Emergent cosmology from matrix theory

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Our work

- **S. Brahma, R. Brandenberger and S. Laliberte, "Emergent cosmology from matrix theory," JHEP 03, 067 (2022) doi:10.1007/JHEP03(2022)067 [arXiv:2107.11512 [hep-th]].**

- S. Brahma, R. Brandenberger and S. Laliberte, "Emergent metric space-time from matrix theory," JHEP **09**, 031 (2022) doi:10.1007/JHEP09(2022)031 [arXiv:2206.12468 [hep-th]].

- **S. Brahma, R. Brandenberger and S. Laliberte, "Spontaneous symmetry breaking in the BFSS model: analytical results using the Gaussian expansion method," Eur. Phys. J. C 83, no.10, 904 (2023) doi:10.1140/epjc/s10052-023-12082-w [arXiv:2209.01255 [hep-th]].**

- S. Laliberte and S. Brahma, "IKKT thermodynamics and early universe cosmology," JHEP **11**, 161 (2023) doi:10.1007/JHEP11(2023)161 [arXiv:2304.10509 [hep-th]].

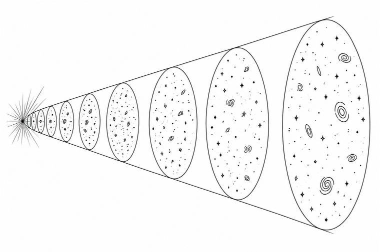
- S. Laliberte, "Effective mass and symmetry breaking in the Ishibashi-Kawai-Kitazawa-Tsuchiya matrix model from compactification," Phys. Rev. D **110**, no.2, 026024 (2024) doi:10.1103/PhysRevD.110.026024 [arXiv:2401.16401 [hep-th]].

Context

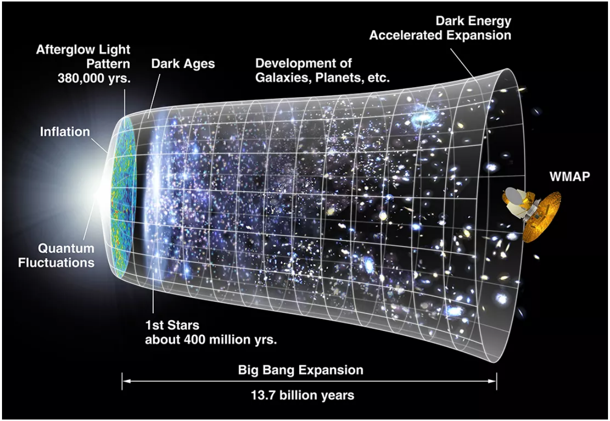
Goal: Find a theory that can describe the early universe self-consistently.

- This theory should go beyond perturbative Einstein gravity.
- It should be consistent with observations.
- It should also explain why we live in a four dimensional universe.

Cosmology



Our universe



Source: https://science.nasa.gov/mission/wmap/wmap-overview/

Cosmic microwave background

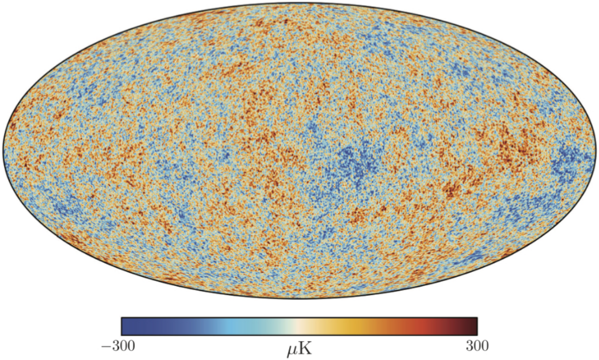


Figure: Planck measurement of the temperature variations in the CMB.

