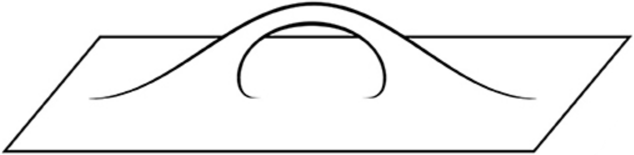
Wormholes & The Imaginary Distance Bound



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Wormholes

This is a talk about Wormholes.

I *don't* mean "traversable" wormholes, which are Lorentzian geometries that violate causality and are difficult to construct.

I mean *Euclidean* wormholes, which are solutions to equations of motion in $(++++)$ signature. They:

- Are easy to construct.
- Still can be problematic, but in a more subtle way.

Let's see why.

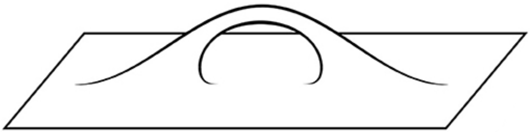
The Implications of Euclidean Wormholes

Consider a low energy effective theory for a field $\Phi(x)$

$$Z = \int D\Phi \exp\left\{ \int dx\, \mathcal{L}(\Phi(x), \partial\Phi(x), \dots) \right\}$$

governed by some Lagrangian $\mathcal{L}(x)$ in flat space.

Now add a wormhole that connects distant points in space:



This induces a coupling $\mathcal{O}_i(x)\mathcal{O}_i(y)$ between the endpoints, where $\mathcal{O}_i$ is an operator built out of Φ.

Disorder Averaging

How does this affect the low energy theory?

Integrating over the endpoints of the wormholes:

$$Z = \int D\Phi \exp\left\{ \int dx \mathcal{L}(x) + \sum_i \int \mathcal{O}_i(x)\mathcal{O}_i(y) dxdy \right\}$$
$$= \int d\alpha^i P(\alpha_i) \int D\Phi \exp\left\{ \int dx \left(\mathcal{L}(x) - \alpha_i \mathcal{O}_i(x) \right) \right\}$$

adds $\mathcal{O}_i$ to the Lagrangian, with a Gaussian random coupling

$$P(\alpha^i) \sim e^{-(\alpha^i)^2}$$

It is a theory with *random couplings*, like a spin glass.

This is no longer a standard, unitary quantum theory!

Coleman; Giddings & Strominger,...

The Imaginary Distance Bound

So, Euclidean Wormholes:

1. Seem to violate the unitarity of quantum mechanics
2. Seem to be ubiquitous in low energy EFTs with gravity.

Our Proposal: The Imaginary Distance Bound (IDB).

- A "swampland" constraint on effective field theories,
- That rules out wormholes, except in particular circumstances.

We will provide some evidence for the IDB from string theory.

The IDB is a generalization of

- The weak gravity conjecture, and
- The Kontsevich-Segal-Witten bound on complex metrics.

The Theory

Take a low energy EFT with real massless scalars ϕ^α

$$S = -\frac{1}{2\kappa^2}\int\sqrt{g}\left(R + \frac{d(d-1)}{2\ell^2} - \frac{1}{2}h_{\alpha\beta}(\phi)\nabla_\mu\phi^\alpha\nabla^\mu\phi^\beta\right)$$

that parameterize a target space $\mathcal{M}$ with metric $h_{\alpha\beta}(\phi)$.

In Lorentzian space-time, the metric $h_{\alpha\beta}(\phi)$ is Euclidean.

But in Euclidean space-time, $h_{\alpha\beta}(\phi)$ can have mixed signature.

This happens if we complexify the ϕ^α

- E.g. for axions we usually take $\phi \to i\phi$ under $t \to it$.

The resulting stress tensor supports a wormhole.

Moduli Space

The asymptotic values of the scalars $\langle \phi^\alpha \rangle$ are coupling constants.

In AdS, they are the couplings of the dual CFT:

- $\mathcal{M}$ is a moduli space of CFTs.
- $h_{\alpha\beta}(\phi)$ is the Zamolodchikov metric.

