

Primordial Black Holes

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CAP Theory, Montreal

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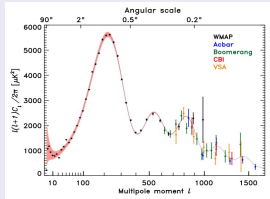
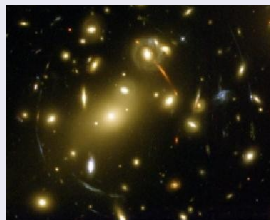
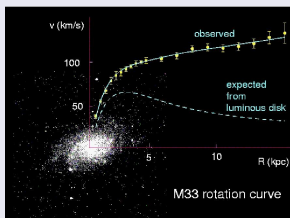
JCAP09(2022)034 [arXiv:2205.08906]

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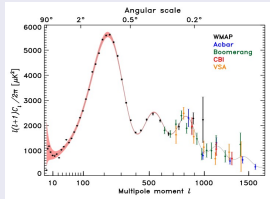
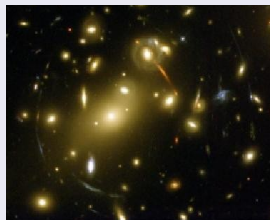
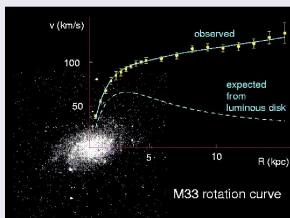
Dark Matter

Evidences



Dark Matter

Evidences



How much DM?

Planck.XIII

$$\Omega_{\text{DM}} h^2 = 0.1188 \pm 0.0010$$

Properties

- stable
- neutral
- weakly interacting
- right relic density

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Candidates

- Axions
- Sterile neutrinos
- WIMPs
- ...

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Any candidate in Standard Model?

Primordial Black Holes (PBHs)

Definition

A PBH is a type of black hole that is **not** formed by the gravitational collapse of a star, but by the extreme density of matter present during the Universe's early expansion.

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PBHs properties

Mass: $M_{\text{BH}} = 10^{15} \left(\frac{t}{10^{-23} \text{ s}} \right) \text{ g}$

$$M_{\odot} \simeq 2 \times 10^{33} \text{ g}$$

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$$\text{Planck scale} \longrightarrow 10^{-5} \text{ g}$$

$$\text{GUT scale} \longrightarrow 10^3 \text{ g}$$

$$\text{EW scale} \longrightarrow 10^{28} \text{ g}$$

$$\text{QCD scale} \longrightarrow 10^{32} \text{ g}$$

Hawking radiation

Temperature: $T_{\text{BH}} \approx 10^{-7} \left(\frac{M}{M_{\odot}} \right)^{-1} \text{ K}$

	$M > 10^{17} \text{ g}$	massless particles
10^{15} g	$\lesssim M \lesssim 10^{17} \text{ g}$	electrons
10^{14} g	$\lesssim M \lesssim 10^{15} \text{ g}$	muons
	$M < 10^{14} \text{ g}$	hadrons

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$$\text{Lifetime: } \tau_{\text{BH}} \approx 10^{64} \left(\frac{M}{M_{\odot}} \right)^3 \text{ y}$$

M_{BH}	τ_{BH}
A man	10^{-12} s
A building	1 s
10^{15} g	10^{10} y
The Earth	10^{49} y
The Sun	10^{66} y
The Galaxy	10^{99} y

Why PBHs are useful?

- PBHs as a probe of the early Universe ($M < 10^{15}$ g)
- PBHs as a probe of gravitational collapse ($M > 10^{15}$ g) ✓
DM candidates $\Omega_{\text{PBH}}^0 \lesssim \Omega_{\text{CDM}}^0 (= 0.23)$
- PBHs as a probe of High Energy Physics ($M \sim 10^{15}$ g)
- PBHs as a probe of quantum gravity ($M \sim 10^{-5}$ g)
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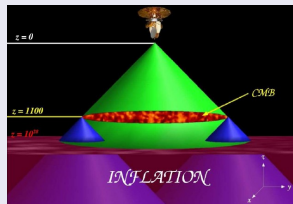
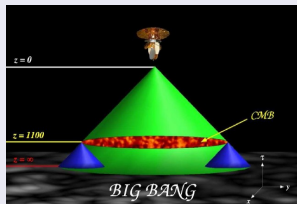
How PBHs form?

