# Unconventional Pairing in Interacting Ladders

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20/06/2026

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# Synthetic Quantum Platforms in search of exotic phases of matter

**Physical Review X**

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## Observation of Magnon Bound States in the Long-Range, Anisotropic Heisenberg Model

Florian Kranzl[1,2,*], Stefan Birnkammer[3,4,*], Manoj K. Joshi[1], Alvise Bastianello[3,4], Rainer Blatt[1,2], Michael Knap[3,4,†], and Christian F. Roos[1,2,‡]

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Phys. Rev. X 13, 031017 – **Published 11 August, 2023**

DOI: https://doi.org/10.1103/PhysRevX.13.031017

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## Programmable quantum simulations of spin systems with trapped ions

C. Monroe*, W. C. Campbell, L.-M. Duan, Z.-X. Gong, A. V. Gorshkov, P. W. Hess, R. Islam, K. Kim, N. M. Linke *et al.*

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## Strongly interacting Meissner phases in large bosonic flux ladders

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Jad C. Halimeh, Monika Aidelsburger, Fabian Grusdt, Philipp Hauke & Bing Yang

*Nature Physics* 21, 25–36 (2025) | Cite this article

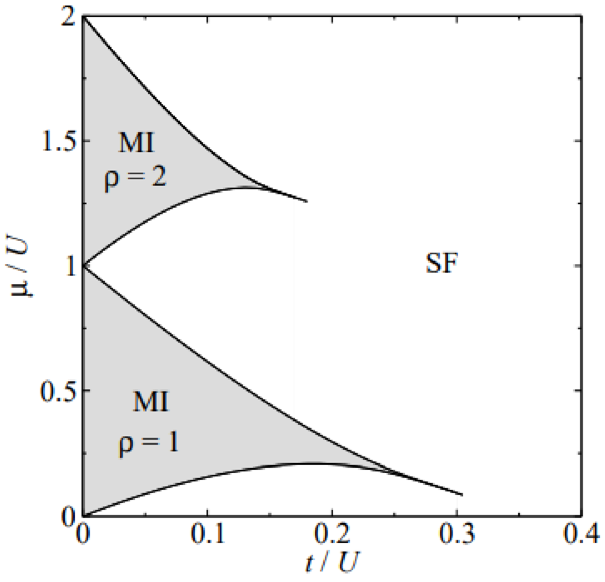
- We focus on **Bosonic systems**, described by **low-dimensional interacting systems** (e.g. – *two leg ladder* ).
- We try to go **beyond 'Flow-no-Flow' binary** of Bose-Hubbard model.

**Phases of interacting bosons from Bose-Hubbard Model ( Phys. Rev. A 85, 053644, 2012 )**

$$\hat{H} = t\hat{T} + U\hat{D} \,,$$

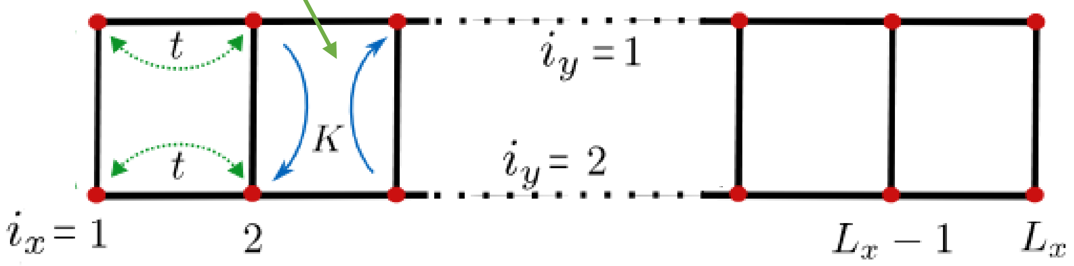
$$\hat{T} = -\sum_{j=1}^{L}(\hat{b}_j^\dagger \hat{b}_{j+1} + \hat{b}_{j+1}^\dagger \hat{b}_j)$$

$$\hat{D} = \frac{1}{2}\sum_{j=1}^{L}\hat{n}_j(\hat{n}_j - 1) \,.$$



# We introduce **Ring-Exchange** processes to obtain non-trivial pairing

$$H_K = K \sum_{i_x} b^{\dagger}_{i_x,2} b_{i_x,1} b^{\dagger}_{i_x+1,1} b_{i_x+1,2}.$$




**The bare minimum of a reduced Two-Dimensional systems: quasi-one-dimensional Ladder.**

**The Ring Exchange requires another dimension. The physics may not remain effectively one-dimensional !!**

# The Importance of Ring Exchange has been studied before !!!

PRL **106,** 046402 (2011) PHYSICAL REVIEW LETTERS week ending 28 JANUARY 2011

**Exotic Gapless Mott Insulators of Bosons on Multileg Ladders**

Matthew S. Block,[1] Ryan V. Mishmash,[1] Ribhu K. Kaul,[2] D. N. Sheng,[3]
Olexei I. Motrunich,[4] and Matthew P. A. Fisher[1,4]

[2]*Dep*
[3]*Depart*
[4]*D*

PHYSICAL REVIEW B **78**, 054520 (2008)

**Strong-coupling phases of frustrated bosons on a two-leg ladder with ring exchange**

D. N. Sheng,[1] Olexei I. Motrunich,[2] Simon Trebst,[3] Emanuel Gull,[4] and Matthew P. A. Fisher[3]
[1]*Department of Physics and Astronomy, California State University, Northridge, California 91330, USA*
[2]*Department of Physics, California Institute of Technology, Pasadena, California 91125, USA*
[3]*Microsoft Research, Station Q, University of California, Santa Barbara, California 93106, USA*

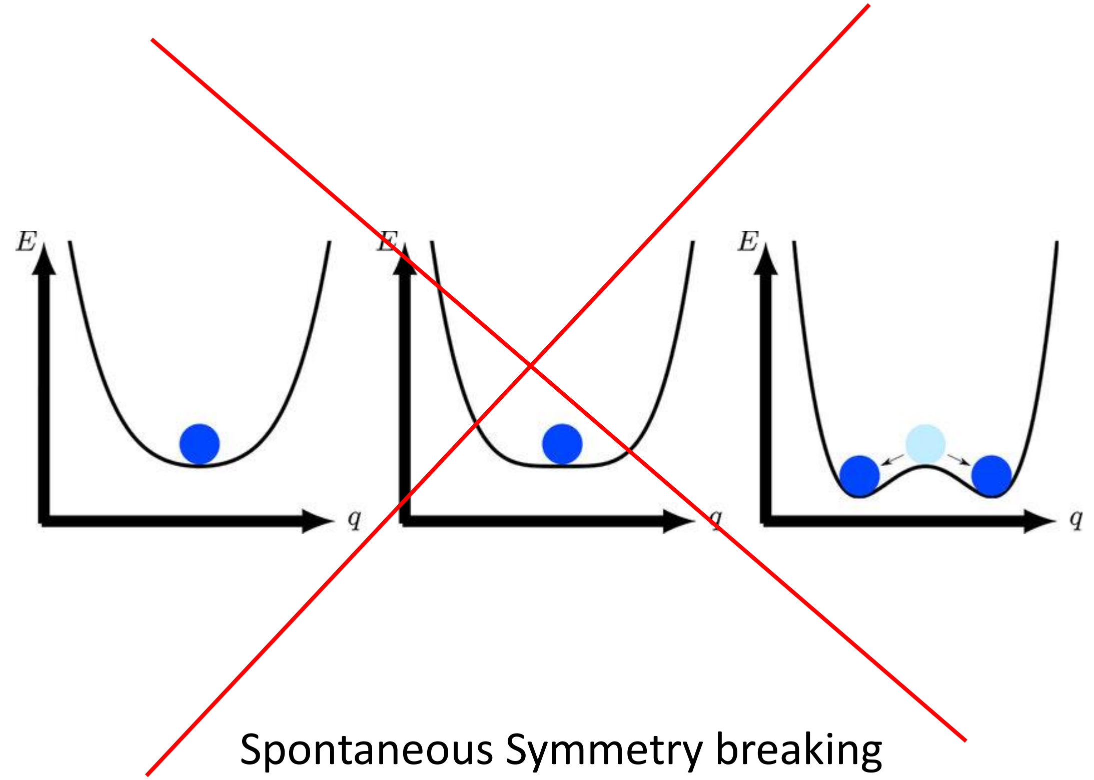
PHYSICAL REVIEW B **75**, 235116 (2007)