Source: [Planck Collaboration 2015]

Temperature Anisotropies

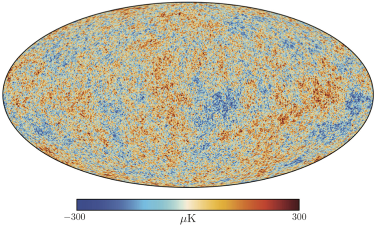


Figure: Planck measurement of the temperature variations in the CMB.

Harmonic expansion

$$\Theta(\hat{n}) = \frac{\Delta T(\hat{n})}{T_0} = \sum_{lm} a_{lm} Y_{lm}(\hat{n})$$

Angular power spectrum

$$C_l^{TT} = \frac{1}{2l+1} \sum_m \langle a^*_{lm} a_{lm} \rangle$$

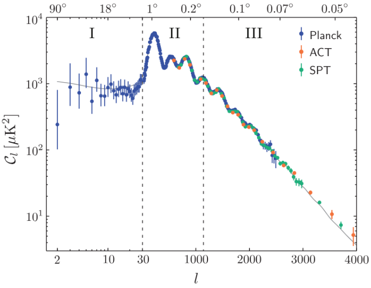


Figure: Angular spectrum of CMB temperature fluctuations.

Features:

- **Region I :** Fluctuations are frozen and scale invariant.
- **Region II :** Oscillations of matter at recombination.
- **Region III :** Fluctuations are damped (wavelengths smaller than mean free path of the photons).

Inflation

Original work: [Guth 80]

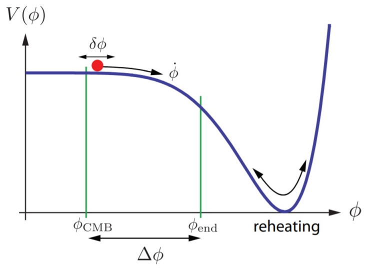


Figure: Example of inflaton potential.

Single field slow-roll action

$$S = \frac{1}{2}\int dx^4 \sqrt{-g}\left[R - (\partial_\mu \phi)^2 - 2V(\phi)\right]$$

Equation of state parameter

$$\omega_\phi \equiv \frac{p_\phi}{\rho_\phi} = \frac{\dot{\phi}^2 - 2V}{\dot{\phi}^2 + 2V} \approx -1$$

Slow roll conditions

$$\dot{\phi}^2 \ll V(\phi)$$

$$|\ddot{\phi}| \ll |3H\dot{\phi}|, |V, \phi|$$

Inflation and structure formation

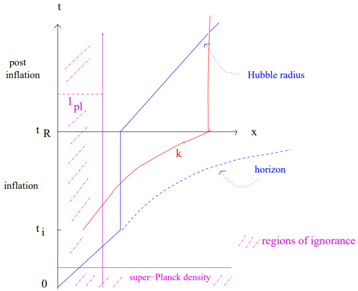


Figure: Propagation of modes during inflation.

Second order expansion of the action (comoving gauge)

$$S = \frac{1}{2}\int dx^4 a^3 \frac{\dot{\phi}^2}{H^2}\left[\dot{\mathcal{R}}^2 - a^{-2}(\partial_i \mathcal{R})^2\right]$$

$$\delta\phi = 0 \quad , \quad g_{ij} = a^2[(1-2\mathcal{R})\delta_{ij}]$$

Relation to the power spectrum

$$\langle \mathcal{R}_k \mathcal{R}_{k'} \rangle = (2\pi)^3 \delta(k+k') P_{\mathcal{R}}(k)$$

Inflationary power spectrum

$$\Delta^2_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2} P_{\mathcal{R}}(k) = \frac{H_*^2}{(2\pi)^2}\frac{H_*^2}{\dot{\phi}_*^2}$$

Scale invariant!

Problems with Inflation

1. **Difficult to realize in string theory:**

   - **Refined de Sitter conjecture [Obied/Ooguri/Spodyneiko/Vafa 18]:** The scalar potential of a theory coupled to gravity must satisfy one of the conditions

     $$|\nabla V| \geq cV \quad , \quad \min(\nabla_i \nabla_j V) \leq -c' V$$

     where $c, c' > 0$ are of order one, to be consistent with string theory.