- Soft equation of state
- Bubble collisions
- Collapse of cosmic loops
- Fluctuations by inflation ✓

Accelerated expansion of the Universe $\ddot{a} > 0$

Why inflation?

- Flatness problem
- Horizon problem



$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \xrightarrow{\ddot{a} > 0} w < -\frac{1}{3}$$

Scenario

Equation of motion $\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \xrightarrow{V(\phi) \gg \dot{\phi}^2} 3H\dot{\phi} \simeq -V'(\phi)$

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Inflation Parameters

Power Spectrum

$$\mathcal{P}_{\mathcal{R}_c} = \frac{1}{12\pi^2 M_{\text{P}}^6} \frac{V^3}{V'^2}$$

$$\mathcal{P}_{\mathcal{R}_c}(k) = \mathcal{P}_{\mathcal{R}_c}(k_0) \left(\frac{k}{k_0} \right)^{n_s(k) - 1}$$

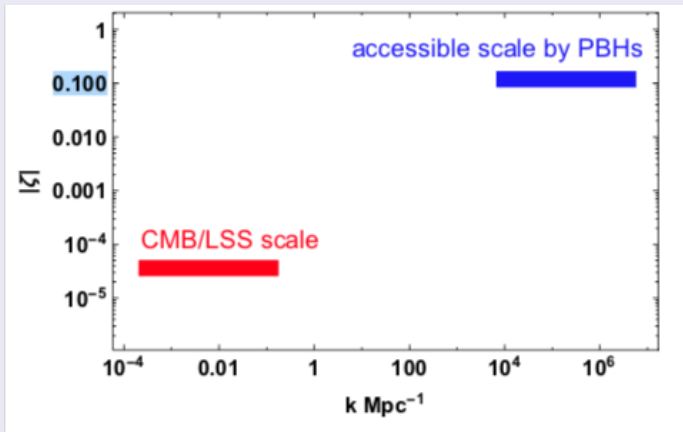
$$n_s(k_0) \equiv \frac{d \ln \mathcal{P}_{\mathcal{R}_c}}{d \ln k}$$

$$n_s = 1 - 3 \left(\frac{V'}{V} \right)^2 + 2 \frac{V''}{V}$$

$$\ln(10^{10} \mathcal{P}_{\mathcal{R}_c}(k_0)) = 3.094 \pm 0.034$$

$$n_s = 0.9645 \pm 0.0049$$

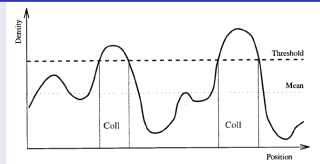
$$k_0 = 0.05 \text{ Mpc}^{-1}$$



Press-Schechter Formalism

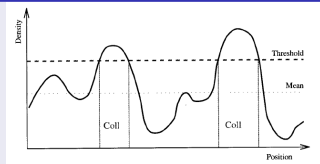
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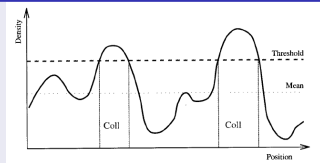
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$$\delta_{\text{th}} = 0.47$$

$$\text{Gaussian PDF: } P_G(\delta; R) = \frac{1}{\sqrt{2\pi}\sigma_\delta(R)} \exp\left(-\frac{\delta^2(R)}{2\sigma_\delta^2(R)}\right)$$

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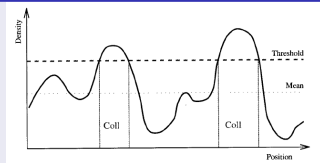
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$$w = 1/3$$

