Tunnelling Across a Trapped Region and Out of a Black Hole

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Physical Black Holes

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The maximally extended Schwarzschild solution of classical general relativity is not a physical black hole. What is there about the Schwarzschild solution that is unphysical?

- Maximal extension:
  - Mathematical result for eternal vacuum black holes.
- Central singularity:
  - Expected to be resolved by quantum gravity.

I am interested in a non-singular black hole that forms from gravitational collapse (not an eternal black hole).

Non-Singular Black Holes

If quantum gravity does indeed resolve the singularity, there seem to be two main classes of possible solutions:

- To a reasonable approximation, there is an effective geometry everywhere in the spacetime, where the state $|\Psi\rangle$ for a black hole spacetime is such that

$$\frac{(\Delta g_{ab})_\Psi}{|\langle \hat{g}_{ab} \rangle_\Psi|} \ll 1,$$

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I will focus on the first possibility, and apply QFT on a fixed background spacetime geometry.

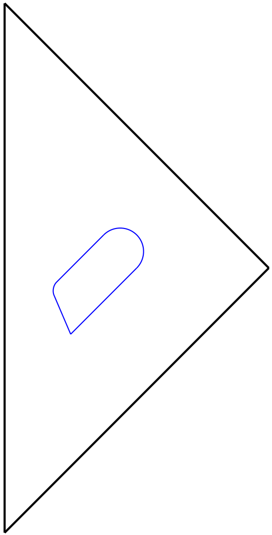
Inner Horizons

Most black holes have an inner horizon, including the astrophysically relevant rotating black holes.

Even in spherical symmetry, an inner horizon can be present.

- During collapse, inner and outer horizons form in pairs; the only reason the inner horizon eventually disappears classically is that it is swallowed by the singularity.
- In a non-singular geometry, we can also expect there to be an inner horizon [Hayward, 2006].

Conformal Diagram Sketch



This is a simple sketch of the conformal diagram for a physical black hole, formed by gravitational collapse, and eventually dying, perhaps through Hawking evaporation.

The time that the outer and inner horizon are present can be very long: I will focus on this portion of the spacetime.

The Metric

Working in two dimensions for simplicity,

$$ds^2 = -dt^2 + (dx - v\,dt)^2.$$

Here x is a Cartesian coordinate, and $v(x)$ can be any function of x. Following the usual convention, I take $v \le 0$.

There is a horizon wherever $v = -1$; if v is a smooth function and the spacetime is asymptotically flat so $v \to 0$ as $x \to \pm\infty$, then (excluding extremal horizons) there is an equal number of inner and outer horizons.

$\Rightarrow$ I will assume here v is such that there is one trapped region (where $v < -1$), with an outer and an inner horizon.

Scalar Field

Consider a massless scalar field on that 2D background, with action

$$S = -\frac{1}{2}\int d^2x\,\sqrt{-g}\,g^{ab}\,(\partial_a\phi)\,(\partial_b\phi).$$

This has the advantage of being a conformally invariant theory, so for any explicitly conformally flat metric $g^{(C)}_{ab} = \Omega^2\eta_{ab}$, the Klein-Gordon equation is simply

$$\eta^{ab}\partial_a\partial_b\phi = 0.$$

In this case, the quantization of ϕ is direct: it is the same as on Minkowski space.

Null Coordinates

To study the quantum theory, we want to put the metric in an explicitly conformal form. This can be done by introducing null coordinates

$$u = t - \int \frac{dx}{1+v}, \qquad V = t + \int \frac{dx}{1-v},$$

in terms of which

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Since $v \leq 0$, it follows that V is well-defined everywhere, but u diverges at the horizons where $v = -1$. u can be used in any one of the three regions (outer, trapped, inner), but not all three.

Therefore, we want to introduce a different null coordinate U that is also well-defined everywhere (like V) and does not diverge at horizons.

Null Coordinate U

To avoid coordinate singularities, we want $dU = f(U)du$ such that $(1 - v^2)/f(U)$ is everywhere strictly positive, so $f(U)$ must go to zero at exactly the same rate as $1 + v$ at the horizons.