   - **Trans-Planckian Censorship Conjecture [Bedroya/Vafa 20]:** Sub-Planckian quantum fluctuations should remain quantum.

     $$T \leq H^{-1} \ln(H^{-1})$$

     This imposes a bound on the duration of inflation (see above).

2. **Dimensionality of space is imposed, not explained.**

String Gas Cosmology

String Gas Cosmology

Idea [Brandenberger/Vafa 88]: Describe the universe using a gas of strings living on a space which is fully compactified on T^9.

String mass spectrum:

$$M = \underbrace{\left(\frac{n}{R}\right)^2}_{\text{KK modes}} + \underbrace{\left(\frac{mR}{\alpha'}\right)^2}_{\text{Winding modes}} + \underbrace{\frac{2}{\alpha'}\left(N + \tilde{N} - 2\right)}_{\text{Oscillatory modes}}$$

Important property (T-duality):

$$R \longleftrightarrow \frac{\alpha'}{R} \quad , \quad n \longleftrightarrow m$$

String thermodynamics

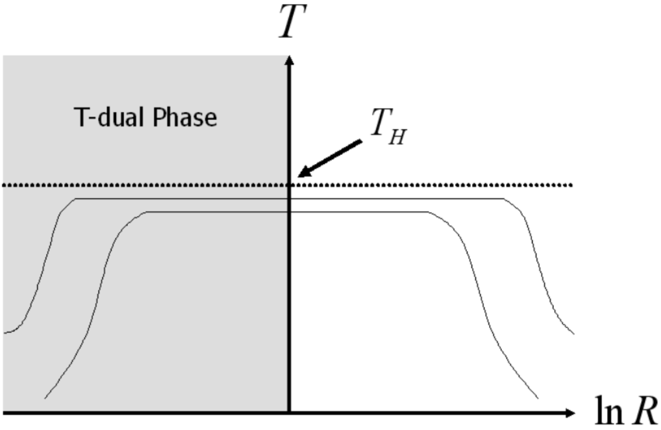


Figure: The temperature (vertical axis) as a function of radius (horizontal axis) of a gas of closed strings in thermal equilibrium.

Emergence of three spatial dimensions

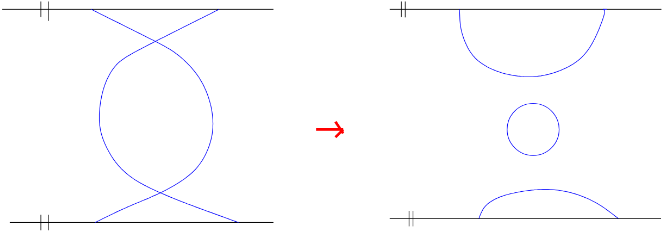


Figure: The process by which string loops are produced via the intersection of winding strings. The top and bottom lines are identified and the space between these lines represents space with one toroidal dimension un-wrapped.

Background evolution in String Gas Cosmology

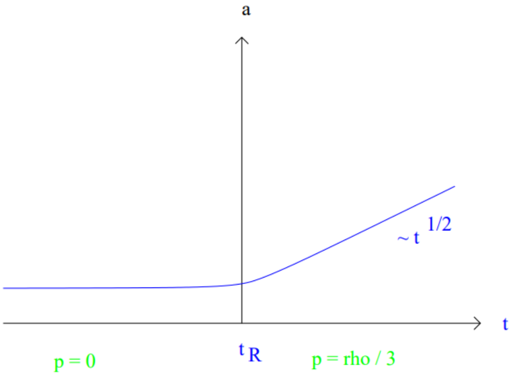


Figure: The dynamics of string gas cosmology. The vertical axis represents the scale factor of the universe, the horizontal axis is time.

