Monodromy defects in Chern-Simons

Symmetries, defects, and holography

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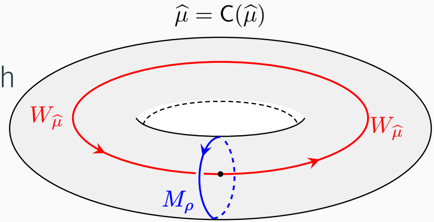
Theory Canada 18, Montreal

Based on work to appear soon with
Jaume Gomis & Suriyah Kannagi



Topological theory in 2+1 d

Gauge group G , level k

$$S_{\mathrm{CS}}[A] = \frac{k}{4\pi}\int_{\mathcal{M}^{(3)}} \mathrm{Tr}\left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A\right)$$

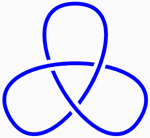
No local operators

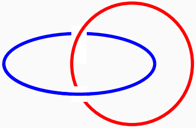
Line operators : $W_R(\mathcal{K}) = \mathrm{Tr}_R \, \mathcal{P} \exp \oint_{\mathcal{K}} A$

Topological lines $\leftrightarrow$ 1-form symmetry $\mathcal{Z}(G)$

Link and knots

Interesting observables on $\mathcal{M}^{(3)} = S^3$

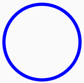
Knots $\leftrightarrow$ VEV $\quad \langle W_R(\mathcal{K}) \rangle_{S^3} = \Big\langle$  $\Big\rangle$

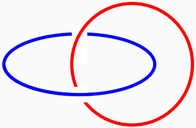
Links $\quad \langle W_{R_1}(\mathcal{K}_1) W_{R_2}(\mathcal{K}_2) \rangle_{S^3} = \Big\langle$  $\Big\rangle$

3D TFT- 2d CFT correspondance

Computed by 2d CFT!

$$\boxed{\mathrm{CS}(G)_k \longleftrightarrow G_k \quad \mathrm{WZW{-}model}}$$

(Un)knots $\quad \langle W_R(S^1)\rangle = \Big\langle$  $\Big\rangle = \boxed{\dfrac{S_{R0}}{S_{00}}}$

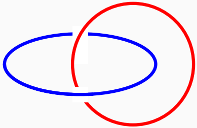
Hopf $\quad \langle W_{R_1}(S^1) W_{R_2}(S^1)\rangle = \Big\langle$  $\Big\rangle = \boxed{\dfrac{S_{R_1,R_2}}{S_{00}}}$

$$S_{ij} = \mathrm{Modular\,S{-}Matrix\ of\ } \widehat{\mathfrak{g}}_k^{(1)}$$

Charge conjugation symmetry

Symmetries of CS theory

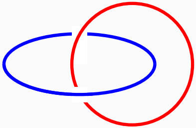
1-form symmetry $\longleftrightarrow$ Topological lines



Charge conjugation symmetry

Symmetries of CS theory

1-form symmetry $\longleftrightarrow$ Topological lines



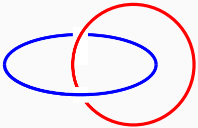
Today: **Charge conjugation**

0-form symmetry $\longleftrightarrow$ Topological surfaces!

Charge conjugation symmetry

Symmetries of CS theory

1-form symmetry $\longleftrightarrow$ Topological lines



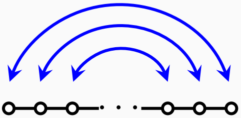
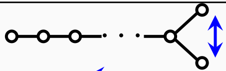
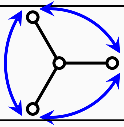
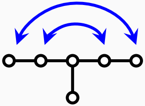
Today: **Charge conjugation**

0-form symmetry $\longleftrightarrow$ Topological surfaces!

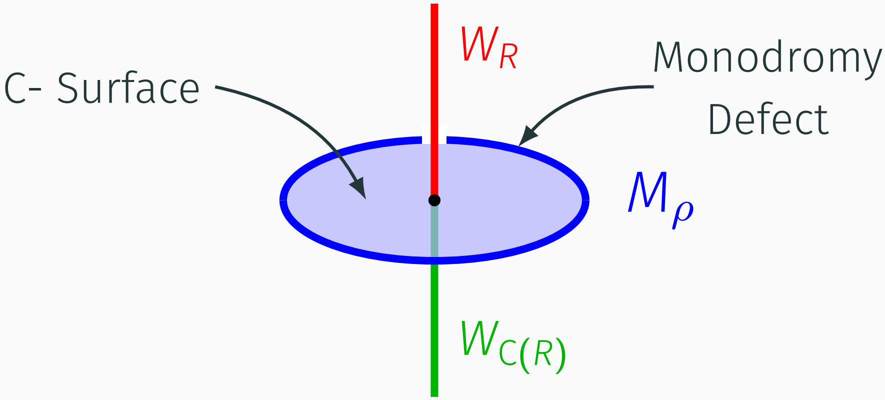
New observables in CS theory!

