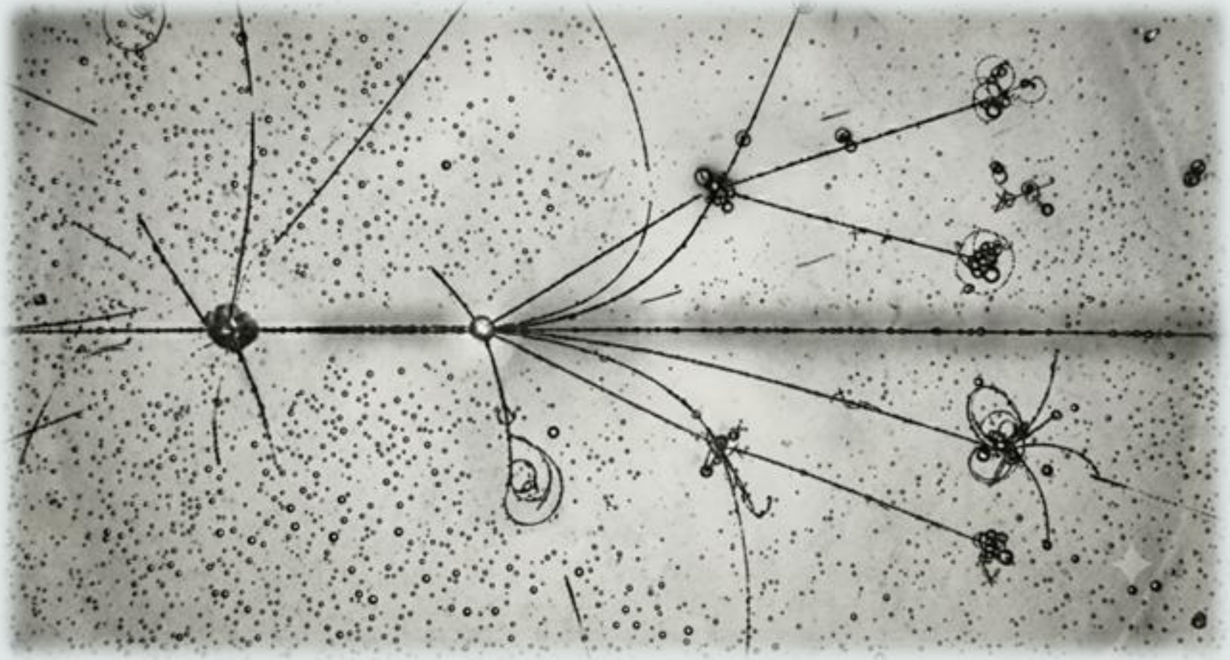
SU(3)-flavour breaking in $\bar{B} \to D(\bar{D})P$ decays

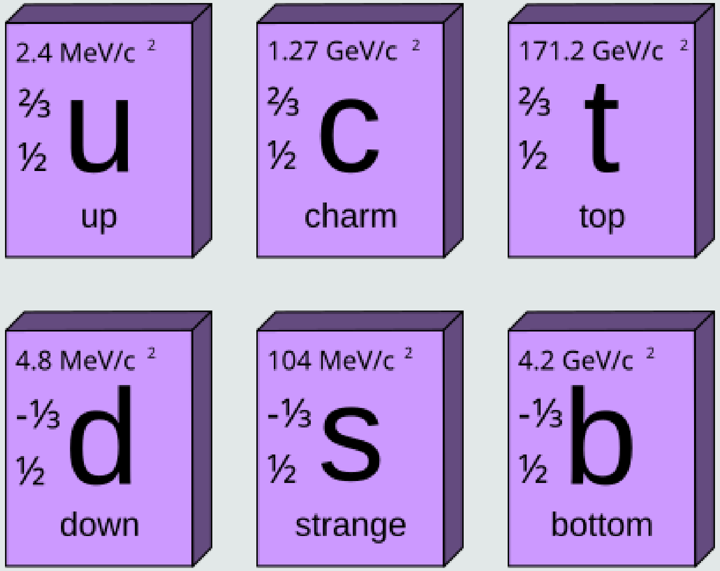


Work done in collaboration with:

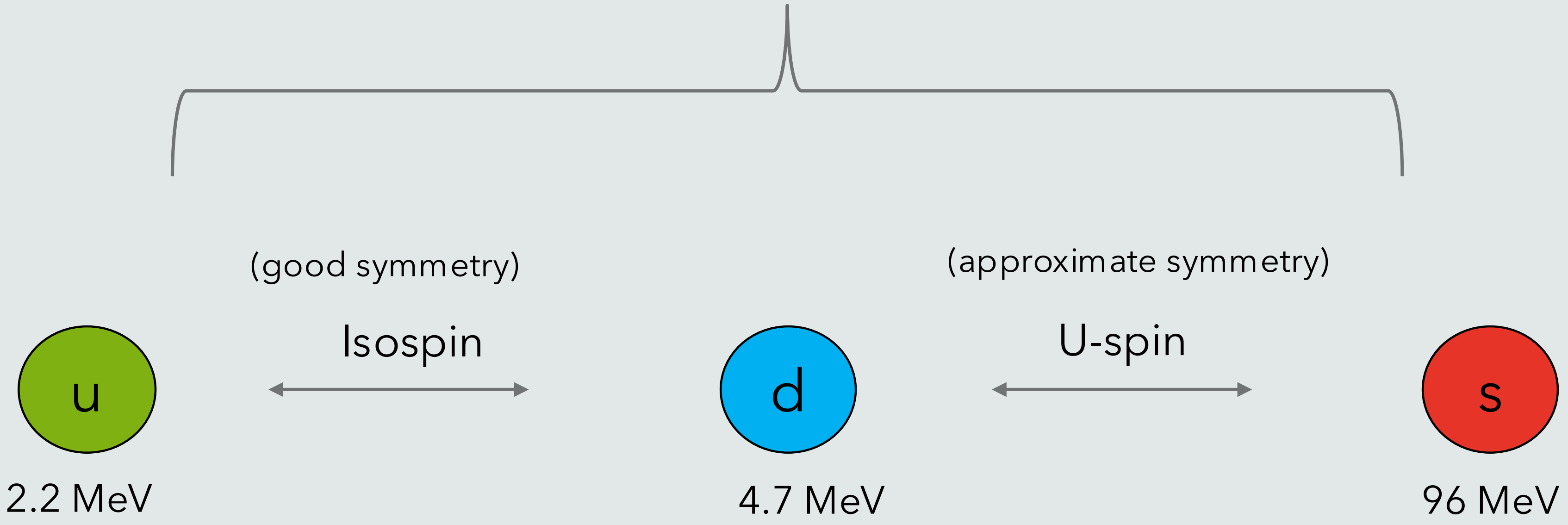
Bhubanjyoti Bhattacharya & David London

RIDA BOUMRIS

SU(3)-flavour symmetry



SU(3)-flavour symmetry



$m_u \sim m_d \sim m_s$: Symmetry under the strong force

Standard model (SM)

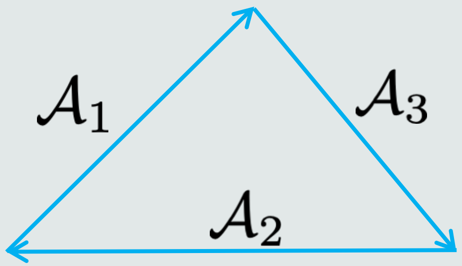
→ The symmetry breaking is due to the mass difference between the s-quark & u,d-quarks $\approx 30\%$

Sum rules

- Using the SU(3) symmetry one could find relations between several decay amplitudes.
- Any deviation from these sum-rules hints at SU(3) breaking .

$$\mathcal{A}(\bar{B}^0 \to \pi^+ D_s^-) = \sqrt{2}\,\mathcal{A}(B^- \to \pi^0 D_s^-)$$ ⟹ Related by isospin

$$\mathcal{A}(B^- \to K^- D^0) = \mathcal{A}(\bar{B}^0 \to K^- D^+) \;+\; \mathcal{A}(\bar{B}^0 \to \bar{K}^0 D^0)$$ ⟹ Related by SU(3)



Why study SU(3) breaking in $\bar{B} \to D(\bar{D})P$ decays?

- SU(3) symmetry breaking higher than 30% seen in B -> PP decays.
- QCD-factorization discrepancies in certain B -> DP decays.