***d*-wave correlated critical Bose liquids in two dimensions**

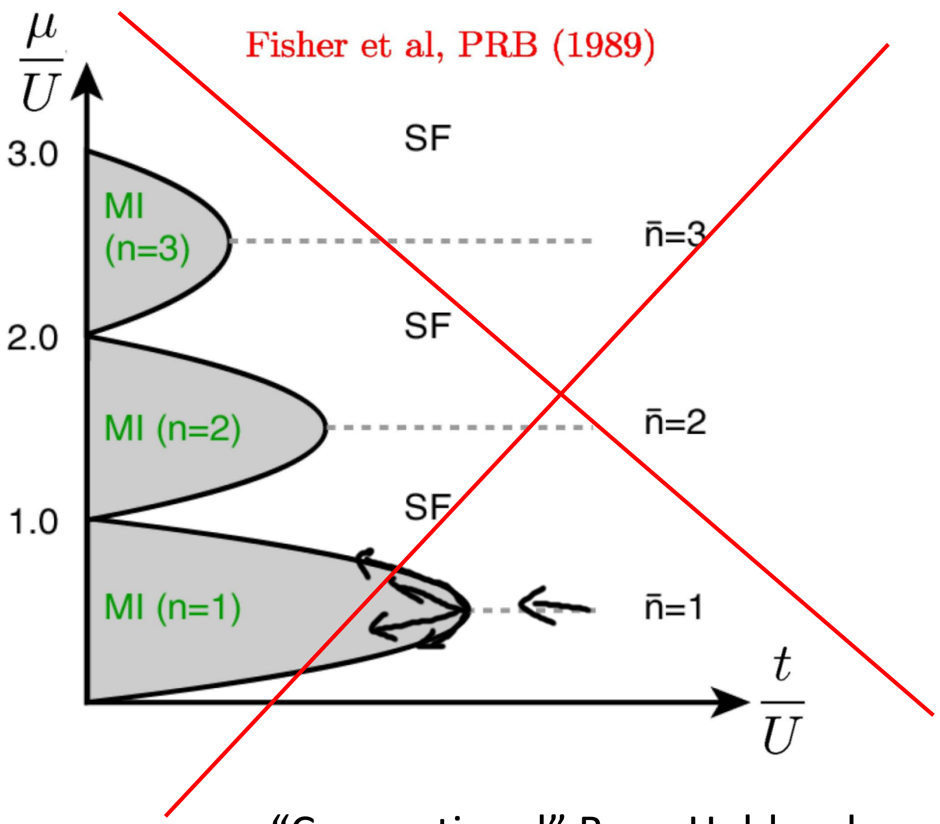
Olexei I. Motrunich
*Department of Physics, California Institute of Technology, Pasadena, California 91125, USA*

Matthew P. A. Fisher
*Kavli Institute for Theoretical Physics, University of California, Santa Barbara, California 93106, USA*

# It enables us to investigate exotic phases where :




Spontaneous Symmetry breaking




"Conventional" Bose-Hubbard

## INSTEAD,

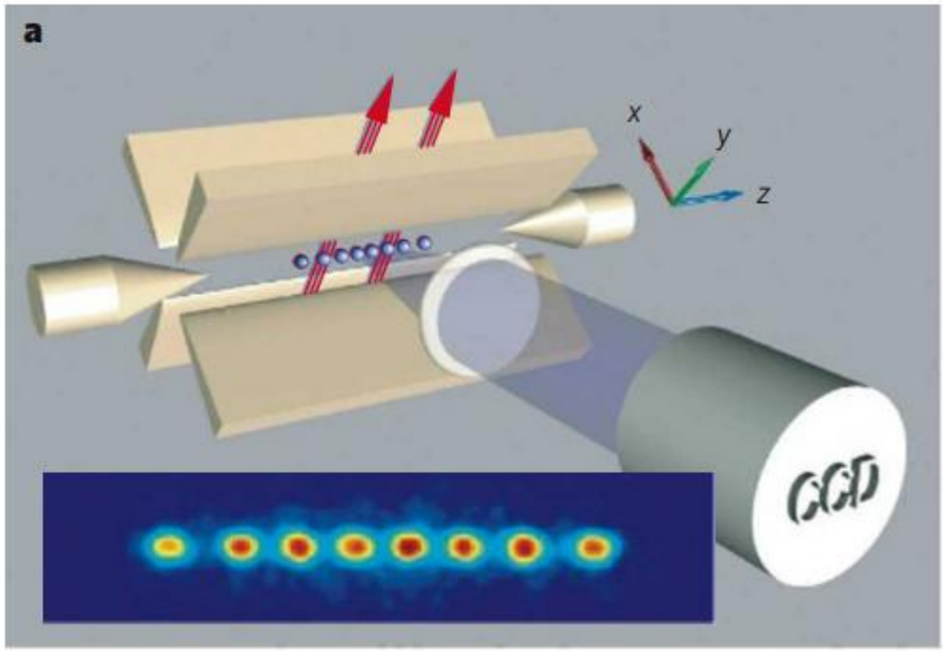
A gapless/metallic state with Fermi-surface like singularity can be achieved in a bosonic system. **It emerges as a gapless many-body state with 'd-wave pairing' due to Ring-Exchange!!**

# But How to Realize them !!?

# A chain of trapped three-level Ions can be an answer !

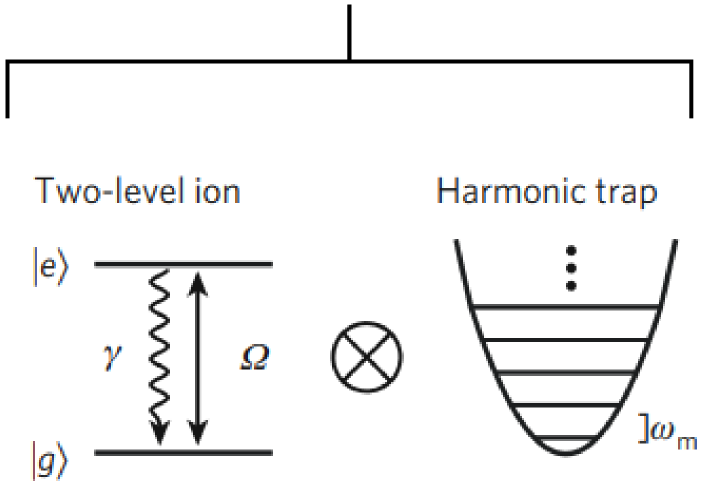
# A bird eye view :

