Conformal Extension of Higgs and Freeze-in Dark Matter

Radiative Symmetry Breaking, Constraints, and Reopen of Parameter Space

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Outline

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Hierarchy problem → Conformal symmetry → Radiative symmetry breaking

Part II · Model

Effective potential → Mass eigenstates → Mixing angles → Degrees of freedom

Part III · Constraints

UV stability → Collider bounds → Direct detection → Relic density

Part IV · Freeze-in Reopening Parameter space

Low reheating temperature → Reopened parameter space → EFT considerations

Motivation

Hierarchy Problem · Conformal Symmetry · Radiative Breaking

The Higgs Mass Hierarchy Problem

The Naturalness Puzzle

Higgs mass gets a quadratic correction from heavy degrees of freedom, much larger than Higgs mass.

Extricate cancellation between bare mass and quantum corrections.

$$\delta m^2 \sim \Lambda^2 \gg m_H^2$$

The Essence of the Problem

Electroweak scale much smaller than Planck scale

$$v = 246\mathrm{GeV} \ll \mathrm{m_{Pl}} = 10^{19}\mathrm{GeV}$$

Possible solutions

Supersymmetry: Fermion and boson loops cancel with each other.

Conformal symmetry: Mass generated radiatively

Conformal Symmetry as a Solution

Conformal Symmetry Forbids mass scales

Ex. Dilatation

$$\mathscr{L} = \partial_\mu\phi\partial^\mu\phi - \lambda\phi^4$$

$$x^\mu \to a x^\mu,\ \phi \to \phi/a,\ L \to L$$

Reasons for conformal symmetry

1. Standard model is conformal symmetric (tree level) except Higgs mass.

Is conformal symmetry fundamental?

2. If it is fundamental, the Higgs mass must be generated dynamically at symmetry breaking.

— Radiative symmetry breaking

Radiative Symmetry Breaking

Radiative corrections generating scales

With absent tree-level mass, quantum corrections can generate physical scales.

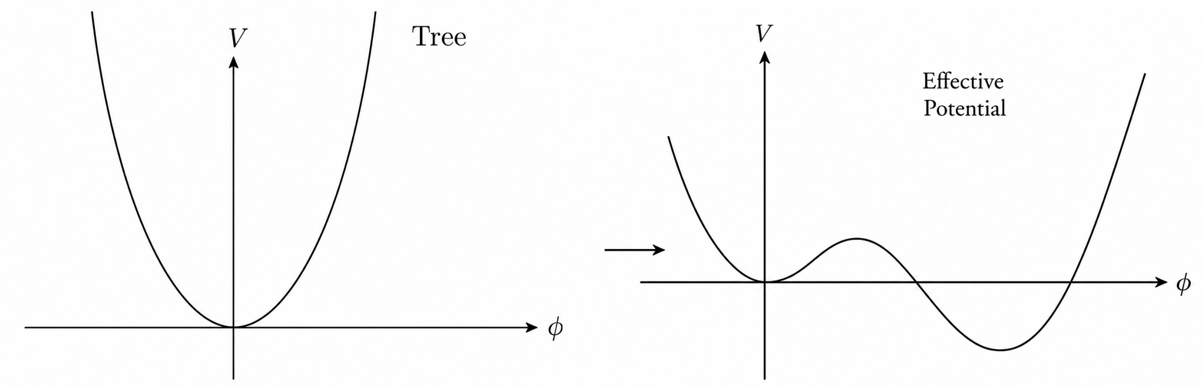
Generally require an occasional condition, so that quantum correction can be comparable to tree level.

Ex: Colemann-Weinberg mechanism

In scalar QED, the effective potential

$$V = \frac{\lambda}{4!}\varphi_c^4 + \frac{3e^4}{64\pi^2}\varphi_c^4\left(\ln\frac{\varphi_c^2}{\mu^2} - \frac{25}{6}\right)$$

If $\lambda \sim e^4/\pi^2$ at a specific μ_0, then $V'(\langle\varphi\rangle) = 0$ is achieved for non-zero $\langle\varphi\rangle$. Scale generated.