Mechanism for structure formation

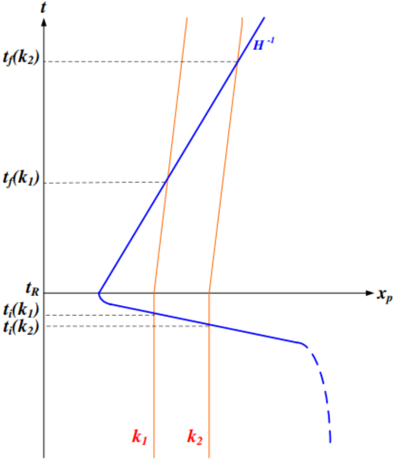


Figure: Mechanism for structure formation in String Gas Cosmology.

Extracting metric fluctuations

Ansatz for the metric including cosmological perturbations and gravitational waves:

$$ds^2 = (1 + 2\Phi)dt^2 - a(t)^2 \left((1 - 2\Phi)\delta_{ij} + h_{ij}\right) dx^i dx^j$$

Inserting into the perturbed Einstein equations yields

$$\langle|\Phi^2(k)|^2\rangle = 16\pi^2 G^2 k^{-4} \langle\delta T^0_0(k)\delta T^0_0(k)\rangle$$

$$\langle|h^2(k)|^2\rangle = 16\pi^2 G^2 k^{-4} \langle\delta T^i_j(k)\delta T^i_j(k)\rangle$$

Power spectrum of scalar perturbations

Results from [Nayeri/Brandenberger/Vafa 06]:

Density perturbations sourced by a thermal system in a square box of lenght R:

$$\langle \delta T^0_0(k) \delta T^0_0(k) \rangle = \langle \rho^2 \rangle = \frac{T^2}{R^6} C_V$$

Heat capacity of a string gas in a square box of lenght R:

$$C_V \approx 2 \frac{R^2 / l_s^3}{T(1 - T/T_H)}$$

Power spectrum of scalar perturbations:

$$P_\Phi \sim k^3 \langle |\Phi^2(k)|^2 \rangle = 8 G^2 \frac{T}{l_s^3} \frac{1}{1 - T/T_H}$$

Scale invariant!

Power spectrum of tensor perturbations

Results from [Brandenberger/Nayeri/Patil/Vafa 07]: Density perturbations sourced by a thermal system in a square box of lenght R:

$$\langle \delta T^i_j(k) \delta T^i_j(k) \rangle = \frac{T}{R^2} \frac{\partial p}{\partial R}$$

Thermal pressure of the system:

$$p = -\frac{1}{V} \frac{\partial F}{\partial \ln R}$$

Power spectrum of scalar perturbations:

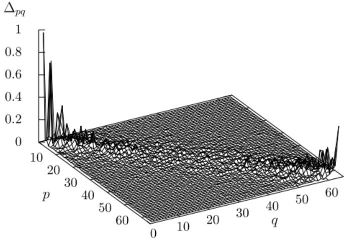
$$P_h \sim k^3 \langle |h^2(k)|^2 \rangle = 16\pi^2 G^2 \frac{T}{l_s^3} (1 - T/T_H)$$

Scale invariant!

One important problem of String Gas Cosmology:

- Lacks a complete and non-perturbation description in the Hagedorn phase.

Matrix Theory



BFSS matrix model

BFSS matrix model action **[Banks/Fischler/Shenker/Susskind 97]**:

$$S_{BFSS} = \frac{1}{2g^2} \int dt \mathrm{Tr} \left((D_t X^i)^2 - \frac{1}{2}[X^i, X^j]^2 + \bar{\psi}\Gamma^0 D_t \Psi - i\bar{\Psi}\Gamma^i [X_i, \Psi] \right) .$$

Why the BFSS matrix model?

- It is a conjectured to be a non-perturbative definition of M-theory, and therefore provides a interesting candidate framework from which spacetime itself can emerge.

