Stability of R-Charged Membranes

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Based on A.Buchel and R.Monten, ”Near-extremal membranes in M-theory”, arXiv: 2511.01974

Consider IIB SUGRA on S^5, dual to $\mathcal{N} = 4$, SU(N_c) SYM:

$\Longrightarrow$ Low temperature limit violates third law: $\frac{s}{c}\big|_{T=0} \propto \mu^3$, $\qquad c = \frac{N_c}{4}$

Known instability resolutions:

- Superconducting (s-c)
- Thermodynamic/Hydrodynamic[a]
- Renormalization (RN)
- Fermi-Seasickness
- Quantum corrections

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[a] Thermo and Hydro instabilities are equivalent for SYM plasmas: Gladden-Ivo-Kovtun-Starinets (GIKS), Instability in N=4 supersymmetric Yang-Mills theory at finite density, arXiv:2412.12353

$\Longrightarrow$ The quest for no s-c instabilities and extremal horizon led to

- “Baryonic Black Branes” (Herzog-Klebanov-Pufu-Tesileanu), hep-th/0911.0400
  - has non-perturbative brane nucleation instability
  - has perturbative charge clumping instability, (AB arXiv:2502.05971)

- **”Baryonic Black Membranes”**, (Klebanov-Pufu-Tesileanu), hep-th/1004.0413
  - no non-perturbative brane nucleation instability
  - no s-c instabilities

Alternative formulation: 11D supergravity truncated over the Sasaki-Einstein coset $Q^{1,1,1} = \frac{\mathrm{SU}(2)^3}{\mathrm{U}(1)^2}$ with fluxes[a]:

$$S_{CKV} = \frac{1}{\kappa_4^2} \int_{\mathcal{M}_4} [\ \frac{1}{2} R \star 1 - \left\{ (\partial\phi)^2 + g_{ij} \partial t^i \partial \bar{t}^j \right\} \star 1 - \frac{1}{4} e^{-4\phi} dB \wedge \star dB$$

$$+ \frac{1}{4} \mathrm{Im} \mathcal{N}_{IJ} F^I \wedge \star F^J + \frac{1}{4} \mathrm{Re} \mathcal{N}_{IJ} F^I \wedge F^J - \frac{1}{2} e_0 dB \wedge A^0 - V_{CKV}(\phi, b_i, v_i) \star 1]$$

$$F^0 = \mathcal{F}^0 \qquad F^i = \mathcal{F}^i - 2B \qquad \mathcal{F}^I = dA^I$$

Truncation terms: v_i, ϕ, A^0

Flux terms: b_i, A^i, $B \rightarrow A^H = 4e^{4\phi} \star dB$

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[a] D. Cassani, P. Koerber and O. Varela, All homogeneous $N = 2$ M-theory truncations with supersymmetric AdS4 vacua, JHEP 11 (2012) 173, [1208.1262]

Sub-truncation to holographic $\mathcal{N}=2$ from $Q^{1,1,1} \to M^{1,1,0} = \frac{\mathrm{SU}(3)\times\mathrm{SU}(2)}{\mathrm{SU}(2)\times\mathrm{U}(1)}$:

$$A^3 \equiv A^1 \qquad v_3 \equiv v_1 \qquad b_3 \equiv b_1$$

Reproduce KPT model using

$$b_i = 0 \qquad B = 0 \qquad \sum_i \frac{\mathcal{F}^i}{v_i^2} = 0$$

Two charged background solutions: R-charge and baryonic charge. R-charge background found by truncate to minimal $\mathcal{N}=4$ gauged supergravity:

$$v_i = 1 \quad b_i = 0 \quad A^H = 0 \quad \Longleftrightarrow \quad \mathcal{F}^1 = \mathcal{F}^2 = \star\mathcal{F}^0$$

