A UNIFIED FRAMEWORK FOR LOCALLY STABLE PHASES

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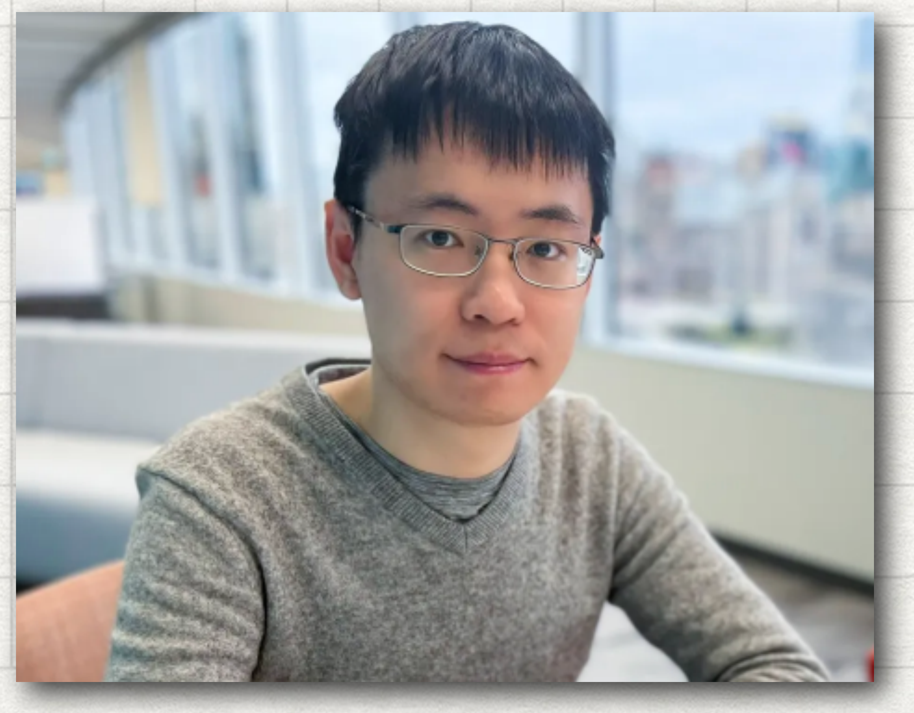
JOINT WORK WITH ZHI LI AND TIM HSIEH

20/06/2026

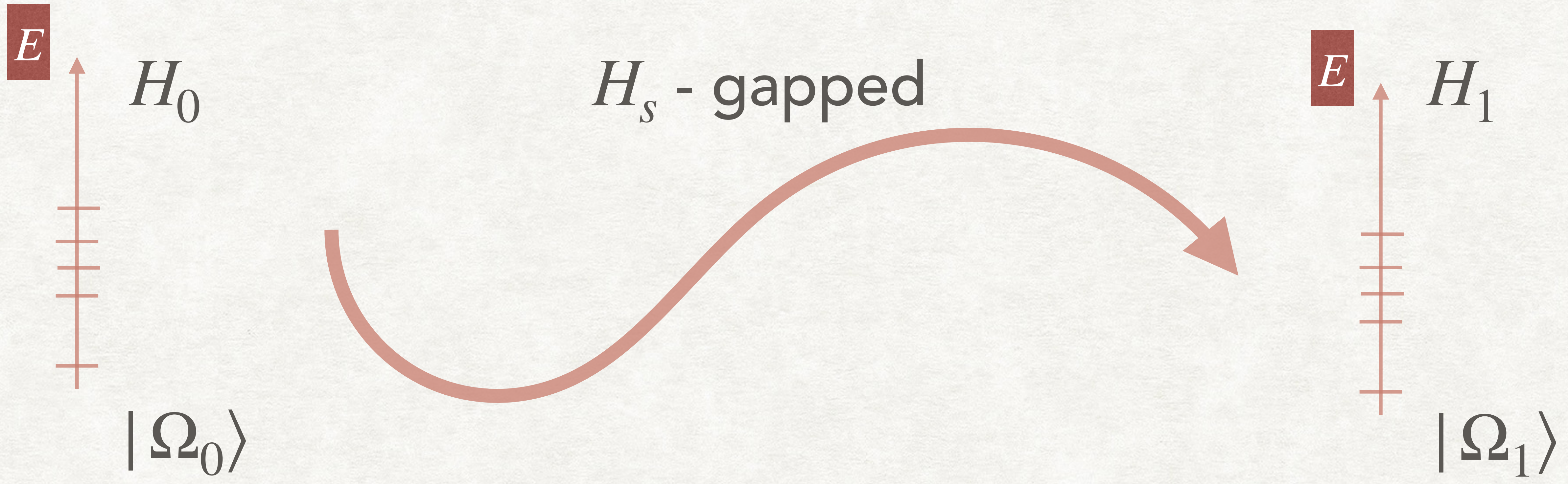
Based on arXiv:2605.00088

PERIMETER INSTITUTE



PURE STATE PHASES

- Let $|\Omega_0\rangle, |\Omega_1\rangle$ be ground states of gapped local Hamiltonians H_0, H_1.
- $|\Omega_0\rangle, |\Omega_1\rangle$ are in the same phase if there is a path



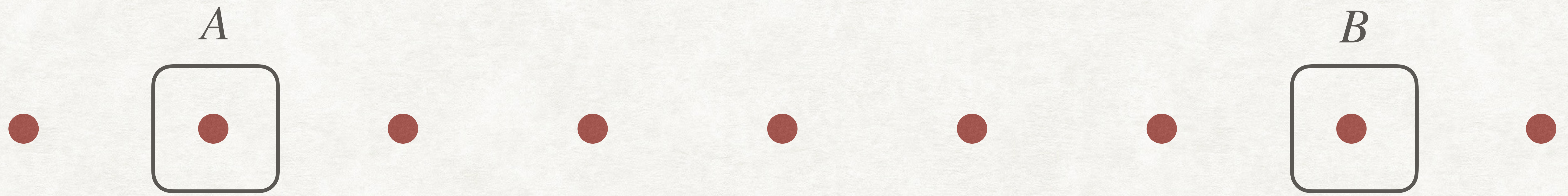
- Quasi-adiabatic continuation -

  [Chen , Gun & Wen 10]

$$|\Omega_1\rangle = e^{-i\int_0^1 \tilde{H}_g \, dg} \, |\Omega_0\rangle$$

CORRELATION LENGTH

- Locallity features of gapped ground states: decay of correlations, area laws,…
- Correlation length is associated with the Hamiltonian’s gap [Hastings 03, Hastings & Koma 05]:



$\rho = |\Omega\rangle\langle\Omega|$

- $$\langle O\rangle \triangleq \mathrm{Tr}[\rho \cdot O]$$
- $$|\langle O_A O_B\rangle - \langle O_A\rangle\langle O_B\rangle| \leq f(|A|,|B|)\|O_A\|\|O_B\|\, e^{-\frac{\mathrm{dist}(A,B)}{\xi}}$$
- $$\xi \leq \mathcal{O}(1/\Delta E)$$

EQUILIBRIUM PHASES

- Thermal state: $\sigma \propto e^{-\beta H}, \quad H = \sum H_i, \quad \beta = 1/T.$
- Phases have been extensively studied in the literature as well.

1. Spontaneous symmetry breaking (SSB)
2. Lindbladian (Gibbs sampler) gap [Kastoryano & Brandao 14, Bergamaschi & Chen 25,...].
3. Local Stability [Brandao & Kastoryano 19, Capel et al. 25]
4. Analyticity of partition function [Araki 69,...]
5. 2-way connectivity by a channel [Hastings 11,...]

$\mathscr{L} = \sum_i \mathscr{L}_i, \quad \mathscr{L}(\sigma) = 0$

$\mathscr{L}$ is gapped

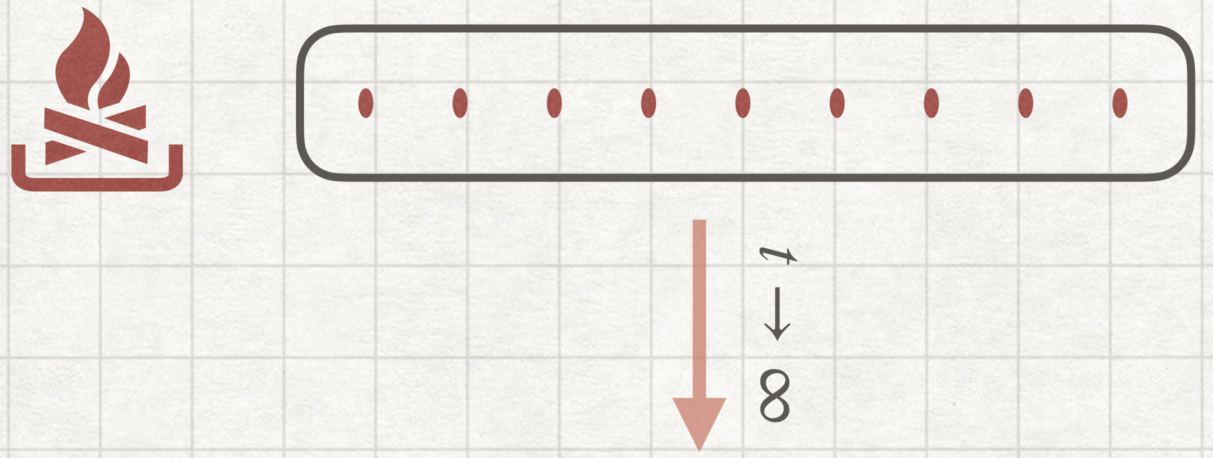
↓

σ has decay of correlations

WHAT ABOUT NON-EQUILIBRIUM?