Eur. Phys. J. C (2020) 80:951
https://doi.org/10.1140/epjc/s10052-020-08512-8
THE EUROPEAN PHYSICAL JOURNAL C
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Regular Article - Theoretical Physics

A puzzle in $\bar{B}^0_{(s)} \to D^{(*)+}_{(s)}\{\pi^-, K^-\}$ decays and extraction of the f_s/f_d fragmentation fraction

Marzia Bordone[1,a], Nico Gubernari[2,b], Tobias Huber[1,c], Martin Jung[3,d], Danny van Dyk[2,e]

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PHYSICAL REVIEW LETTERS **133,** 211802 (2024)
Featured in Physics

Anomalies in Hadronic *B* Decays

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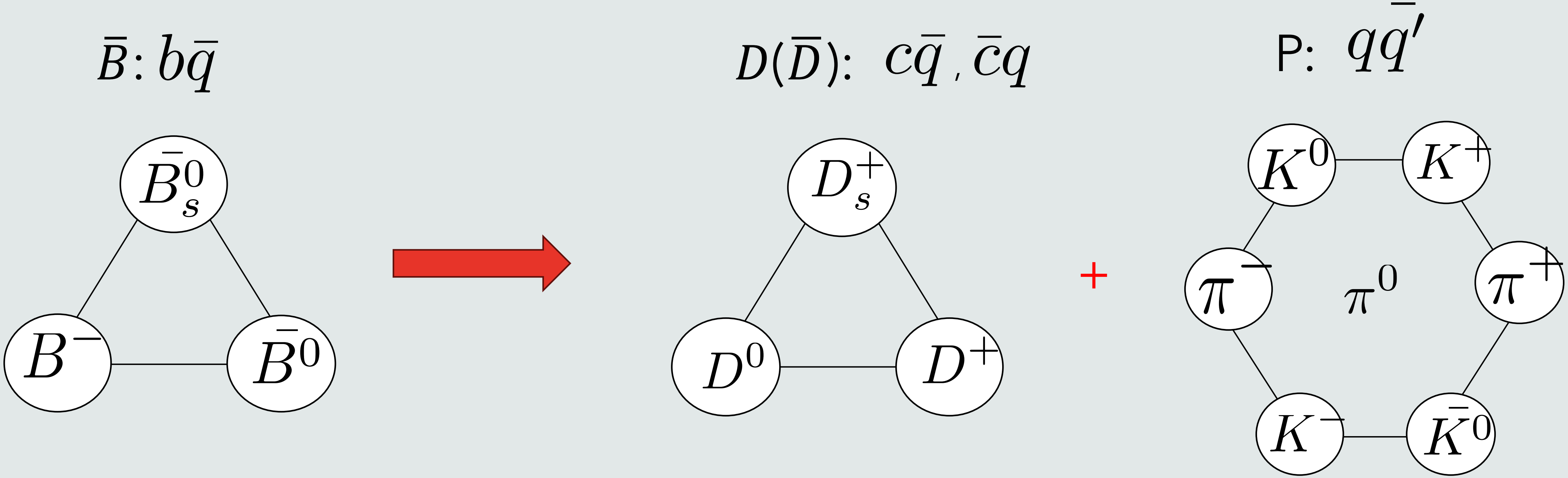
(Received 12 December 2023; accepted 9 October 2024; published 21 November 2024)

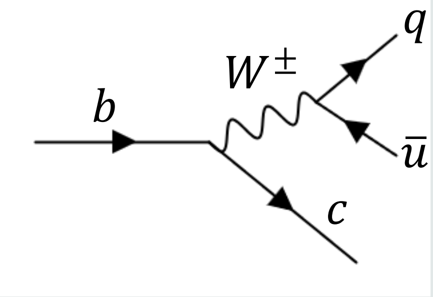
$\bar{B} \to D\,(\bar{D})P$ system under SU(3)

- Under SU(3)-flavour: B forms a triplet **3**, D forms a triplet **3** and P is an octet **8**.
- We have a total of 26 decays to consider.