Connection to IKKT matrix model

In the BFSS model, the zero modes of the theory are related to the IKKT matrix matrix model [Ishibashi/Kawai/Kitazawa/Tsuchiya 97]

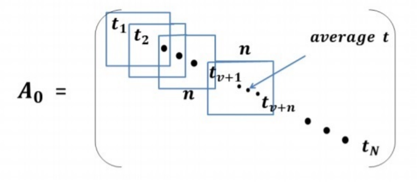
$$S_{BFSS} = \underbrace{S_0}_{Background} + \underbrace{S_{n\neq 0}}_{Fluctuations}$$

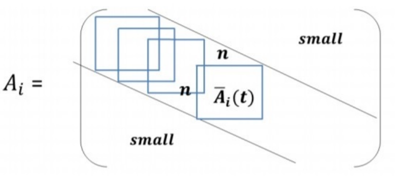
$$S_0 = S_{IKKT} = -\frac{1}{4g^2}\mathrm{Tr}[X^\mu, X^\nu]^2 - \frac{1}{2g^2}\mathrm{Tr}\Psi\Gamma^\mu[X_\mu, \Psi]$$

This model is known to have interesting emergent universe solutions.

Emergent universe scenario in the IKKT model

Work from [Kim/Nishimura/Tsuchiya 12]:





Time parameter

$$t \equiv \frac{1}{n}\sum_{a=1}^{n} t_{\nu+a}$$

Time evolving matrix element

$$\bar{A}_i^{ab}(t) \equiv \langle t_{\nu+a}|A_i|t_{\nu+b}\rangle$$

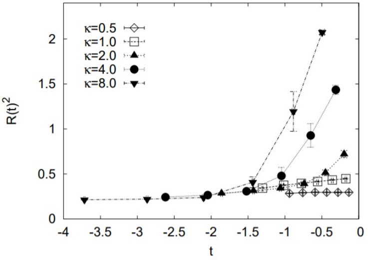


Figure: The extent of space $R(t)^2$ becomes large at a critical time t_c.

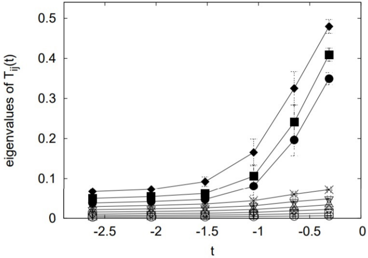


Figure: 3 out of 9 eigenvalues of T_{ij} become large at t_c.

Extent of space parameter

$$R(t)^2 \equiv \frac{1}{n}\mathrm{Tr}\bar{A}_i(t)^2$$

Moment of inertia tensor

$$T_{ij}(t) \equiv \frac{1}{n}\mathrm{Tr}\{\bar{A}_i(t)\bar{A}_j(t)\}$$

Our scenario

Work from [Brahma/Brandenberger/Laliberte 22] :

1. Consider the BFSS matrix model at finite temperature as a description of space-time.

$$S_{BFSS} = \frac{1}{2g^2}\int_0^\beta d\tau \mathrm{Tr}\left((D_\tau X^i)^2 - \frac{1}{2}[X^i, X^j]^2 + \bar{\psi}\Gamma^0 D_\tau \Psi - i\bar{\Psi}\Gamma^i[X_i, \Psi]\right) .$$

2. Expand around the zero modes, treat the zero modes as a description of the emergent space, and the non-zero modes as fluctuations.

$$S_{BFSS} = \underbrace{S_0}_{Background} + \underbrace{S_{n\neq 0}}_{Fluctuations}$$

3. Since the zero modes are describe the IKKT model, we expect a (3+1)-dimensional background to emerge as a solution given the numerics presented in [Kim/Nishimura/Tsuchiya 12].

Question: Can the fluctuations give us a spectrum comparable with observations?