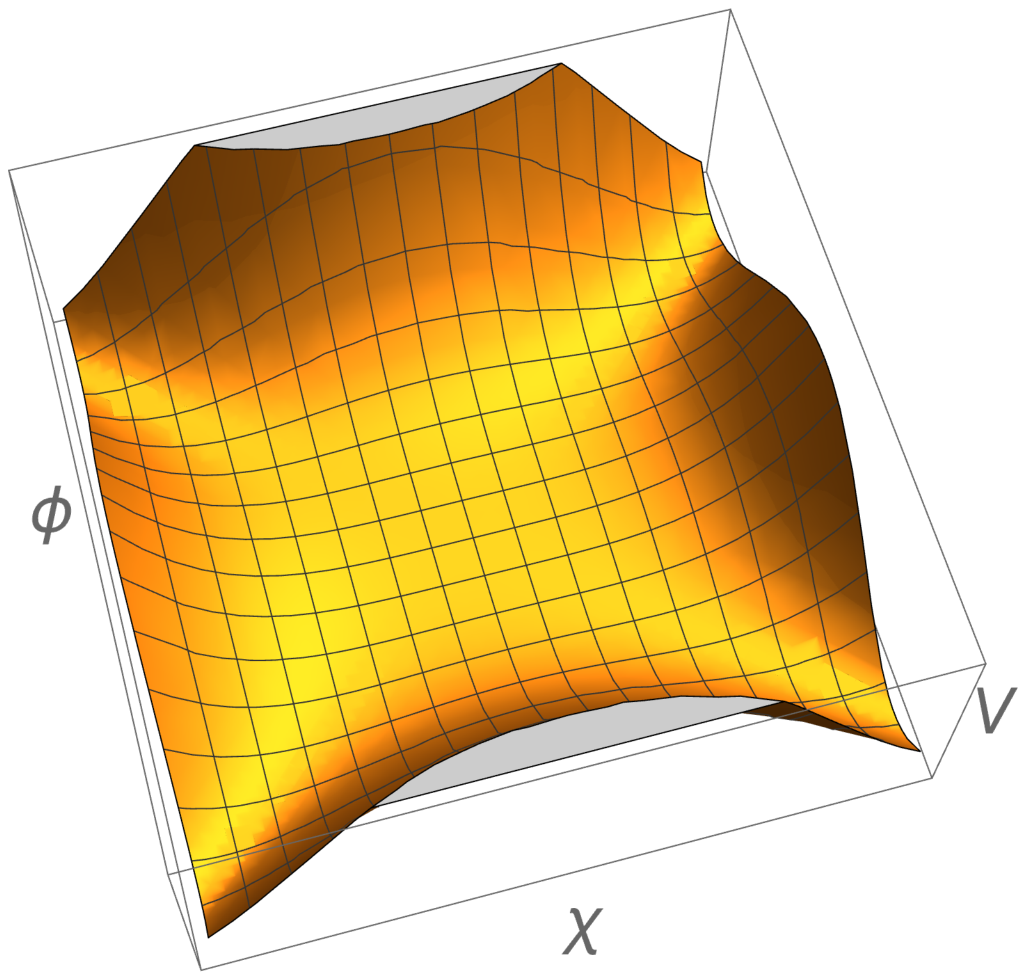
Radiative Symmetry Breaking

Ex: Gildener-Weinberg mechanism

Gildener-Weinberg mechanism work on multi-field model, assuming a flat direction:

$$V_{tree} = (\lambda_h h^4 + \lambda_{hS} h^2 S^2 + \lambda_S S^4)/4$$
$$= (\sqrt{\lambda_h} h^2 - \sqrt{\lambda_S} S^2)^2/4$$

Similarly, quantum correction generates a non-zero VEV on the flat direction.



Radiative Symmetry Breaking

Ex: Gildener-Weinberg mechanism

Gildener-Weinberg mechanism work on multi-field model, assuming a flat direction:

$$
\begin{aligned}
V_{tree} &= (\lambda_h h^4 + \lambda_{hS} h^2 S^2 + \lambda_S S^4)/4 \\
&= (\sqrt{\lambda_h} h^2 - \sqrt{\lambda_S} S^2)^2/4
\end{aligned}
$$

Similarly, quantum correction generates a non-zero VEV on the flat direction.

Temperature evolution

High $T > T_c$: Thermal terms dominates effective potential

↓

Universe cools: thermal effects decouple

↓

T → 0: Radiative minimum emerges — symmetry broken

Model

Effective Potential · Mass Eigenstates · Mixing Angles · Degrees of Freedom

Effective Potential

Effective Potential Definition

Effective potential takes into account quantum corrections (sometimes also thermal effects) to the vacuum.

$$V'_{eff}(\langle\phi\rangle) = 0$$

- $\langle\phi\rangle$ is vacuum expectation value;
- $V_{eff}(\langle\phi\rangle)$ is vacuum energy density.
- $V''_{eff}(\langle\phi\rangle)$ is effective mass at vacuum.