Steady states of Lindbladians

$\mathscr{L}(\sigma) = 0$



σ

Thermal states

$\sigma \propto e^{-\beta H}$

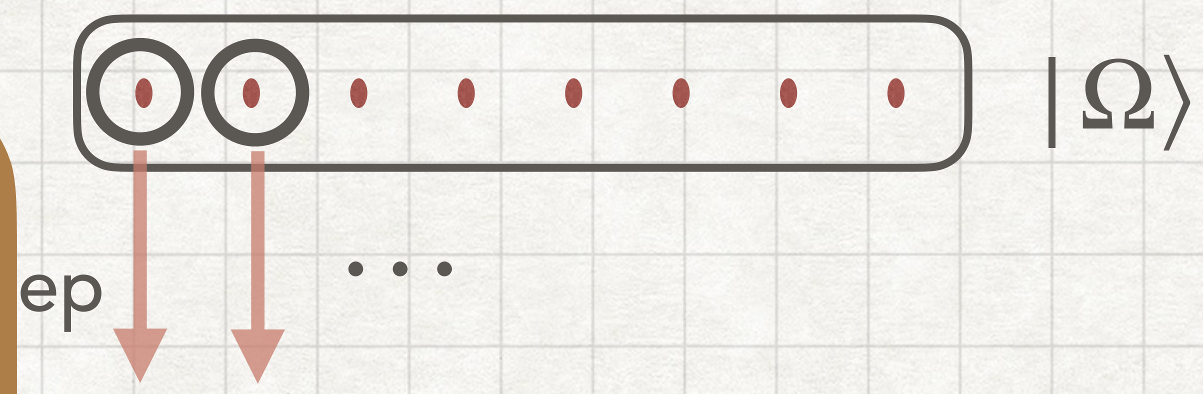
Metastable

$\|\mathscr{L}(\sigma)\| \leq \varepsilon$

"σ IS A "GAPPED PHA HIBITS EXPONENTIAL
DECAY TIONS"

Decohered states

$\sigma = \mathscr{E}(|\Omega\rangle\langle\Omega|)$



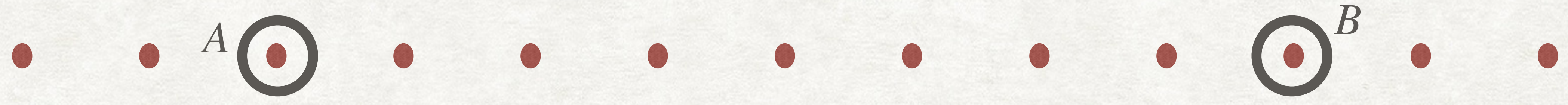
EXAMPLE - STRONG-WEAK SSB

- Define the n-qubit state

$$\rho_{Ising} = \frac{1}{2^n}\left(\mathbb{1} + Z_1 \ldots Z_n\right).$$

- Note that for any A and B:
  - Regular correlation - $\langle X_A X_B \rangle = 0$
  - Fidelity correlation - $\langle X_A X_B \rangle_F \triangleq \mathrm{F}\left(\rho, X_A X_B \rho X_A X_B\right) = \mathrm{Tr}\left[\sqrt{\sqrt{\rho} X_A X_B \rho X_A X_B \sqrt{\rho}}\right] = 1$

Analog to $\langle \psi | X_A X_B | \psi \rangle$

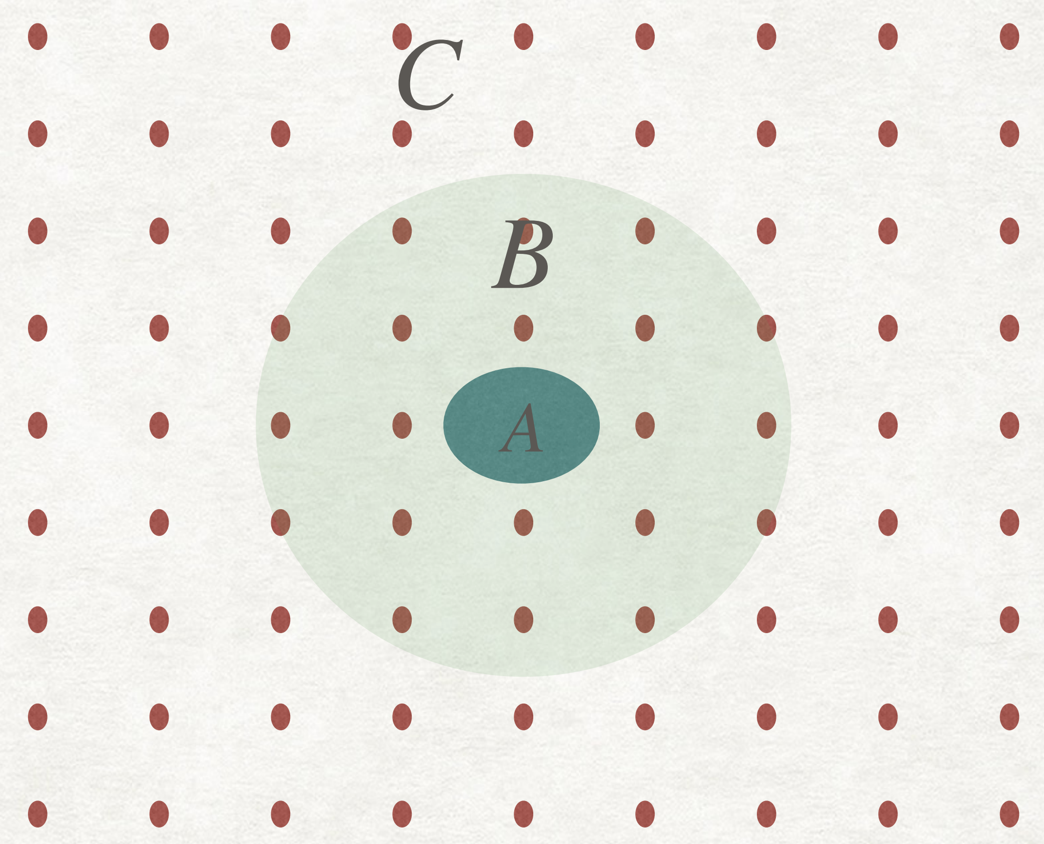


$C_{p,q} = \left(X_A X_B\right) \geq const$

$\langle X_A X_B \rangle \longrightarrow 0$

CONDITIONAL MUTUAL INFORMATION (CMI)

$\left(S(\rho) = -\operatorname{Tr}[\rho \log \rho]\right)$



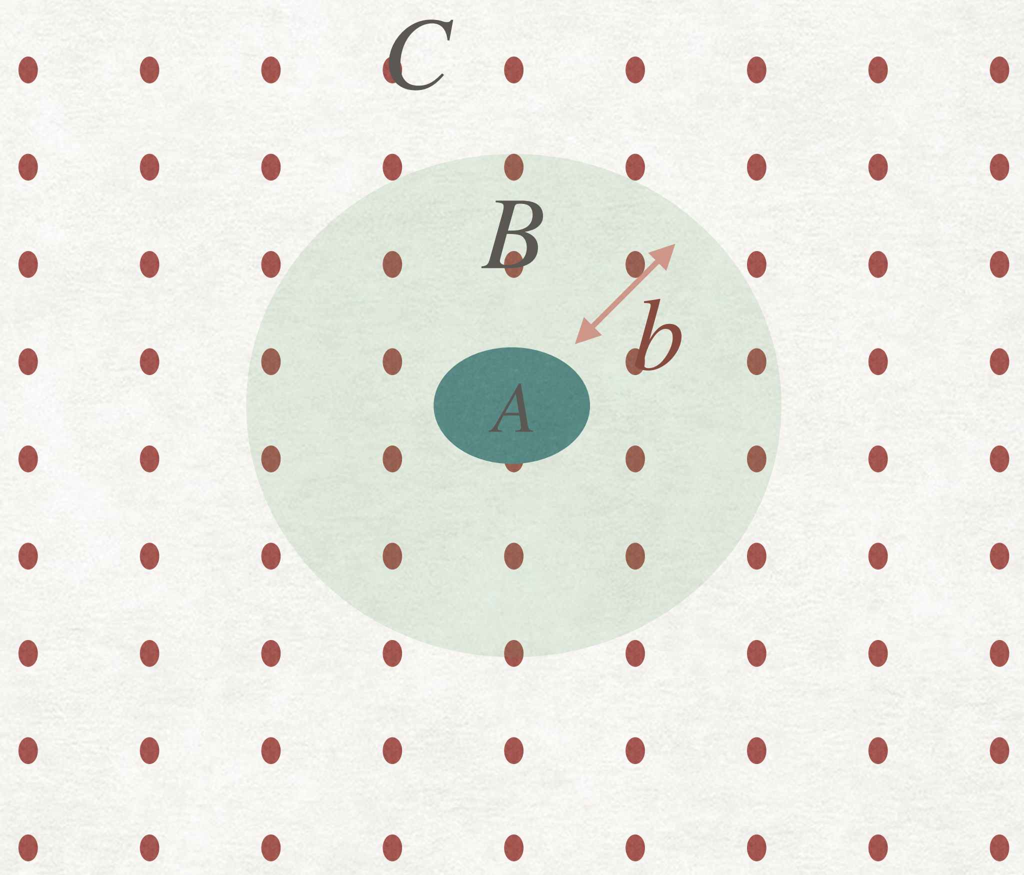
- Correlations are not enough.
- Introduce Conditional Mutual Information (CMI):
- For a tripartition $A:B:C$, B shields A from C,

$$\mathrm{I}(A:C|B) \triangleq \mathrm{S}(AB) + \mathrm{S}(BC) - \mathrm{S}(ABC) - \mathrm{S}(B)$$