*ref*: Entangled states of trapped atomic ions, Blatt-Wineland, Rainer Blatt & David Wineland, Nature 2008




A Schematic Set up

The Hilbert Space




Ions

Motional DoF, as vibrations/phonons

# Ring-exchange physics in a chain of three-level ions

Quantum 9, 1683 (2025)

By

*S. Biswas*, E. Rico, and T. Grass

## Remember !!

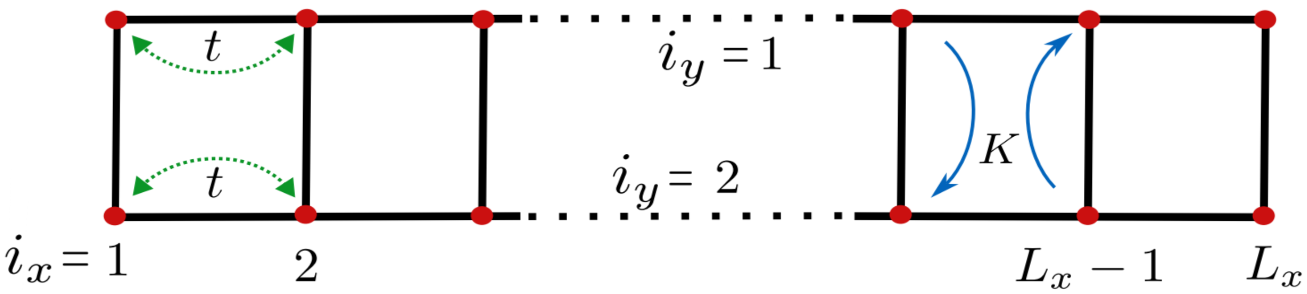
Long range processes occur naturally in trapped-ions. They are inevitable due to phonons.

# We think of a two-leg ladder, with itinerant hardcore bosons, under ring exchange !

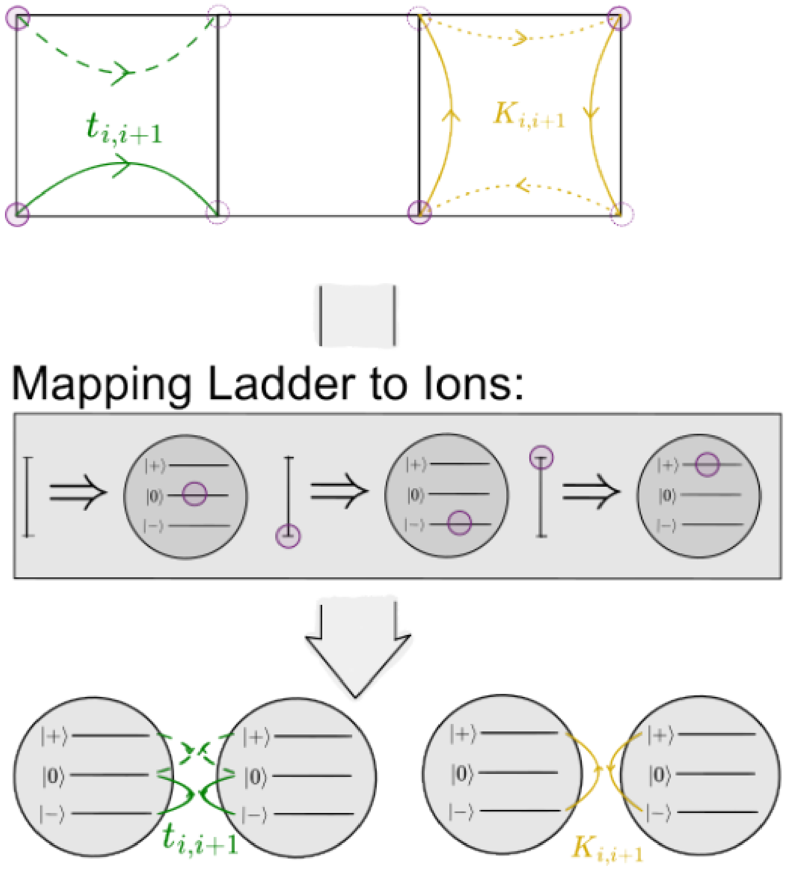
$$H = \sum_{j>i} \left( \sum_{\sigma} -t^{\sigma}_{ij} a^{\dagger}_{j\sigma} a_{i\sigma} + K_{ij} a^{\dagger}_{i\downarrow} a^{\dagger}_{j\uparrow} a_{j\downarrow} a_{i\uparrow} \right) + \text{h.c.}$$

*Notation:*

$\uparrow$: $Leg - 1$
$\downarrow$: $Leg - 2$




We begin by *NN + NNN….*

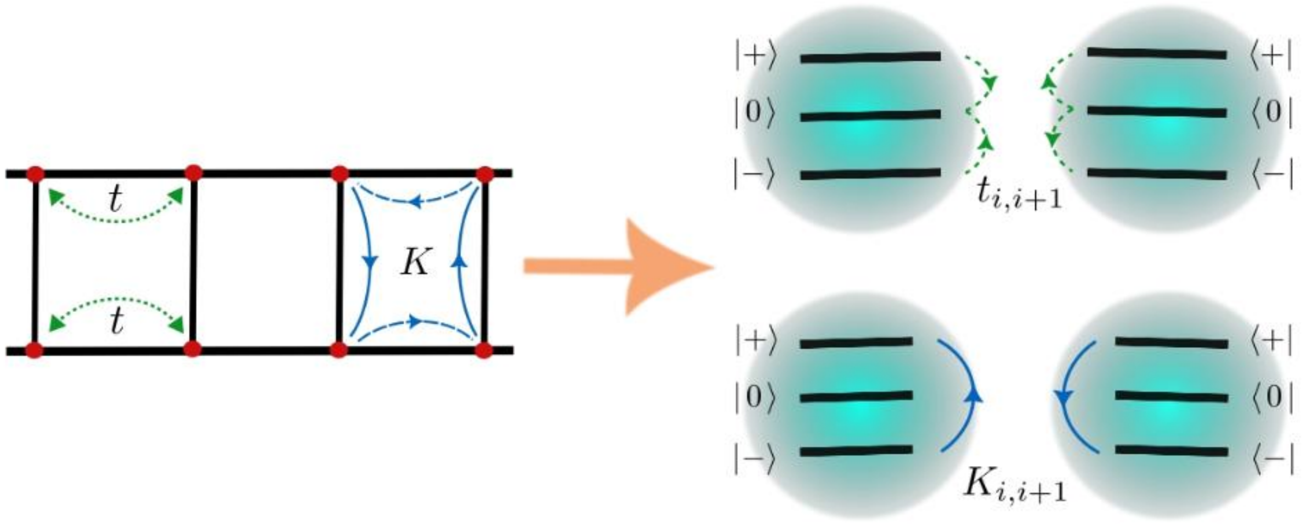



$$H = \sum_{i>j,\sigma} \left( -t^{\sigma}_{ij} a^{\dagger}_{i\sigma} a_{j\sigma} + K_{ij} a^{\dagger}_{i\bar{\sigma}} a^{\dagger}_{j\sigma} a_{j\bar{\sigma}} a_{i\sigma} + \text{h.c.} \right)$$

*the claim is*:

** **This system can host a phase of interacting bosons, which does not condense but show non-trivial correlations.** **

Two leg ladder is known to host such a phase, theoretically.

