in magnetic field





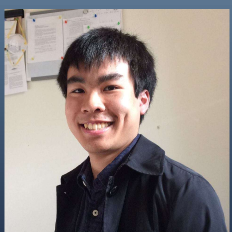
Levitan and TPB,
PRR **2**, 033327 (2020)
Levitan, Goutte and TPB,
PRB **104**, 125105 (2021)

Outlook

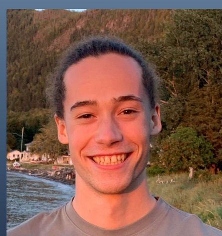
- Add interactions get more interesting phenomena (fractional (spin) quantum Hall, inter valley coherence in bilayer graphene etc)
- Add disorder – test robustness, find surprises (Anderson topological states)
- Design new materials and devices – proximity effect, charge transfer, twistronics
- Many analogue systems (sound waves, optical lattices, resonator arrays, magnons)

Thanks

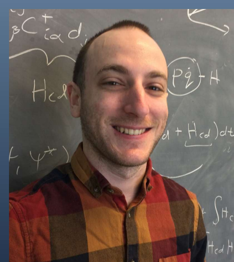
Bill Truong



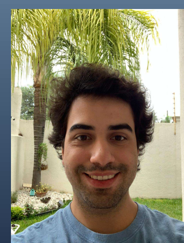
Clement Fortin



Ben Levitan



Ivan Martinez Berumen

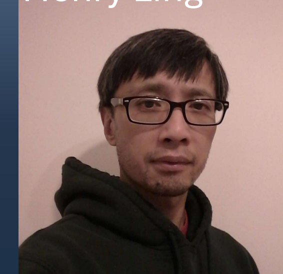


Bill Coish

Kartiek Agarwal

Dganit Meidan

Henry Ling



Aditi Mitra