- Operationally: Petz recovery map $\mathscr{P}: B \to AB$,

$$\mathscr{P}_{B\to AB}(\rho_{BC}) = \rho_{ABC} \quad \Leftrightarrow \quad \mathrm{I}(A:C|B) = 0$$

CONDITIONAL MUTUAL INFORMATION (CMI)



- Operationally: Petz recovery map $\mathscr{P} : B \to AB$,

$$\mathscr{P}_{B\to AB}(\rho_{BC}) = \rho_{ABC} \qquad \Leftrightarrow \qquad \mathrm{I}(A : C|B) = 0$$

- Also works for approximate analog: $\exists\tilde{\mathscr{P}} : B \to AB$,

$$\left\| \tilde{\mathscr{P}}_{B\to AB}(\rho_{BC}) - \rho_{ABC} \right\|_1 \approx 0 \qquad \Leftrightarrow \qquad \mathrm{I}(A : C|B) \approx 0$$

- <u>**Definition**</u>: A state σ has a finite Markov length $\eta > 0$ if

$$\mathrm{I}(A : C|B) \leq f(A)\, e^{-\frac{b}{\eta}}$$

CONDITIONAL MUTUAL INFORMATION (CMI)

- **<u>Definition</u>**: A state σ has a finite Markov length $\eta > 0$ if

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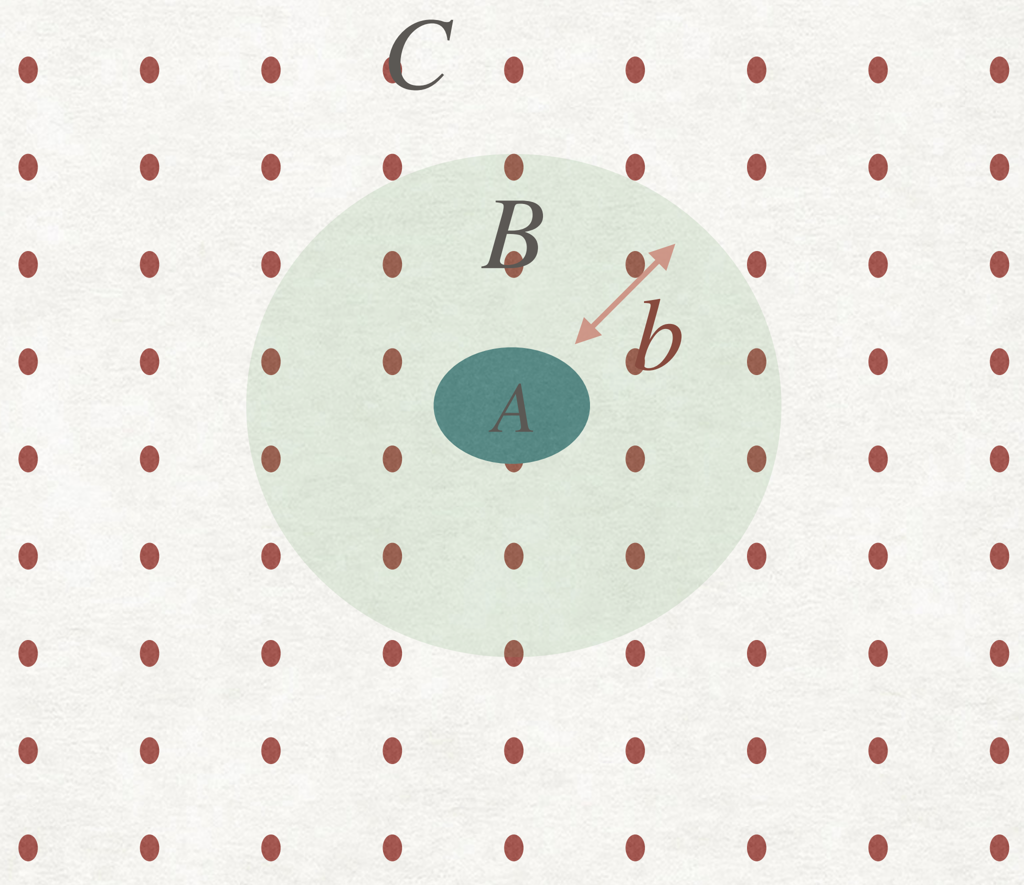


- Non-equilibrium transitions involve loss of Markov length

  [Sang & Hsieh 25]

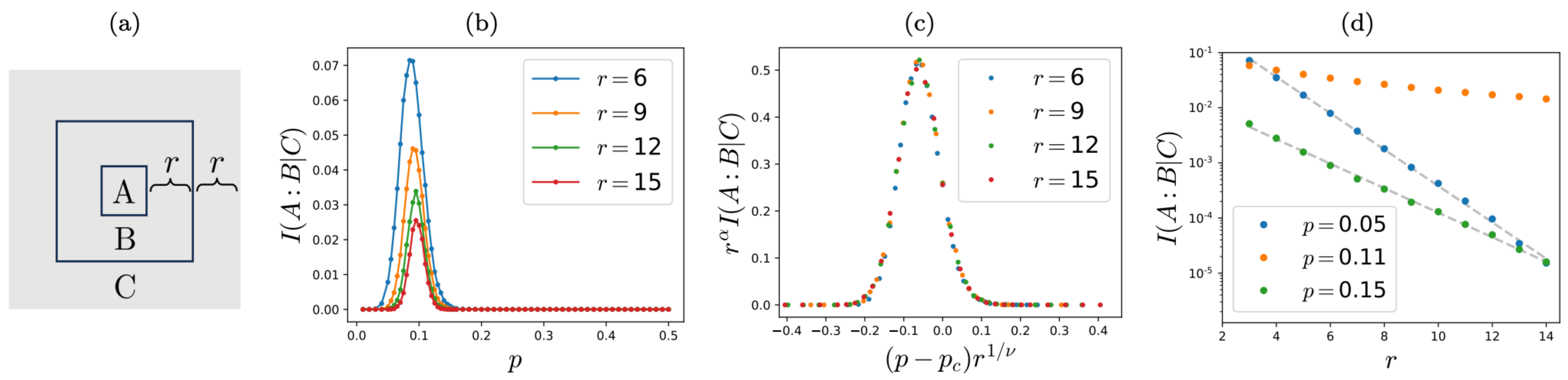


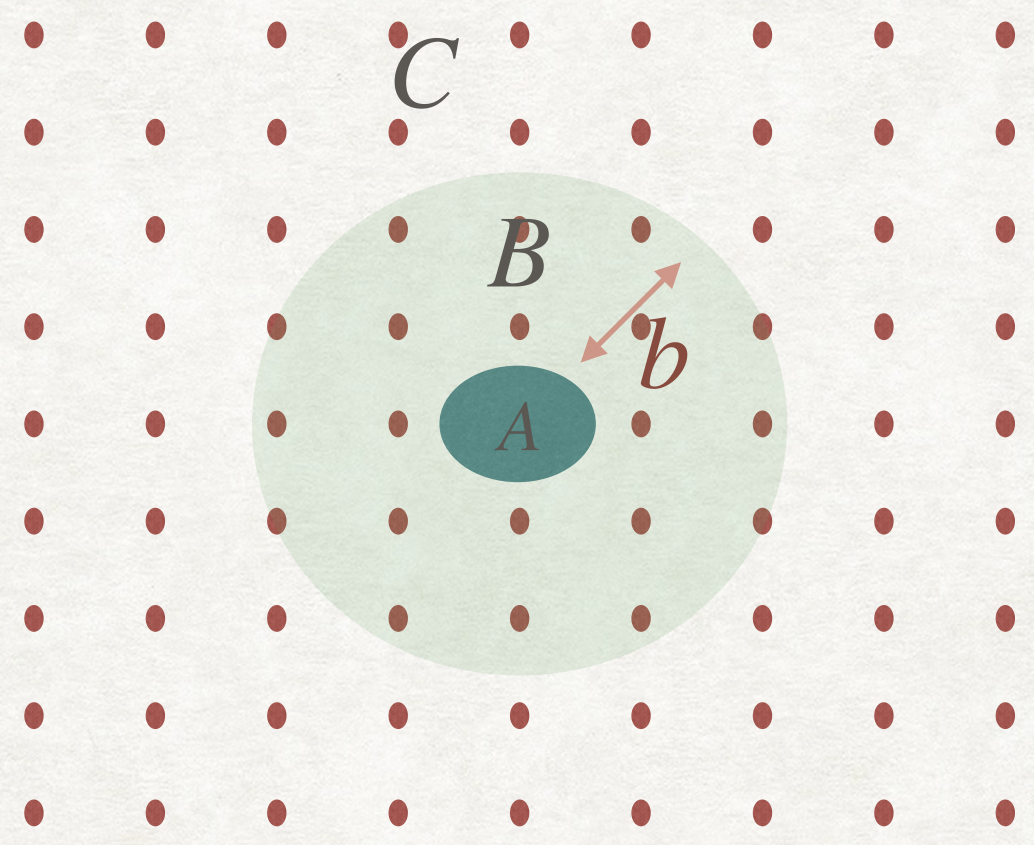
Figure 3. **CMI of dephased toric code state** — **(a)** Partition with A fixed and varying r (width of B, C). **(b)**