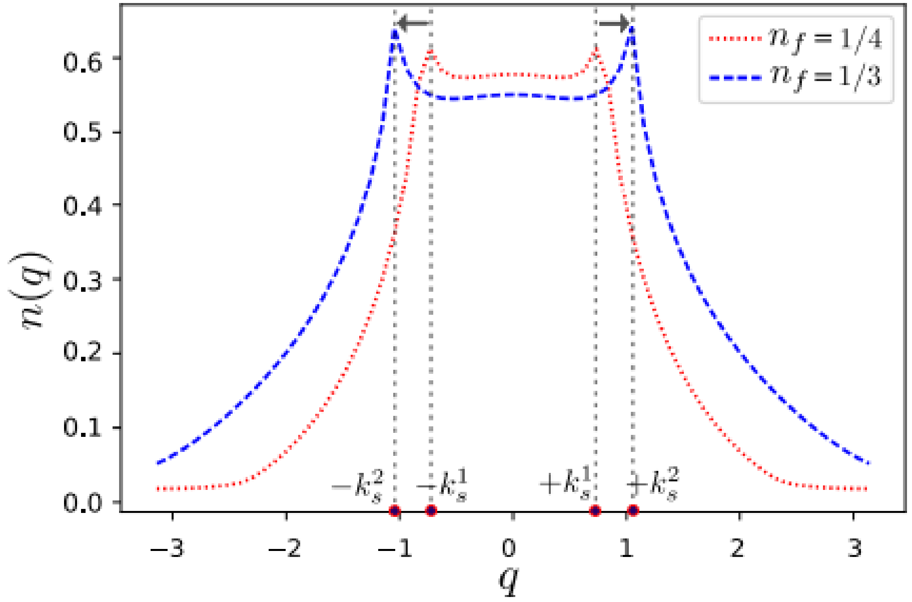


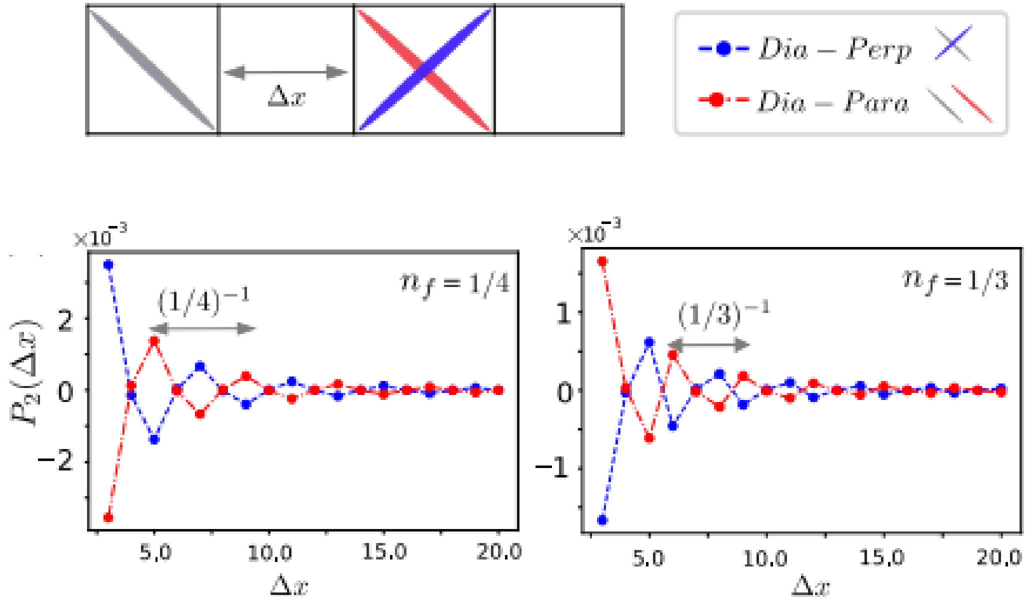

We want to utilize synthetic platform in order to realize it

**The physics of frustrated bosons $\{a_{j\sigma}\}$ can also be understood**

**in terms of three level spins: $\tau^i_{\alpha\beta} \equiv |i,\alpha\rangle\langle i,\beta|$ , such that**

$$H^{\alpha\beta}_{\text{eff}} = \sum_{i\neq j} J^{ij}_{\alpha\beta}\tau^i_{\alpha\beta}\tau^j_{\beta\alpha}$$ with, $$t^{\uparrow}_{ij} = -J^{ij}_{0+} \text{ and } t^{\downarrow}_{ij} = -J^{ij}_{0-}$$ & $$K_{ij} = J^{ij}_{-+}$$




Momentum distribution:

$$n(q) = \sum_{j_1, j_2} \sum_{\sigma=\uparrow,\downarrow} \exp(-iq(j_1 - j_2)) \langle a^\dagger_{j_1,\sigma} a_{j_2,\sigma} \rangle / L$$

Pair correlation:

$$P_2(\Delta x) = \langle a^\dagger_{1,\uparrow} a^\dagger_{2,\downarrow} a_{\Delta x+\gamma,\uparrow} a_{\Delta x+\eta,\downarrow} \rangle$$

*Notation:*

$\uparrow$: $Leg - 1$
$\downarrow$: $Leg - 2$

## D-wave Bose liquid (DBL)

# But that cannot be all……

Two-point correlation/Order parameter:

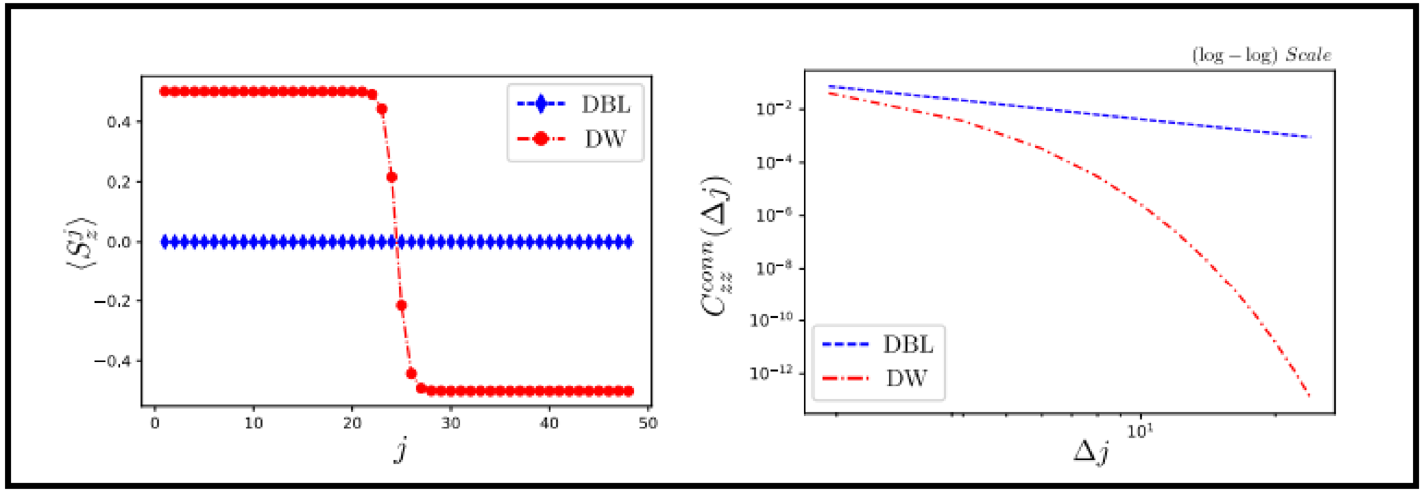
$$C_{zz}(i,j) = \langle S_z^i S_z^j \rangle$$