Colemann-Weinberg Potential

Colemann-Weinberg potential

1-loop zero-temperature correction written in terms of effective masses.

$$V_{CW} = \frac{1}{64\pi^2}\sum_i n_i m_i(\phi)^4(\log\frac{m_i^2(\phi)}{\mu^2} - c_i)$$
$$= \frac{1}{64\pi^2\langle\phi\rangle^4}(A\phi^4 + B\phi^4\log\frac{\phi^2}{\mu^2})$$

Here A, B are constant, simplified due to $m_i(\phi)^2 \propto \phi^2$

Generated vacuum

- Minimum

$$v_\phi = \langle\phi\rangle = \mu\exp(-\frac{1}{4} - \frac{A}{2B})$$

- Generated mass

$$\Delta m_\phi^2 = \frac{B}{8\pi^2 v_\phi^2}$$

- $B > 0$ required for stable vacuum
- $B_{SM} < 0$ due to heavy m_t

Need extra bosonic degree of freedom.

Minimal Conformal Extension of Higgs

Minimal extension involves 2 scalars

$$V = \frac{1}{4}(\sqrt{\lambda_h}h^2 - \sqrt{\lambda_S}S^2)^2 + \frac{1}{2}\eta_h h^2 R^2 + \frac{1}{2}\eta_S S^2 R^2 + \frac{1}{4}\lambda_R R^4$$

- h: Standard Model Higgs
- S achieves VEV v_s, mixes with Higgs
- R has zero VEV, odd under Z_2 symmetry, acts as dark matter,

Mass eigenstates

Symmetry breaking gives h, S, R masses, and the mass eigenstates

$$h' = h\cos\theta - S\sin\theta$$
$$\varphi = h\sin\theta + S\cos\theta$$

Here $m_{h'} = 125$ GeV, $v = 246$ GeV.

m_φ^2 vanishes only at tree level, the loop contribution $m_\varphi^2 = \dfrac{B}{8\pi^2(v^2 + v_S^2)}$, so it is pseudo-Goldstone boson.

Mixing Angles

Flat Direction Angle: β

$$\tan\beta = v_s/v = \sqrt{\lambda_h/\lambda_S}$$

Mass Mixing Angle: θ

$$h' = h\cos\theta - S\sin\theta$$
$$\varphi = h\sin\theta + S\cos\theta$$

Approximate Correspondence

$\beta = \theta$ if we neglect loop correction to $m_{h'}$

Degrees of Freedom

Parameter Inputs

$m_{h'}^2, v, m_S^2, m_R^2, \eta_h$

solve →

Relations

$$\lambda_{hS} + \sqrt{\lambda_h \lambda_S} = 0$$
$$v_s / v = \sqrt{\lambda_h / \lambda_S}$$
$$m_R^2 = \eta_h v^2 + \eta_S v_S^2$$
$$m_{h'}^2 = v^2(2\lambda_h - \lambda_{hS})$$
$$m_S^2 = B/(8\pi^2(v^2 + v_S^2))$$

Derive →

Derived Couplings

$\lambda_h, \lambda_{hS}, \lambda_S, \eta_S, v_S$

Different Parametrization, see ArXiv: 2511.11367

η_h can be replaced by v_S as input parameter.

Constraints

UV Stability · Collider · Direct Detection · Relic Density

UV Stability

The Conformal Anomaly Condition

Conformal symmetry being fundamental means symmetry should not have anomaly.

↓

However, in our low-energy theory it is broken by quantum corrections.