$\rho_{Ising} \propto \mathbb{I} + Z_1 \ldots Z_n,$

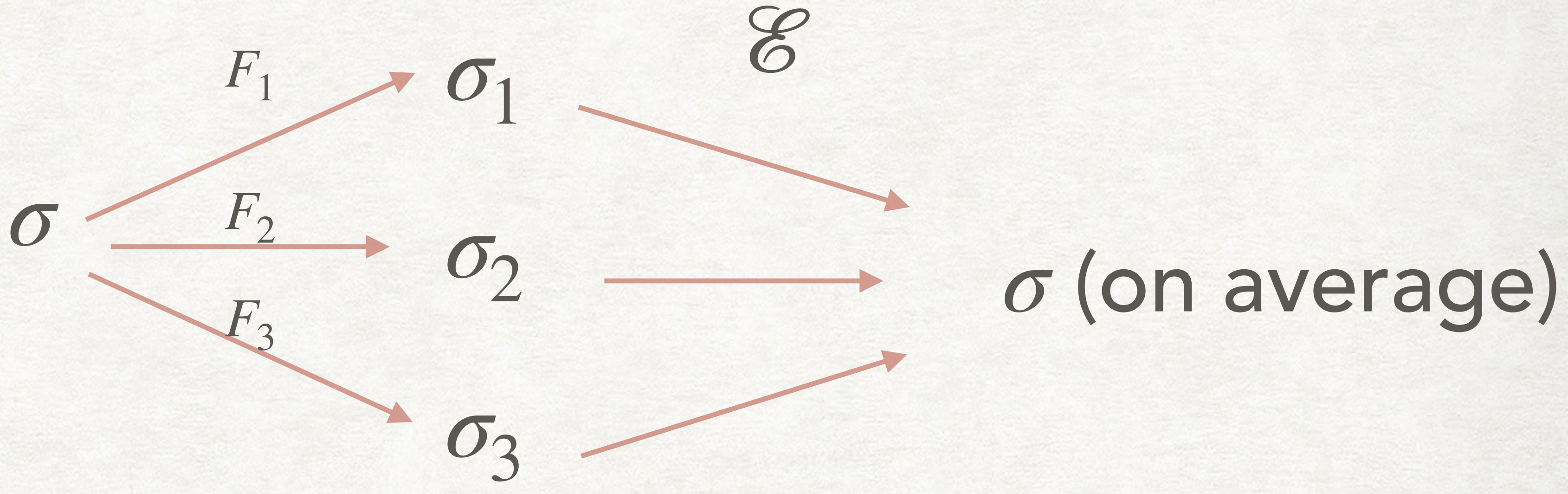
$\mathrm{I}(A : C|B)_{\rho_{Ising}} = 1$

DEFINING A GAPPED MIXED STATE

- Use both correlations & CMI.
- <u>**Definition**</u>: A state σ is short-range correlated (SRC) if
    - Correlation length: $|\langle O_A O_B \rangle - \langle O_A \rangle \langle O_B \rangle| \leq f(A)\, e^{-\frac{b}{\xi}}$,
    - Markov length: $\mathrm{I}(A : C | B) \leq g(A)\, e^{-b/\eta}$.
    - $f, g = e^{O(A)}$.
  - For pure states, $\mathrm{I}(A : C | B) = \mathrm{I}(A : C)$ (CMI=correlations)
  - Both are invariant of a phase [Yi et al. 26]
- We would like an operational characterization! (locality/stability).



OUR RESULTS



SRC ⟷ LOCAL STABILITY

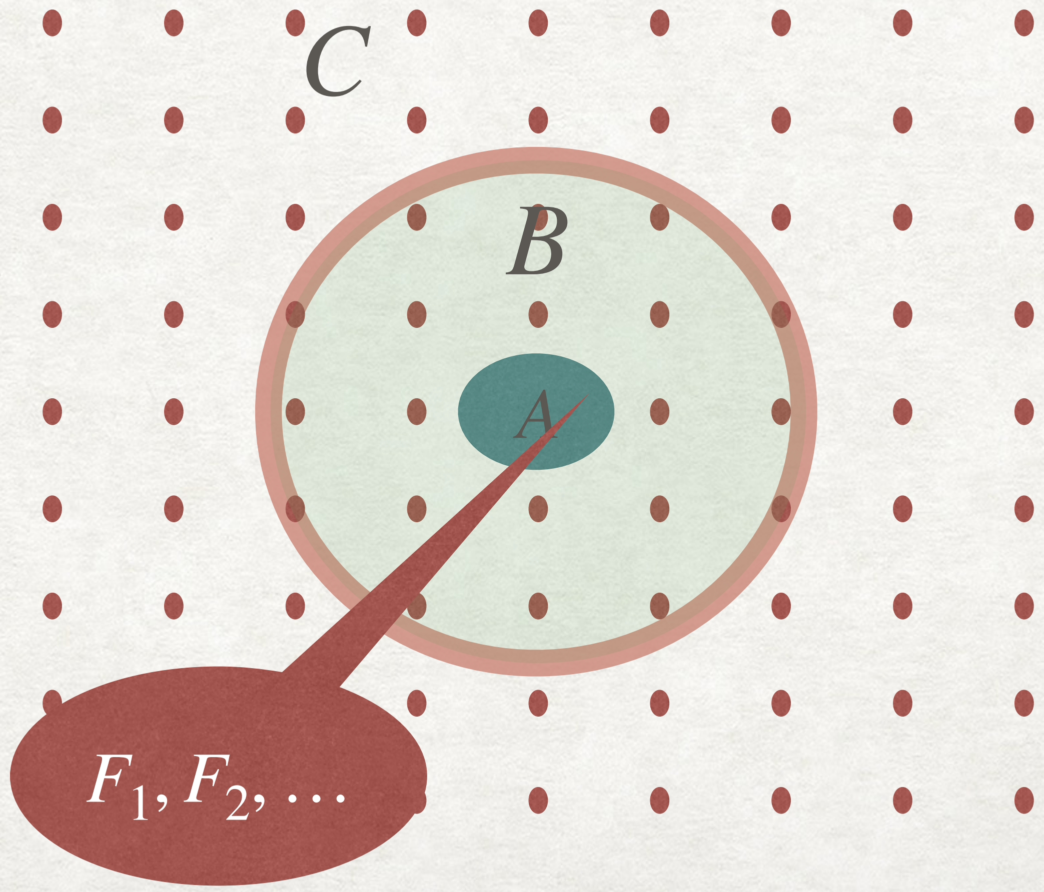
GAPPED DYNAMICS → LOCAL STABILITY

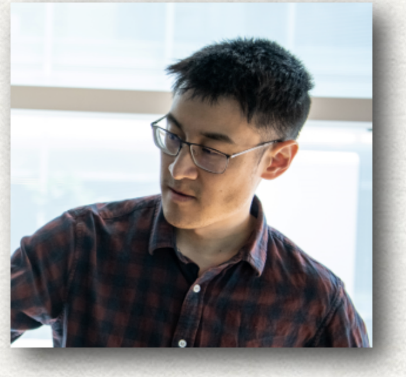
SRC → IMPLEMENTABILITY OF POST-SELECTION