Charge conjugation symmetry

Charge conjugation C $\leftrightarrow \mathrm{Out}(\mathfrak{g})$

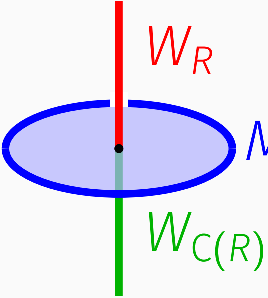
$\mathfrak{g}$	$\mathrm{Out}(\mathfrak{g})$	Dynkin diagram
$\mathfrak{su}(N)\quad N>2$	$\mathbb{Z}_2$	
$\mathfrak{so}(2N)$	$\mathbb{Z}_2$	
$\mathfrak{so}(8)$	S_3	
$\mathfrak{e}_6$	$\mathbb{Z}_2$	

Topological operators of charge conjugation



Monodromy defects! (Cod 2)

Monodromy defects:



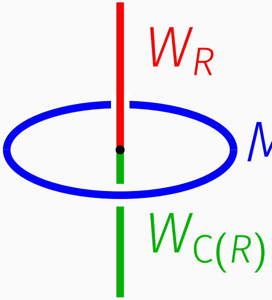
- Monodromy around M_ρ:

  $A_\mu(\phi + 2\pi i) = C \cdot A_\mu(\phi)$
- M_ρ line defects of CS

  → **How to compute with them?**
- CS theory ↔ Top Strings

  → **Fit in the duality?**

Monodromy defects:

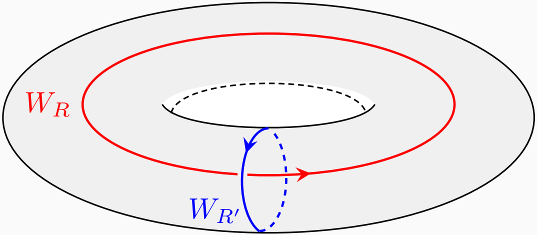


- ⋆ Monodromy around M_ρ: $A_\mu(\phi + 2\pi i) = \mathrm{C} \cdot A_\mu(\phi)$
- ⋆ M_ρ line defects of CS
  - → **How to compute with them?**
- ⋆ CS theory ↔ Top Strings
  - → **Fit in the duality?**

Monodromy defects from twisted 2d CFT

Quantization of CS on solid T^2

No monodromy defects:



$\mathcal{H}^{(1,1)} \leftrightarrow \widehat{\mathfrak{g}}_k^{(1)}$

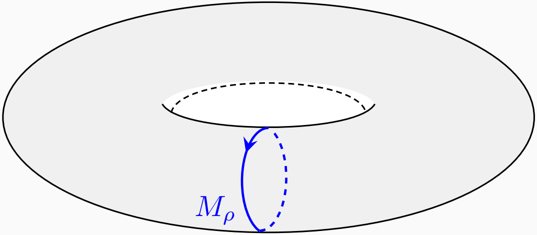
$R, R' \subset$ Int. rep $(\widehat{\mathfrak{g}}_k^{(1)})$

Modular-S $(\widehat{\mathfrak{g}}_k^{(1)})$

Monodromy defects from twisted 2d CFT

Quantization of CS on solid T^2

Monodromy defect wrapping A-cycle



$\mathcal{H}^{(C,1)} \leftrightarrow \widehat{\mathfrak{g}}_k^{(2)}$

$\simeq$ Conf.blocks$(\mathfrak{g}_k^{(2)})$

C-twisted boundary conditions on B cycle

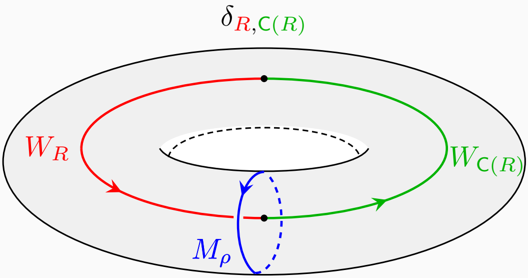
$$\mathcal{J}^\alpha(z\, e^{2\pi i}) = C(\mathcal{J}^\alpha(z))$$