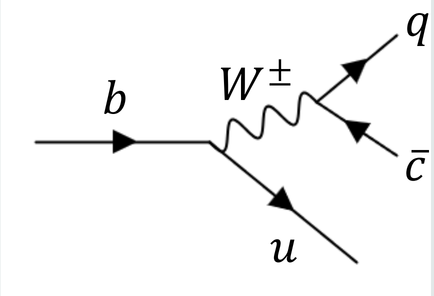


$\bar{B} \to D\,(\bar{D})P$

$\bar{B} \to DP$ → $b \to c\bar{u}q \propto V_{cb}V_{uq}^*$



$\bar{B} \to \bar{D}P$ → $b \to u\bar{c}q \propto V_{ub}V_{cq}^* \quad \sim e^{-i\gamma}$



- The second channel has a weak phase: γ
- Hamiltonians transform differently under SU(3).
- Interference between the two channels gives rise to CP-violations

- RMEs are defined using representations of the initial and final states, as well as the Hamiltonian.
- We use Wigner-Eckart theorem to express decay amplitudes.

$$RME \equiv \langle DP || \mathcal{H} || B \rangle$$

representations

$\bar{B} \to DP$ — 3 RMEs

$$O_{\bar{15}} \equiv \langle \bar{15} || 8 || \bar{3} \rangle \qquad O_{\bar{3}} \equiv \langle \bar{3} || 8 || \bar{3} \rangle \qquad O_6 \equiv \langle 6 || 8 || \bar{3} \rangle$$

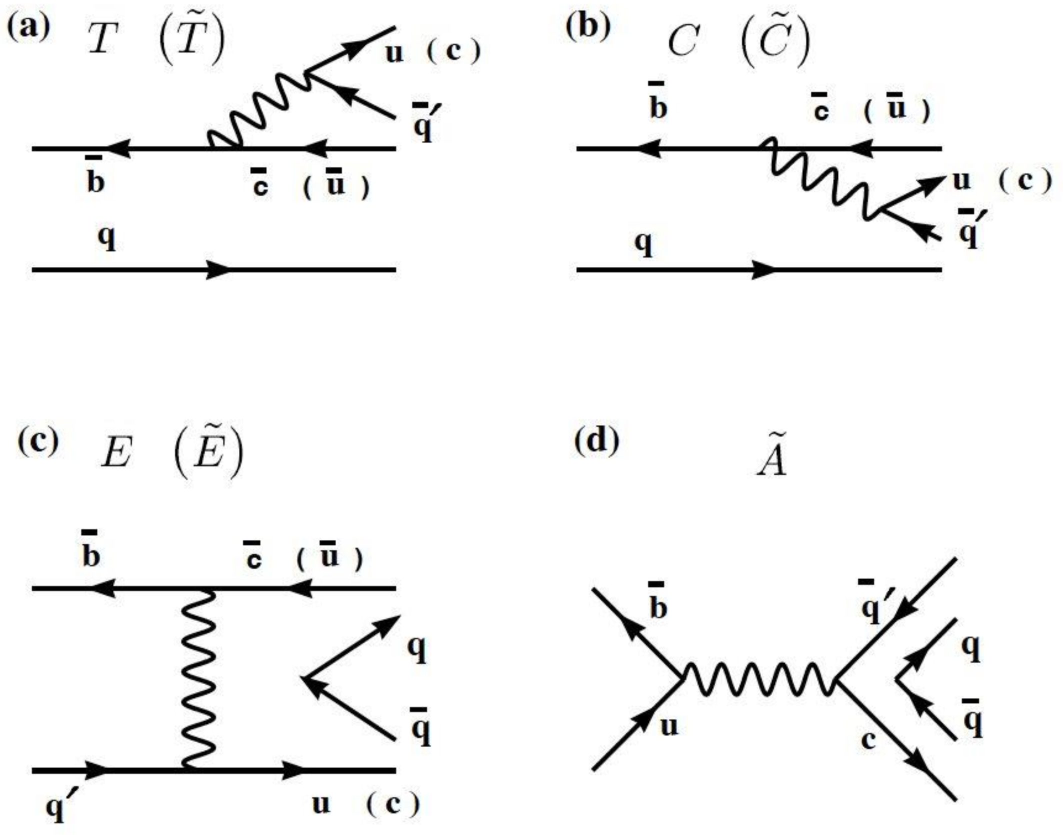
$\bar{B} \to \bar{D}P$ — 4 RMEs

$$S_{15} \equiv \langle 15 || 6 || \bar{3} \rangle \qquad S_3 \equiv \langle 3 || 6 || \bar{3} \rangle$$

$$T_{\bar{6}} \equiv \langle \bar{6} || \bar{3} || \bar{3} \rangle \qquad T_3 \equiv \langle 3 || \bar{3} || \bar{3} \rangle$$

How to express decay amplitudes?

