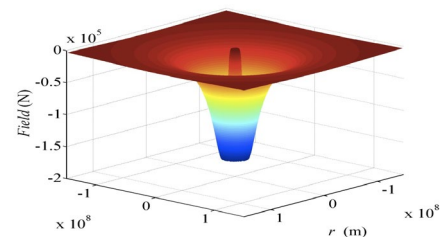


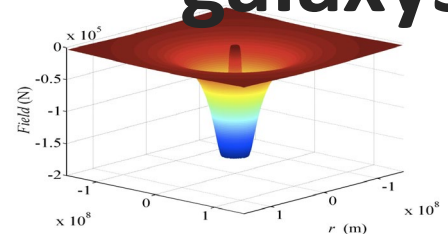
A SOLUTION TO THE HUBBLE TENSION PROBLEM?

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BIG MYSTERIES SURVEY

Among the ten foundational and controversial topics in contemporary physics, as reported in the large-scale survey conducted through the American Physical Society's *Physics Magazine* in 2024-2025, stands **the Hubble tension problem**, the difference between the measured values of the Hubble constant, based on gauging the distance to star and supernovae in the local universe using Doppler measurements or using measurements of more distant signals coming from the Cosmic Microwave Background (CMB) and large scale galaxy surveys.



What is the most likely explanation for this « Hubble tension »?

Hubble Tension

The current cosmic expansion rate, or Hubble parameter, can be measured in two distinct ways. One involves gauging the distance to stars and supernovae in the local universe—corresponding to “late times” in cosmic history. The second involves measurements of more distant signals coming from the CMB and large-scale galaxy surveys—corresponding to “early times.” These two methods give different results, with the statistical significance of this difference increasing as more data come in. In your opinion, what is the most likely explanation for this “Hubble tension”?

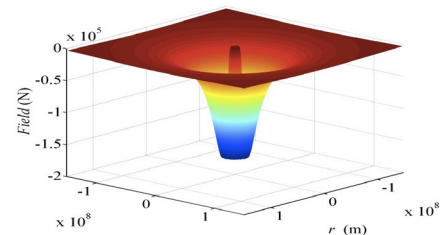
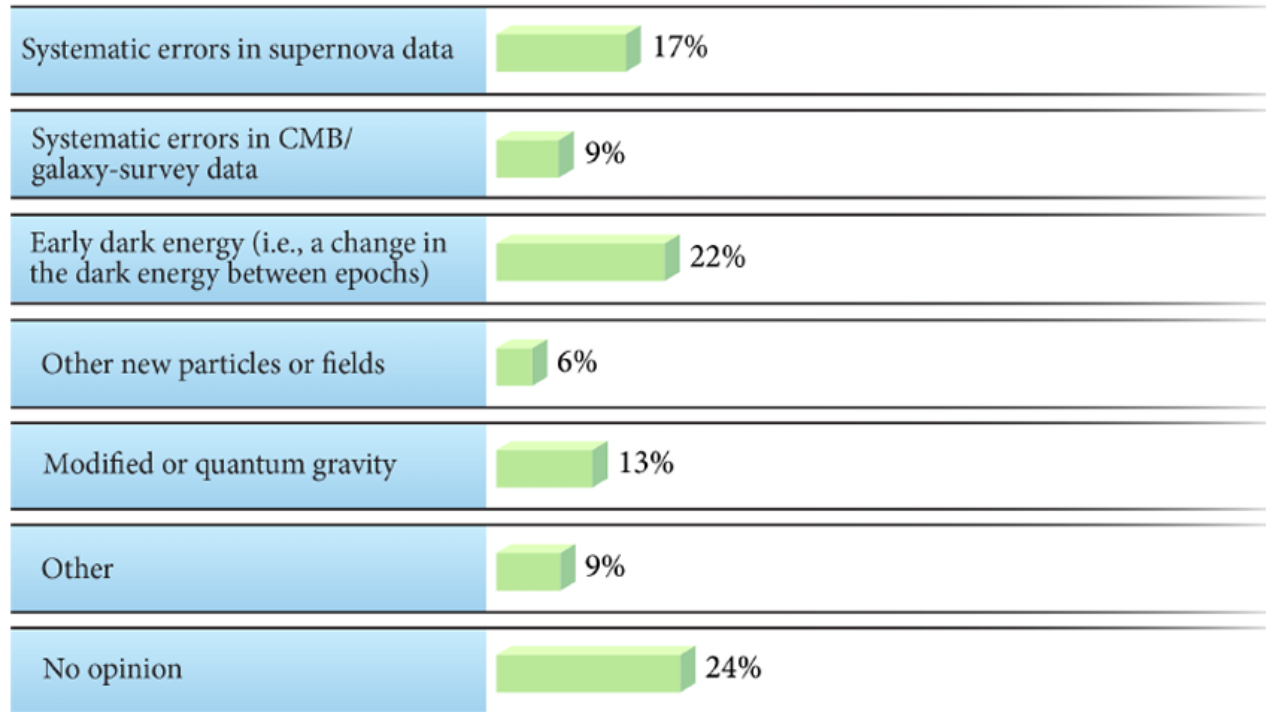
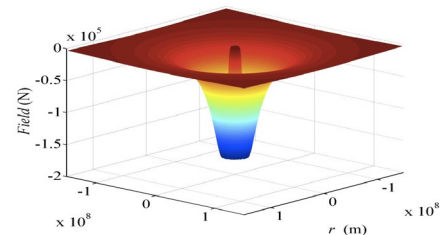


FIG. 6. Big Mysteries Survey (1,675 respondents): Most likely explanation for the Hubble tension.

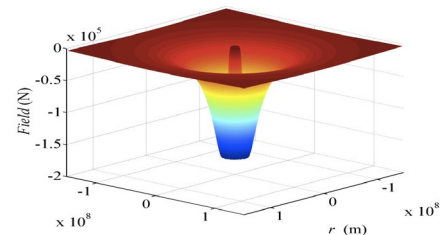
CONDITIONS FOR MODIFYING GRAVITY

If Newton's laws have to be modified, the resulting modified gravity shall be an **emergent, universal, scale independent paradigm directly embedded in Einstein's General Relativity theory**
(no adhoc plug-in)



ROAD MAP

- 1-Emergence of a Modified Newton's Law
- 2-The *erfc* Metrics
- 3-Space time Expansion
- 4-Cosmic Microwave Background
- 5- Concluding remarks



ROAD MAP

1-Emergence of a Modified Newton's Law

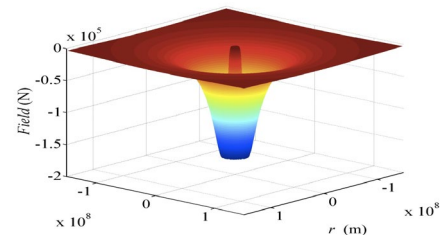
Plamondon, R., (2021) **What does the Central Limit Theorem Have to Say About General Relativity?**, in Quantum Theory and Symmetries, Proceedings of the 11th International Symposium, Montréal, Canada, , Paranjape, M.B., MacKenzie, R, Thomova, Z., Winternitz, P., WitczKrempa, W., (EDS), Springer, CRM Series in Mathematical Physics, 503-511.

2-The *erfc* Metrics

3-Space time Expansion

4-Cosmic Microwave Background

5- Concluding remarks

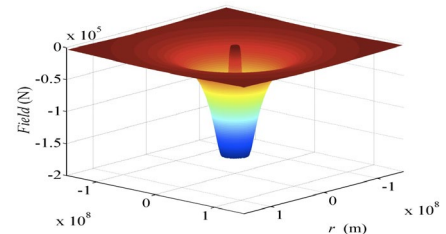


AN ANALOGY BETWEEN EINSTEIN'S LAW

$$G_{\mu\nu} = KT_{\mu\nu}$$

AND BAYES'S LAW

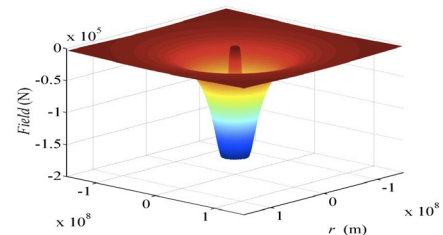
$$P(G_{\mu\nu} / T_{\mu\nu}) = P(T_{\mu\nu} / G_{\mu\nu}) \times \frac{P(G_{\mu\nu})}{P(T_{\mu\nu})}$$



FOR A WEAK FIELD, LOW SPEED, SYMMETRIC STATIC SYSTEM

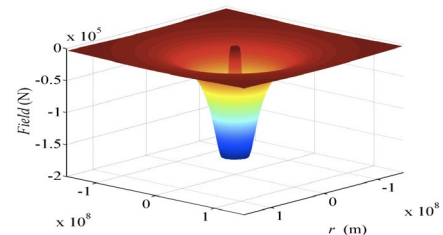
$$G_{00} \Leftrightarrow f(G_{00}/T_{00})$$

$$KT_{00} \Leftrightarrow \frac{f(G_{00})}{f(T_{00})} \times f(T_{00}/G_{00})$$



How can we take into account the
probability of presence
of the energy-momentum
at a given curvature?

$$f(T_{00} / G_{00}) ?$$



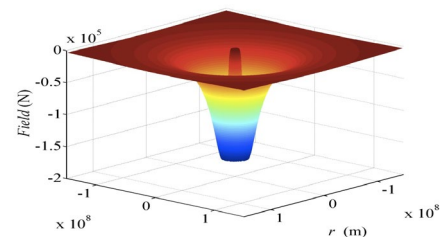
Modelling the slow process of a star formation
from the addition of chunks of energy-momentum
considered as random variables

(or of a galaxy from the addition of stars)

(or a universe from the addition of galaxies)

and **using the Central Limit Theorem**
projected in a curved spacetime, we obtain:

$$f(T_{00} / G_{00}) = \frac{1}{4\pi^2 \sigma^2 \hat{r}^2} \exp\left(-\frac{\sigma^2}{2r^2}\right)$$



AN EMERGENT PARAMETER

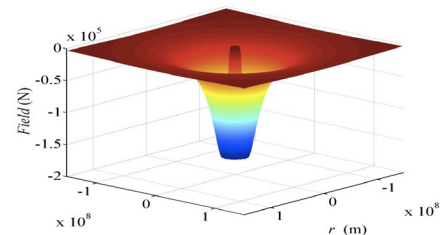
$$f(T_{00} / G_{00}) = \frac{1}{4\pi^2 \sigma^2 \hat{r}^2} \exp\left(-\frac{\sigma^2}{2r^2}\right)$$

$\sigma =$ Lorentz Scalar

An intrinsic and **emergent feature** of the central limit process.

The system reference proper length.

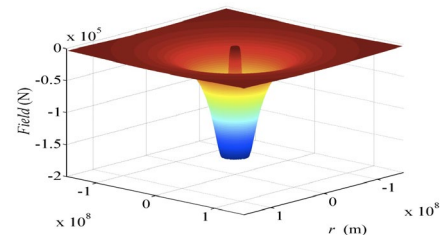
An intrinsic scale factor.