Monodromy = New line defects $\leftrightarrow$ Int. Rep$(\widehat{\mathfrak{g}}^{(2)})$

Monodromy defects from twisted 2d CFT

Quantization of CS on solid T^2

Monodromy defect wrapping A-cycle, Wilson on B



$\mathcal{H}^{(\mathsf{C},1)} \leftrightarrow \widehat{\mathfrak{g}}_k^{(2)}$

$\simeq$ Conf.blocks$(\widehat{\mathfrak{g}}_k^{(2)})$

$\neq 0$ iff $R = \mathsf{C}(R)$

C-twisted boundary conditions on B cycle

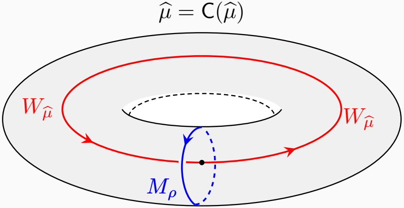
$$\mathcal{J}^\alpha(z\, e^{2\pi i}) = \mathsf{C}(\mathcal{J}^\alpha(z))$$

Monodromy = New line defects $\leftrightarrow$ Int. Rep$(\widehat{\mathfrak{g}}^{(2)})$

Twisted affine Lie algebras

$$\mathcal{H}^{(\mathsf{C},1)} \simeq \text{Conf.blocks}(\widehat{\mathfrak{g}}_k^{(2)})$$

Twisted affine $\widehat{\mathfrak{g}}_k^{(2)}$ areclassified[Fuchs, Schweigert][Gaberdiel, Gannon][Kac]

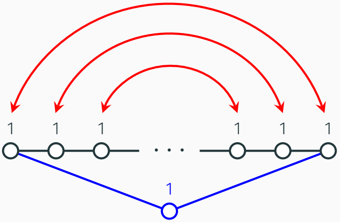


$= S^{(\mathsf{C},1)}_{\rho,\widehat{\mu}}$

- $\widehat{\mu}$: C-invariant weight of $\widehat{\mathfrak{g}}^{(1)}$ $\sim$ Wilson Lines that can link
- ρ: Int. Rep($\widehat{\mathfrak{g}}_k^{(2)}$) $\sim$ Monodromy defects

$\widehat{\mathfrak{g}}^{(2)}$ obtained by folding

Twisted affine Lie algebras: SU(N) example

$A_k^{(1)} \simeq$ 

Fold by automorphism:

$A_{2m-1}^{(1)} \mapsto D_{m+1}^{(2)}$: 

$A_{2m}^{(1)} \mapsto A_{2m}^{(2)}$: 

Some more details

$\mathfrak{g}^{(1)}$	$(1,1)$	$(C,1)$	$(1,C)$	(C,C)
$A^{(1)}_{2n-1}$ $(n \geq 2)$	$A^{(1)}_{2n-1}$	$A^{(2)}_{2n-1}$	$D^{(2)}_{n+1}$	$B^{(1)}_{n}$
$A^{(1)}_{2n}$ $(n \geq 1)$	$A^{(1)}_{2n}$	$A^{(2)}_{2n}$	$A^{(2)}_{2n}$	$A^{(2)}_{2n}$
$D^{(1)}_{n}$ $(n \geq 4)$	$D^{(1)}_{n}$	$D^{(2)}_{n}$	$A^{(2)}_{2n-3}$	$C^{(1)}_{n-1}$
$E^{(1)}_{6}$	$E^{(1)}_{6}$	$E^{(2)}_{6}$	$E^{(2)}_{6}$	$F^{(1)}_{4}$

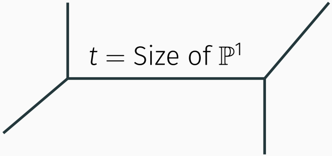
With exact results comes hard questions

Now we know how to compute exactly with monodromy defects

What can we learn about holography?