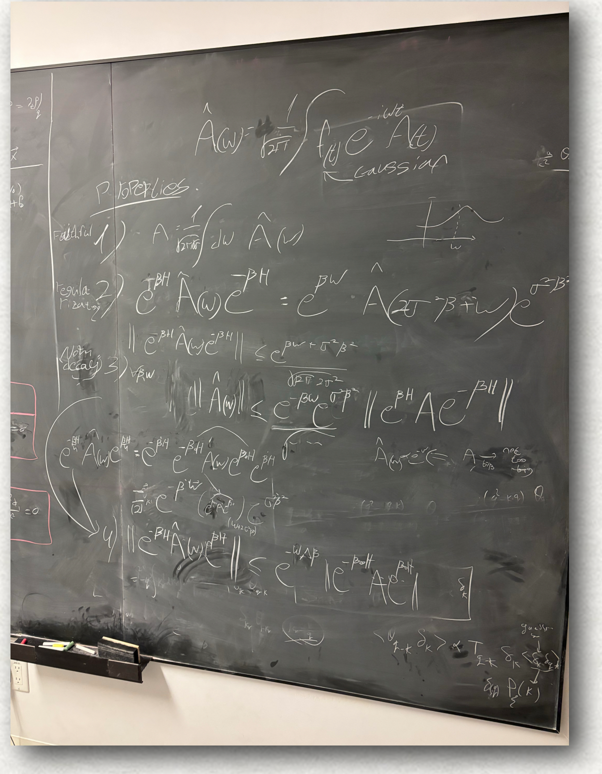
SRC → DECAY OF NON-LINEAR CORRELATIONS

DECAY OF NON-LINEAR CORRELATIONS → SRC CANONICAL PURIFICATION

$\exists \mathscr{E} : AB \longrightarrow AB$

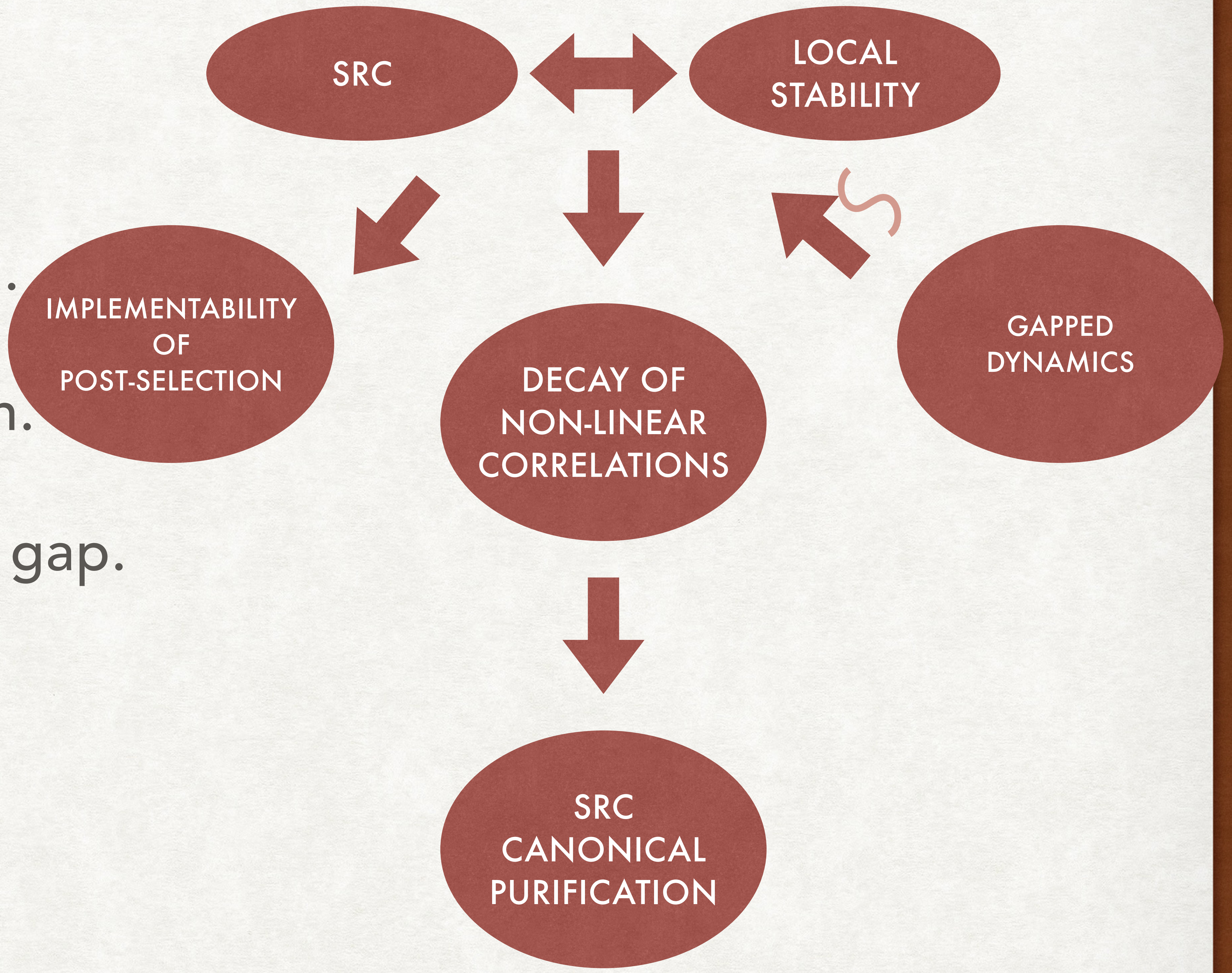






CONCLUSIONS

- Future Research:
  - Improve the canonical purification bound.
  - Establish nonlinear fluctuation-dissipation.
  - Refine the connection to the Lindbladian gap.
  - Understand the inverses of the diagram.

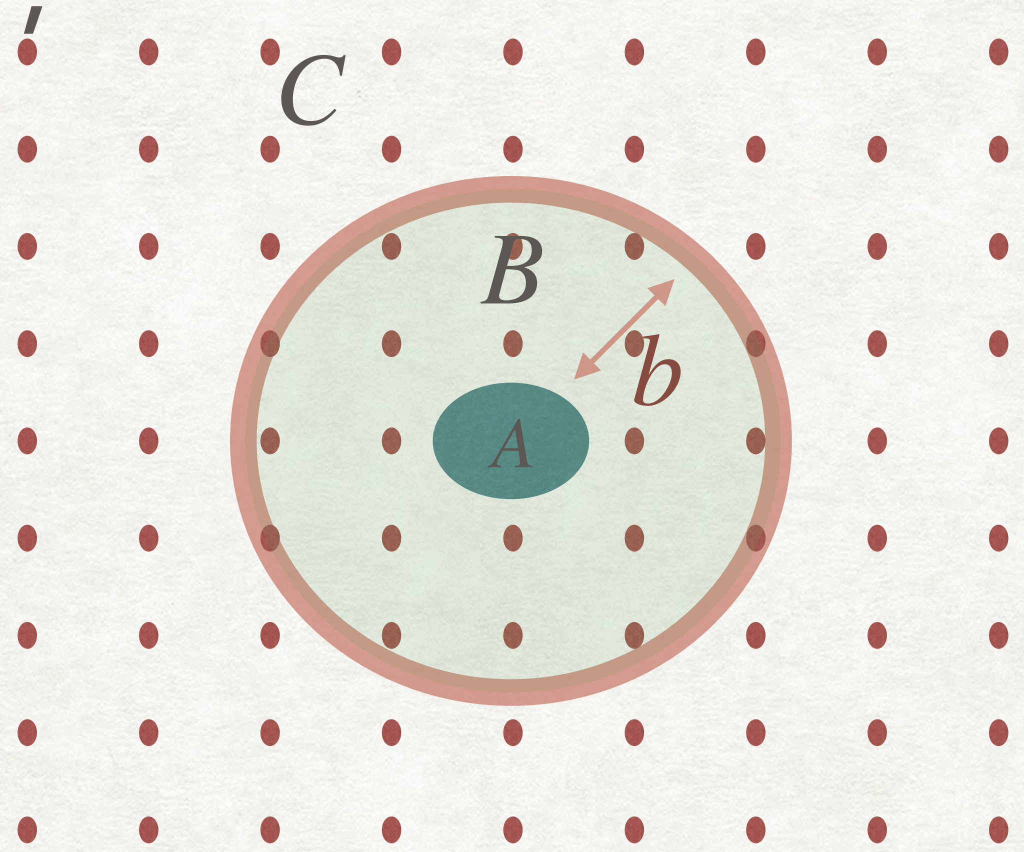


TTHANK YOU
STAY CORRELATED

PROOFS AND DEFINITIONS

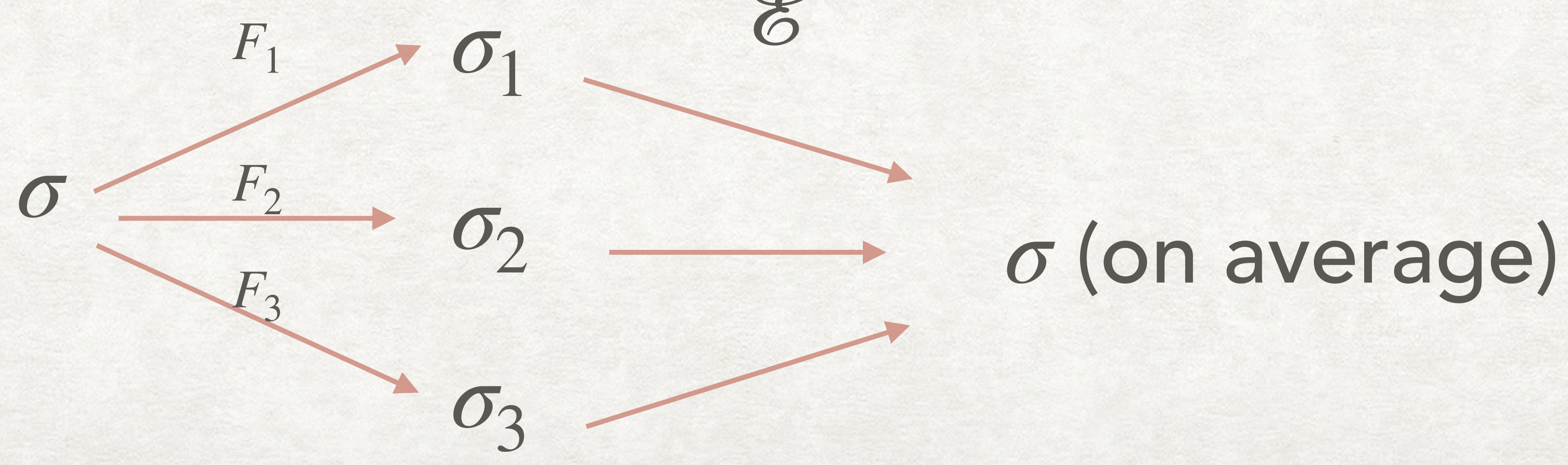
LOCAL STABILITY

- We would like a definition that demonstrates locality/stability.
- Recall: low CMI implies Petz recovery $B \to AB$.
- **Definition:** σ has local stability if there is a channel $\mathscr{E} : AB \to AB$ such that for any measurement $\mathscr{F} : A \to A$, $\mathscr{F}(\cdot) = \sum_k F_k(\cdot)F_k^\dagger$,



$$p_k = \mathrm{Tr}[F_k \sigma F_k^\dagger] \qquad \sum_k p_k \|\mathscr{E}(\sigma_k) - \sigma\|_1 \le h(A)\, e^{-\frac{b}{\zeta}}$$