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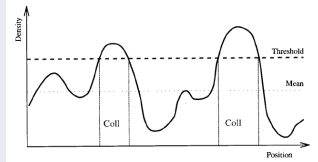
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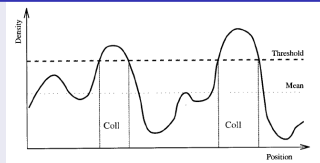
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$$\sigma_\delta^2(R) = \int_0^\infty W^2(kR) \mathcal{P}_\delta(k) \frac{dk}{k}$$

$$W(kR) = \exp(-k^2 R^2 / 2)$$

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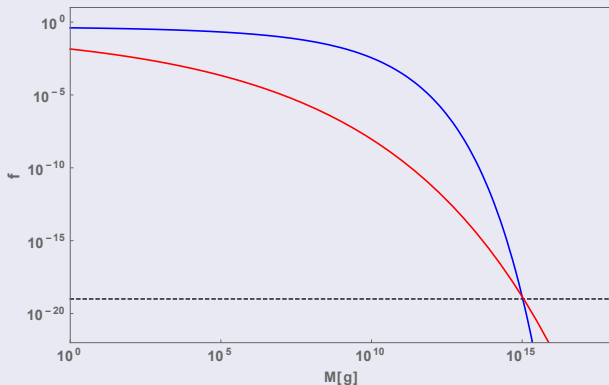
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$$M_{\text{PBH}} \simeq M_{\text{PH}} \rightarrow \frac{R}{1 \text{ Mpc}} = 5.5 \times 10^{-24} \left(\frac{M_{\text{PBH}}}{1 \text{ g}}\right)^{1/2} \left(\frac{g_*}{3.36}\right)^{1/6}$$