E.g. *IIB* SUGRA has an axio-dilaton (φ, C_o) with target space:

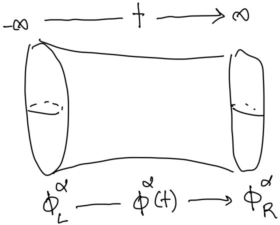
$$ds^2_{\mathcal{M}} \equiv h_{\alpha\beta} d\phi^\alpha d\phi^\beta = d\varphi^2 + e^{2\varphi} dC_o^2 \quad \underset{t \to it}{\longrightarrow} \quad d\varphi^2 - e^{2\varphi} dC_o^2$$

The asymptotic values are the couplings (g_{YM}, θ) of the $\mathcal{N} = 4$ Yang-Mills dual to AdS_5 supergravity:

$$e^{-\phi} = \frac{4\pi^2}{g^2_{YM}} \quad \& \quad C_o = \theta$$

Scalar Wormholes in Flat Space ($\ell = \infty$)

We can find $S^d \times \mathbb{R}_t$ wormholes that interpolate between different values of couplings:



$$ds^2 = dt^2 + a(t)^2\ d\Omega_d^2$$

The scalars $\phi^\alpha(t)$ follow a *Lorentzian* geodesic through $\mathcal{M}$, with endpoints $(\phi^\alpha_L, \phi^\alpha_R)$ separated by a distance

$$(\Delta s)^2_{\mathcal{M}} = -\frac{2d}{d-1}\pi^2$$

Solutions only exist for this particular "time-like" separation in $\mathcal{M}$.

But they exist for any $a_o = a(0)$ and have zero action.

The Imaginary Distance Bound

What is the implication?

Imagine starting with real couplings $\langle \phi^\alpha \rangle$ and complexifying

- Eventually we reach a point a wormhole (of any size, and zero action) exists.

We interpret this as a breakdown of the effective field theory.

Proposal: we cannot displace $\langle \phi^\alpha \rangle$ away from real values by an imaginary distance more than

$$\tau_{IDB} = \pi \sqrt{\frac{d}{2(d-1)}}$$

Some effect renders the low energy EFT unstable before this point.

This is a bound in a *single* theory, not a pair of theories. It bounds any low energy EFT coming from a UV complete theory of quantum gravity.

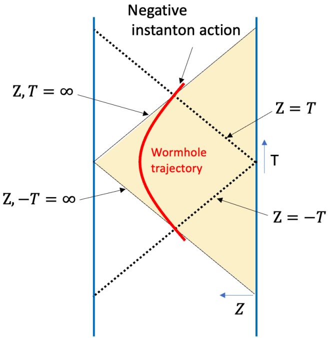
AdS Imaginary Distance Bound

Similarly, for any moduli space of CFTs, with couplings ϕ^α, the theory must break down if we make the couplings too complex.

Let's check in $\mathcal{N} = 4$ Yang-Mills. Instantons are weighted by

$$q = \exp\left\{ i\theta - \frac{4\pi^2}{g_{YM}^2} \right\}$$

The instanton sum diverges if $|q| \to 1$, which happens if θ or g_{YM} are made too imaginary.



Dimensional Reduction

The Imaginary Distance Bound generalizes other prior bounds.

If we dimensionally reduce from $D+1 \to D$ on a circle:

$$ds^2 = e^{-\frac{2\nu}{D-2}} ds_D^2 + e^{2\nu} d\sigma^2$$

we obtain a theory

$$S = \frac{1}{16\pi G_D} \int d^D x \sqrt{g} \left(R - \frac{1}{2} (\nabla \nu)^2 \right)$$

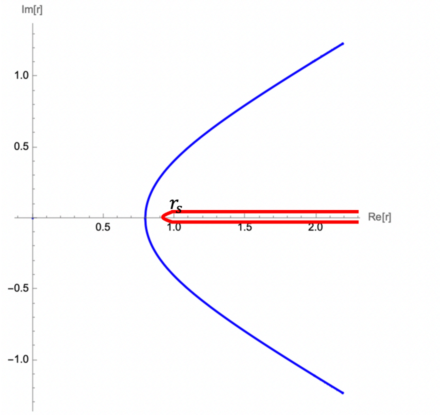
that allows wormholes for complex ν.