$U(1)_R$-charged RN background solution in Poincaré coordinates:

$$ds_4^2 = -\tfrac{4^2 f}{r^2}\, dt^2 + \tfrac{4\alpha^2}{r^2}\left(dx_1^2 + dx_2^2\right) + \tfrac{1}{4r^2 f}\, dr^2, \qquad f = 1 - (1+q^2)r^3 + q^2 r^4$$

$$a_0 = 2q\alpha(1-r) \qquad \mathcal{F}^0 = a_0'\, dr \wedge dt \qquad \mathcal{F}^1 = \mathcal{F}^2 = -8\alpha^2 q\, dx_1 \wedge dx_2$$

- Horizon at $r = 1^-$
- Boundary at $r = 0^+$
- Chemical potential $\mu_R = 2q\alpha$
- Temperature $T = \alpha(3 - q^2)/\pi$
- $q_{crit} = \sqrt{3}$

Linearized fluctuations about the background solution with A^i turned on electrically:

$$A^0 = a_0\,dt + \delta A^0_{x_1}\,dx_1 + \delta A^0_r\,dr, \qquad A^H = \delta A^H_{x_1}\,dx_1$$
$$\delta A^i = \delta A^i_t\,dt + \delta A^i_{x_2}\,dx_2 + \delta A^i_r\,dr, \qquad b_i = \delta b_i$$

Electrical fluctuations in the x_2-direction:

$$\delta A^{I,H}(r,t,x_2) = e^{-iwt+ikx_2}\mathcal{A}^{I,H}(r)$$
$$\delta b_i(r,t,x_2) = e^{-iwt+ikx_2}\mathcal{B}_i(r)$$

Table 1: Holographic spectroscopy of the pseudoscalars

mass eigenstate	m^2L^2	Δ	$U(1)$ R-charge
$b_1 - b_2$	-2	$(2,1)$	0
$2b_1 + b_2$	10	5	0

Focusing solely on fluctuations coupling to $\mathcal{A}_t^i$, EOM reduce to:

- $B \equiv \mathcal{B}_1 - \mathcal{B}_2$
- $Z \equiv \hat{k}\left(\mathcal{A}_t^1 - \mathcal{A}_t^2\right) + \hat{w}\left(\mathcal{A}_{x_2}^1 - \mathcal{A}_{x_2}^2\right)$
- $\hat{k} = \frac{k}{2\pi T}$, $\hat{w} = \frac{w}{2\pi T}$

$\Longrightarrow$

$$Z'' = \frac{\hat{w}^2 f'}{f\left(\hat{k}^2 f - \hat{w}^2\right)} Z' + \frac{T^2 \pi^2 \left(\hat{k}^2 f - \hat{w}^2\right)}{4 f^2} Z - 4q\hat{k}B' + \frac{4q\,\hat{w}^2 \hat{k} f'}{f\left(\hat{k}^2 f - \hat{w}^2\right)} B$$

$$B'' = \left(\frac{2}{r} - \frac{f'}{f}\right) B' - \frac{\hat{k} q\, r^2}{\hat{k}^2 f - \hat{w}^2} Z'$$

$$+ \frac{\left(\hat{k}^2 \left(T^2 \hat{k}^2 \pi^2 r^2 - 8q^2 r^4 - 8\right) f^2 - 2\hat{w}^2 \left(T^2 \hat{k}^2 \pi^2 r^2 + 4q^2 r^4 - 4\right) f + \pi^2 T^2 r^2 \hat{w}^4\right)}{4 r^2 f^2 \left(\hat{k}^2 f - \hat{w}^2\right)} B$$

Imposing normalizability at the boundary and selecting the $Re[\hat{w}] = 0$ diffusive branch gives:

$$Z = (1-r)^{-i\hat{w}/2}\, z, \quad B = (1-r)^{-i\hat{w}/2}\, b, \quad \hat{w} = -i\hat{k} \cdot v(\hat{k})$$

The baryonic-charge diffusion constant is then given by:

$$\mathcal{D} \equiv \left.\frac{dv}{d\hat{k}}\right|_{\hat{k}=0}$$

An analytic solution for $\mathcal{D}$ is possible at $q = 0$.