THE EMERGING LAPLACIAN

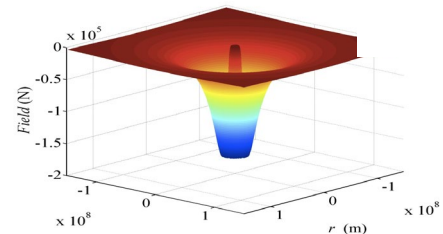
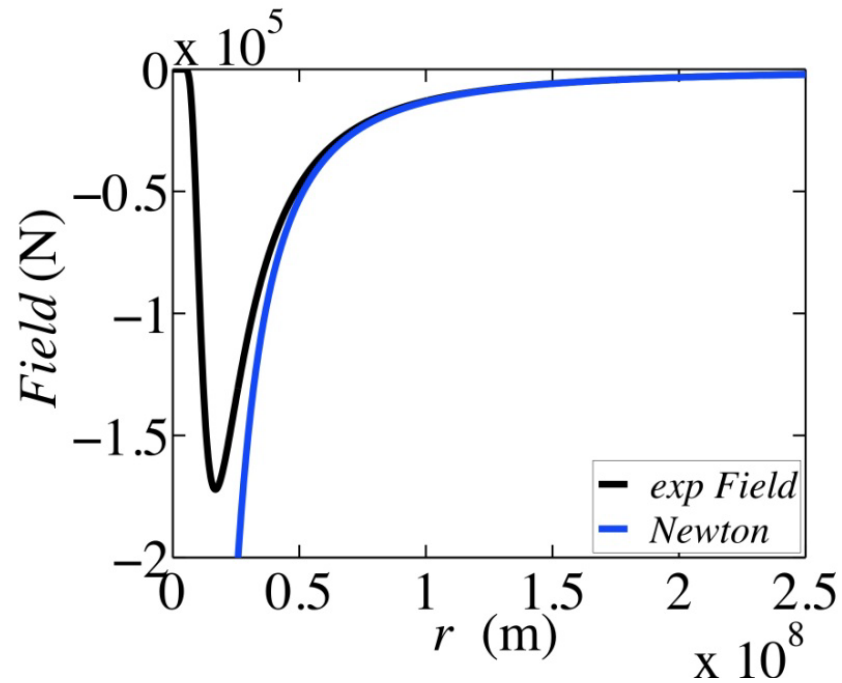
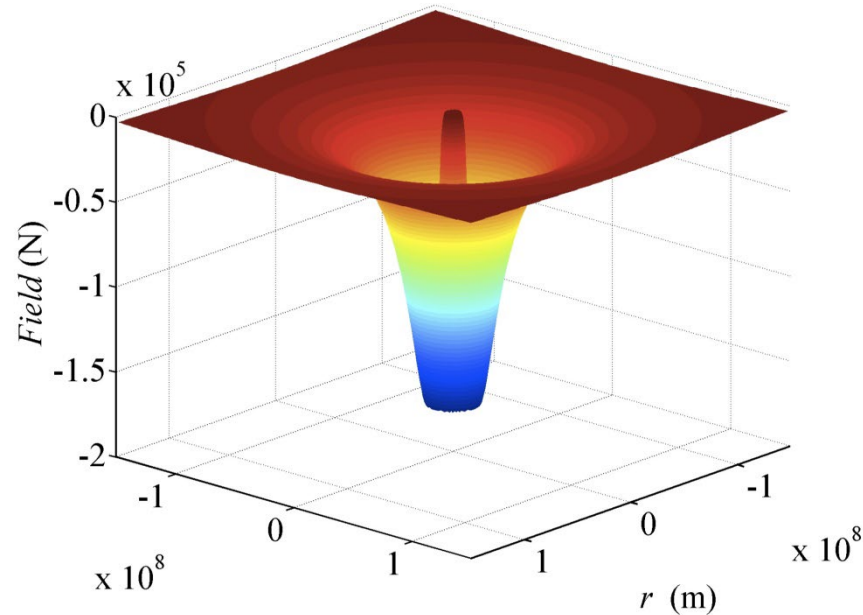
$$R_{00} \cong \frac{1}{c^2} \nabla^2 \Phi = \frac{1}{2} K T_{00} f(T_{00} / G_{00})$$

$$\nabla^2 \Phi = \frac{GM\sigma^2}{r^5} \exp\left(-\frac{\sigma^2}{2r^2}\right)$$



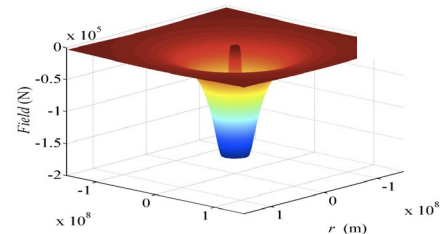
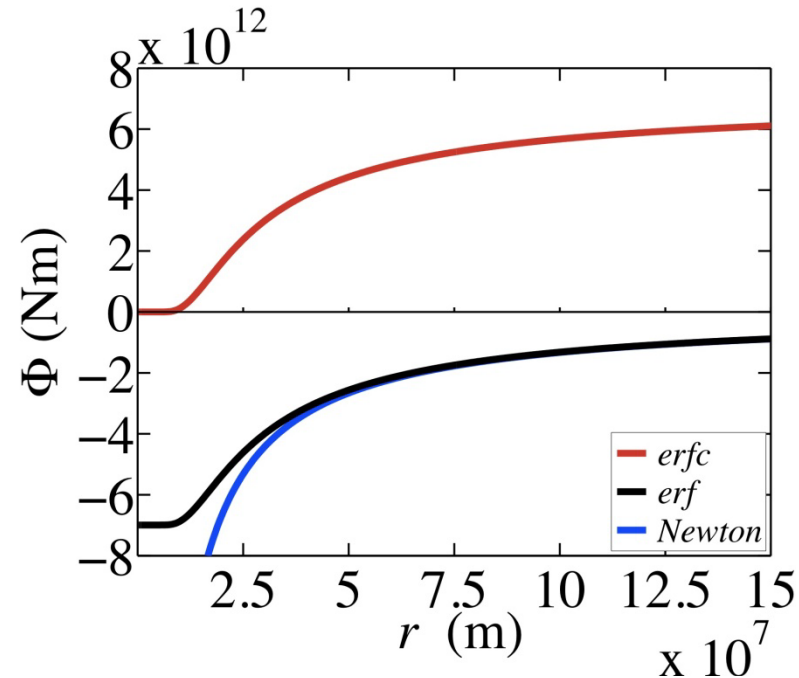
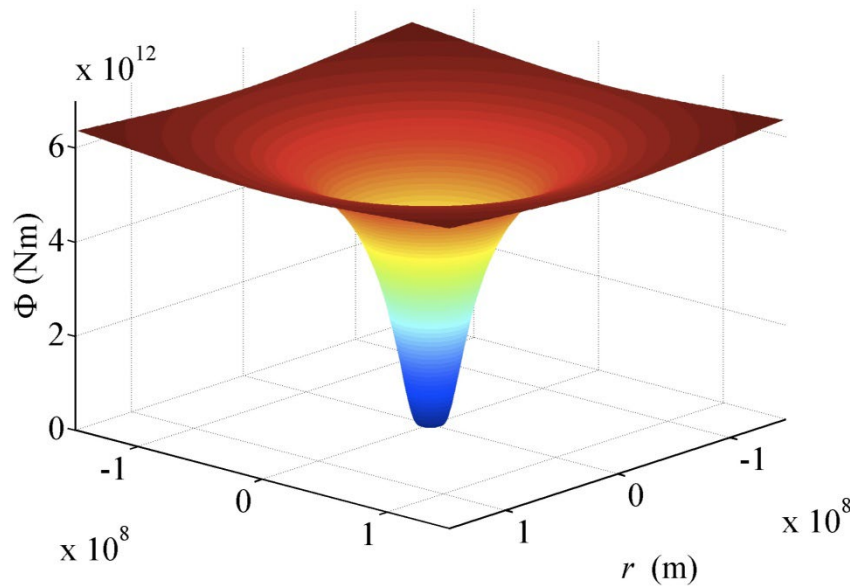
THE EMERGING FIELD

$$g(r) = -\left|\vec{\nabla}\Phi(r)\right| = -\frac{GM}{r^2} \exp\left(-\frac{\sigma^2}{2r^2}\right) \Rightarrow \approx \frac{GM}{r^2} \Big|_{r \rightarrow \infty}$$



THE EMERGING POTENTIAL

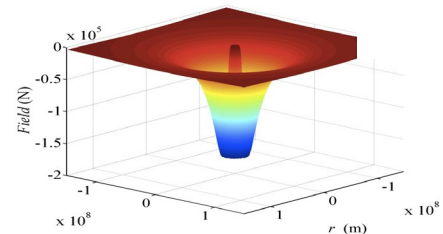
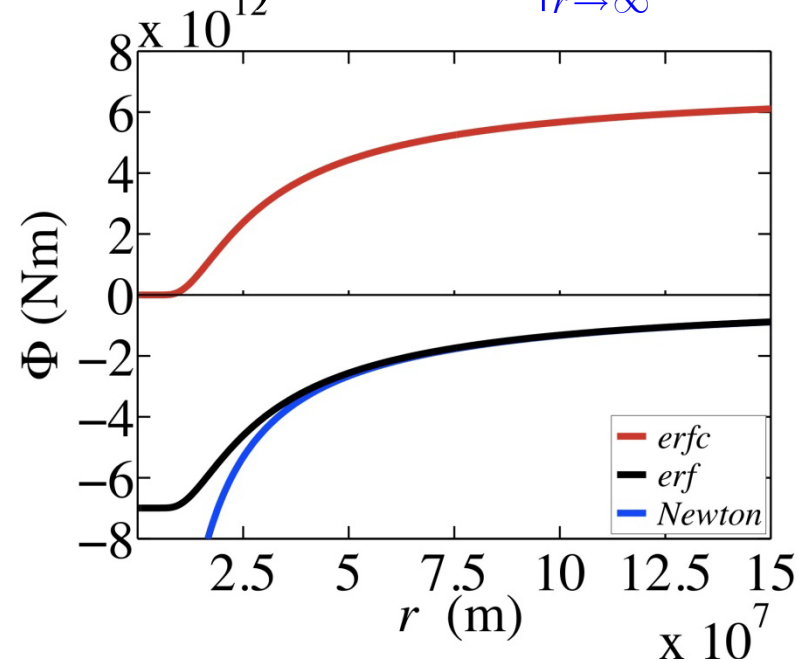
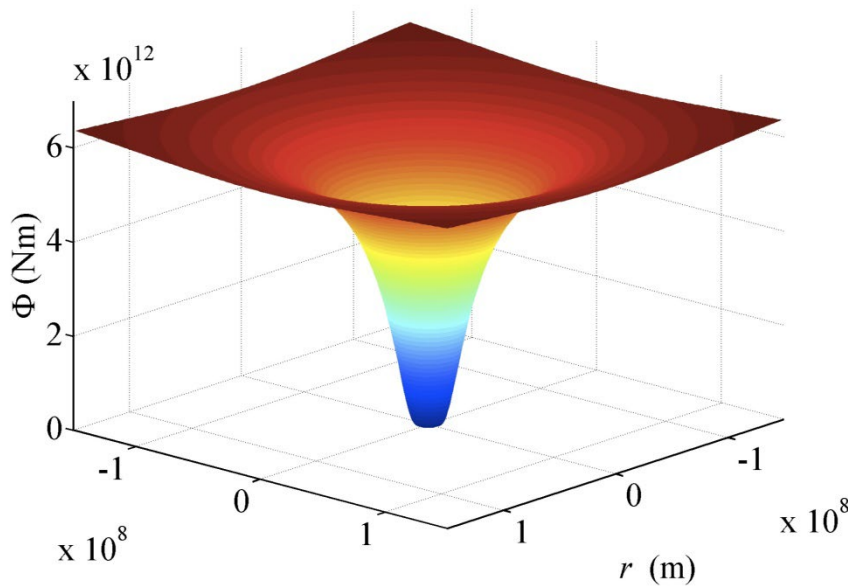
$$\Phi_{erfc}(r) = GM \left(\frac{\sqrt{\pi}}{\sqrt{2}\sigma} \right) \text{erfc} \left(\frac{\sigma}{\sqrt{2}r} \right)$$