The womholes are complex slices of the Schwarzchild black hole!

- Let $e^{2\nu} = -(1 - R/r)$ and think of σ as "Schwarschild-time".

Double-Cone

The wormhole is a slice of complex Schwarszchild:



Similar to the "double cone" which computes the ramp in the spectral form factor $\mathrm{Tr}\, e^{-(\beta-iT)H}\ \mathrm{Tr}\, e^{-(\beta+iT)H}$.

Saad, Shenker & Stanford

Charged Black holes and the Weak Gravity Conjecture

Charged black holes are wormholes as well. Reducing

$$ds^2 = e^{-\frac{2\nu}{D-2}} ds_D^2 + e^{2\nu} d\sigma^2, \qquad A = \alpha d\sigma$$

we obtain a theory that allows wormholes:

$$S = \frac{1}{16\pi G_D} \int d^D x \sqrt{g} \left(R - \frac{1}{2} \left((\nabla \nu)^2 + e^{2\nu} (\nabla \alpha)^2 \right) \right)$$

A charged particle with worldline wrapping σ is an instanton that will correct the low energy EFT. This renders the wormhole invalid when the particle has

$$q/m > 1$$

So the IDB coincides with the weak gravity conjecture!

Complex Metrics & Kontsevich-Segal-Witten

When you dimensionally reduce general relativity (e.g. on a T^N)

$$ds^2 = ds_D^2 + ds_{int}^2$$

the parameters in the internal metric show up as scalars ϕ^α in the dimensionally reduced theory:

$$S = -\frac{1}{2\kappa^2}\int \sqrt{g}\left(R - \frac{1}{2}h_{\alpha\beta}(\phi)\nabla_\mu\phi^\alpha\nabla^\mu\phi^\beta\right)$$

where $h_{\alpha\beta}$ is a metric on the moduli space of internal metrics ds_{int}^2.

Wormholes correspond to *complex* internal metrics.

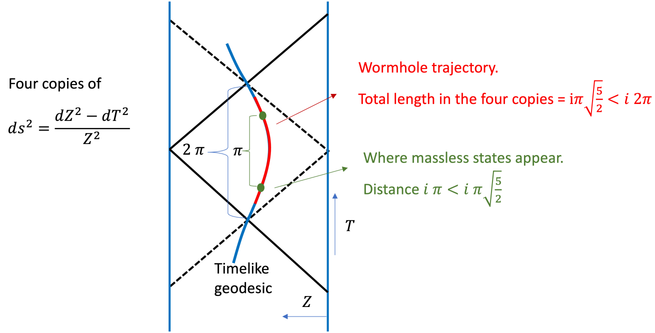
The IDB coincides with the Kontsevich-Segal-Witten criterion that determines which complex metrics are stable:

$$\sum_{i=1}^{N} |Arg(g_{ii}^{int})| < \pi$$

A UV Obstruction to Wormholes

Our conjecture: in any low energy EFT coming from a UV complete theory of gravity, some quantum gravity instanton effect will render the low energy EFT invalid before you get to τ_{IDB}.

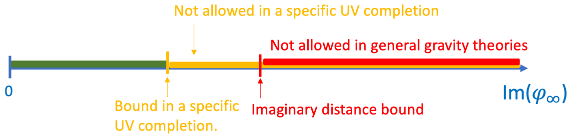
We can check in string theory compactifications (e.g. T^4).



Arkani-Hamed, Orgera & Polchinski

Conclusions

Wormholes imply an upper bound on the analytic continuation of the coupling constants of any theory:



This bound seems to be true in string theory. It is a

- Generalization of weak gravity conjecture,
- Generalization of KSW bound.

Thank you!