STRONGER THAN CHANNEL RECOVERY

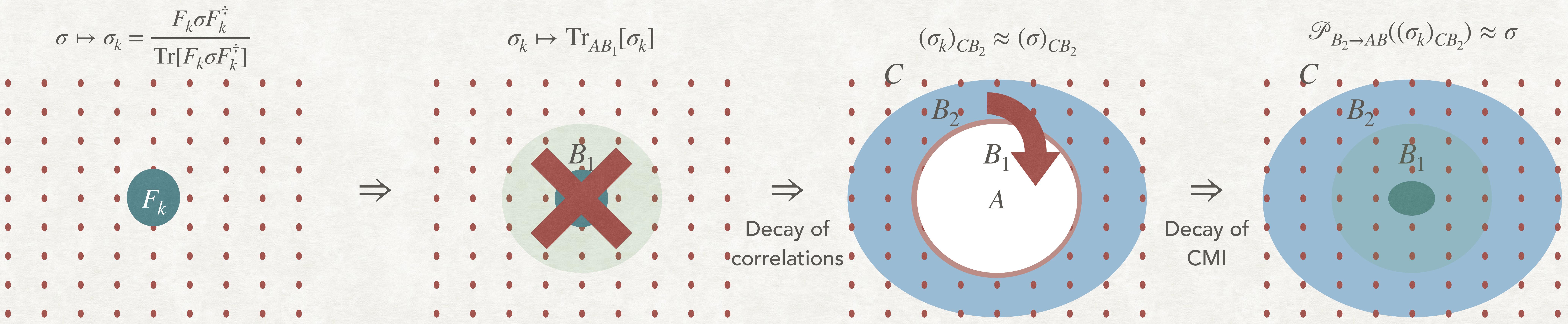


LOCAL STABILITY

$$\sum_k \|\mathcal{E}(F_k \rho F_k^\dagger) - p_k \sigma\|_1 \leq h(A)\, e^{-\frac{b}{\zeta}}$$

- **Theorem**: σ is locally stable $\Leftrightarrow$ σ is short-range correlated.

- Proof Idea ($\Leftarrow$): Trace out and recover

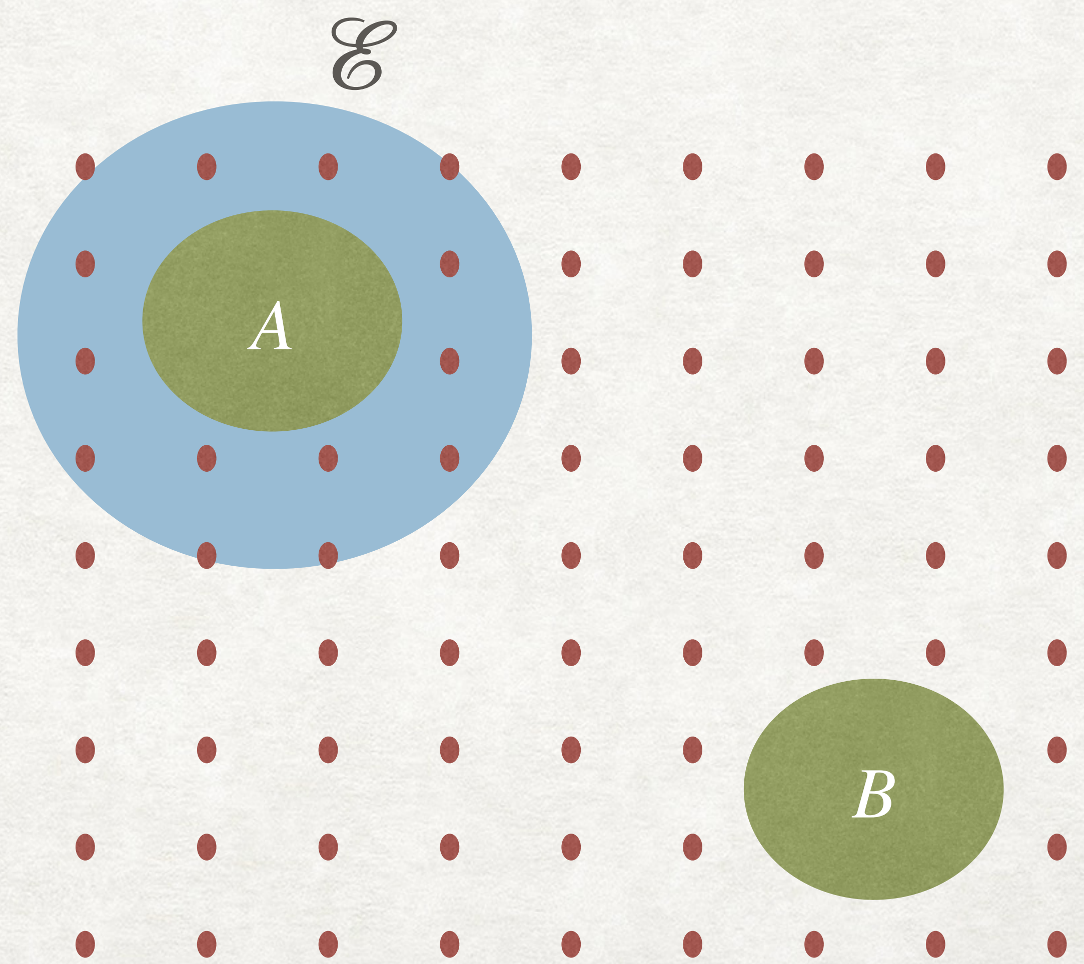


LOCAL STABILITY

$$\sum_k \|\mathcal{E}(F_k \rho F_k^\dagger) - p_k \sigma\|_1 \leq h(A)\, e^{-\frac{b}{\zeta}}$$

- **<u>Theorem</u>**: σ is locally stable $\Leftrightarrow$ σ is short-range correlated.
- Proof Idea ($\Rightarrow$):
  - CMI decay - by using $\mathcal{E}$ as a Petz recovery
  - Correlations decay - recover the effect of one observable.
  - $$\begin{aligned} \mathrm{Tr}[\sigma O_A O_B] &= \mathrm{Tr}[\left(\sqrt{O_A}\sigma\sqrt{O_A}\right) O_B] \\ &= \mathrm{Tr}[\left(\sqrt{O_A}\sigma\sqrt{O_A}\right)\mathcal{E}^*(O_B)] \\ &= \mathrm{Tr}[\mathcal{E}\left(\sqrt{O_A}\sigma\sqrt{O_A}\right) O_B] \approx \mathrm{Tr}[\sigma O_A]\mathrm{Tr}[\sigma O_B] \end{aligned}$$
    $$\approx \mathrm{Tr}[\sigma O_A]\sigma$$
-

$\mathrm{Tr}[\sigma O_A] \cdot$



NON-LINEAR CORRELATIONS

- Introduce a family of correlators for $p, q \in (0,1]$:

$$C_{p,q}(O) = \left\| \rho^{p/2} O \rho^{q/2} \right\|_{\frac{2}{p+q}}$$

- Includes most known nonlinear correlators.

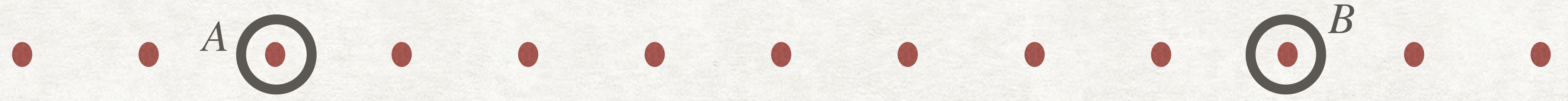
	p	q	Form
Fidelity	1	1	$\mathrm{F}\left(O\rho O^{\dagger}, \rho\right)$
Wightman	$\frac{1}{2}$	$\frac{1}{2}$	$\mathrm{Tr}\left[\sqrt{\rho}O\sqrt{\rho}O^{\dagger}\right]$
α- Rényi	α	α	$D_{\alpha}(O\rho O^{\dagger}\\|\rho)$
α sanwiched Rényi	α	1	$\tilde{D}_{\alpha}(O\rho O^{\dagger}\\|\rho)$

DIAGNOSTIC OF STRONG-WEAK SSB

[Lessa et al. 25, Weinstein 25, Liu et al. 25, Zhang et al. 26, ...]