Spectrum of fluctuations from the BFSS model

We computed the spectrum of fluctuations exactly as in String Gas Cosmology, but from the thermal partition function of the BFSS model

$$Z = \int DXD\Psi e^{-\beta S_{BFSS}} .$$

Power spectrum of scalar perturbations:

$$P_\Phi = 16\pi^2 G^2 k^2 T^2 (kR)^{-6} C_V$$

Power spectrum of tensor perturbations:

$$P_h = 16\pi^2 G^2 k^{-4} \alpha \frac{T}{R^2} \frac{\partial \tilde{p}}{\partial R}$$

We found the following results:

Power spectrum of scalar perturbations:

$$P_\Phi(k) = 16\pi^2 (kR)^{-4} \left(\frac{1}{l_s m_{pl}}\right)^4 \left(\frac{3}{8}\right) \left(\frac{d-1}{12} - \frac{p}{8}\right) \left(\frac{(d-1)^2}{d}\left(1 - \frac{1}{N^2}\right) - 4\right)$$

Power spectrum of tensor perturbations:

$$P_h(k) = \alpha\, 16\pi^2 (kR)^{-4} \left(\frac{1}{l_s m_{pl}}\right)^4 \left(\frac{3}{8}\right) \left(\frac{d-1}{12} - \frac{p}{8}\right) \left(\frac{(d-1)^2}{d}\left(1 - \frac{1}{N^2}\right) - 4\right)$$

Both scale invariant!

Why is a three-dimensional space-time emergent?

Hypothesis: There is an energy argument that explains this.

Gaussian expansion method

Idea behind the method [Kabat/Lifschytz 00]:

1. Add and substract a quadratic symmetric breaking piece:

$$S = \underbrace{S_{matrix}}_{\text{Matrix model}} + \underbrace{S_0 - S_0}_{\text{Symmetry breaking piece}}$$

2. Expand around the added piece and treat the rest as a perturbation:

$$Z = \int DA^\mu e^{-S} = \int DA^\mu \left(e^{-(S_{matrix} - S_0)} \right) e^{-S_0}$$

3. Find the symmetry that minimises the free energy $F = -\ln Z$.

Gaussian expansion of the IKKT model

Work from [Nishimura/Sugino 02]:

Gaussian expansion ansatz:

$$S = \underbrace{S_{matrix}}_{\text{Matrix model}} + \underbrace{S_0 - S_0}_{\text{Symmetry breaking piece}}$$

Matrix model:

$$S_{matrix} = -\frac{N}{4}\mathrm{Tr}[X^\mu, X^\nu]^2 - \frac{i}{2}N\mathrm{Tr}\left(\psi_\alpha(\Gamma^\mu)_{\alpha\beta}[X_\mu, \psi_\beta]\right)$$

Symmetry breaking piece:

$$S_0 = \sum_{\mu=1}^{D}\frac{N}{v_\mu}\mathrm{Tr}\left(A_\mu A_\mu\right) + \frac{N}{2}\sum_{a=1}^{N^2-1}\psi_\alpha^a \mathcal{A}_{\alpha\beta}\psi_\beta^b \quad , \quad v_\mu > 0\,,$$

v_μ and $\mathcal{A}_{\alpha\beta} \equiv$ are symmetry breaking parameters

Symmetry breaking in the IKKT model

Self-consistency equations (first order):

$$\frac{\partial F}{\partial v_\mu} = 0 \implies \frac{-1}{v_\mu} + \frac{1}{4}\sum_{\mu\neq\nu} v_\nu - \frac{1}{4}\rho_\mu + \ldots = 0 \quad , \quad \rho_\mu = \frac{1}{4}\mathrm{Tr}\left[(\mathcal{A}^{-1}\Gamma^\mu)^2\right]$$

$$\frac{\partial F}{\partial \mathcal{A}_{\alpha\beta}} = 0 \implies -\frac{1}{2}\mathrm{Tr}(\mathcal{A}^{-1}\mathcal{B}_{\mu\nu\lambda}) + \frac{1}{8}\sum_{\mu} v_\mu \mathrm{Tr}\left((\mathcal{A}^{-1}\Gamma^\mu)^2\mathcal{A}^{-1}\mathcal{B}_{\mu\nu\lambda}\right) + \ldots = 0$$

Here, the vector ρ^μ breaks the SO(10) symmetry of the system. Solving these self-consistency equations at third order yields:

$$v_0 = v_1 = v_4 = v_3 \quad , \quad v_5 = \ldots = v_{10}$$

$SO(4) \times SO(6)$ is the preferred symmetry of this system!