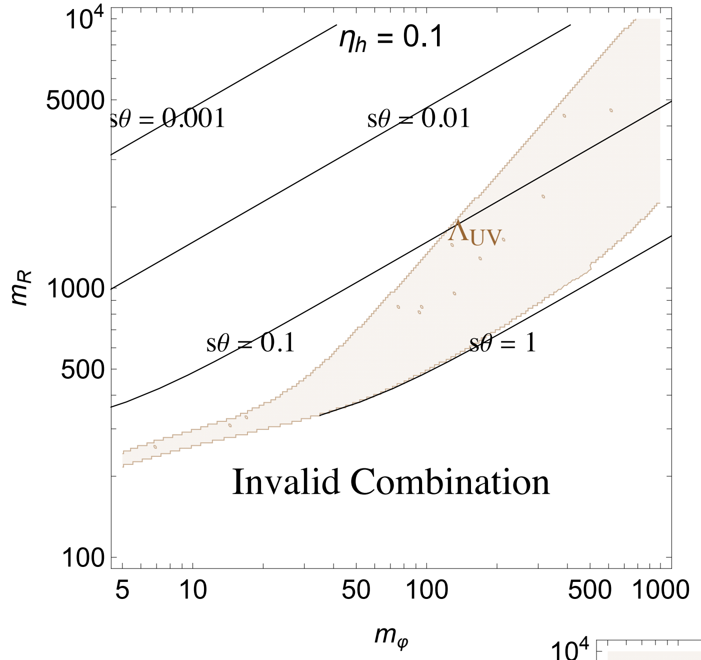
↓

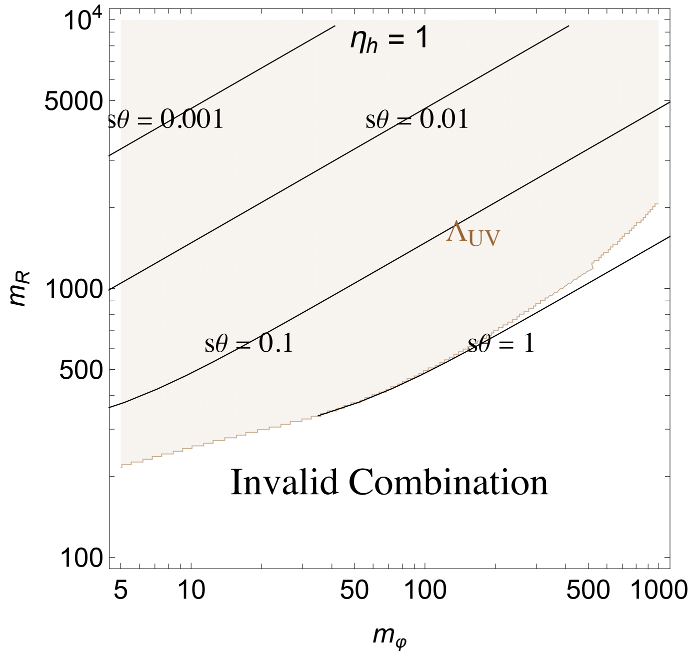
We need high-energy theory (gravity) to cancel the “trace anomaly”.

$$j^\mu = x_\mu T^{\mu\nu},\ 0 = \partial_\mu j^\mu = T^\mu_\mu$$

↓

The model should remain stable to the Planck scale, $\Lambda_{UV} > \Lambda_{Pl}$.