- For Ising symmetry, $O = X_A X_B$

  ρ strongly symmetric



$C_{p,q} = \left(X_A X_B\right) \geq const$

$\langle X_A X_B \rangle \longrightarrow 0$

NON-LINEAR CORRELATOR

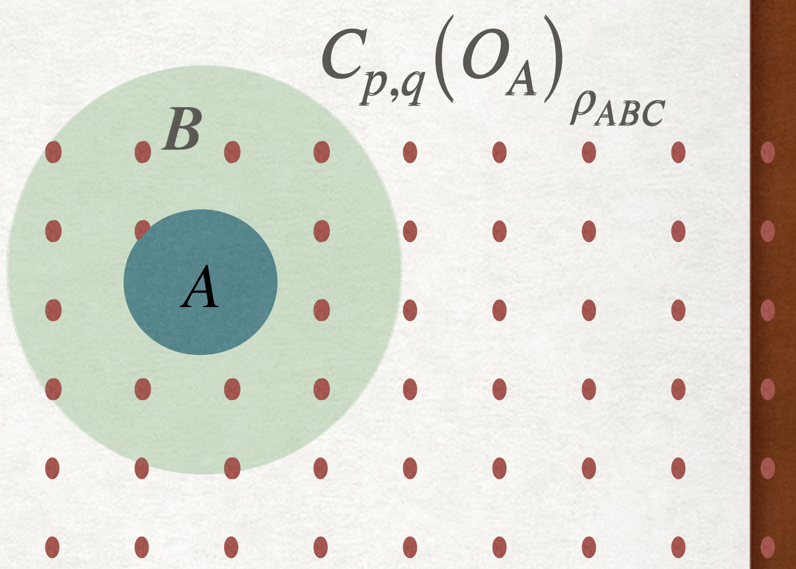
- Criticality is extended to the non-symmetric regime via the connected correlator

$$\hat{C}_{p,q}(O_A, O_B) = \left| C_{p,q}(O_A \cdot O_B) - C_{p,q}(O_A)C_{p,q}(O_B) \right|$$

- Presents richer correlations than $\langle O_A O_B \rangle - \langle O_A \rangle \langle O_B \rangle$.
  - e.g., $\mathrm{Tr}[\rho O_A] = \mathrm{Tr}[\rho_A O_A]$, but $C_{p,q}(O_A)_\rho \neq C_{p,q}(O_A)_{\rho_A}$.
- What happens in the non-symmetric case? How to control $\hat{C}_{p,q}$?

NON-LINEAR CORRELATOR

$$C_{p,q}(O)_\rho = \left\| \rho^{p/2} O \rho^{q/2} \right\|_{\frac{2}{p+q}}$$



- **Theorem**: For a short-range correlated state σ (correlations ≈ 0, CMI ≈ 0),

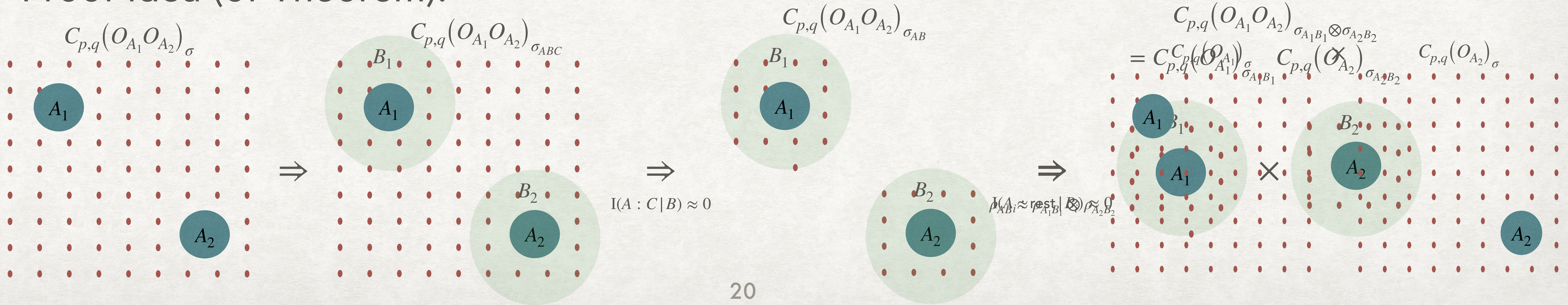
$$\hat{C}_{p,q}(O_{A_1}, O_{A_2}) \approx 0$$

OPERATOR INTERPRETATION OF CMI

- **Supporting claim:** $\mathrm{I}(A:C|B) = 0 \quad * \Leftrightarrow \quad C_{p,q}(O_A)_{\rho_{ABC}} = C_{p,q}(O_A)_{\rho_{AB}}$

  $\mathrm{I}(A:C|B) \approx 0 \quad \Rightarrow \quad |C_{p,q}(O_A)_{\rho_{ABC}} - C_{p,q}(O_A)_{\rho_{AB}}| \approx 0$

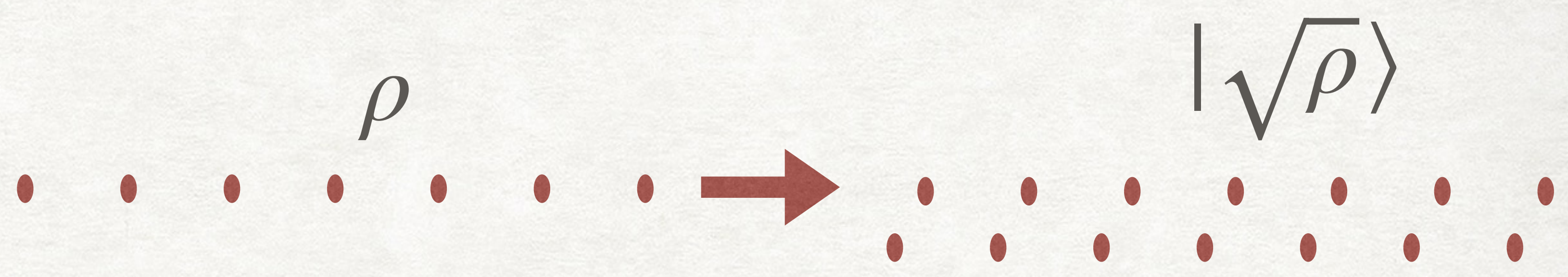
- Proof idea (of Theorem):



CANONICAL PURIFICATION

- The canonical purification of ρ is defined by

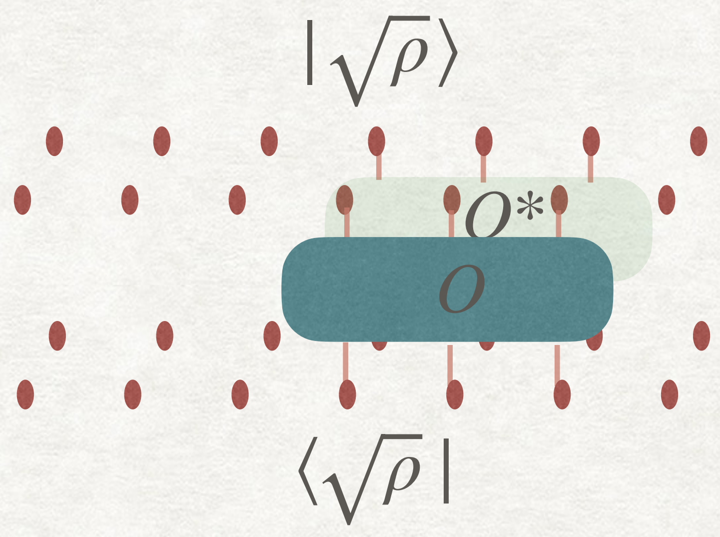
$$|\sqrt{\rho}\rangle \triangleq \sum_{i,j} (\sqrt{\rho})_{ij} |i, j\rangle$$



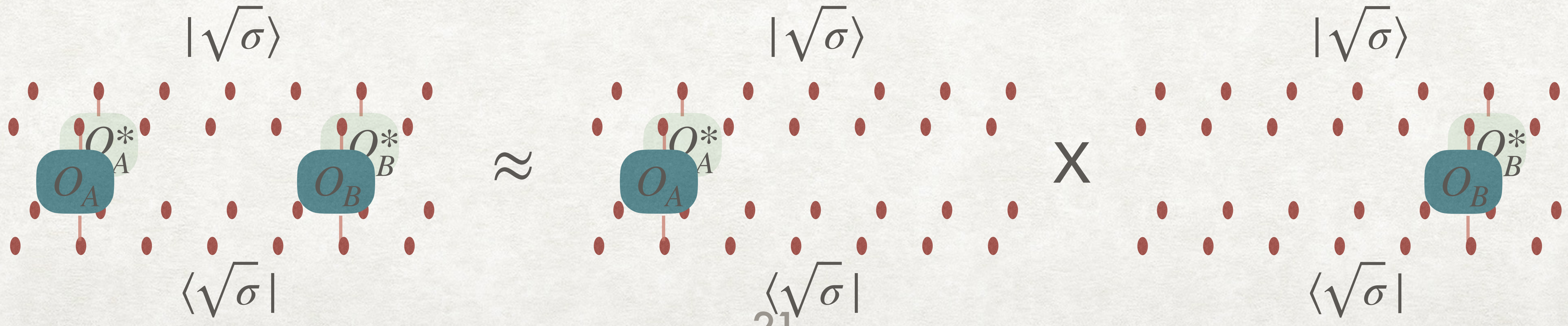
- Generally, ρ weakly correlated $\not\Rightarrow |\sqrt{\rho}\rangle$ weakly correlated.