$SU(N)_k$ CS on S^3 $\leftrightarrow$ Top. Strings on Resolved Conifold

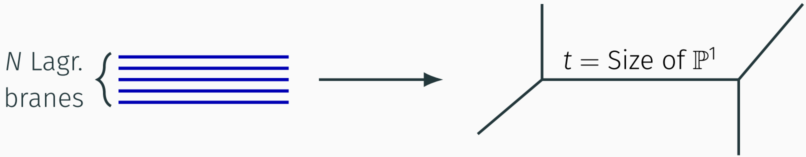
$$g_s = \frac{-2\pi i}{k+N}, \qquad \text{'t Hooft: } t = N g_s \quad \text{fixed}$$



Originally at Large N, even at finite!

Holographic duality

N Langrangian branes wrapping $S^3 \to$ geometry



$$\log S_{0,0}^{SU(N)} = (\text{non-per}) + \sum_{m>0} \frac{e^{-mt}}{m[m]^2} = \log \mathcal{Z}_{\text{con}}$$

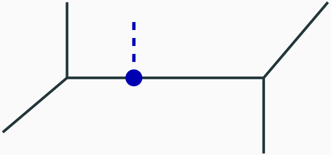
Partition function of the Resolved Conifold!

Count maps from $\Sigma_g \to \mathrm{CY}_3$

Holographic duality

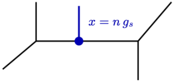
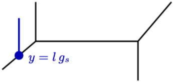
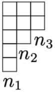
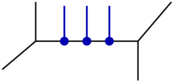
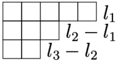
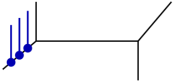
N Langrangian branes wrapping $S^3 \to$ geometry

We can add branes to the geometry:



$$N \to N+1 \quad t \to t + g_s$$
$$x \sim n$$

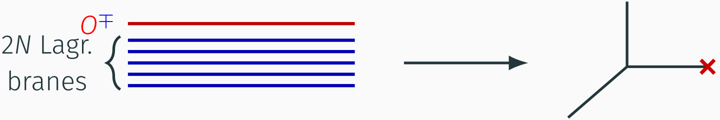
$$\log S^{SU(N)}_{\Lambda^n,0} = \sum_{m>0} \frac{e^{-m\widehat{t}}}{m[m]^2} + \sum_{m>0} \frac{e^{-mx} + e^{-m(\hat{t}-x)}}{m[m]} = \log \mathcal{Z}_{\mathrm{con}} + \mathcal{Z}_{\mathrm{open}}$$

$\langle W_R \rangle = S_{R0}$	Branes A-model	Toric diagram
Antisymm rank n	Brane in internal leg at $x = ng_s$ $\widehat{t} = t + g_s$	
Symm rank l	Anti-brane in external leg at $y = lg_s$ $\widehat{t} = t - g_s$	
	P branes in internal leg at $(x_1, \cdots, x_P) = (n_1, \cdots, n_P)g_s$ $\widehat{t} = t + Pg_s$	
	M Anti-branes in external leg at $(y_1, \cdots, y_M) = (l_1, \cdots, l_M)g_s$ $\widehat{t} = t - Mg_s$	

Adding orientifold planes

Projection by $\underbrace{\Omega}_{\text{Worldsheet parity}} \circ \underbrace{\sigma_-^{SO/Sp}}_{\text{Geometric involution}}$ [Hori, Vafa][Sinha,Vafa]

$\sigma_- : CY_3 \to CY_3$ target anti-hol inv. $\omega(z) = -1/\omega(\bar{z})$ on $\mathbb{P}^1$



$$\log S_{00}^{SO/Sp} = \underbrace{\frac{1}{2}\sum_m \frac{e^{-m\hat{t}}}{m[m]^2}}_{\text{oriented}} \mp \underbrace{\sum_{m\,\text{odd}} \frac{e^{-m\hat{t}/2}}{m[m]}}_{\text{not-oriented}}$$

$SU\,CS \to SO/Sp\,CS \qquad (\hat{t} = t \mp g_s)$

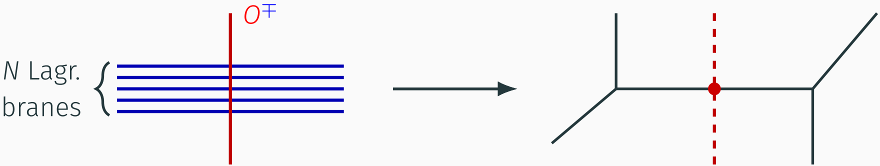
maps $\Sigma \to \mathrm{CY}_3$ and unoriented equivariant $\Sigma/\sigma_- \to \mathrm{CY}_3$ (e.g $\mathbb{RP}^2 \to \mathrm{CY}_3 \ldots$)