$f(\geq M)$ diagram for the mass range $10^0 - 10^{20}$ g



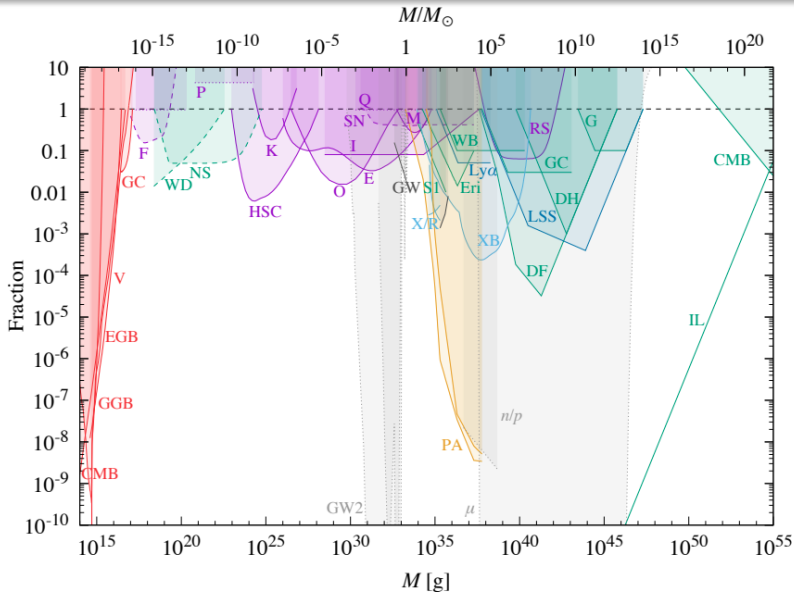
Result

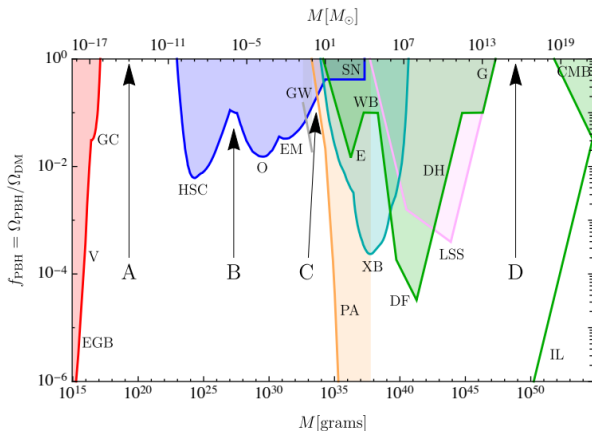
$$\beta = \frac{1}{2} \operatorname{erfc} \left(\delta_c / \sqrt{2\sigma_\delta^2(R)} \right)$$

$$n_s(k_{\text{PBH}}) \geq 1.418 \quad \Rightarrow \quad \mathcal{P}_{\mathcal{R}_c} \simeq 2 \times 10^{-2} \quad \text{for Gaussian PDF}$$

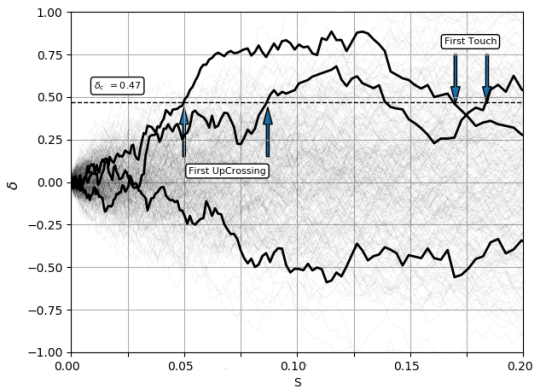
$$\beta \simeq 3.7 \times 10^{-9} \left(\frac{g_{*,i}}{10.75} \right)^{1/4} \left(\frac{M_{\text{PBH}}}{M_{\odot}} \right)^{1/2} f_{\text{PBH}},$$

$$f_{\text{PBH}} \equiv \frac{\Omega_{\text{PBH}}}{\Omega_{\text{DM}}}$$



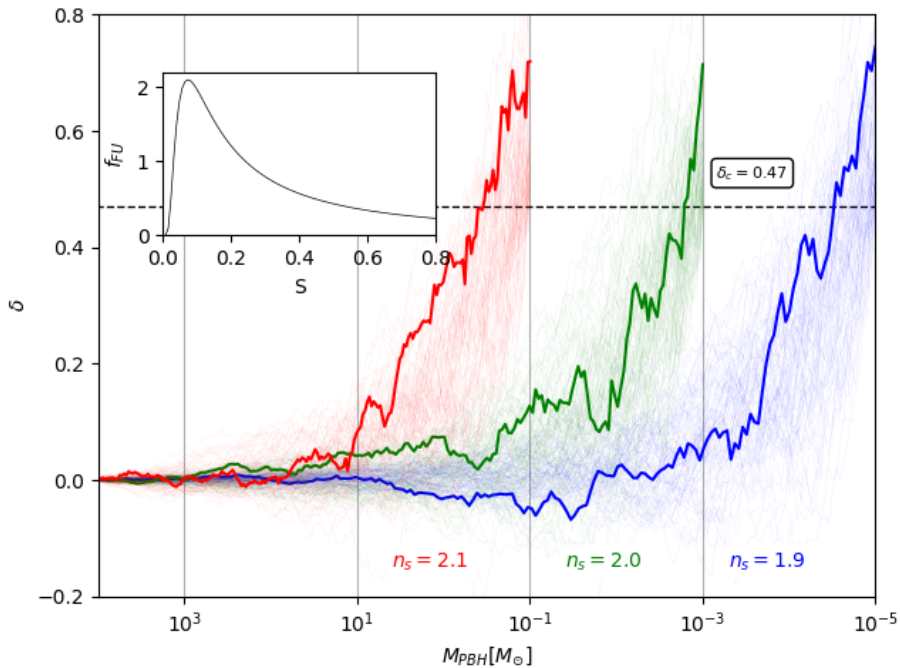


- asteroid mass range $10^{16} \text{ g} \lesssim M_{\text{PBH}} \lesssim 10^{17} \text{ g}$,
- sublunar mass range $10^{20} \text{ g} \lesssim M_{\text{PBH}} \lesssim 10^{24} \text{ g}$,
- intermediate mass range $10 M_{\odot} \lesssim M_{\text{PBH}} \lesssim 10^3 M_{\odot}$,
- stupendously large black holes (SLABs) $M_{\text{PBH}} \geq 10^{11} M_{\odot}$.