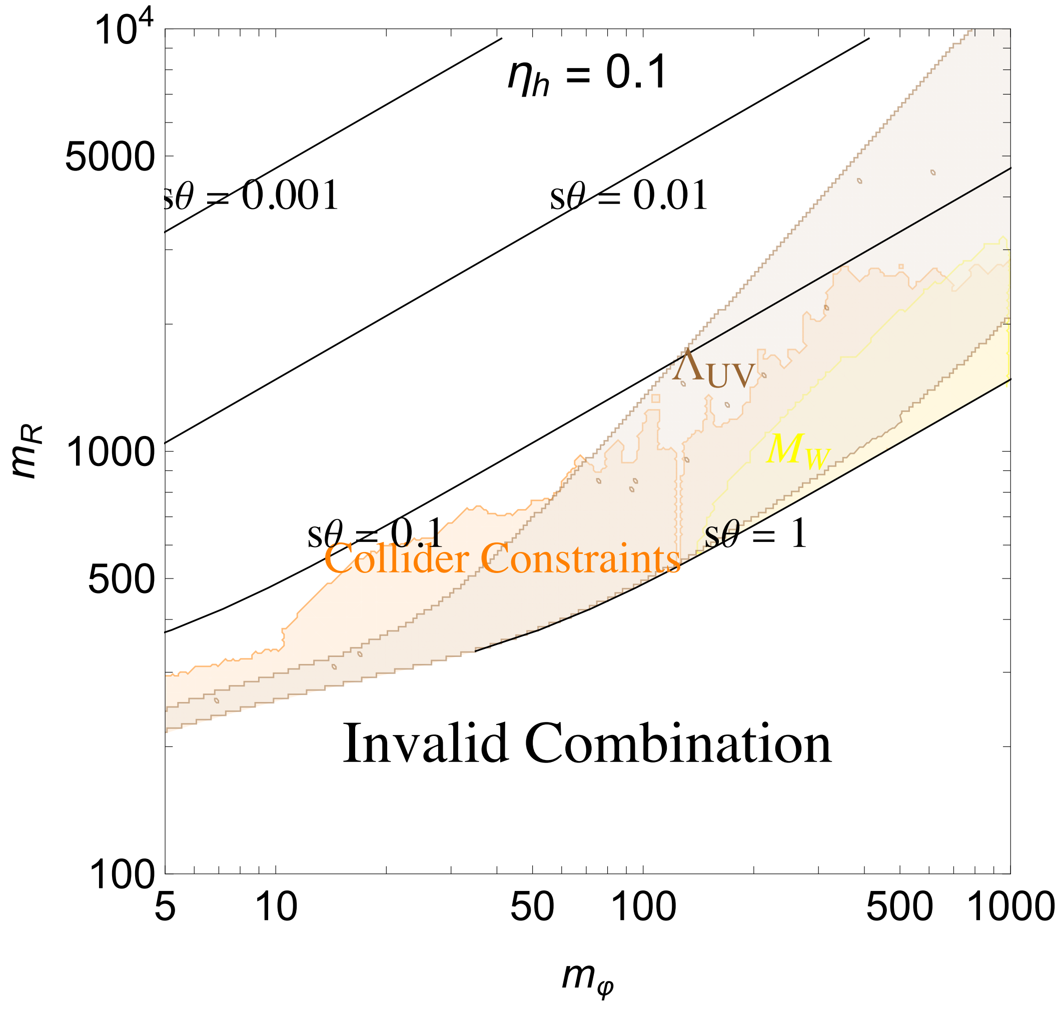


Higgs Mixing Constraints

Collider Constraint

Collider constraints
- Higgs searches
- Higgs signal

W boson mass constraint



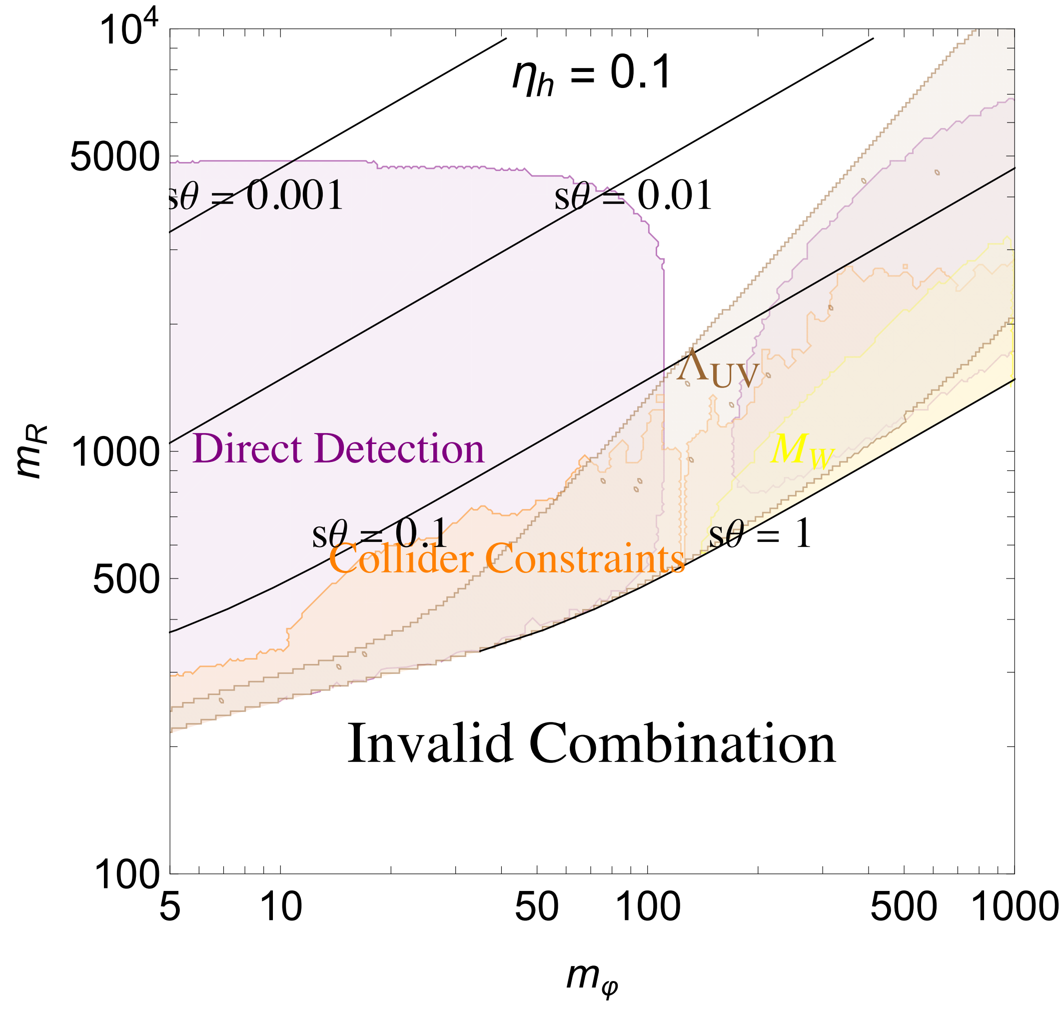
Direct Detection

Spin-Independent Cross Section

DM-nucleon scattering via exchange of h', φ.