Adding monodromy defects

What is the dual to monodromy defects? We find:

$$\text{Projection by } \Omega \circ \sigma_{+}^{SO/Sp}$$

$\sigma_+ : CY_3 \to CY_3$ target anti-hol inv. $\omega(z) = 1/\omega(\bar{z})$
Fixed locus $S^1 \subset S^3$ ($\mathbb{RP}^1 \subset \mathbb{P}^1$): support of M_0



$$\log S_{00}^{(C,1)} = \underbrace{\frac{1}{2}\sum_m \frac{e^{-m\hat{t}}}{m[m]^2}}_{\text{oriented}} \mp \underbrace{\sum_{m\ \text{even}} \frac{e^{-m\hat{t}/2}}{m[m]}}_{\text{not-oriented}}$$

Monodromy defects add a **non-oriented sector!**

Tricky signs

Theory		Background
$SU(2N+1)_k$,	k odd	$\Omega\sigma_+^{\mathrm{SO}} \oplus D$
$SU(2N+1)_k$,	k even	$\Omega\sigma_+^{\mathrm{SO}}$
$SU(2N)_k$,	k odd	$\Omega\sigma_+^{\mathrm{Sp}}$
$SU(2N)_k$,	k even, $N \leq k$	$\Omega\sigma_+^{\mathrm{Sp}}$
$SU(2N)_k$,	k even, $2N > k+2$	$\Omega\sigma_+^{\mathrm{SO}}$

Very non trivial dependence on k, N

New feature of monodromy defects

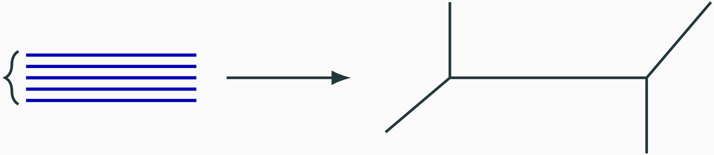
You can ask about it if interested

Monodromy defects of $SO(2N)$

SO(2N) has "chirality flip":

Start with $SU(2N)$: $t = 2Ng_s$

$2N$ Lagr. branes



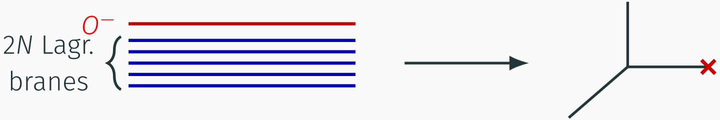
Using $D_N^{(2)}$ we find:

$$\log S_{00}^{(C,1)} = \underbrace{\frac{1}{2}\sum_m \frac{e^{-m\hat{t}}}{m[m]^2}}_{\text{oriented}} \mp \underbrace{\sum_{m\ \text{even}} \frac{e^{-m\hat{t}/2}}{m[m]}}_{\text{not}-\text{oriented}}, \quad \hat{t} = (2N-2)g_s \checkmark$$

Monodromy defects of $SO(2N)$

SO(2N) has "chirality flip":

Insert O^-: $t \to (2N-1)g_s$



$$\{1, \Omega\sigma_-\}$$

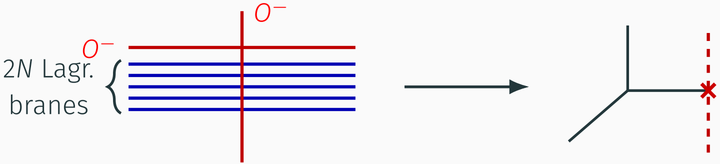
Using $D_N^{(2)}$ we find:

$$\log S_{00}^{(C,1)} = \underbrace{\frac{1}{2}\sum_{m} \frac{e^{-m\hat{t}}}{m[m]^2}}_{\text{oriented}} \mp \underbrace{\sum_{m \text{ even}} \frac{e^{-m\hat{t}/2}}{m[m]}}_{\text{not}-\text{oriented}}, \quad \hat{t} = (2N-2)g_s \checkmark$$

Monodromy defects of $SO(2N)$

SO(2N) has “chirality flip”: 