THE EMERGING POTENTIAL

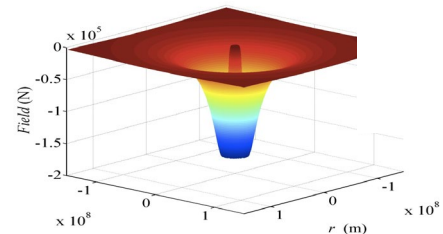
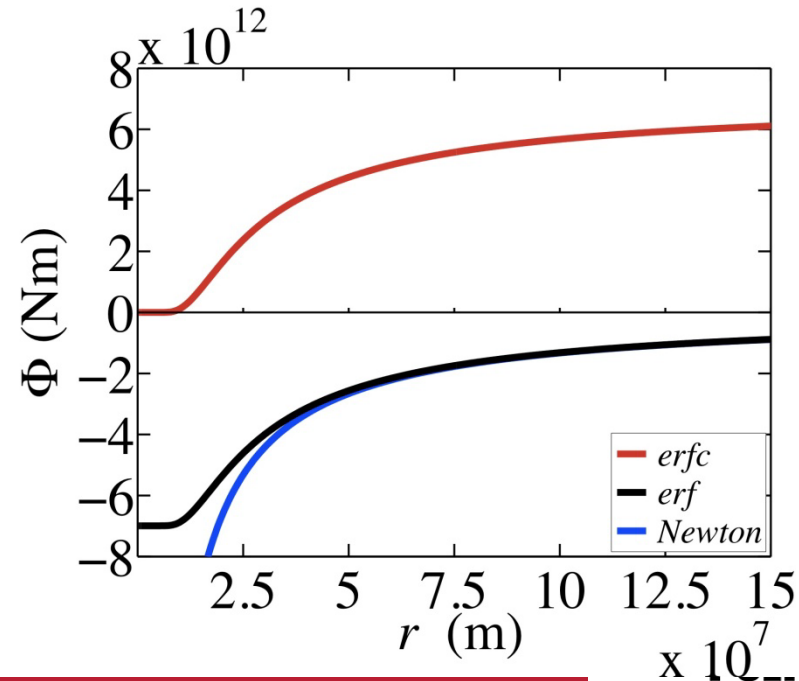
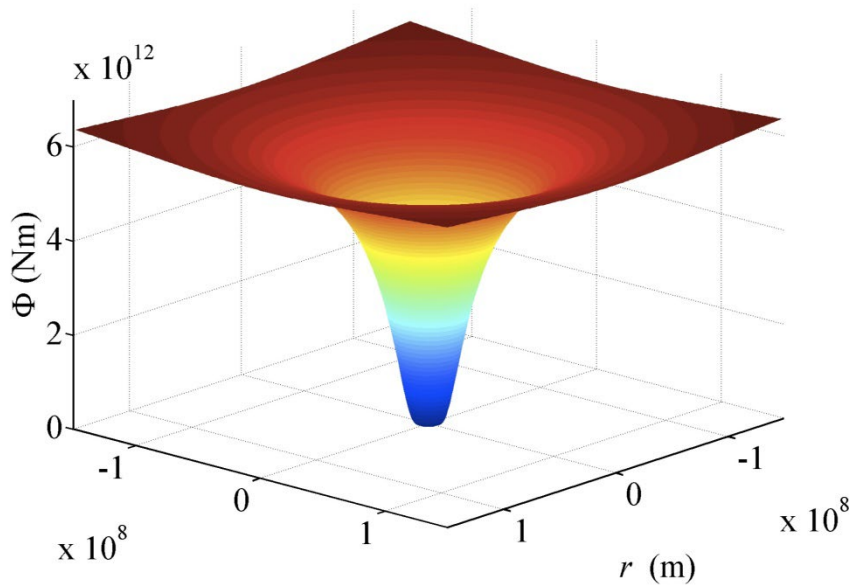
$$\Phi_{erfc}(r) = GM \left(\frac{\sqrt{\pi}}{\sqrt{2\sigma}} \right) - GM \left(\frac{\sqrt{\pi}}{\sqrt{2\sigma}} \right) \operatorname{erf} \left(\frac{\sigma}{\sqrt{2}r} \right)$$

$$\Phi_{erf}(r) = -GM \left(\frac{1}{r} - \frac{1}{6r^3} + \frac{1}{40r^5} - \dots \right) \approx -\frac{GM}{r} \Big|_{r \rightarrow \infty}$$



The Model Predicts the Emergence of a Constant Baryonic Potential

$$\Phi_{erfc}(r) = GM \left(\frac{\sqrt{\pi}}{\sqrt{2\sigma}} \right) - GM \left(\frac{\sqrt{\pi}}{\sqrt{2\sigma}} \right) \operatorname{erf} \left(\frac{\sigma}{\sqrt{2}r} \right)$$



ROAD MAP

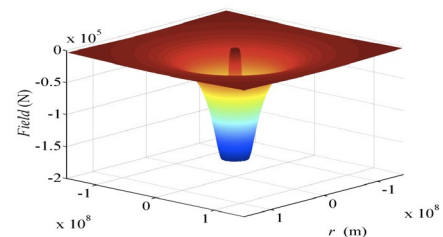
1-Emergence of a Modified Newton's Law

2-The *erfc* Metrics

3-Space time Expansion

4-Cosmic Background

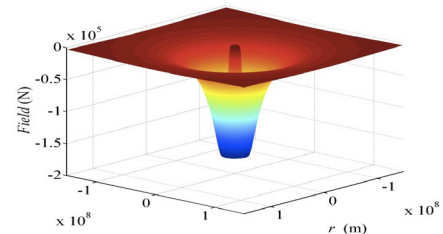
5- Concluding remarks



The *erfc* metric

$$ds^2 = \left[1 + \frac{\sqrt{2\pi GM}}{\sigma c_{th}^2} \operatorname{erfc}\left(\frac{\sigma}{\sqrt{2}r}\right) \right] c_{th}^2 dt^2 - \left[1 + \frac{\sqrt{2\pi GM}}{\sigma c_{th}^2} \operatorname{erfc}\left(\frac{\sigma}{\sqrt{2}r}\right) \right]^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2$$

This diagonal metric is a solution to the Einstein's field equations for a spherically symmetric system from $r=0$ to $r=\infty$. There is no null vacuum solution. Moreover, the relativistic tensors $R_{\mu\mu}$, $G_{\mu\mu}$, the Ricci and the Kretschmann scalars, all extend from $0 \leq r \leq \infty$ **without any singularity.**



Plamondon, R., (2018), *General Relativity: an erfc metric*, Results in Physics, 9, 456-462.

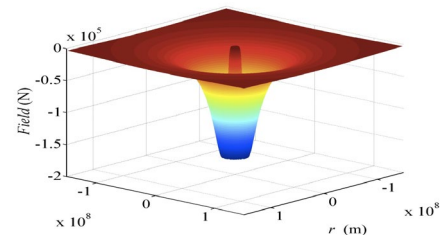
The far distant gravitational solution

Whatever the value of σ , at $r = \infty$

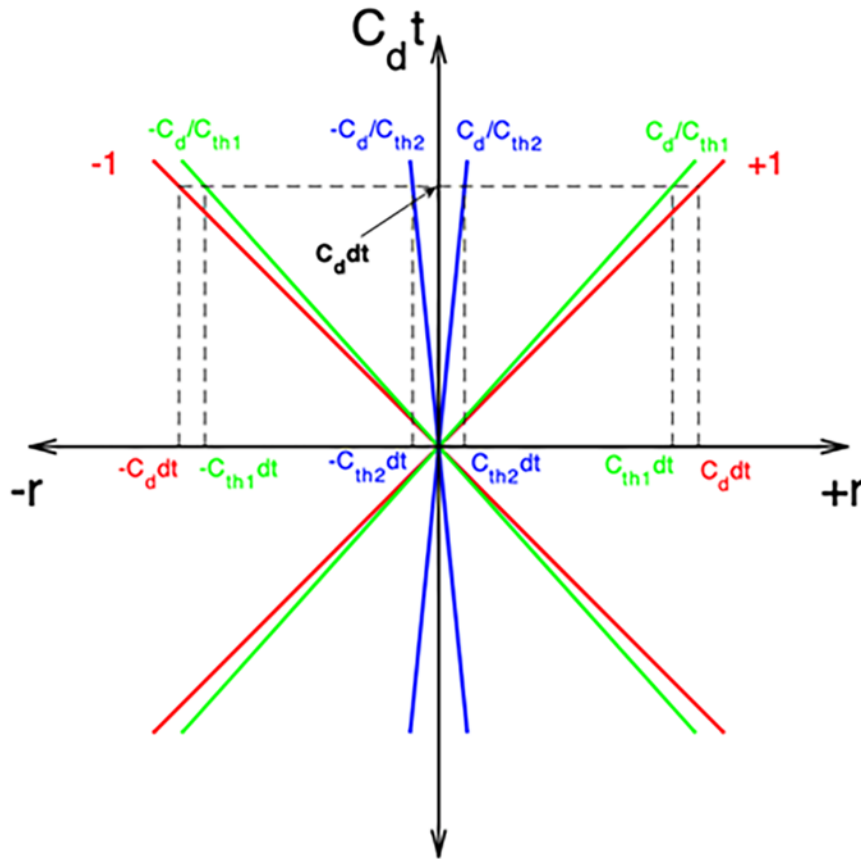
$$ds^2 = \left[1 + \frac{\sqrt{2\pi GM}}{\sigma c_{th}^2} \right] c_{th}^2 dt^2 - \left[1 + \frac{\sqrt{2\pi GM}}{\sigma c_{th}^2} \right]^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2$$

Equivalent to a Minkowski metric
if this binomial condition is respected:

$$c_{th}^2 - c_{th} c_d + \frac{\sqrt{2\pi GM}}{\sigma} = 0$$



The real solutions $\Delta > 0$

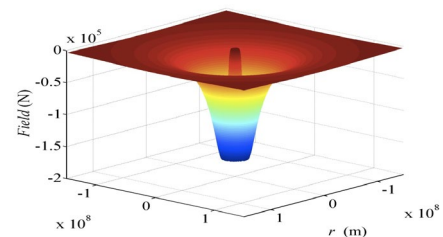


$$C_{th1,2} = \frac{C_d}{2} \pm \sqrt{\frac{C_d^2}{4} - \frac{\sqrt{2\pi GM}}{\sigma}}$$