Solving the a perturbative expansion for $z = z_0 + z_1 \hat{k} + z_2 \hat{k}^2 \cdots$

$$z_2(r) = \frac{\left(\mathcal{D}^2 - \frac{9r}{4}\right)\sqrt{3}\,\arctan\!\left(\frac{(2r+1)\sqrt{3}}{3}\right)}{3} + \frac{\left(4\mathcal{D}^2 + 9r + 18\right)\ln\!\left(r^2 + r + 1\right)}{24}$$
$$+ \frac{\left(-4\mathcal{D}^2 + 9\right)\ln(r-1)}{12} + \frac{(2\mathcal{D} - 3)\ln(r-1)\,r}{4} + \frac{(1-r)\,\mathcal{D}}{2} + \frac{9\ln(4/3)}{16} - \frac{3}{4}$$

Solution must be regular at the horizon

$\Longrightarrow\ \mathcal{D} = \frac{3}{2}$

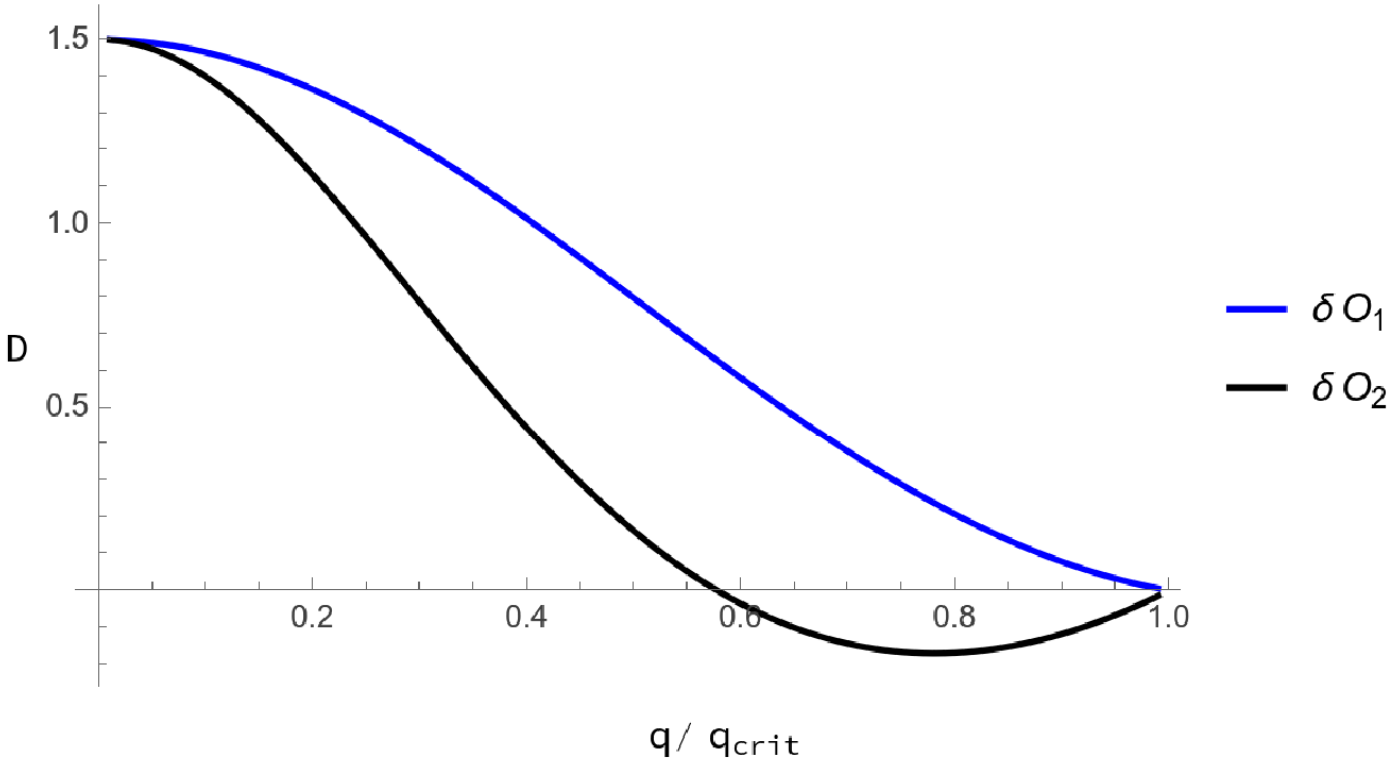
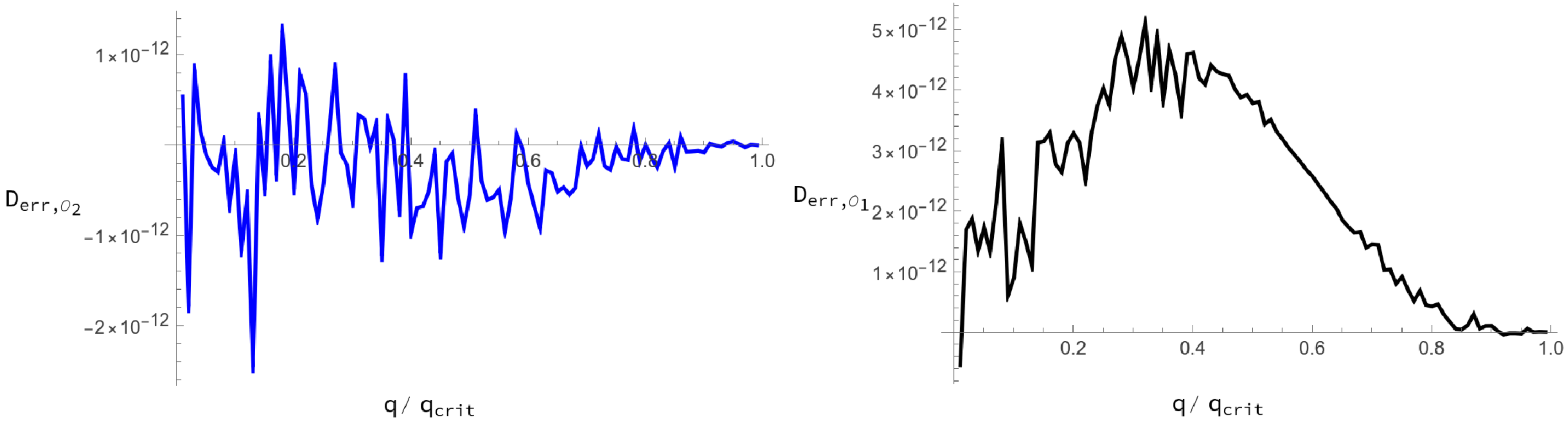