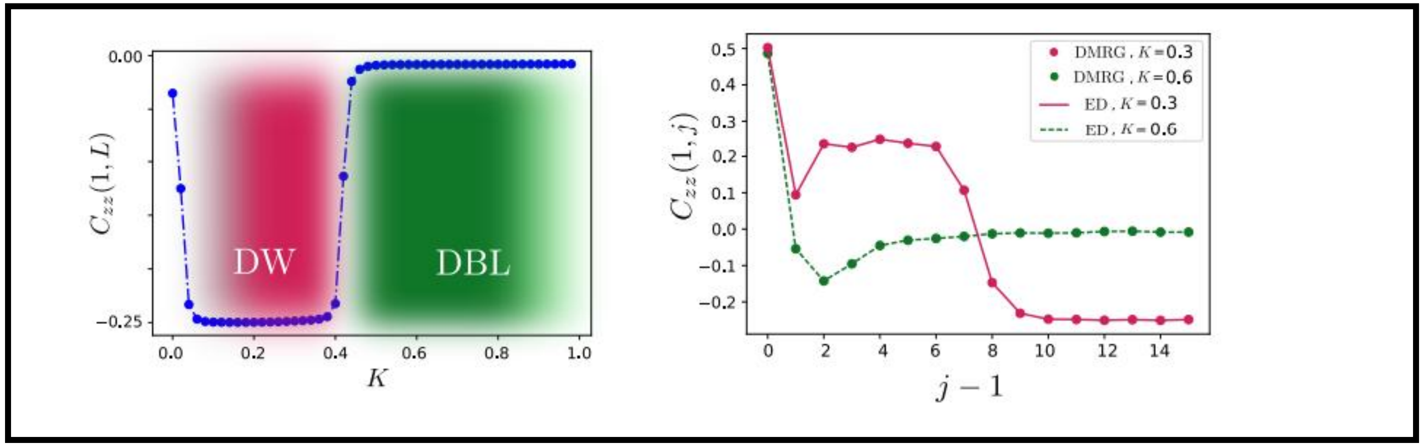
Connected Correlator:

$$C_{zz}^{conn}(\Delta j) = \langle S_z^{L/2+\Delta j} S_z^{L/2-\Delta j} \rangle - \langle S_z^{L/2+\Delta j} \rangle \langle S_z^{L/2-\Delta j} \rangle$$

**DW** : Domain Wall

**DBL** : D-wave Bose liquid

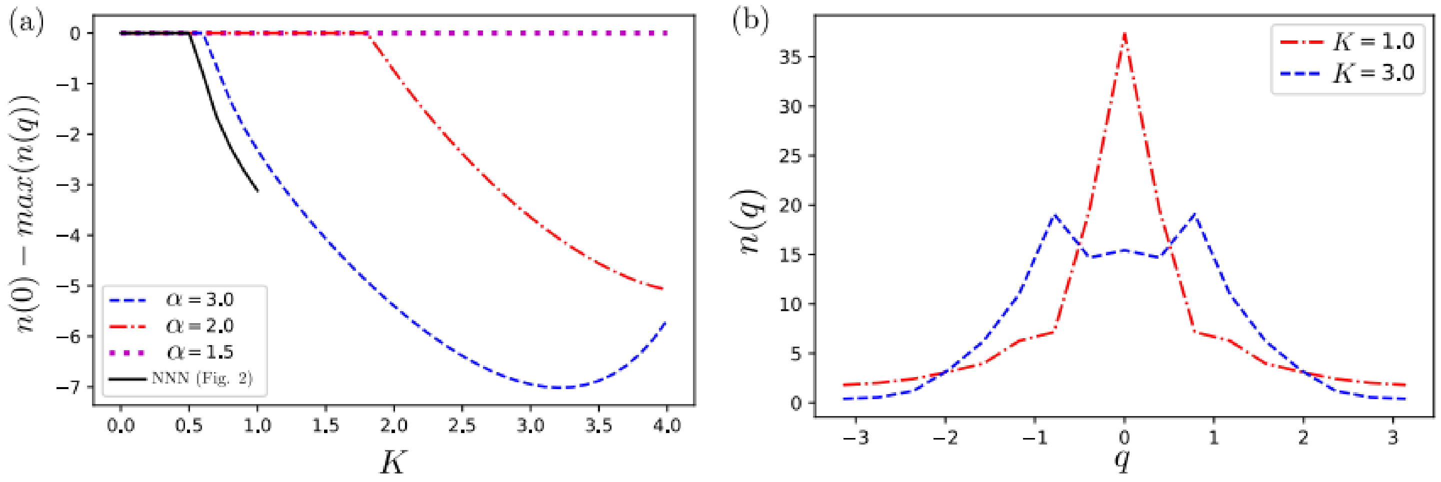


# But what lies beyond *NN + NNN* ???

We use **Exact diagonalization (ED)** method with 16 three level ions.