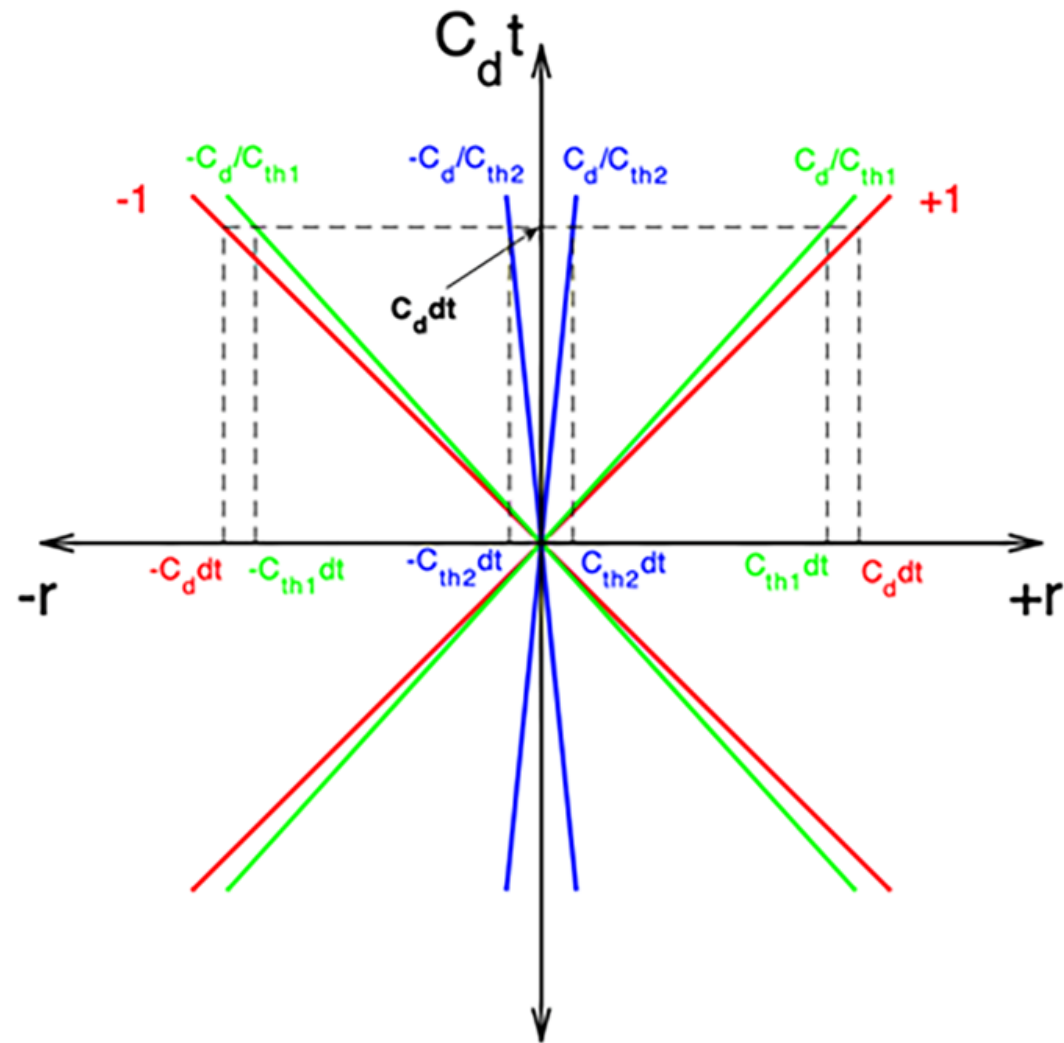
$$C_{th1} + C_{th2} = C_d$$

$$C_{th1} C_{th2} = \frac{\sqrt{2\pi GM}}{\sigma}$$

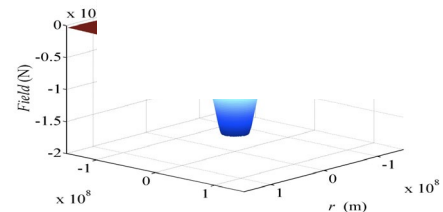
$$ds_d^2 = ds_{th1}^2 + ds_{th2}^2$$



The real solutions $\Delta > 0$



where the c_{th1} component can be seen as the speed of light with respect to a fixed spacetime and the c_{th2} component as a spacetime expansion.



ROAD MAP

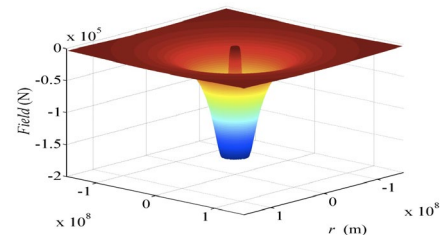
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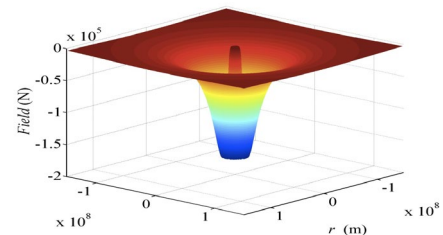


A Doppler Interpretation

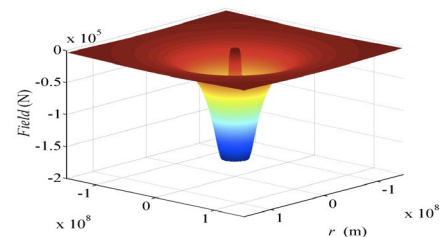
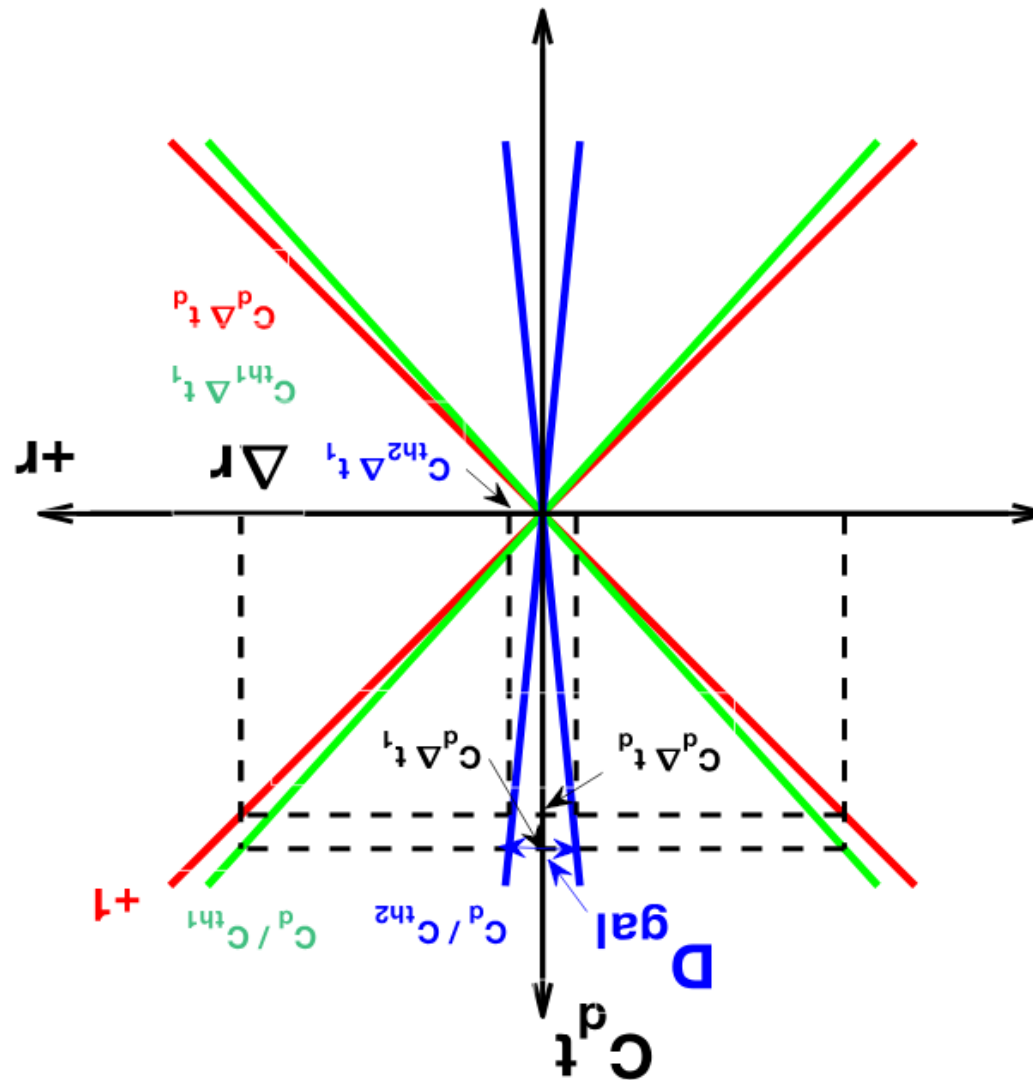
$$\frac{c_{th_2}}{c_d} = \frac{\lambda_d - \lambda_{th_2}}{\lambda_d} = \frac{\Delta\lambda}{\lambda_d}$$

For an observer working with c_{th_1} in the solar system and exploring a galaxy at a distance D_{Gal} in the outer space will conclude that the corresponding space-time is expanding:

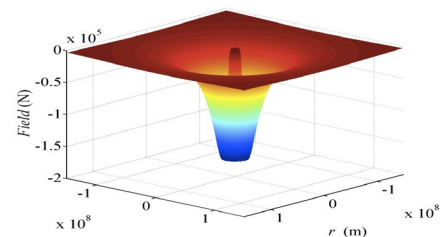
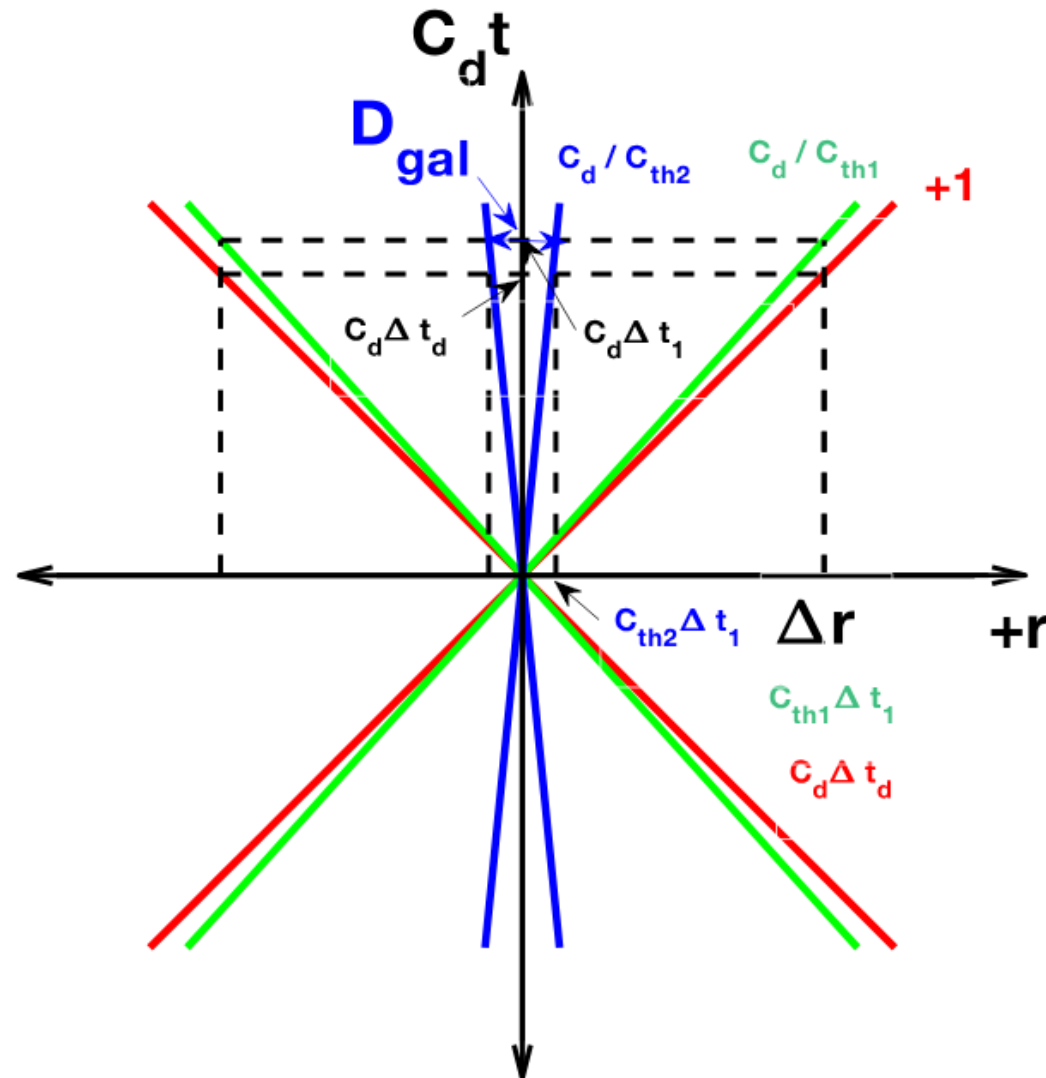
$$V_{stex} = c_{th_2}$$