Insert other O^- plane with fixed locus: $t \to (2N-2)g_s$

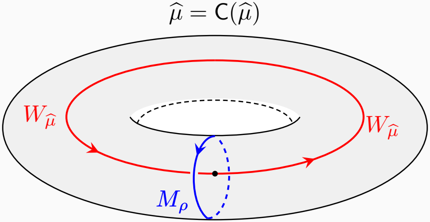


$$\{1, \Omega\sigma_-, \Omega\sigma_+, \underbrace{\sigma_-\sigma_+}_{\text{orbifold}}\} \simeq \mathbb{Z}_2 \times \mathbb{Z}_2$$

Using $D_N^{(2)}$ we find:

$$\log S_{00}^{(C,1)} = \underbrace{\frac{1}{2}\sum_m \frac{e^{-m\hat{t}}}{m[m]^2}}_{\text{oriented}} \mp \underbrace{\sum_{m\,\text{even}} \frac{e^{-m\hat{t}/2}}{m[m]}}_{\text{not}-\text{oriented}}, \quad \hat{t} = (2N-2)g_s \checkmark$$

Summary



$$= S^{(\mathsf{C},1)}_{\rho,\widehat{\mu}}$$

- Monodromy = ∂ Charge conjugation defects
- New line defects of CS:
    - Wilson lines $\leftrightarrow \mathrm{Int}(\widehat{\mathfrak{g}}_k^{(1)})$
    - Monodromy defects $\leftrightarrow \mathrm{Int}(\widehat{\mathfrak{g}}_k^{(2)})$
- Non-oriented sector in topological strings!
- Test of holography at finite N and k
- New Gromov-Witten invariants, new knots invariants?

Resolved conifold

Simplest CY_3: $\mathcal{O}(-1) \oplus \mathcal{O}(-1) \to \mathbb{P}^1$:

$$\{\underbrace{|z_1|^2 + |z_4|^2}_{\mathbb{P}^1 \text{ size t}} - |z_2|^2 - |z_3|^2 = t\}$$

Start from $\quad T^*S^3 \simeq xy - uv = a \sim 1/N$

$a \to 0 \quad$ singular, resolve by $\quad x = \lambda v, u = \lambda y$

$$x = z_1 z_3, \quad y = z_2 z_4, \quad u = z_1 z_2, \quad v = z_2 z_3, \quad \mathbb{P}^1 \ni \lambda = z_1/z_4$$

gives exactly the resolved conifold. S^3 is Lagrangian in T^*S^3
branes wrapping
Transition replaces S^3 by $\mathbb{P}^1$

Involutions

$$\sigma_- : \{z_1, z_2, z_3, z_4\} \mapsto \{\overline{z_4}, -\overline{z_3}, -\overline{z_2}, \overline{z_1}\}$$

Fixed locus : $z_1 z_4 - z_2 z_3 = |z_1|^2 + |z_2|^2 = t \leftrightarrow S^3$

On the $\mathbb{P}^1 \ni z$ after transition $f(z) \mapsto -f(1/\bar{z})$

$$\sigma_- : \{z_1, z_2, z_3, z_4\} \mapsto \{\overline{z_1}, \overline{z_2}, \overline{z_3}, \overline{z_4}\}$$

Fixed locus : $z_i \in \mathbb{R}$: real quartic $z_1 z_4 - z_2 z_3 = t$ $f(z) = 1/f(\bar{z})$

$$\mathrm{Fix}(\sigma_+) \cap \mathbb{P}^1 \simeq S^1$$

$$\text{Clearly :} \qquad \sigma_\pm^2 = 1 = \Omega^2$$

$$\text{Orbifold :} \qquad \sigma_+\sigma_- : \{z_1, z_2, z_3, z_4\} \mapsto \{z_4, -z_3, -z_2, z_1\} \quad \text{holomorphic}$$

Action of charge conjugation

$\mathfrak{su}(N)$:

$$A_\mu(ze^{2\pi i}) = \mathsf{C}\, A_\mu(z) = A_\mu^*(z) = -A_\mu^T$$
$$W_R \to W_{\bar{R}}$$

$\mathfrak{so}(2N)$:

$$A_\mu(ze^{2\pi i}) = \mathsf{C}\, A_\mu(z) = MA_\mu(z)M^T$$
$$M \in O(2N)/SO(2N), \qquad W_{S^+} \to W_{S^-}$$

Non trivial result: $M \leftrightarrow$ odd partition of $2N$

$$\mathrm{Inv\,algebra} := SO(2N-1) \to SO(N_1) \times SO(N_2)$$
$$N_1 + N_2 = 2N - 1$$

Resulting $\widehat{\mathfrak{g}}_k^2$ are isomoprhic