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Do a Taylor series expansion at the (outer) horizon,

$$v = -1 + \alpha_o(x - x_o), \quad \Rightarrow u = \alpha_o^{-1} \ln\big(\alpha_o(x - x_o)\big),$$

and the same for the inner horizon. The coordinate change

$$U = \begin{cases} -\alpha_o^{-1}(e^{-\alpha_o u_{(o)}} + 1), & \text{outer region}, \\ \alpha_o^{-1}(e^{-\alpha_o u_{(t)}} - 1), & \text{trapped region}, u_{(t)} > 0, \\ -\alpha_i^{-1}(e^{\alpha_i u_{(t)}} - 1), & \text{trapped region}, u_{(t)} < 0, \\ \alpha_i^{-1}(e^{\alpha_i u_{(i)}} + 1), & \text{inner region}, \end{cases}$$

does the trick, with U covering the whole spacetime and, for example,

$$\left.\frac{1 + v}{f(U)}\right|_{x_o} = e^{\alpha_o t}.$$

Quantization

The quantization is now direct for a massless scalar field on the 2D black hole spacetime with the explicitly conformally flat metric

$$ds^2 = -\frac{(1-v^2)}{f(U)}\, dU\, dV\,,$$

or in terms of $T = (U+V)/2$ and $X = (V-U)/2$.

The field operator is simply

$$\hat{\phi} = \frac{1}{2\pi}\int_0^\infty \frac{dk}{\sqrt{2k}}\left(e^{-ikU}\hat{a}_k + e^{ikU}\hat{a}_k^\dagger + e^{-ikV}\hat{a}_{-k} + e^{ikV}\hat{a}_{-k}^\dagger\right),$$

and the Fock vacuum $|0\rangle$ is the state annihhilated by all a_k.

Note that the mode expansion in the field operator is expressed in terms of the null coordinates U, V, which cover all three regions.

Correlations

Can a particle tunnel out of a black hole, from the inner to the outer region?

Recall that in standard quantum field theory a particle can 'tunnel' outside of its lightcone [Hegerfeldt, 1974]. . .

A first indication that tunnelling may be possible is given by the two-point correlation function

$$\langle 0|\, \hat{\phi}(X_2, T_2)\, \hat{\phi}(X_1, T_1)\, |0\rangle \sim \ln\left(\frac{1}{-\Delta T^2 + \Delta X^2}\right),$$

which shows correlations outside the lightcone, including between points in the inner and outer regions that are causally disconnected.

Localized States

For a tunnelling calculation, consider localized states

$$|X_1, T_1\rangle = \frac{1}{2\pi}\int_{-\infty}^{\infty} dk\, e^{-i|k|T_1 + ikX_1} a_k^\dagger |0\rangle.$$

These are single-particle states, with a wave packet that at $T = T_1$ is a delta function: $\psi(X) = \delta(X - X_1)$.

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These states have a simple form, but the drawback of the delta-function localization is that the state is not normalizable.

(It is also possible to consider other states that are normalizable.)

Tunnelling Transition Amplitude

In general, the transition amplitude between two delta-function localized states is

$$\langle X_2, T_2 | X_1, T_1 \rangle = \frac{i\,\Delta T}{\pi \Delta s^2},$$

which is only polynomially suppressed with respect to Δs^2.

Taking (X_1, T_1) to be located just inside the inner horizon and (X_2, T_2) located just outside the outer horizon; in terms of Painlevé-Gullstrand coordinates

$$\langle X_2, T_2 | X_1, T_1 \rangle = \frac{i}{2\pi}\left(\frac{1}{\alpha_i^{-1}+\alpha_o^{-1}} - \frac{1}{\Delta t + \Delta x/(1-v_t)}\right),$$

where v_t is the typical value of v in the trapped region.

$\Rightarrow$ Particles can tunnel out of a black hole, from the inner region.

Information Loss Problem

Does this sort of tunnelling have an impact on the information loss problem?

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Work in progress: Does the black hole lose mass more rapidly

- from Hawking radiation (thermal emission), or
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- from Hawking radiation (thermal emission), or
- from tunnelling (unitary process)?

Simplistic order of magnitude estimates suggest the two effects occur at approximately the same rate—if this is correct, then perhaps tunnelling can help resolve the information loss problem, but more work is needed. . .

Summary

The calculation:

- Consider a 2D black hole model with outer and inner horizons,
- Use null coordinates U, V that cover the entire space, and for which the metric is explicitly conformally flat,
- Quantize the (conformally invariant) massless scalar field,
- Compute the transition amplitude between:
  - a state with 1 particle localized inside the inner horizon, and
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Thank you for your attention!