Looking back in the past



Flipping for a better visualization



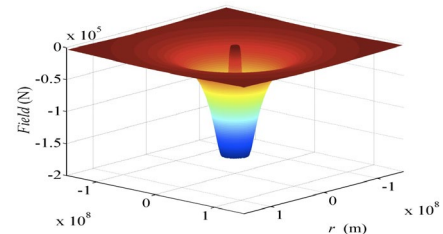
Along the Perpendicular Axis

Equivalence along the radial axis:

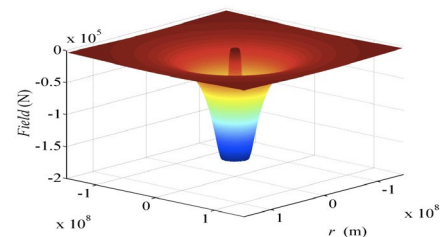
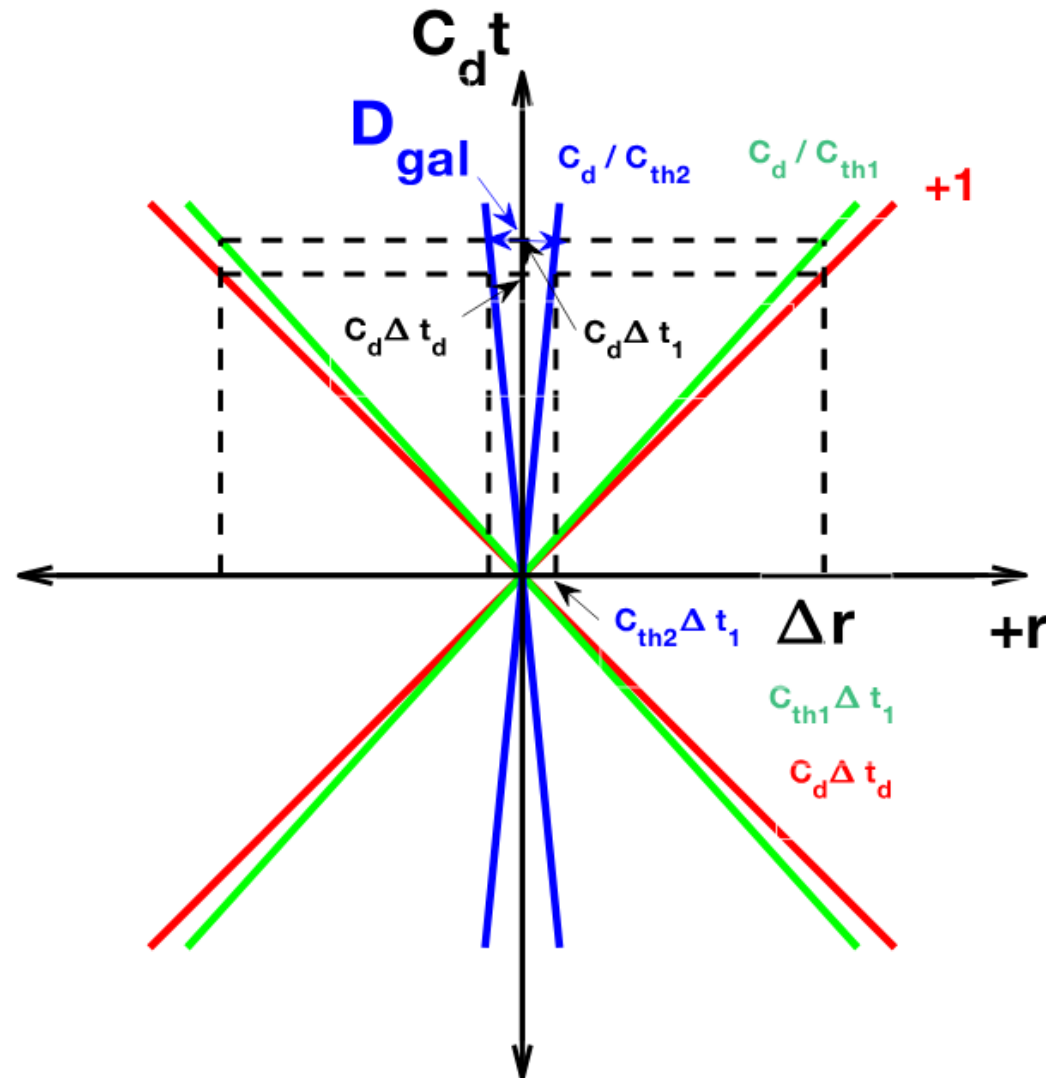
$$\Delta r = c_d \Delta t_d = c_{th_1} \Delta t_1$$

Difference along the time axis:

$$\Rightarrow \Delta t_d = \frac{c_{th_1} \Delta t_1}{c_d} \Rightarrow \Delta t_d < \Delta t_1$$



Flipping for a better visualization

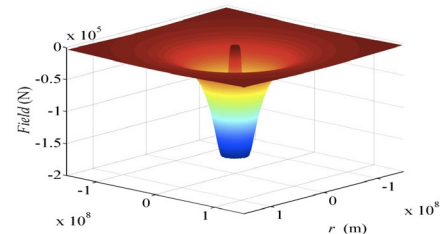


Along the Oblique Axis

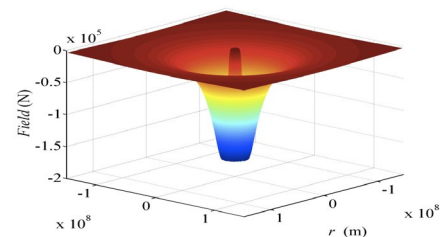
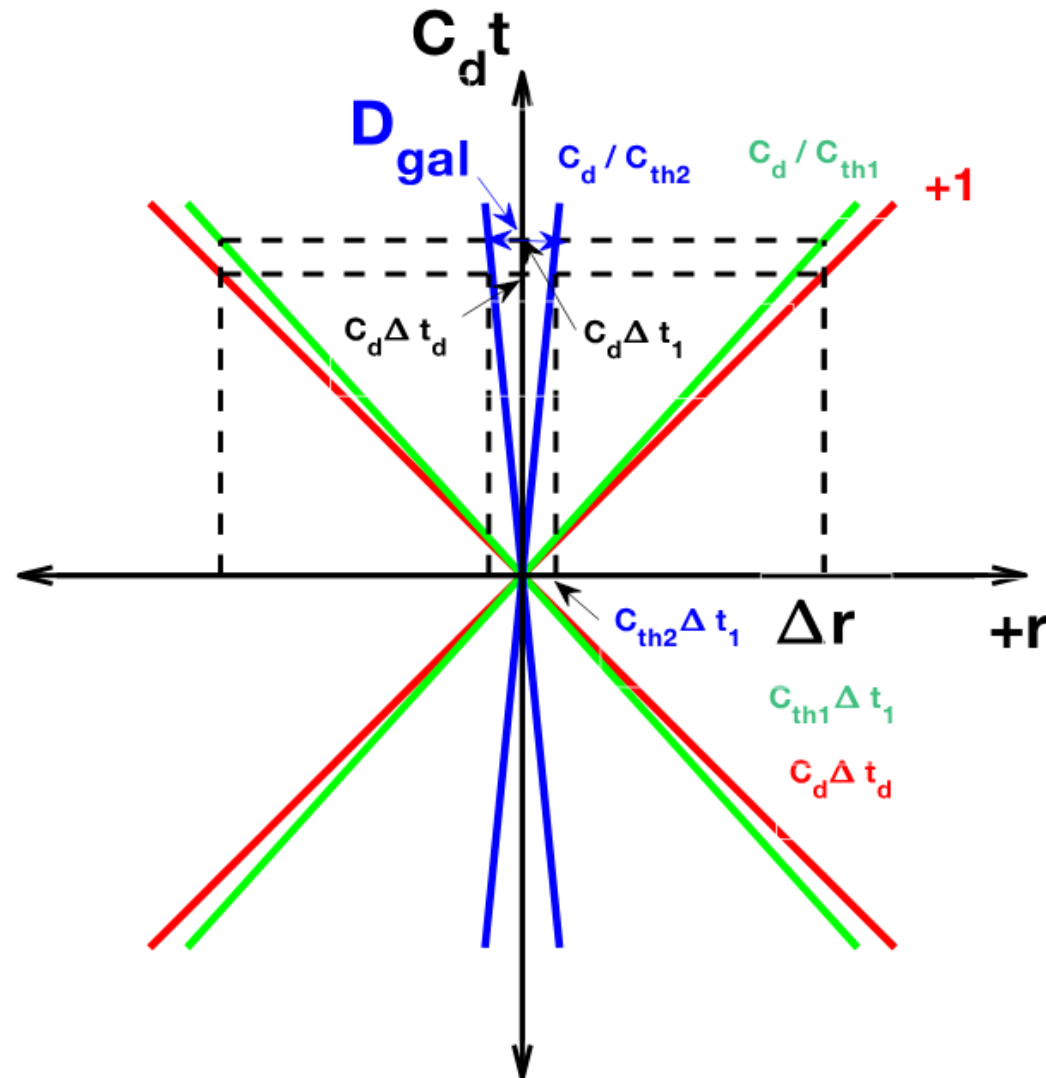
$$V_{Gal} = \frac{D_{Gal}}{\Delta t_1} = \frac{D_{Gal} c_{th1}}{c_d \Delta t_d} \text{ along the } c_{th1} \text{ axis}$$

$$D_{Gal} = 2c_{th2} \Delta t_1 \text{ along the } c_{th2} \text{ axis}$$

$$\text{Combining: } V_{Gal} = \frac{2c_{th2} c_{th1} \Delta t_1}{c_d \Delta t_d} = \frac{2c_{th2} D}{D_{ref}} \text{ along the time axis}$$



Flipping for a better visualization

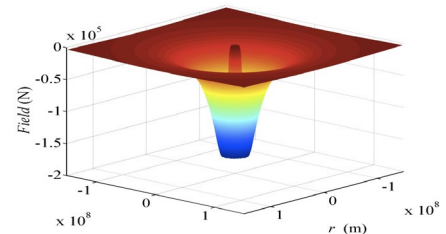


A Doppler Interpretation

with D_{ref} and D in Mpc

and $c_{th2} \Rightarrow \frac{c_{th2}}{3.26}$ in km/s

$$V_{Gal} = \frac{2c_{th2}D}{D_{ref}} = \frac{2c_{th2}}{3.26 \times 1 \text{Mpc}} D_{GalMpc}$$



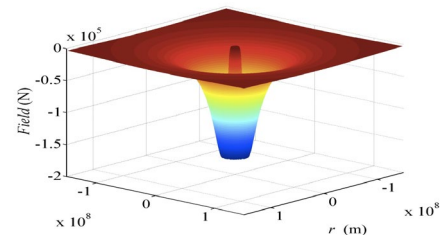
In the solar system

$$ds^2 = \left[1 + \frac{\sqrt{2\pi GM_{Sun}}}{\sigma_{Sun} c_{th}^2} \right] c_{th}^2 dt^2 - \left[1 + \frac{\sqrt{2\pi GM_{Sun}}}{\sigma_{Sun} c_{th}^2} \right]^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2$$

$$\sigma_{Sun} \frac{4\pi r_{Sun}^2}{(4/3)\pi r_{Sun}^3} = \left[\frac{\rho_{ref}}{\rho_{Sun}} \right] \left[\frac{u_m}{m_{s1}} \right] \Rightarrow \sigma_{Sun} = \frac{r_{Sun} \rho_{ref} u_m}{3\rho_P m_{s1}}$$

With $\sigma_{Sun} = \frac{r_{Sun} \rho_{ref} u_m}{3\rho_{Sun} m_{H_2O}} = 9.1508(20) \times 10^6 \text{ m}$

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In the solar system at $r \rightarrow \infty$

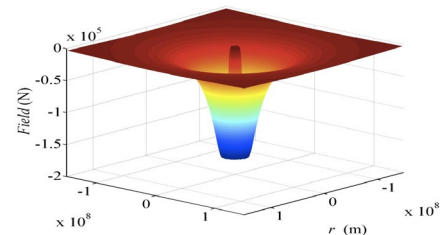
$$\sigma_{Sun} = 9.1508(20) \times 10^6 \text{ m}$$

$$c_{th1} = 299671152(27) \text{ m/s}$$

$$c_{th2} = 121306(27) \text{ m/s}$$

$$c_d = c_{th1} + c_{th2} = 299792458 \text{ m/s}$$

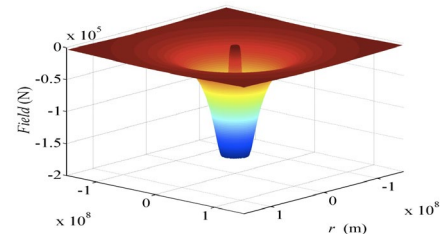
$$\Rightarrow c_{th2} = c_d - c_{th1} = \Delta c_{Sun}$$



Plamondon, R., (2017), Solar System Anomalies: Revisiting Hubble's law, Physics Essays, 30(4), 403-411.