Figure 1: Numerical evaluation of the baryonic charge diffusion coefficient $\mathcal{D}$ for different quantizations of the gravitational dual pseudoscalar $(b_1 - b_2)$: $\{\delta\mathcal{O}_2\}$ (black), $\{\delta\mathcal{O}_1\}$ (blue)

More generally, we can expand around $r = 1$ for each perturbative term.

- z_0, z_1, b_0, b_1, b_2 contain taylor series expansions
- $z_2, b_3 \cdots$ require additional $\ln(r-1) \cdot (r-1)^n$ terms
- $\ln(r-1)$ must vanish in z_2 expansion

$$\Longrightarrow \mathcal{D} = \frac{1}{2} \cdot \frac{q\left(3-q^2\right) z_1(1)}{\left(3-q^2\right) b_0'(1) - 2\left(1-q^2\right) b_0(1)}$$



Error between using the approximate horizon parameterization D for quantizations of the mode $\ln[v_1 v_2^{-1}]$: $\mathcal{O}_2$ (black) and $\mathcal{O}_1$ (blue)

Conclusion:

- Alternate Quantization $b_1 - b_2 \Longleftrightarrow \mathcal{O}_1$ contains a hydrodynamic instability at $q = 1$
- Normal Quantization $\mathcal{O}_2$ has no know instabilities
- Diffusion coefficients can be found perturbatively in terms of values at the black membrane → consistent with membrane paradigm

Thank you!