$$t_{ij} \sim |i-j|^{\alpha} \;\; \& \;\; K_{ij} \sim |i-j|^{\alpha}$$

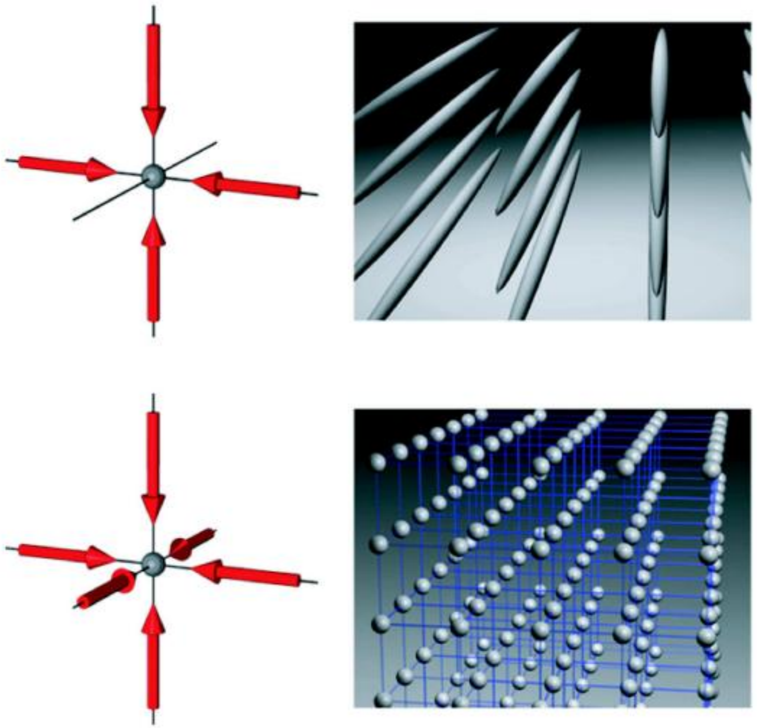



(a) The quantity $\Delta n = n(0) - \max(n(q))$ is plotted vs. $K$ to identify DBL formation through the characteristic features of $n(q)$, using ED for a chain with $L = 16$. The zero value of the quantity implies that the maximum of $n(q)$ corresponds to $q = 0$. A finite negative value, with increasing $K$, denotes the appearance of a momenta peak at a non-zero value, indicating peak splitting which is typical of the DBL phase. We plot the same function for different $\alpha$. For $\alpha \geq 2$, the DBL transition is observed, whereas for $\alpha = 1.5$ it is not. We have also shown the $\Delta n$, corresponding to the data set of Fig. 2 of the main text, for the sake of completeness. (b) We show $n(q)$ for $\alpha = 2$, to illustrate the splitting of peaks at sufficiently large ring exchange ($K = 3.0$, red dashed-dotted line), in contrast to a single peak for sufficiently small ring exchange ($K = 1.0$, blue dashed line).

**Trapped ion is not the only synthetic matter platform one can think of for realizing this………**

# Optical lattice

*ref*: *[Many-Body Physics with Ultra Cold Gases, Bloch et al, RoMP, 2007]*



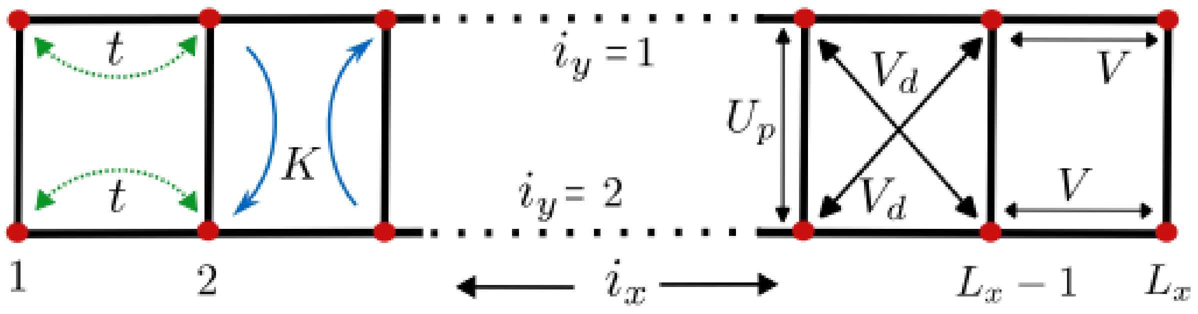
Two-dimensional **(a)** and three-dimensional **(b)** optical lattice potentials formed by superimposing two or three orthogonal standing waves. For a two-dimensional optical lattice, the atoms are confined to an array of tightly confining one-dimensional potential tubes, whereas in the three-dimensional case the optical lattice an be approximated by a three dimensional simple cubic array of tightly confining harmonic oscillator potentials at each lattice site.

# Frustrated Bose ladder with extended range density-density interaction

*Phys. Rev. B* 112, 115122 (2025)

By

*S. Biswas*, E. Rico, and T. Grass




$$*V_d = \frac{V}{(\sqrt{2})^3}$$

# ORDER PARAMETERs

*momentum distribution*

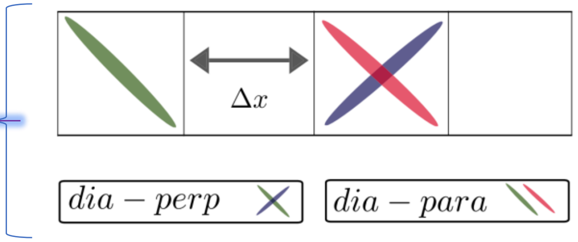
$$n(q_x, q_y) = \frac{1}{L}\sum_{i,j} e^{-iq_x(i_x-j_x)-iq_y(i_y-j_y)}\langle b^{\dagger}_{i_x,i_y} b_{j_x,j_y}\rangle$$

*momentum peak position*

$$\tilde{n}(q_x, 0) = \frac{n(q_x,0)}{\max[n(q_x,0)]}$$

*pair correlator*

$$P_2(\Delta x) = \langle b^{\dagger}_{1,1} b^{\dagger}_{2,2} b_{\delta_1+\Delta x,1} b_{\delta_2+\Delta x,2}\rangle$$




*density wave order*

$$\mathcal{O}^{j_y}_{\mathrm{DW}_2} = \sum_{j=1}^{L_x/2}(-1)^j (n_{2j-1,j_y} + n_{2j,j_y}) \rightarrow O_{\mathrm{DW}_2} = \sum_{j_y} |\mathcal{O}^{j_y}_{\mathrm{DW}_2}|$$

*superfluid stiffness*

$$O_{\mathrm{twist}} = L_x \times (E^{0}_{\mathrm{GS}} - E^{\pi}_{\mathrm{GS}})$$

***Superfluid***

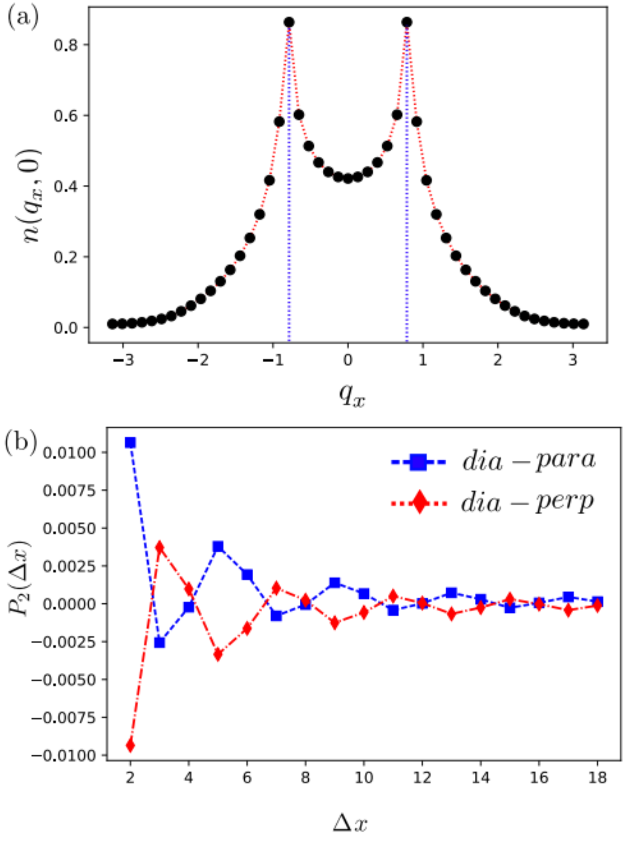
***Density modulated S-wave pairing***

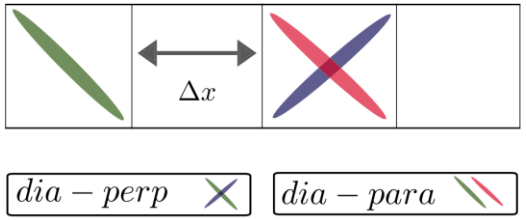
***D-wave correlated Bose liquid***

| | SF | DMSP | DBL |
|---|---|---|---|
| $\max[n(q_x, 0)]$ | $q_x = 0$ | $q_x = 0$ | $q_x = \pi n_f$ |
| $O_{\mathrm{DW}_2}$ | 0 | $\neq 0$ | 0 |
| $O_{\mathrm{twist}}$ | $\neq 0$ | 0 | $\times$ |
| $\mathrm{sgn}(P_2^{\mathrm{perp}})/\mathrm{sgn}(P_2^{\mathrm{para}})$ | 1 | 1 | $-1$ |