The Doppler Hubble constant

$$V_{Gal} = \frac{2c_{th_2} D}{D_{ref}} = \frac{2c_{th_2}}{3.26 \times 1 \text{Mpc}} D_{GalMpc}$$

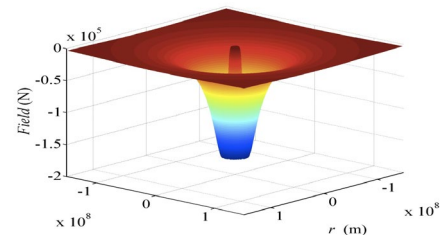


The Doppler Hubble constant

$$V_{Gal} = \frac{2c_{th_2} D}{D_{ref}} = \frac{2c_{th_2}}{3.26 \times 1 \text{Mpc}} D_{GalMpc}$$

$$V_{gal} = H_{0Doppler} D_{Mpc}$$

where $H_{0Doppler} = \frac{2c_{th_2}}{3.26 \text{Mpc}} = 74.42(.02) (\text{km/s} \cdot \text{Mpc})$



ROAD MAP

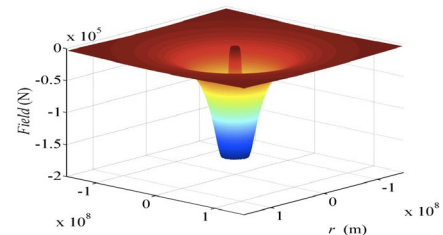
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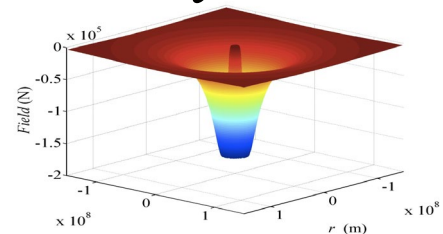


A Cosmological model

Hypothesis: the *erfc* potential is a source of baryonic matter energy density, describing the total energy available in the Universe:

$$\Omega_m + \Omega_r + \Omega_k = \operatorname{erfc} \left(\frac{\sigma_U}{\sqrt{2}r} \right) = 1 - \operatorname{erf} \left(\frac{\sigma_U}{\sqrt{2}r} \right)$$

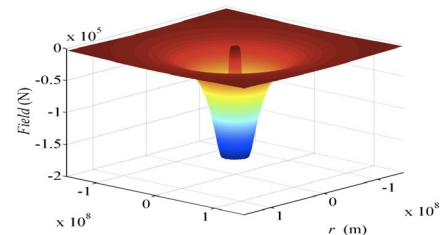
where Ω_m , Ω_r , Ω_k are the dimensionless densities, with $\Omega_m + \Omega_r$ representing the total baryonic energy available in the universe and Ω_k the curvature density parameter.



The Λ -CMB model

If we compare the previous equation with the currently accepted Λ -CDM model that incorporates a dark energy component Ω_Λ :

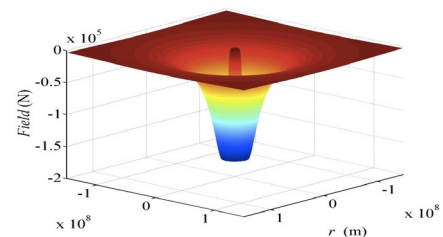
$$\Omega_m + \Omega_r + \Omega_\Lambda + \Omega_k = 1$$



Condition of equivalence

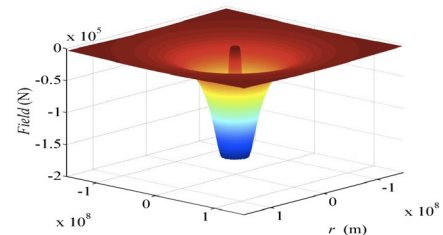
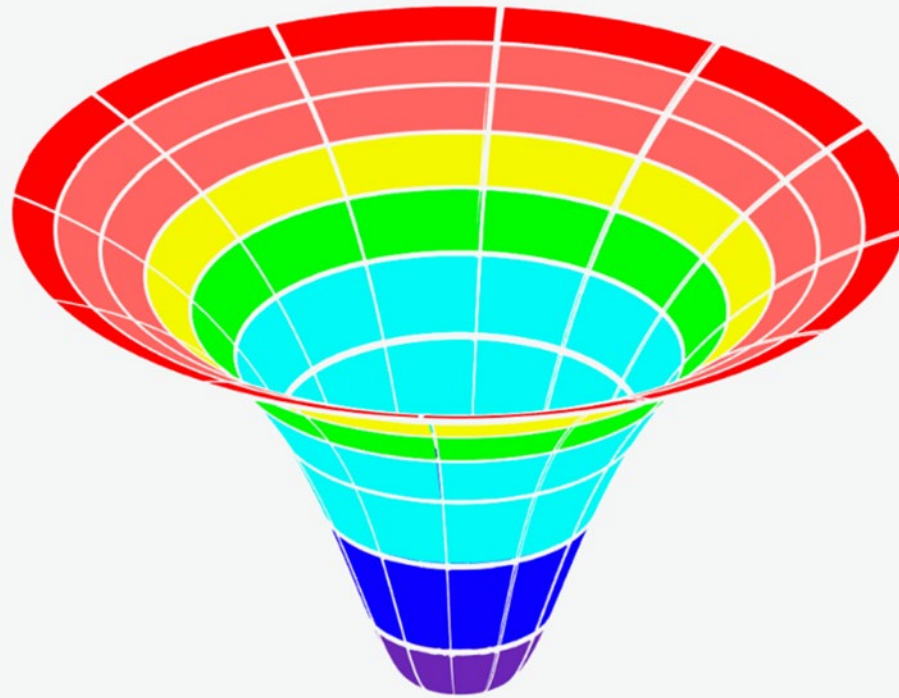
Equivalence if:

$$\Omega_{\Lambda} = \operatorname{erf} \left(\frac{\sigma_U}{\sqrt{2r}} \right)$$



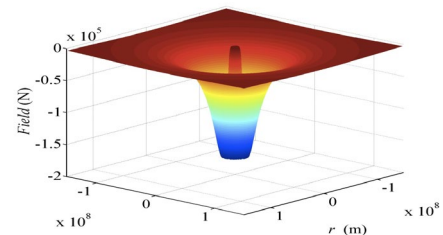
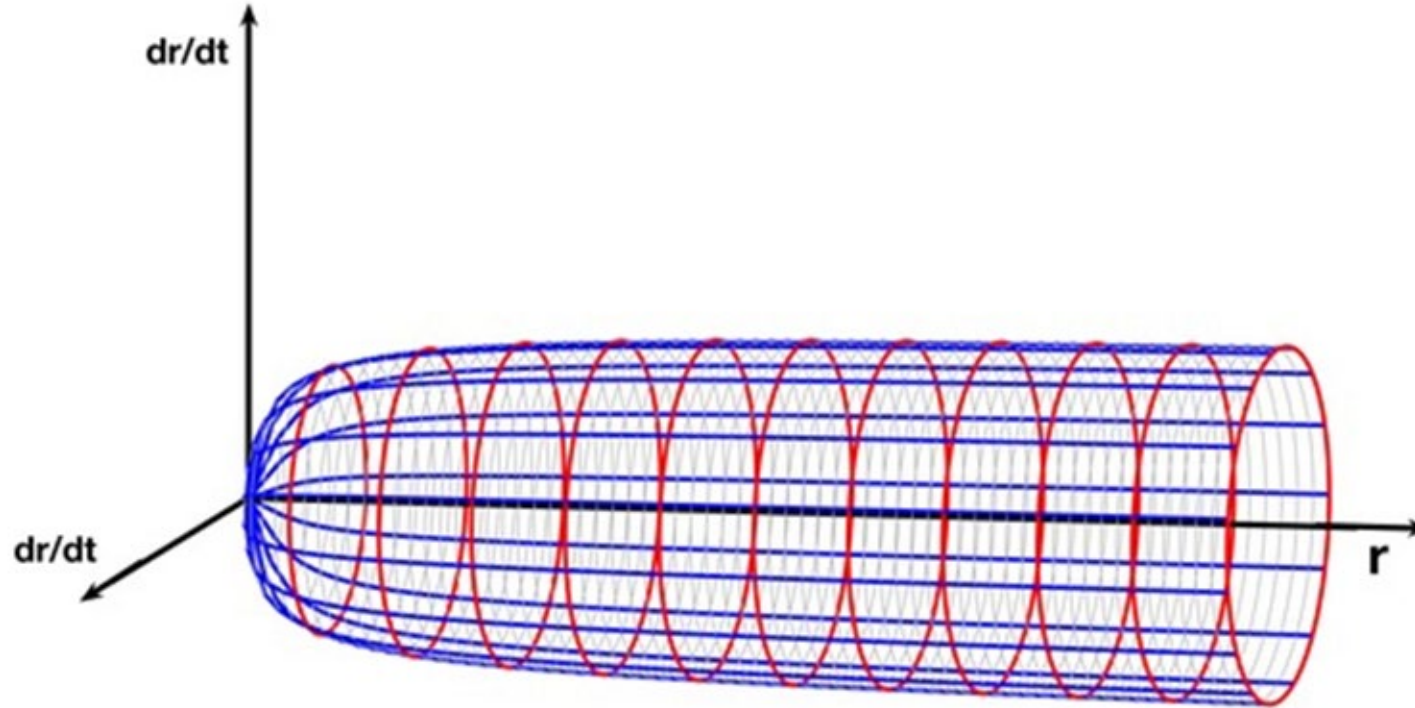
Evolution of an *erfc* universe