$$f_{\text{FU}}(S, \delta_c) = \frac{1}{\sqrt{2\pi}} \frac{\delta_c}{S^{3/2}} \exp\left(-\frac{\delta_c^2}{2S}\right)$$

$$S = \sigma^2(R) = \int \frac{dk}{k} \mathcal{P}_\delta(k) W^2(k, R)$$



Results

	mass range	f_{PBH}	lower bound of mass range	spectral index	
				EST	PS
asteroid mass range	$(10^{16} - 10^{17})$ g	1	10^{16} g	1.490	1.616
sublunar mass range	$(10^{20} - 10^{24})$ g	1	10^{20} g	1.540	1.703
			10^{22} g	1.600	1.756
Subaru HSC	$(10^{-11} - 10^{-6}) M_{\odot}$	10^{-3}	$10^{-10} M_{\odot}$	1.604	1.560
			$10^{-8} M_{\odot}$	1.666	1.605
OGLE	$(10^{-6} - 10^{-3}) M_{\odot}$	10^{-2}	$10^{-6} M_{\odot}$	1.729	1.757
			$10^{-4} M_{\odot}$	1.845	1.835
EROS/MACHO	$(10^{-3} - 10^{-1}) M_{\odot}$	0.04	$10^{-3} M_{\odot}$	1.862	1.947
			$10^{-2} M_{\odot}$	1.942	1.970
sub-Solar mass range	$(0.2 - 1.0) M_{\odot}$	0.02	$0.2 M_{\odot}$	2.018	2.046
			$0.6 M_{\odot}$	2.115	2.078
intermediate mass range	$(10^1 - 10^3) M_{\odot}$	1	$10 M_{\odot}$	2.130	2.408
			$10^2 M_{\odot}$	2.235	2.514
SLABs	$\geq 10^{11} M_{\odot}$	1	$10^{11} M_{\odot}$	5.220	5.598
			$10^{12} M_{\odot}$	6.660	6.891

Dwarf Galaxies

Dwarf Galaxies (DGs) are DM dominated, with mass-to-light ratios $\sim 1000 M_{\odot}/L_{\odot}$

Mass: $M_{\text{DG}} = 10^9 M_{\odot}$ Radius: $R_{\text{DG}} \sim 10 \text{ pc}$

core of DGs: $M_c = 10^5 M_{\odot}$ $R_c = 0.9 \text{ pc}$

number of PBHs:

$$N_{\text{PBH}} = \frac{4\pi}{3} \frac{\rho_{\text{DM}} R_c^3}{m_{\text{PBH}}}$$