$$\sigma_{SI} = \frac{f_N^2 m_N^4}{\pi m_{dm}^2 v^2} \left[\frac{c_\theta}{m_{h'}^2} \lambda_{h'SS} + \frac{s_\theta}{m_\phi^2} \lambda_{\phi SS} \right]^2$$

A cancellation gap appears when two channels have cancellation between each other.



Relic Density

Thermal Freeze-out

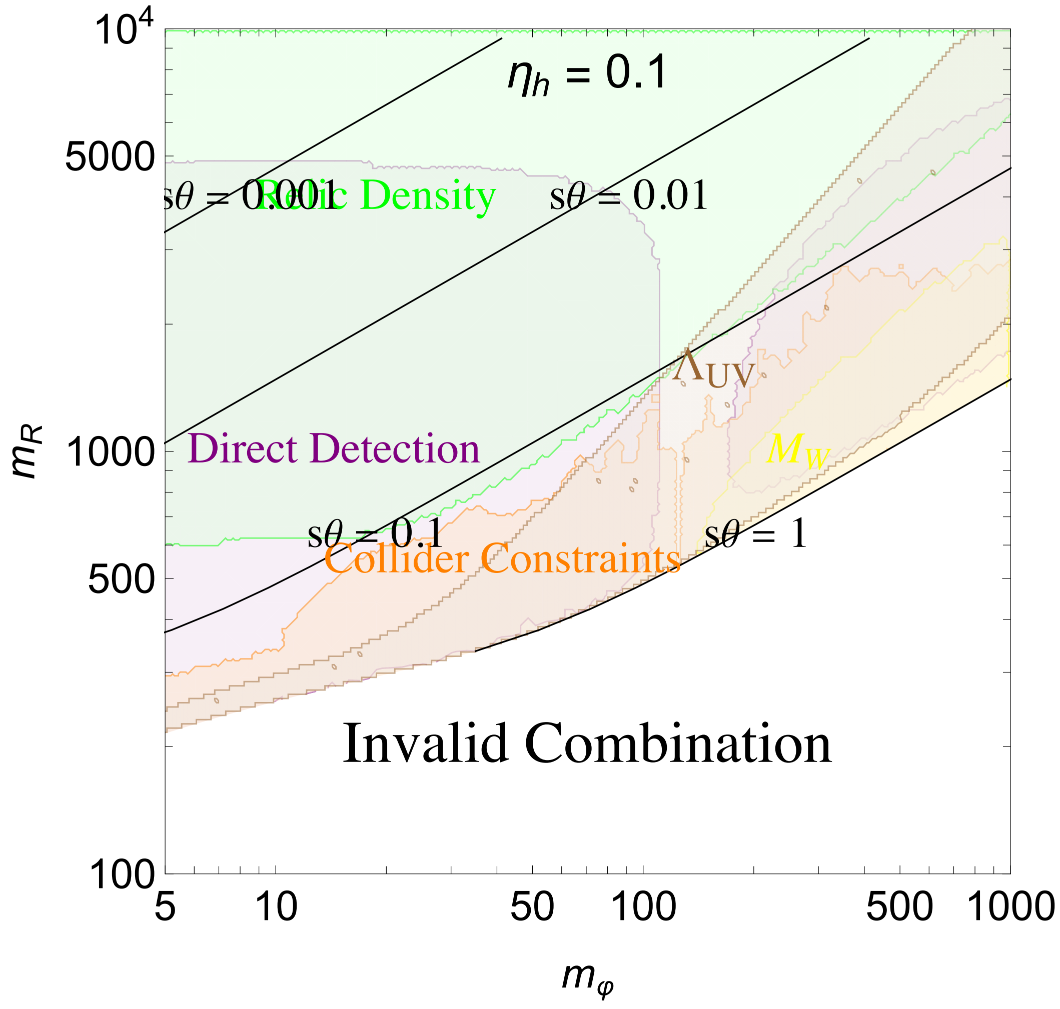
Estimate the freeze-out relic density to be

$$\sigma v \approx \frac{\eta_S^2 + 4\eta_h^2}{16\pi m_R^2}$$

Assuming R can act as part of dark matter

$$\sigma v \geq \frac{1}{M^2}, M \approx 21TeV$$

The parameter space is closed.