An erfc potential cosmos



Photon Velocity in an *erfc* Universe

Photons Speed

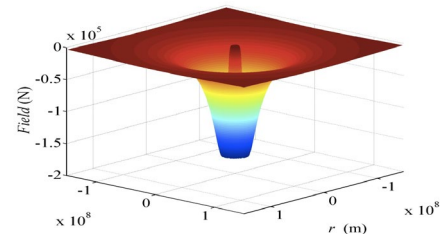


The Cosmological constant

$$\Lambda_{CMB} = 3 \left(\frac{H_{0CMB}}{c_d} \right)^2 \quad \Omega_{\Lambda} = 3 \left(\frac{H_{0CMB}}{c_d} \right)^2 \times \text{erf} \left(\frac{\sigma_U}{\sqrt{2}r} \right)$$

at $r = \sigma_U$ this equation is independent
of the observer position

$$\Lambda_{CMB} = 3 \left(\frac{H_{0CMB}}{c_d} \right)^2 \quad \Omega_{\Lambda} = 3 \left(\frac{H_{0CMB}}{c_d} \right)^2 \times 0.68268949214$$



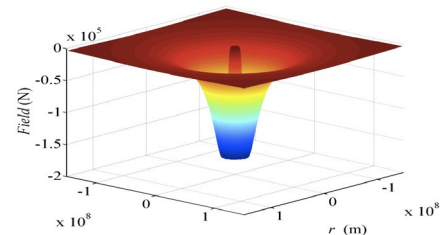
The Doppler vs CMB Hubble constant

Recalling that $H_{0\text{Doppler}}$ has been defined at $r = \infty$

$$\Omega_m + \Omega_r + \Omega_k = \operatorname{erfc} \left(\frac{\sigma_U}{\sqrt{2}r} \right) \Big|_{r=\infty} = 1$$

while $H_{0\text{CMB}}$ is estimated at $r = \sigma_U$,

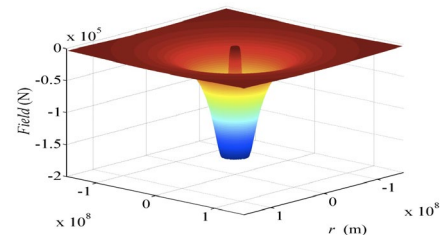
$$\Omega_\Lambda = \operatorname{erf} \left(\frac{\sigma_U}{\sqrt{2}r} \right) \Big|_{r=\sigma_U}$$



The Doppler vs CMB Hubble constant

Both values must be equal when the normalized linear density is taken into account that is;

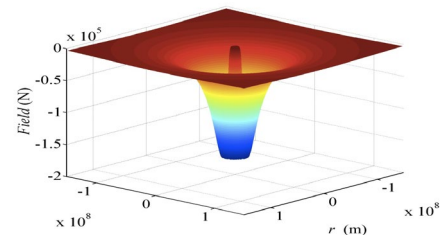
$$\begin{aligned}
 H_{0\text{CMB}} &= H_{0\text{Doppler}} \left(\frac{\Omega_{\Lambda} \Big|_{r=\sigma_U}}{\Omega_m + \Omega_r + \Omega_k} \right)^{1/4} = H_{0\text{Doppler}} \left(\text{erf} \left(\frac{\sigma_U}{\sqrt{2}r} \right) \Big|_{r=\sigma_U} \right)^{1/4} \\
 &= H_{0\text{Doppler}} (0.6827)^{1/4} = H_{0\text{Doppler}} \times 0.909 = 67.64 \frac{\text{km/s}}{\text{Mpc}}
 \end{aligned}$$



The Doppler vs CMB Hubble constant

$$H_{0\text{Doppler}} (0.6827)^{1/4} = H_{0\text{CMB}}$$

$$74.42 \frac{\text{km/s}}{\text{Mpc}} \times 0.909 = 67.64 \frac{\text{km/s}}{\text{Mpc}}$$

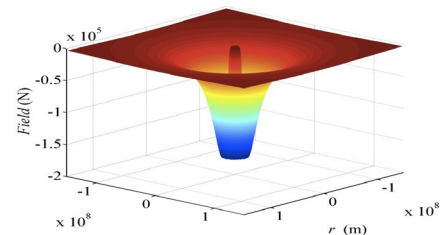


The Doppler vs CMB Hubble constant

$$H_{0\text{Doppler}} (0.6827)^{1/4} = H_{0\text{CMB}}$$

$$74.42 \frac{\text{km/s}}{\text{Mpc}} \times 0.909 = 67.64 \frac{\text{km/s}}{\text{Mpc}}$$

$$\Lambda_{\text{CDM}} = 1.0951 \times 10^{-52} \text{ m}^{-2}$$



ROAD MAP

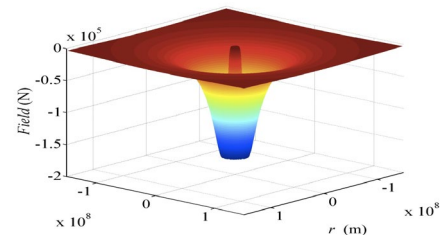
1-Emergence of a Modified Newton's Law

2-The *erfc* Metrics

3-Space time Expansion

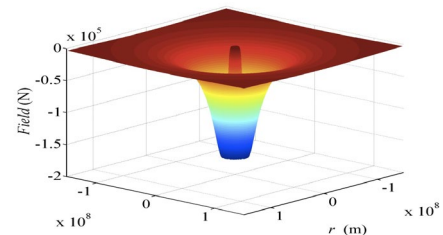
4-Cosmic Background

5- Concluding remarks



TAKE HOME MESSAGE #1

A **modified Newton's Law**
automatically emerges
from General Relativity
when the **probability of presence**
of the energy-momentum density
is taken into account into Einstein's equation.

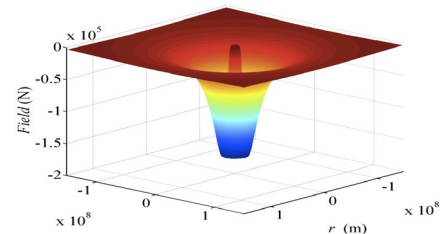


TAKE HOME MESSAGE #2

This modified Newton's Law

$$g(r) = -\frac{GM}{r^2} \exp\left(-\frac{\sigma^2}{2r^2}\right)$$

is characterized by an **emergent Lorentz's scalar σ** that defines the intrinsic reference proper length of any massive system



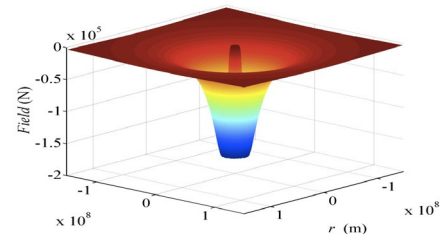
TAKE HOME MESSAGE #3

The emergent modified Newton's field
relies on an *erfc* potential

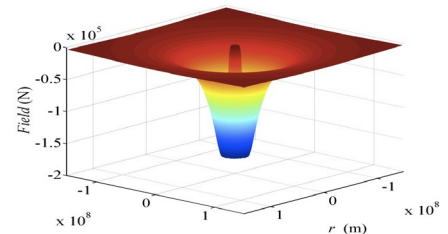
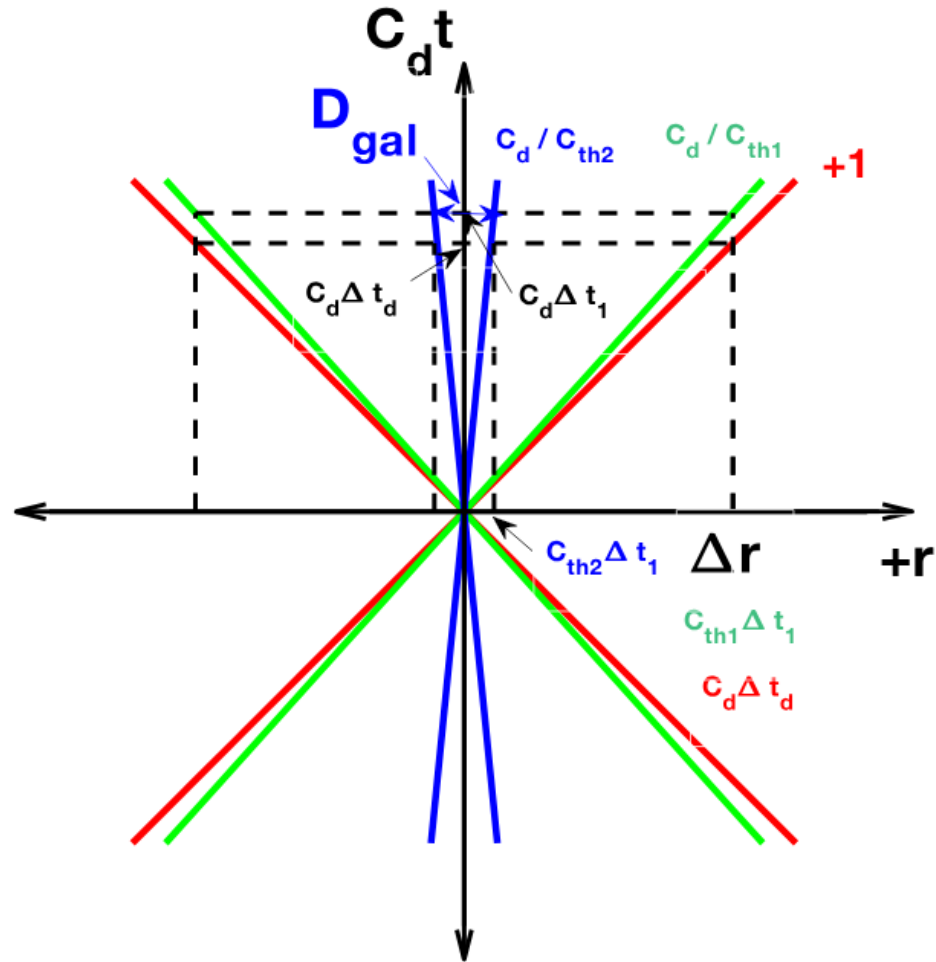
$$\Phi_{erfc}(r) = GM \left(\frac{\sqrt{\pi}}{\sqrt{2\sigma}} \right) - GM \left(\frac{\sqrt{\pi}}{\sqrt{2\sigma}} \right) \operatorname{erf} \left(\frac{\sigma}{\sqrt{2}r} \right)$$

that incorporates

a constant reference component



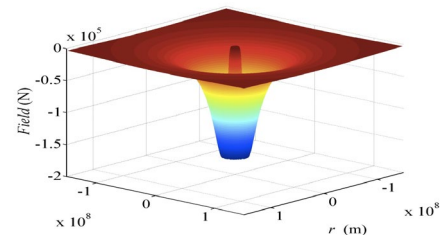
A Worldline Interpretation



A Hubble Doppler effect

$$V_{\text{gal}} = H_{0\text{Doppler}} D_{\text{Mpc}}$$

where $H_{0\text{Doppler}} = \frac{2c_{th_2}}{3.26\text{Mpc}} = 74.42(.02) (\text{km/s} \cdot \text{Mpc})$

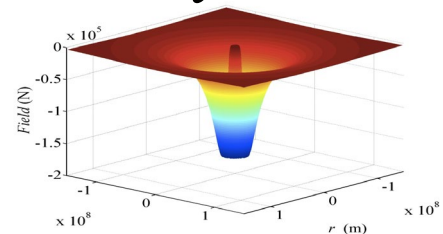


A Cosmological Model Interpretation

Hypothesis: the *erfc* potential is a source of baryonic matter energy density, describing the total energy available in the Universe:

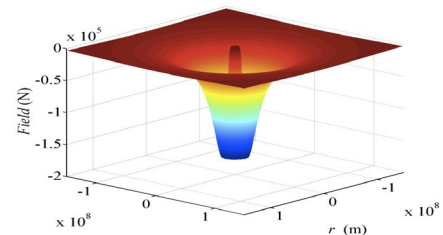
$$\Omega_m + \Omega_r + \Omega_k = \operatorname{erfc} \left(\frac{\sigma_U}{\sqrt{2}r} \right) = 1 - \operatorname{erf} \left(\frac{\sigma_U}{\sqrt{2}r} \right)$$

where Ω_m , Ω_r , Ω_k are the dimensionless densities, with $\Omega_m + \Omega_r$ representing the total baryonic energy available in the universe and Ω_k the curvature density parameter.