central densities of DGs: $\sim (0.08 - 2.1) M_{\odot}/\text{pc}^3$

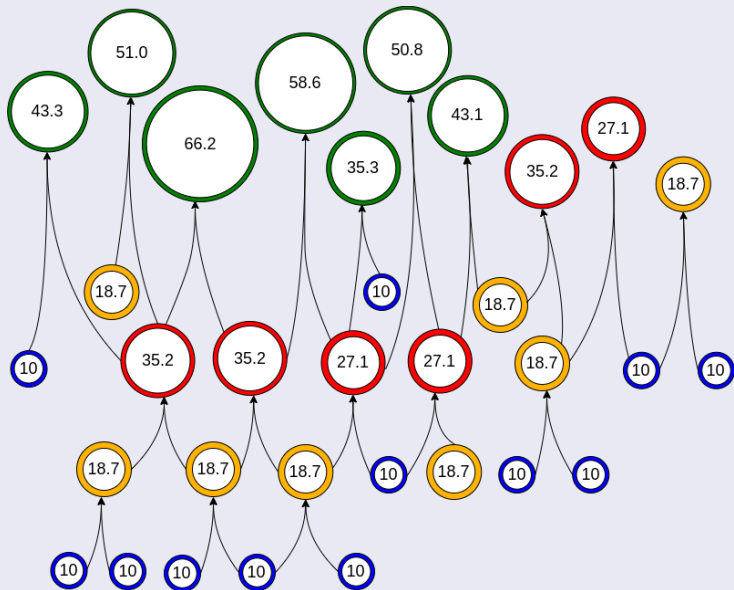
Generations

four different initial masses for PBHs (1st Generation):

1G: $10^{-14} M_{\odot}$ $10^{-2} M_{\odot}$ $1 M_{\odot}$ $10 M_{\odot}$

2G: 1G + 1G

3G: 1G + 2G & 2G + 2G



First Epoch ($z = 20 - 1.88$)

m_i	10
N_i	10^4
v_i	~ 13

Second Epoch ($z = 1.88 - 1.02$)

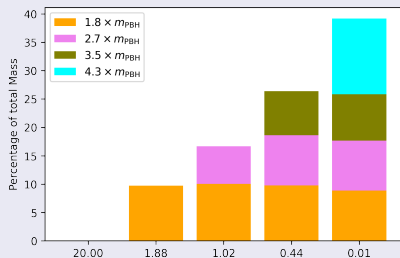
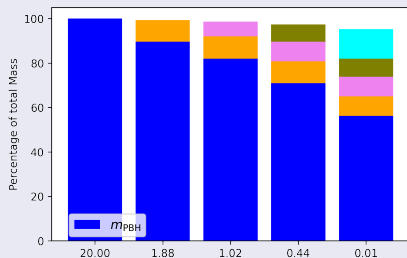
m_i	10	18.7
N_i	6434	1783
v_i	~ 10.6	~ 7.5

Third Epoch ($z = 1.02 - 0.44$)

m_i	10	18.7	27.1	35.2
N_i	4955	994	1092	43
v_i	~ 9	~ 5.6	~ 7	~ 1.5

Fourth Epoch ($z = 0.44 - 0.0$)

m_i	10	18.7	27.1	35.2	35.3	43.1	43.3	50.8	51.0	58.6	66.2
N_i	3154	266	526	12	833	281	19	12	17	7	~ 0
v_i	~ 7.5	~ 2.8	~ 4.8	~ 1	~ 6.8	~ 4.4	~ 1.1	~ 1	~ 1.1	~ 0.77	-



GWs from Merger of PBHs

$$\left\langle \frac{dE}{dt} \right\rangle = -\frac{32}{5} \frac{G^4 (m_i m_j)^2 (m_i + m_j)}{a^5} F(e)$$

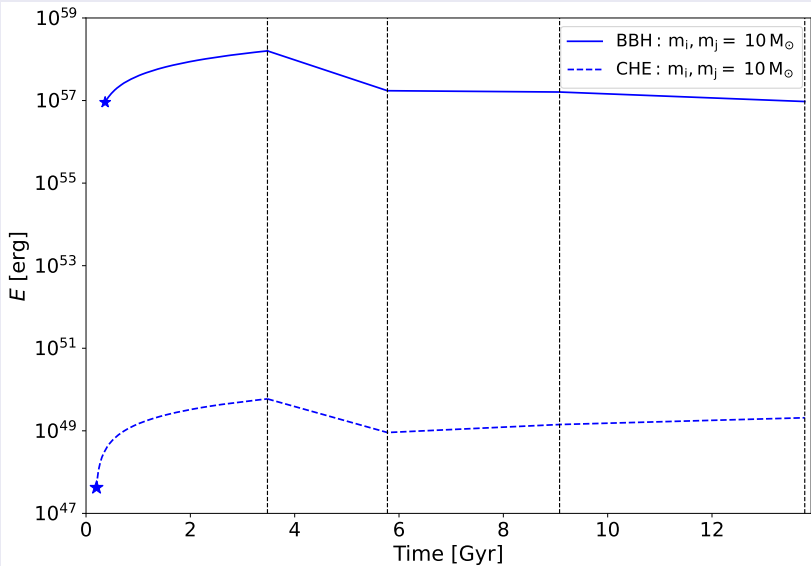
$$F(e) = \frac{1}{(1 - e^2)^{7/2}} \left(1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 \right)$$

