On observers, fixed geometry states, and generalized entanglement wedges

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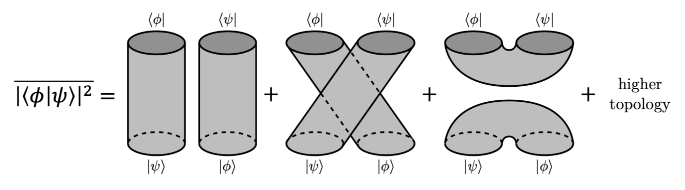
According to various lines of evidence:

$$\boxed{\dim(\mathcal{H}_{\mathrm{CU}}) = 1}\,.$$

Example: Consider the **variance** of the inner product

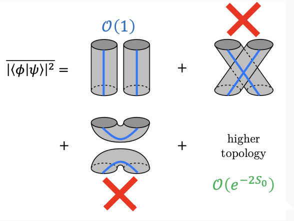
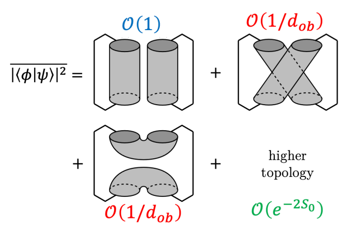
- For a large number of random states in a small Hilbert space, inner product has $\sigma^2 \sim \frac{1}{\dim}$
- **Gravity:** Asymptotic boundary data defines states, **Euclidean path integral** computes inner product, **wormholes** between replicas capture statistical effects



[Penington/Shenker/Stanford/Yang '19], [Usatyuk/Wang/Zhao '24], [Usatyuk/Zhao '24], [Abdalla/Antonini/Iliesiu/Levine '25], [Harlow/Usatyuk/Zhao '25]

Observers to the Rescue

Two proposals to recover a non-trivial Hilbert space in the presence of an **observer**:

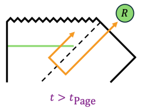
<table>
<tr><th></th><th>Abdalla-Antonini-Iliesiu-Levine</th><th>Harlow-Usatyuk-Zhao</th></tr>
<tr><td>Principle</td><td>Observers must be in the universe where their measurement occurs</td><td>Observers should be fundamentally classical</td></tr>
<tr><td>Rule</td><td>Worldline cannot cross replicas</td><td>Clone observer in the pointer basis</td></tr>
<tr><td>Variance calculation</td><td>
</td><td>
</td></tr>
<tr><td>Dimension</td><td>$\dim = d_{\text{ob}} \times e^{2S_0}$</td><td>$\dim = \min\{d_{\text{ob}}, e^{2S_0}\}$</td></tr>
</table>

[Abdalla/Antonini/Iliesiu/Levine ’25], [Harlow/Usatyuk/Zhao ’25]

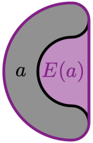
Generalized Entanglement Wedges

Progress on the black hole information problem has also suggested a more general form of holography with **information encoded in a gravitating subregion**

- Hawking radiation reservoir encodes the “entanglement island” in the interior



General proposal due to **Bousso-Penington (BP)** for time-symmetric slices:



$E(a)$ defined as the wedge whose boundary is homologous to a and minimizes $S_{gen}(E(a))$.

- Entropy $S_{\text{gen}}(a) = \frac{\text{Area}[\partial E(a)]}{4G} + S(\rho^{\text{bulk}}_{E(a)})$
- Obeys monotonicity, no-cloning, strong subadditivity, etc.
- Path integral argument (based on tensor network intuition) due to **Kaya-Rath-Ritchie (KRR)**

[Bousso/Penington ’22], [Bousso/Penington ’23], [Kaya/Rath/Ritchie ’25]

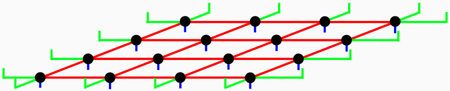
Outline

We will illustrate the close connection between the AAIL observer rules and the path integral rules of KRR for generalized entanglement wedges, helping to clarify the interpretation of the AAIL Hilbert space

1. The rules are equivalent for tensor networks
2. KRR reduces to AAIL for subregion $=$ infinitesimal neighbourhood of worldline

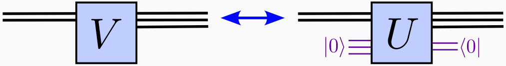
Tensor networks (TNs) are very useful models for holographic maps

- Modular construction which generalizes quantum circuits
- Unitaries replaced by more general tensors (often isometries)



- Black dots = tensors
- *Blue lines* = *tensor factors of input* $\mathcal{H}_{\text{bulk}}$
- *Green lines* = *tensor factors of output* $\mathcal{H}_{\text{bdy}}$
- *Red lines* = *tensor contractions (geometrical)*