Once interpreted in terms of a CMB Baryonic dark energy

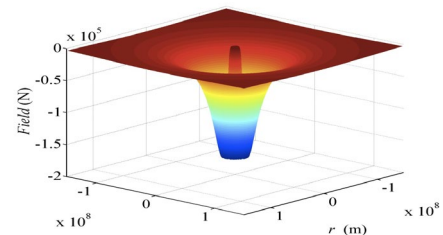
$$H_{0\text{CMB}} = 67.64 \frac{\text{km/s}}{\text{Mpc}}$$



A Plausible Interpretation of the Hubble Tension Problem

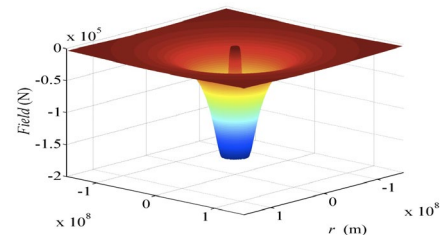
$$H_{0\text{Doppler}} (0.6827)^{1/4} = H_{0\text{CMB}}$$

$$74.42 \frac{\text{km/s}}{\text{Mpc}} \times 0.909 = 67.64 \frac{\text{km/s}}{\text{Mpc}}$$



FINAL TAKE HOME MESSAGE

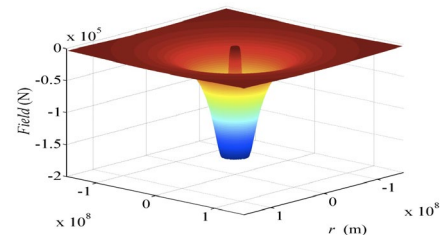
This emergent modelling paradigm raises more questions than the solutions it provides.



FINAL TAKE HOME MESSAGE

This emergent modelling paradigm raises more questions than the solutions it provides.

But it has been useful in predicting Galaxy rotational velocity (CAP2024)



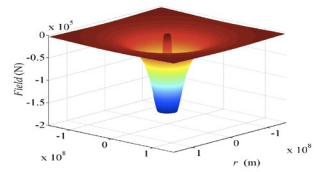
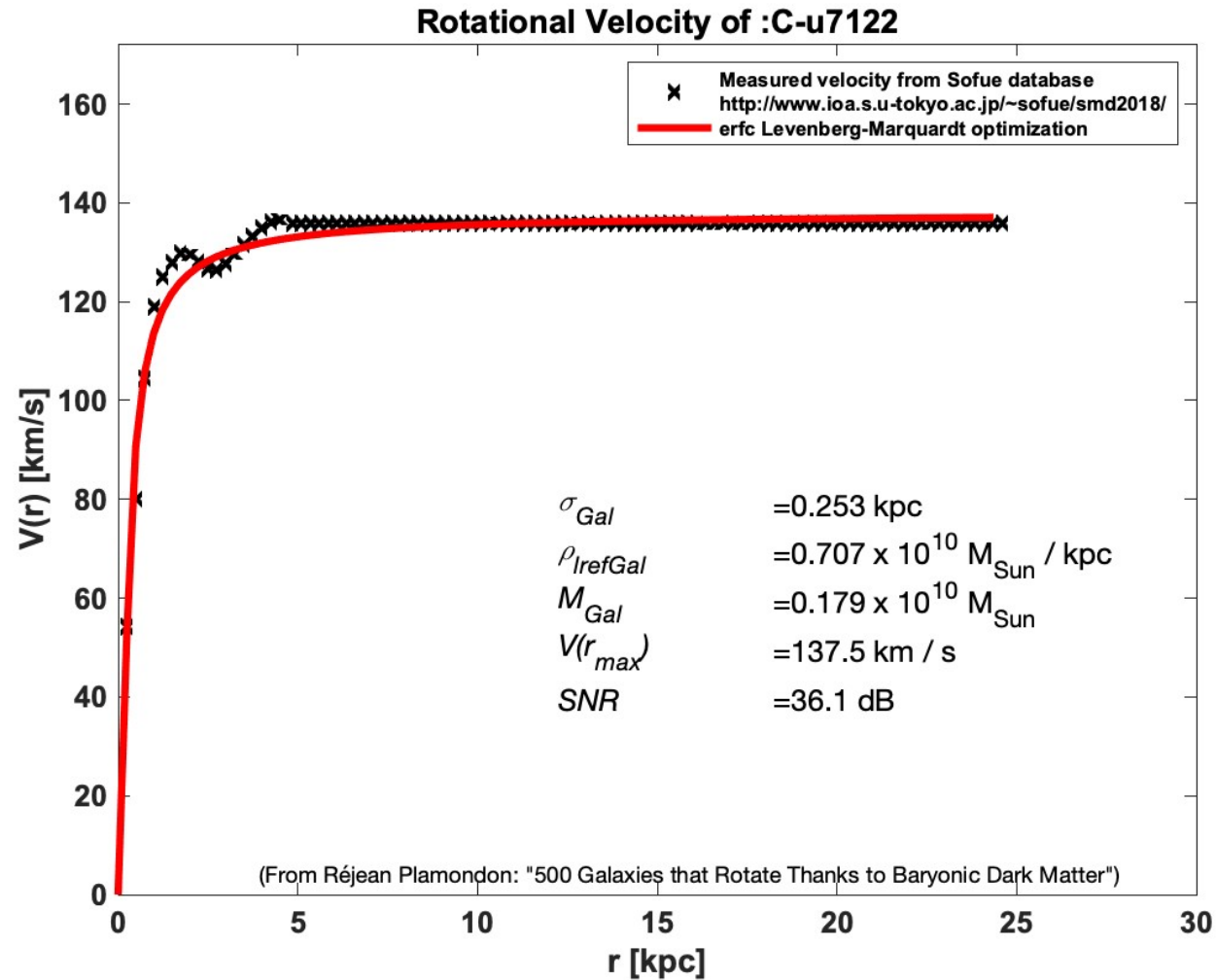


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**500 GALAXIES THAT ROTATE
FROM
BARYONIC DARK MATTER**

**CAP 2024 MEETING LONDON
ONTARIO**

TYPICAL RESULTS >35dB

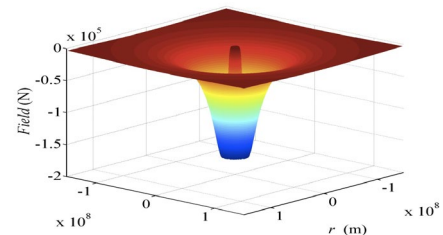


FINAL TAKE HOME MESSAGE

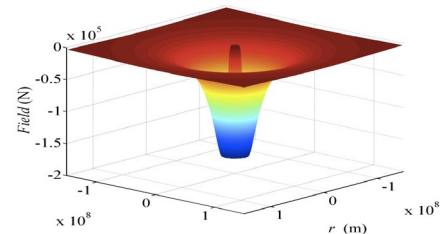
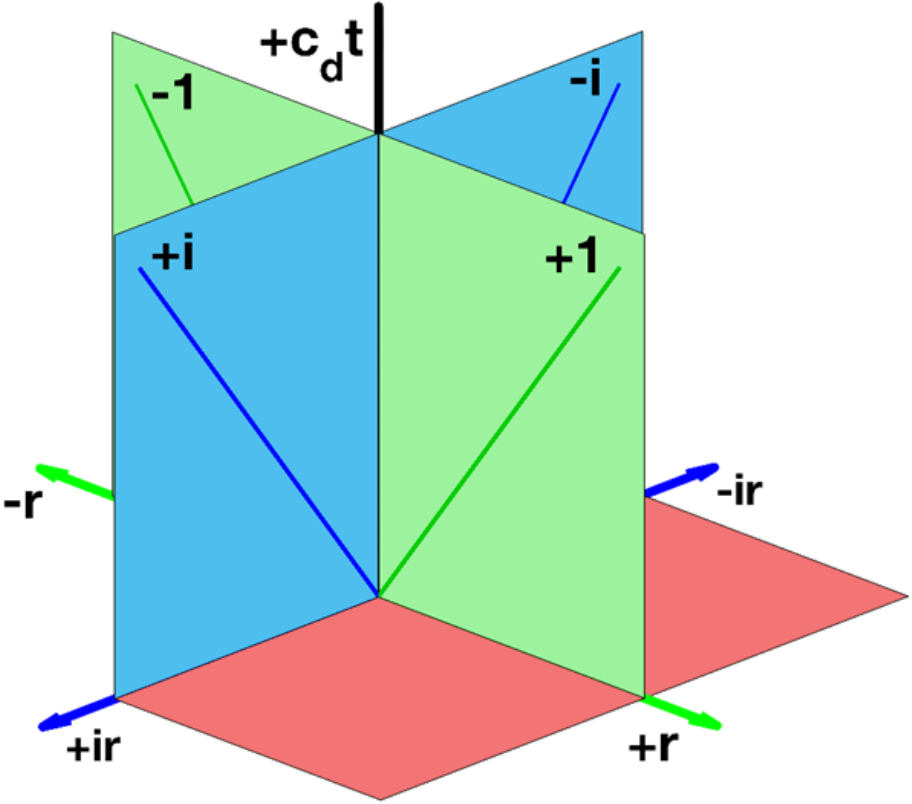
This emergent modelling paradigm raises more questions than the solutions it provides.

But it has been useful in predicting Galaxy rotational velocity (CAP2024)

And it predicts the emergence of a complex Spacetime where photons can go through both slits in a Young experiment (CAP2026)



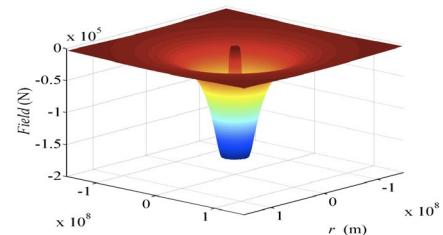
Emergence of a complex spacetime



BRIDGING THE GAP BETWEEN CLASSICAL AND QUANTUM MECHANICS THROUGH GENERAL RELATIVITY

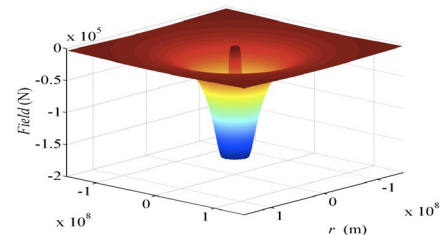
CAP2026
WEDNESDAY
June 24 10H30
SESSION
(DTP) W1-8
Mathematical
Physics and
Quantum Theory

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2. Plamondon, R., (2021) **What does the Central Limit Theorem Have to Say About General Relativity?**, in Quantum Theory and Symmetries, Proceedings of the 11th International Symposium, Montréal, Canada, , Paranjape, M.B., MacKenzie, R, Thomova, Z., Winternitz, P., WitczKrempa, W., (EDS), Springer, CRM Series in Mathematical Physics, 503-511. Available at: <https://publications.polymtl.ca/10588/>
3. Plamondon, R., (2018), **General Relativity: an erfc metric**, Results in Physics, 9, 456-462. Available at : <https://publications.polymtl.ca/3572/>
4. Plamondon, R., (2017), **Solar System Anomalies: Revisiting Hubble's law**, Physics Essays, 30(4), 403-411.



BRIDGING THE GAP BETWEEN CLASSICAL AND QUANTUM MECHANICS THROUGH GENERAL RELATIVITY

QUESTIONS?

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