Gaussian expansion of the BFSS model

Work from [Brahma/Brandenberger/Laliberte 23]:

Gaussian expansion ansatz:

$$S = \underbrace{S_{matrix}}_{\text{Matrix model}} + \underbrace{S_0 - S_0}_{\text{Symmetry breaking piece}}$$

Matrix model:

$$S_{BFSS} = \frac{1}{g^2} \int dt \mathrm{Tr} \left(\frac{1}{2} (D_t X_i)^2 - \frac{1}{4} [X_i, X_j]^2 - \frac{i}{2} \bar{\psi} D_t \psi - \frac{1}{2} \bar{\psi} \Gamma_i [X^i, \psi] \right)$$

Symmetry breaking piece:

$$S_0 = \sum_{l} \sum_{i=1}^{D} \frac{1}{2 v_{l,i}} \mathrm{Tr} \left(X^i_{-l} X^i_l \right) + \sum_{r} \sum_{\alpha\beta} \psi^{\alpha}_r \mathcal{A}_{\alpha\beta} \psi^{\beta}_{-r} \,,$$

v_μ and $\mathcal{A}_{\alpha\beta} \equiv$ are symmetry breaking parameters

Symmetry breaking in the BFSS model

Self-consistency equations (first order):

$$-\frac{1}{v_{l,i}} + \frac{1}{g^2}\left(\frac{2\pi l}{\beta}\right)^2 + \frac{2N}{g^2\beta}\sum_{j\neq i}\sum_{l} v_{l,j} - \frac{N}{g^2\beta}\mathrm{Tr}\left[(\mathcal{A}_l^{-1}\Gamma^i)^2\right] + \ldots = 0\,.$$

Here again, the contribution from the fermions breaks the SO(9) symmetry of the system!

However, we still need to solve the higher order self-consistency equations to show that SO(3) $\times$ SO(6) is the preferred symmetry of the system. (To be done in future work.)

Additional work

Emergent metric space-time from matrix theory [Brahma/Brandenberger/Laliberte 22] :

- We gave a proposal for how to define the space-time metric in the IKKT matrix model.

IKKT thermodynamics and early universe cosmology [Brahma/Laliberte 23] :

- One can realize the present scenario in the IKKT model itself without having to start from the BFSS model.
- In this case, a scale-invariant spectrum of cosmological perturbations is also recovered.

Effective mass and symmetry breaking in the Ishibashi-Kawai-Kitazawa-Tsuchiya matrix model from compactification [Laliberte 24] :

- We showed that symmetry breaking mass term can be recovered in the IKKT model by integrating out non-zero modes following SUSY breaking compactification
- The IKKT model with mass term is known to have emergent cosmological solution. The symmetry breaking mass term could lead to such solution.

Summary of results

To summarize: We have investigated matrix theory as a potential description of String Gas Cosmology.

Our inspiration: Hints of emergent three dimensional universe in numerical simulations.

Our findings:

- **Experimental predictions:** Matrix theory predicts a scale invariant spectrum of cosmological perturbations, just like String Gas Cosmology.
- **Symmetry breaking:** We have found evidence that symmetry breaking can occur in matrix theory, just like in String Gas Cosmology.

Open questions

- Why does tree dimensions become large in matrix cosmology?
- Can we find a better and more complete description of the background?

Recent developments

Symmetry breaking:

- Origin of the $SO(9) \to SO(3) \times SO(6)$ symmetry breaking in the type IIB matrix model [Brandenberger/Pasciecznik 23]

Description of the emergent space:

- Collective field theory of gauged multi-matrix models: integrating out off-diagonal strings [Brahma/Brandenberger/Dasgupta/Lei/Pasciecznik 24]
- Multi-Matrix Quantum Mechanics, Collective Fields and Emergent Space [Lei/Brahma/Brandenberger 26]