Freeze-in Reopening Parameter Space

Alternative DM Production · Reopened Window · EFT Considerations

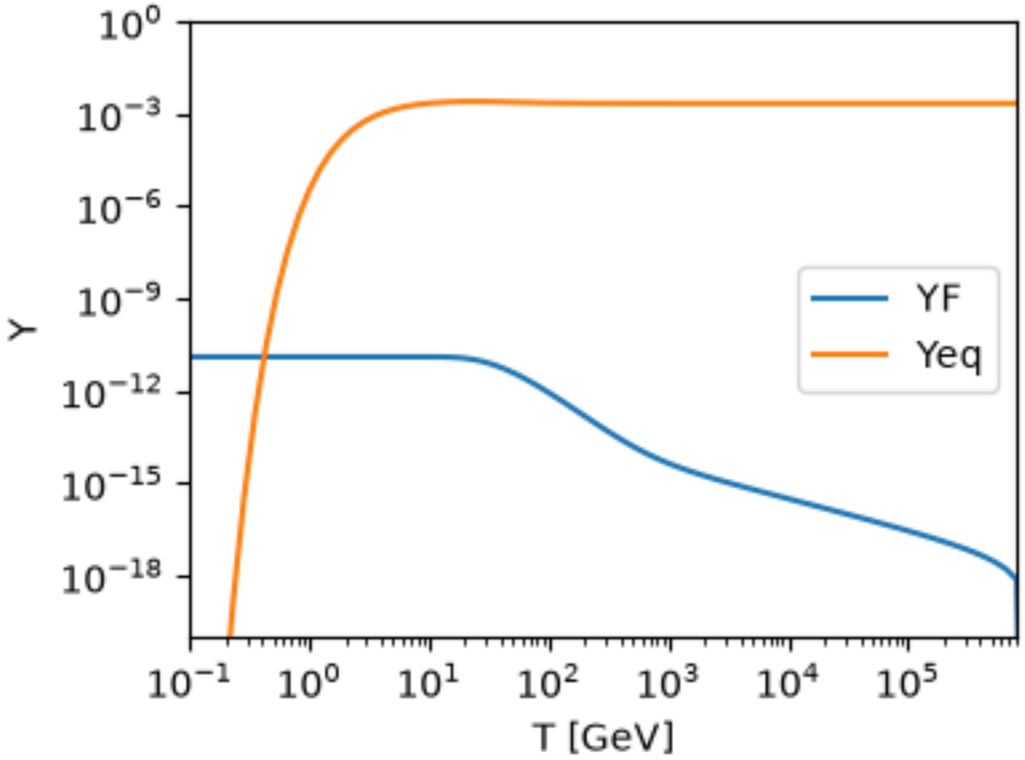
Freeze-in mechanism

Thermal Freeze-in

Consider dark matter couple to other particles so weakly that it never reaches equilibrium until it decouples.

That requires extreme small coupling $\lambda \sim 10^{-11}$.

However, it can explore the parameter space where freeze-out gives over-abundance.



Freeze-in: Reopening the Window

Attempt 1: Small couplings

If $\eta_S, \eta_h \sim 10^{-11}$, we cannot have large $m_R^2 = \eta_h v^2 + \eta_S v_S^2$, which violate direct detection constraints.

Attempt 2: Two-component DM

Consider both φ, R are out of equilibrium, so that η_S can be $\mathcal{O}(1)$. However, that introduces long-lived φ that can lead to more observational constraints.

↓

Resolution: Low Reheating Temperature

Set η_h small, and lower the reheating temperature T_R effectively Boltzmann-suppresses the η_S coupling.