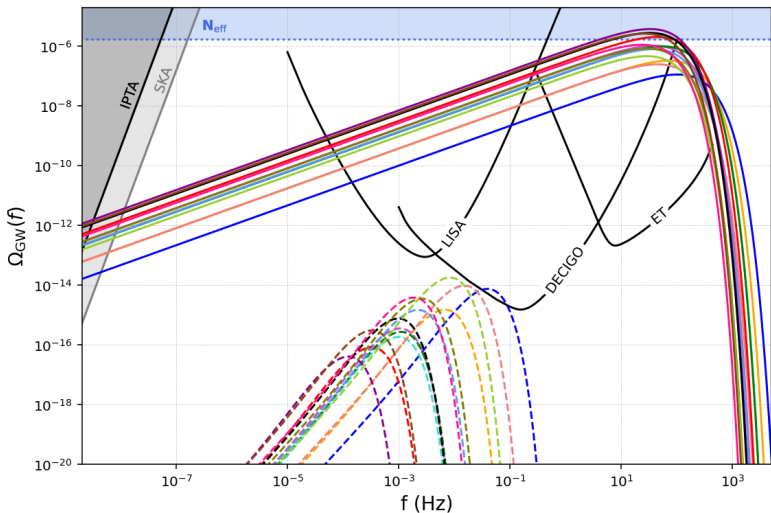
$$\Delta E_{\text{BBH}} = \frac{1}{2} m_i m_j \left(\frac{1}{a_{\text{merge}}} - \frac{1}{a_i} \right)$$

GWs from Close Hyperbolic Encounters of PBHs

$$\sigma_{\text{CHE}}(m_i, m_j) = \pi b^2 = \pi \left(\frac{G (m_i + m_j)}{v_0^2} \right)^2 (e^2 - 1)$$

$$\Delta E_{\text{CHE}} = -\frac{8}{15} \frac{G^{7/2} (m_i + m_j)^{1/2} m_i^2 m_j^2}{c^5 r_{\text{min}}^{7/2}} f(e)$$





— BBH,	- - - CHE: 10 & 10	— BBH,	- - - CHE: 18.7 & 35.2
— BBH,	- - - CHE: 10 & 18.7	— BBH,	- - - CHE: 18.7 & 43.1
— BBH,	- - - CHE: 10 & 27.1	— BBH,	- - - CHE: 27.1 & 35.3
— BBH,	- - - CHE: 10 & 35.2	— BBH,	- - - CHE: 27.1 & 43.1
— BBH,	- - - CHE: 10 & 43.1	— BBH,	- - - CHE: 35.3 & 35.3
— BBH,	- - - CHE: 18.7 & 18.7	— BBH,	- - - CHE: 35.3 & 43.3
— BBH,	- - - CHE: 18.7 & 27.1	— BBH,	- - - CHE: 43.1 & 43.1

- The fluctuation which arise at inflation are the most likely source of PBHs.
- The power spectrum at scale of PBHs formation should be much higher than the one in the observed scales.
- Lowering the collapse threshold increases clustering probability and shifts its peak toward higher PBH masses.
- In DM PBHs dominated DGs, the total mass loss at the DG core from GW emission, and the mass fraction of BH that undergo collisions are mostly independent of the initial PBH mass.

mount Everest mass $\sim 10^{15}$ g

Thank you