Particularly good model for holography: **(Haar) random tensors**

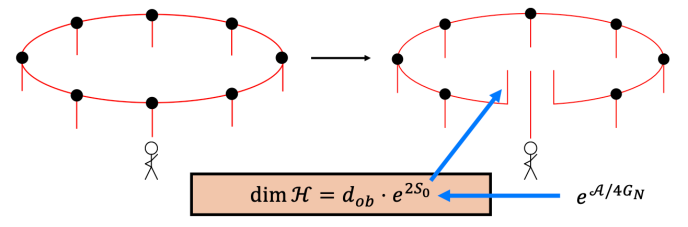


[Swingle '09], [Harlow/Pastawski/Preskill/Yoshida '15], [Hayden et al. '16]

AAIL Observer Rules: Tensor Network Version

A tensor network version of the AAIL observer rules was proposed by **Akers et al.**

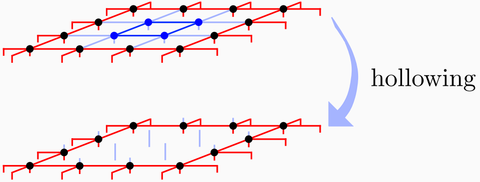
- $\dim(\mathcal{H}_{\text{CU}}) = 1$: The TN has input (bulk) legs but no output (boundary) legs
- Observer rule: Modify TN by **removing tensors acting on the observer**



- The resulting output Hilbert space has d_{ob} internal degrees of freedom for the observer and e^{2S_0} "edge mode" degrees of freedom

[Abdalla/Antonini/Iliesiu/Levine '25], [Akers/Bueller/DeWolfe/Higginbotham/Reinking/Rodriguez '25]

KRR Generalized Entanglement Wedge Rules: Tensor Network Version



KRR propose identical rules for computing the fundamental Hilbert space dimension associated with a bulk subregion

Denote by V_a the "hollowed" tensor network with tensors in subregion a removed

- Fundamental state in the observer's Hilbert space: $\rho_a = V_a(\rho_a^{\text{bulk}} \otimes \rho_{\bar{a}}^{\text{bulk}})V_a^\dagger$
- A calculation with random tensors yields

$$S_{\text{ann}}(\rho_a) = \begin{cases} S(\rho_a^{\text{bulk}}) + S(\rho_{\bar{a}}^{\text{bulk}}) & \ln(d_e) \gg S(\rho_{\bar{a}}^{\text{bulk}}) \\ S(\rho_a^{\text{bulk}}) + \ln(d_e) & \ln(d_e) \ll S(\rho_{\bar{a}}^{\text{bulk}}) \end{cases} .$$

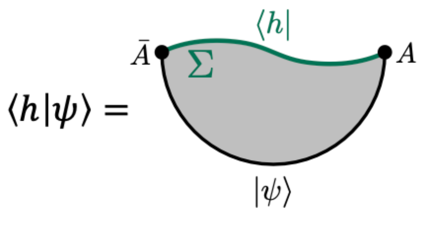
This recovers the RT/BP formula with RT surface $\emptyset$ or ∂a

[Bousso/Penington '22], [Bousso/Penington '23], [Kaya/Rath/Ritchie '25]

From Tensor Networks to the Gravitational Path Integral

Gravity analogue of TN: **Fixed geometry states**

- Roughly: "Position eigenstates" $|h\rangle$ for the spatial metric and fields
- Overlaps computed by a Euclidean path integral with fixed geometry boundary condition



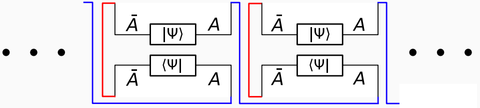
The connection is perhaps unsurprising given the interpretation of the tensor network as a discretization of a bulk spatial geometry

[Akers/Rath '18], [Dong/Harlow/Marolf '18], [Dong/Marolf '19],[Penington/Shenker/Stanford/Yang '19], [Akers/Faulkner/Lin/Rath '21], [Penington/Walter/Witteveen '22], [Dong/Kudler-Flam/Rath '23], [Penington/Rath '24]

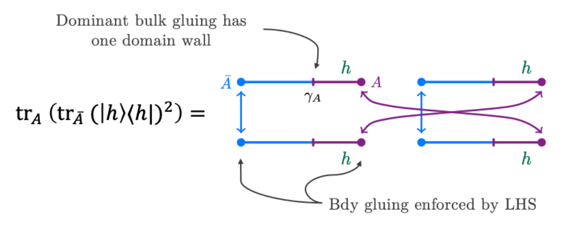
Replica Calculations with Fixed Geometry States

Recall the **replica trick** for von Neumann entropy:

- Rényi entropy: $S_n(\rho_A) = \frac{1}{1-n} \ln \mathrm{tr}(\rho_A^n)$
- VN entropy: $S(\rho_A) = \lim_{n\to 1} S_n(\rho_A)$



Now apply to reduced density matrix $\rho_A(h) = \mathrm{tr}_{\bar{A}}(|h\rangle\langle h|)$ of fixed geometry state $|h\rangle$



- Compute $\mathrm{tr}(\rho_A(h)^n)$ by a path integral
- Involves **2n copies** of h with asymptotic boundaries $A, \bar{A}$ glued appropriately
- $\mathrm{tr}(\rho_A(h)^n) \approx e^{-(n-1)\min_{\gamma_A(h)} \mathrm{Area}[\gamma_A(h)]/4G}$
- $S_n(\rho_A(h)) = \frac{\mathrm{Area}[\gamma_A(h)]}{4G} = \mathrm{const}$

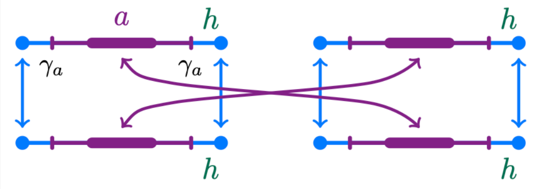
[Lewkowycz/Maldacena '13], [Akers/Rath '18], [Dong/Harlow/Marolf '18], [Dong/Marolf '19],[Penington/Shenker/Stanford/Yang '19], [Akers/Faulkner/Lin/Rath '21], [Penington/Walter/Witteveen '22], [Dong/Kudler-Flam/Rath '23], [Penington/Rath '24]

KRR Generalized Entanglement Wedge Rules: Path Integral Version

KRR rule: Put the bulk subregion a on the same footing as an asymptotic boundary of h

- Gauge-invariantly specify bulk subregion a across all $|h\rangle$
- Compute $\text{tr}(\rho_a(h)^n)$ by appropriate gluing of the asymptotic boundaries and a
- The conical defect is now the boundary of the generalized entanglement wedge

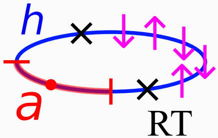
$$\text{tr}(\rho_a(h)^n) =$$



[Kaya/Rath/Ritchie '25]

KRR in a Closed Universe

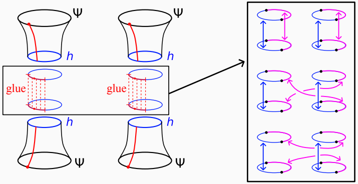
Applying the KRR rule to a fixed geometry state with bulk entropy, we can obtain a **non-vanishing entropy**, indicating a **non-trivial fundamental Hilbert space** for a



$$S(\rho_a) = \frac{\text{Area}[\partial E(a)]}{4G} + S(\rho^{\text{bulk}}_{E(a)})$$

It is instructive to consider how the KRR replica calculation applies to closed universe states defined by asymptotic Euclidean boundaries

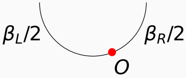
$$\text{tr}(\rho_a^n) \approx \sum_h$$



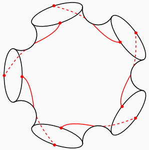
Generalized Entanglement Wedges from the AAIL Observer Rules

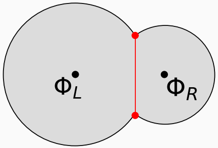
Example: Two-sided black hole in **partially entangled thermal state**

$$|\beta_L, \beta_R, \Delta\rangle = \sum_{m,n} e^{-\beta_L E_L/2} e^{-\beta_R E_R/2} O_{mn} |m\rangle_L |n\rangle_R =$$



- Semi-classical limit $C \equiv \frac{\phi_r}{8\pi G} \gg 1$, heavy operator $\Delta \sim C$
- Observer state $\tilde{\rho}_{\mathrm{ob}}$ formally obtained from $|\beta_L, \beta_R, \Delta\rangle$ by tracing out boundary Hilbert space
- **AAIL:** Lowest topology diagram for $\mathrm{tr}(\tilde{\rho}^n_{\mathrm{ob}})$ is the **"pinwheel"** Z_n
- $S_{\mathrm{ann}}(\rho_{\mathrm{ob}}) \approx \lim_{n\to 1} \frac{1}{1-n} \ln(Z_n/Z_1^n) \approx \underbrace{2\pi(\Phi_0 + \Phi_L)}_{\text{left horizon ``area''}} + \underbrace{2\pi(\Phi_0 + \Phi_R)}_{\text{right horizon ``area''}} + O(\ln C)$

$Z_n =$ 



[Goel/Tam/Turiaci/Verlinde '18], [Boruch/Iliesiu/Lin/Yan '24], [Abdalla/Antonini/Iliesiu/Levine '25]

Merci!