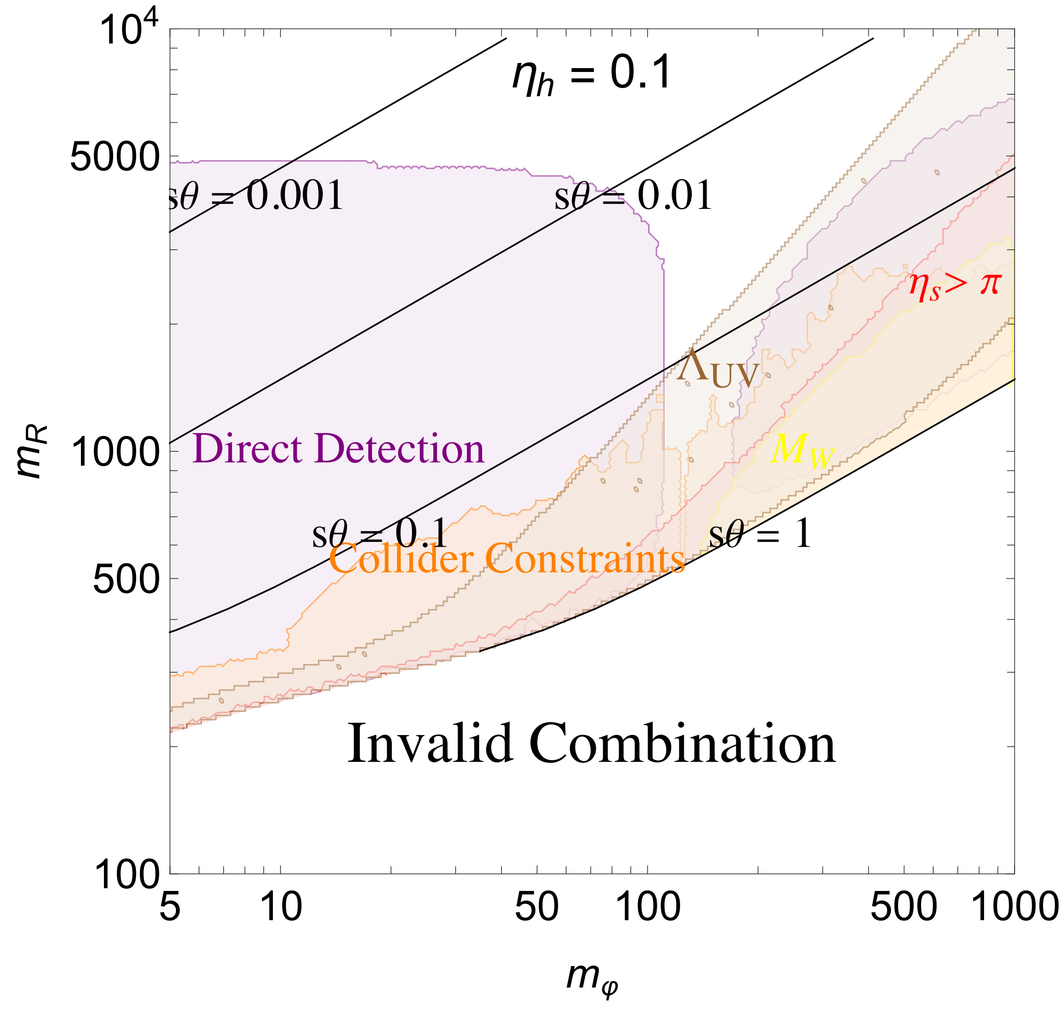
Reopened Parameter Space

Estimate T_R Required

Consider reheating ends at $T_R < m_{dm}$, then the production rate of dark matter is suppressed by $e^{-m_{dm}/T}$.

That suppresses n_{DM} today by e^{-2m_{dm}/T_R}, then to get correct relic density we estimate $T_R \approx m_{dm}/30$

T_R is only bounded >4MeV, so previously closed parameter space is reopened.



EFT Concerns

Mass Hierarchy Calls for EFT.

Small mixing angle induce a mass hierarchy between m_R^2 and $m_\varphi^2, m_{h'}^2$. That will introduce large logarithms thus challenging the perturbation regime.

Thus, in the future, we need effective field theory to properly treat it.

However, according to ArXiv: 2511.11367, the EFT will not change the result too much from only including leading log.

Conclusion

- Conformal symmetry solves the hierarchy problem

  No mass scale at tree level — naturalness is protected by symmetry.
  The Higgs mass emerges dynamically from radiative corrections, not from fine-tuned tree-level parameters.

- Radiative breaking generates physical scales

  Gildener-Weinberg mechanism: a flat direction in the tree-level potential acquires a minimum from quantum effects. All masses and scales are predictions, not inputs.

- Multiple constraints converge on a narrow window

  UV stability, collider Higgs mixing, direct detection, and relic density together define the viable parameter space. Standard freeze-out closes it.

- Freeze-in with low T_R reopens the DM window

  Boltzmann suppression at low reheating temperature enables freeze-in to match the observed relic density.

Thank you