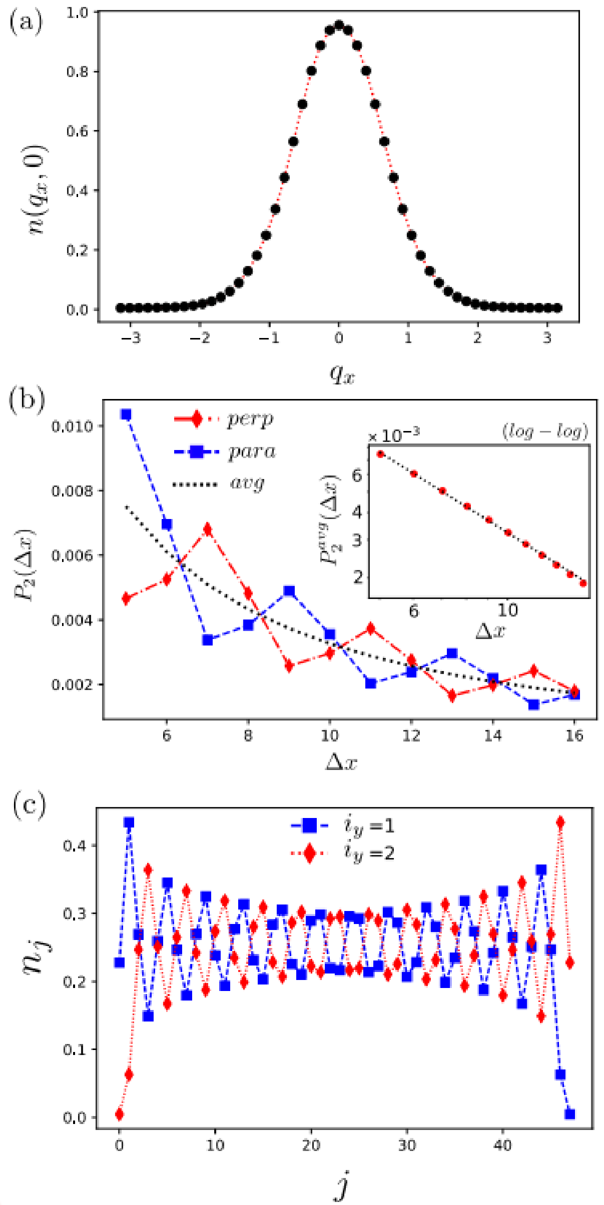
# Two Types of Pairings

DBL

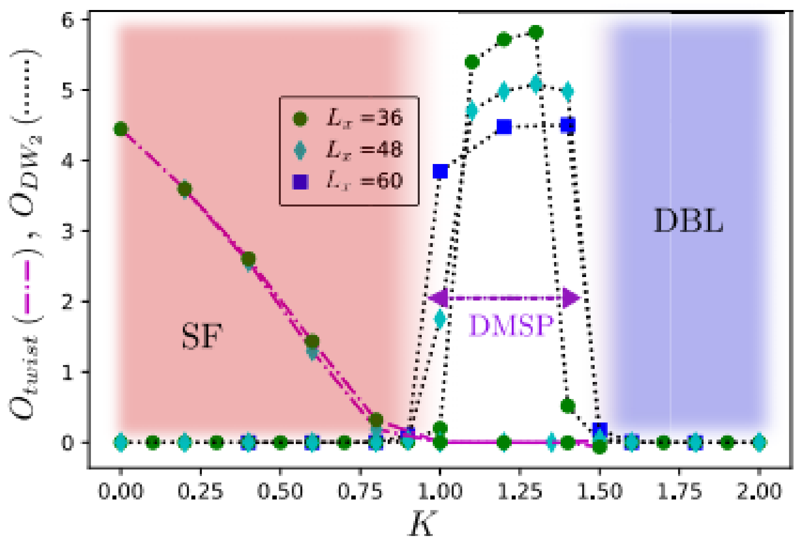


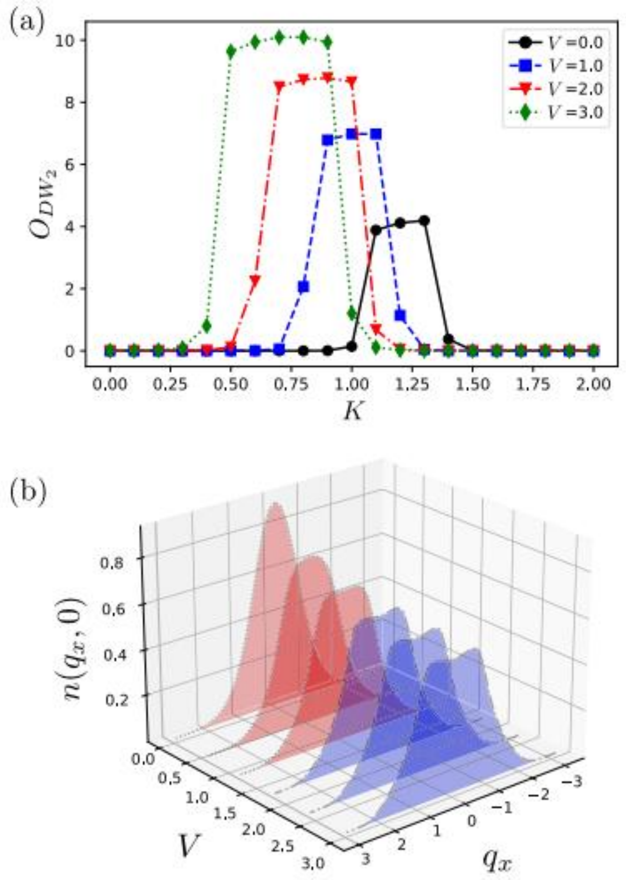

DMSP



$V = U_p = 0$

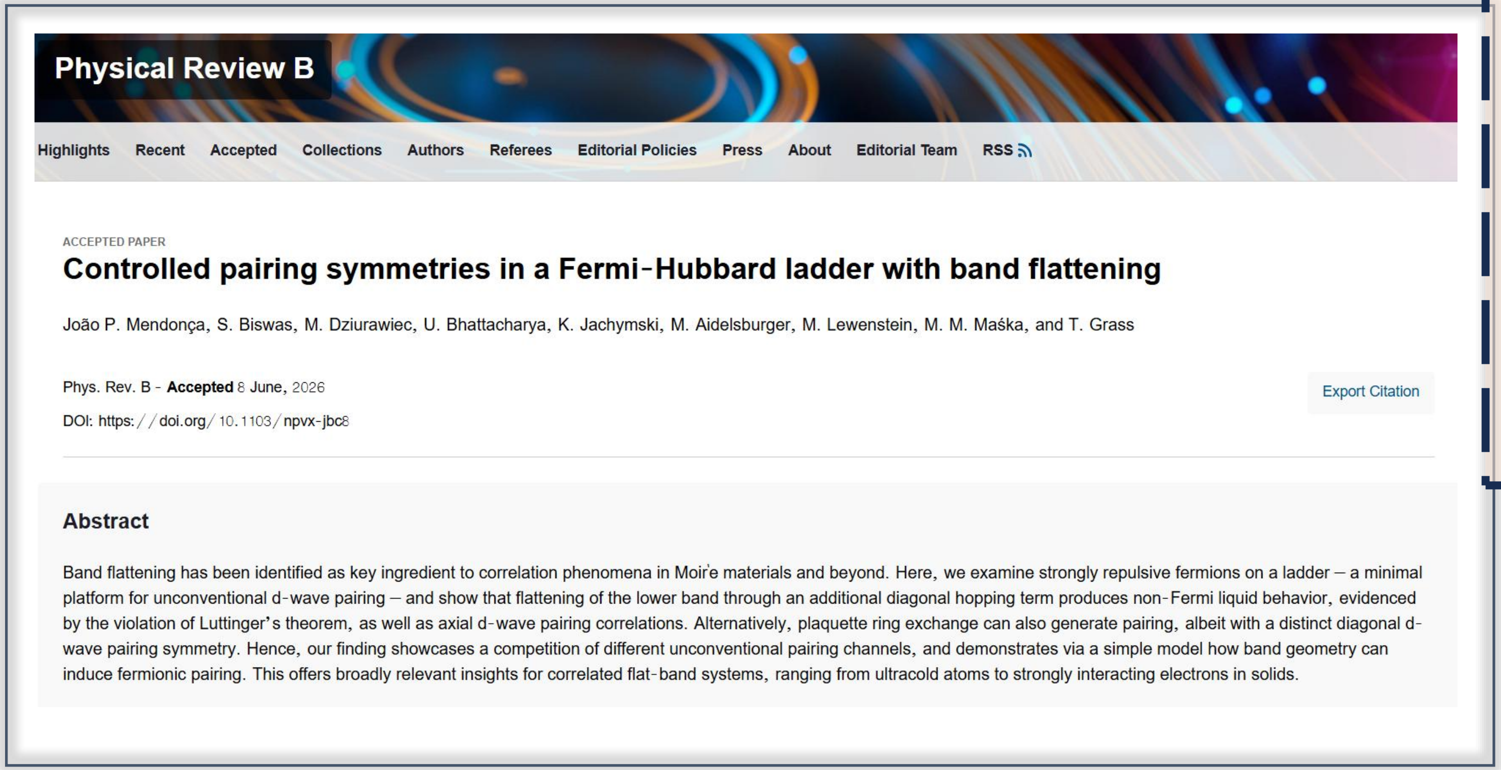



$V = U_p \neq 0$




**Extended Interaction stabilizes DBL**

## Works even for Fermions :

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ACCEPTED PAPER

### Controlled pairing symmetries in a Fermi-Hubbard ladder with band flattening


João P. Mendonça, S. Biswas, M. Dziurawiec, U. Bhattacharya, K. Jachymski, M. Aidelsburger, M. Lewenstein, M. M. Maśka, and T. Grass



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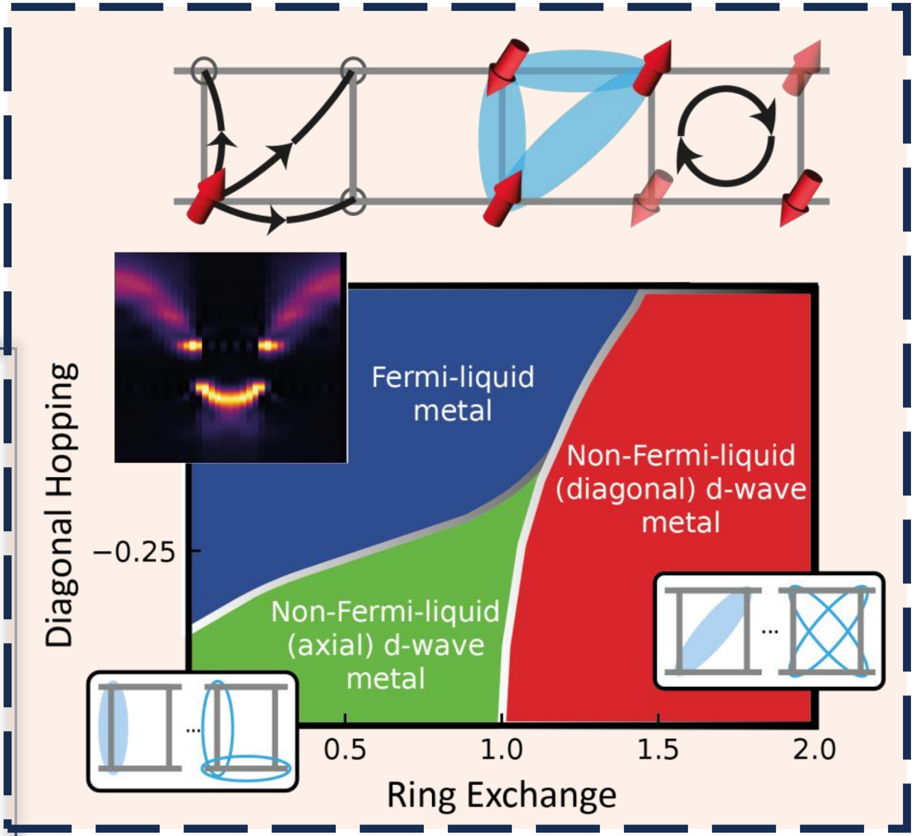
**Abstract**

Band flattening has been identified as key ingredient to correlation phenomena in Moiré materials and beyond. Here, we examine strongly repulsive fermions on a ladder – a minimal platform for unconventional d-wave pairing – and show that flattening of the lower band through an additional diagonal hopping term produces non-Fermi liquid behavior, evidenced by the violation of Luttinger's theorem, as well as axial d-wave pairing correlations. Alternatively, plaquette ring exchange can also generate pairing, albeit with a distinct diagonal d-wave pairing symmetry. Hence, our finding showcases a competition of different unconventional pairing channels, and demonstrates via a simple model how band geometry can induce fermionic pairing. This offers broadly relevant insights for correlated flat-band systems, ranging from ultracold atoms to strongly interacting electrons in solids.

# Conclusions :

- Synthetic quantum platforms offer new possibilities for the application of new quantum technologies in search of exotic phases of matter.
- In the trapped ion set-up, the DBL is obtained as long as the decay exponent is of the order of 2 or more.
- The transition to DBL is discussed in detail, as a function of “K”.
- In a Cold atomic setup, extended interaction is shown to stabilize DBL.
- We report the emergence of a novel DMSP phase.
- Can be extended to Fermions.