- We show that σ is SRC $\Rightarrow$ $|\sqrt{\sigma}\rangle$ has decaying 2-point correlations.

- Setting $p = q = 1/2$, $\quad C_{p,q}(O)_\rho = \mathrm{Tr}[\rho^{1/2} O \rho^{1/2} O^\dagger]^{\frac{1}{2}} = \langle \sqrt{\rho} | O \otimes O^* | \sqrt{\rho} \rangle$



- For SRC state: $\quad |C_{\frac{1}{2},\frac{1}{2}}(O_A O_B)_\sigma - C_{\frac{1}{2},\frac{1}{2}}(O_A)_\sigma \times C_{\frac{1}{2},\frac{1}{2}}(O_B)_\sigma| \approx 0$



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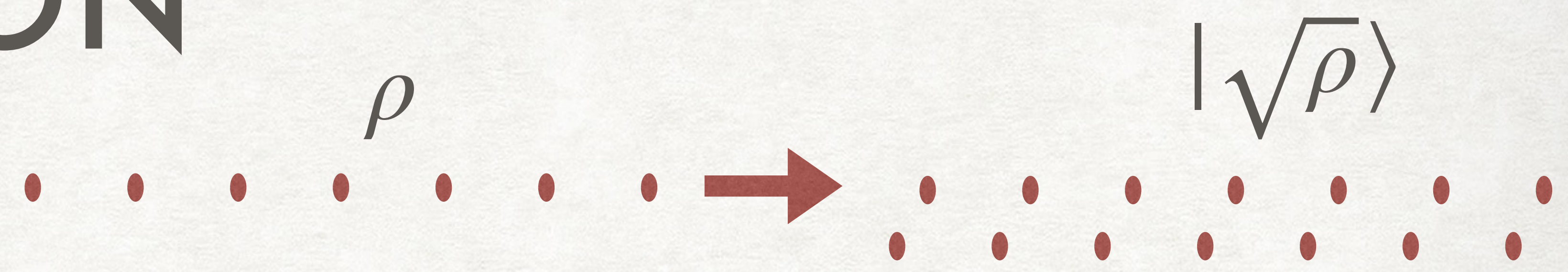
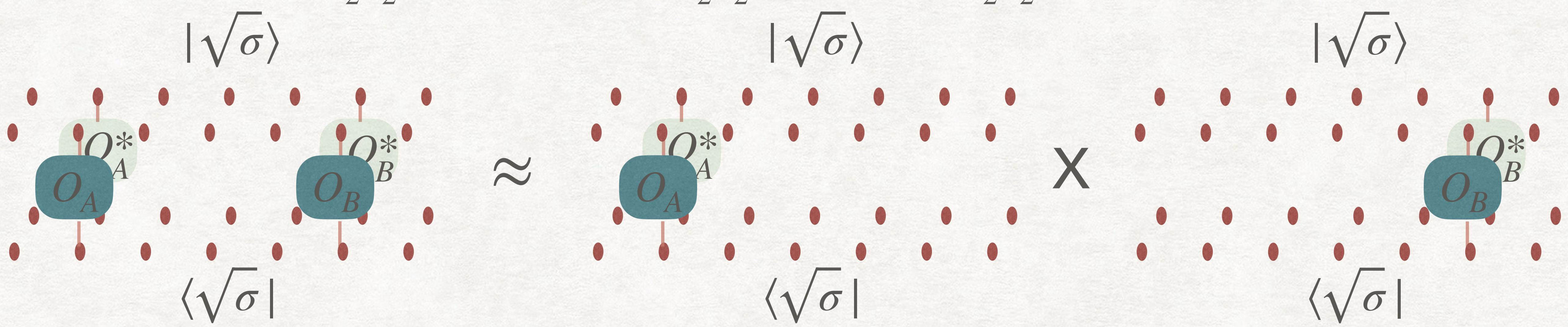
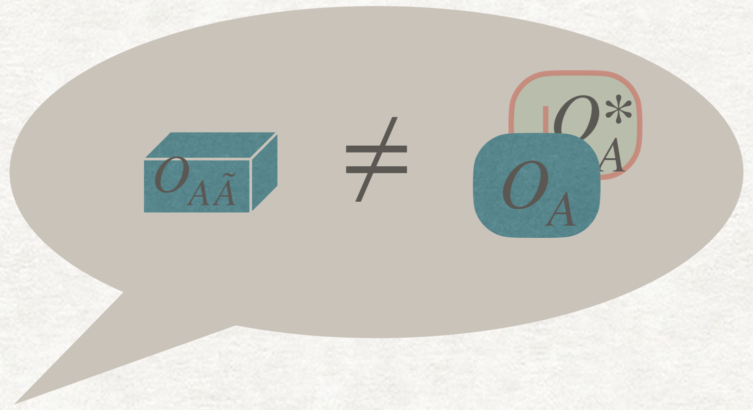


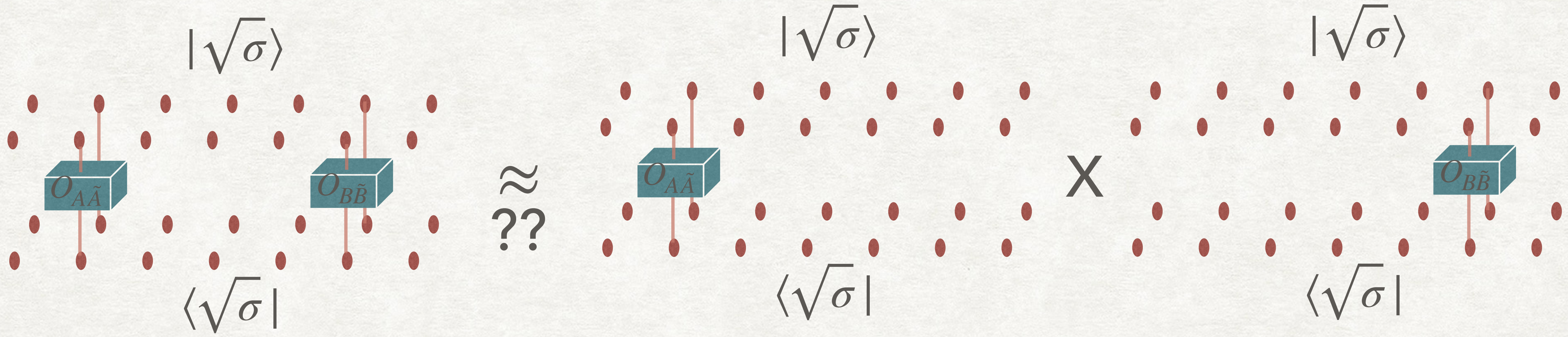
- For SRC state:

$$|C_{\frac{1}{2},\frac{1}{2}}(O_A O_B)_\sigma - C_{\frac{1}{2},\frac{1}{2}}(O_A)_\sigma \times C_{\frac{1}{2},\frac{1}{2}}(O_B)_\sigma| \approx 0$$

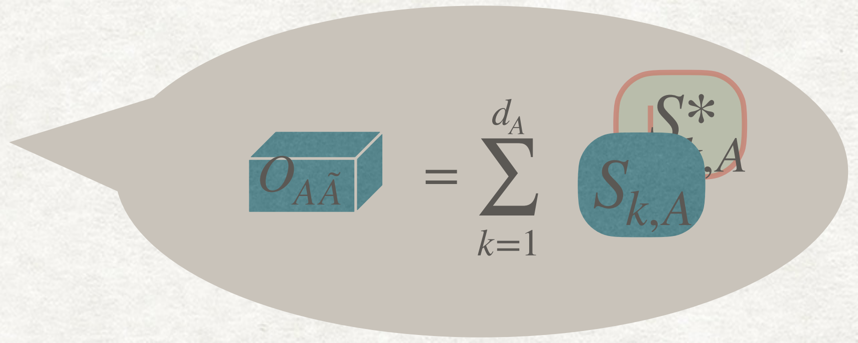


- Decay of correlations refers to general operators





- Writing $O_{A\tilde{A}} = \sum_{a,b} C_{ab} P_a \otimes P_b^* \quad = \sum_k S_{k,A} \otimes S_{k,\tilde{A}}^*$



$\|S_{k,A}\| \lesssim d_A \sqrt{\|O_{A\tilde{A}}\|}$

If C is hermitian $\Rightarrow \quad C = U \begin{pmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \lambda_3 & \\ & & & \ddots \end{pmatrix} U^\dagger$