Topological diagrams



[1]

$$\bar{B} \to DP : \quad T, C, E$$

$$\bar{B} \to \bar{D}P : \quad \tilde{T}, \tilde{C}, \tilde{E}, \tilde{A}$$

Refs : [1] Gronau et al, arXiv:hep-ph/9504326v2

- The amplitude of each decay is expressed in terms of diagrams or RMEs

$$\mathcal{A}(B \to DP) = V_1 V_2^* \sum_k M_k e^{i\delta}$$

CKM factor

Diagrams or RMEs

- The observables are given by:

$$\mathcal{B}(B \to DP) = |\mathcal{A}|^2.\Phi$$

Branching ratio

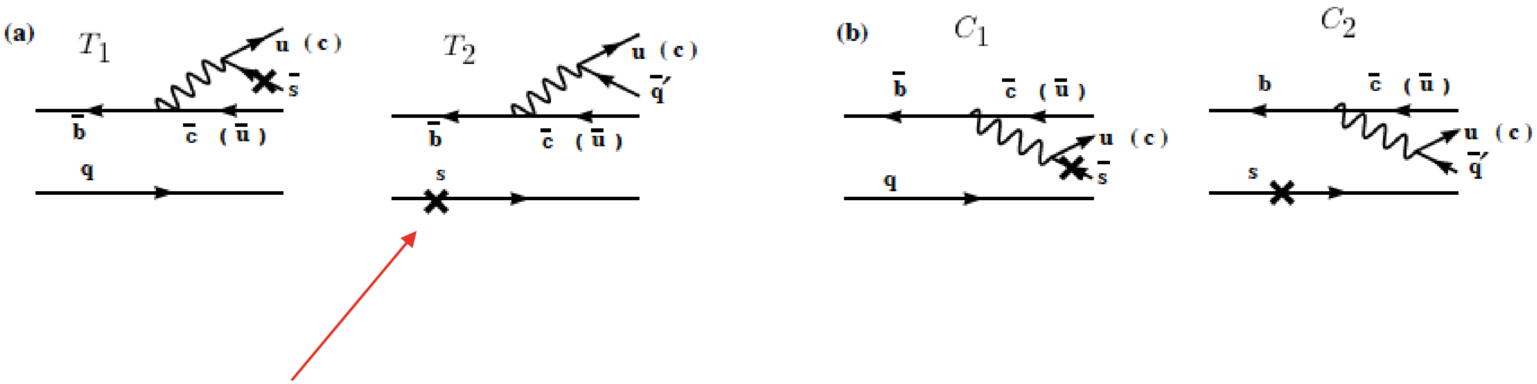
$$C = \frac{|\mathcal{A}|^2 - |\bar{\mathcal{A}}|^2}{|\mathcal{A}|^2 + |\bar{\mathcal{A}}|^2}$$

Time-dependent direct CP-asymmetry

$$(\lambda = \frac{q}{p}\frac{\bar{\mathcal{A}}}{\mathcal{A}}) \quad S \sim 2Im(\lambda)$$

Time-dependent indirect CP-asymmetry

Symmetry breaking in diagrams



Mass difference insertion $(m_s - m_d)/\Lambda$

- Correction diagrams Ti & Ci, when the s-quark is a spectator or when it comes from a vertex.
- This adds additional diagrams to our parameter space
- These corrections are expected to be small: 30%

Running the fits

First channel: $\bar{B} \to DP$

$$\bar{B} \to DP \begin{cases} \Delta S = 0: \text{ 6 decays} \\ \Delta S = -1: \text{ 6 decays} \end{cases}$$

- We have then a total of 12 decays: 10 branching ratios measured
- We have 5 parameters in total: $T, C, E, \delta_C, \delta_E$
- SU(3)-F breaking: $T_i, C_i, E_i, \delta_{T_i}, \delta_{C_i}, \delta_{E_i}$ (12 total)
- We run a χ^2-test.

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PHYSICAL REVIEW D **109,** 113006 (2024)

Reappraisal of SU(3)-flavor breaking in $B \to DP$

Jonathan Davies,[1,*] Stefan Schacht,[1,†] Nicola Skidmore,[2,‡] and Amarjit Soni[3,§]
[1]*Department of Physics and Astronomy, University of Manchester, Manchester M13 9PL, United Kingdom*
[2]*Department of Physics, University of Warwick, Coventry CV4 7AL, United Kingdom*
[3]*Physics Department, Brookhaven National Laboratory, Upton, New York 11973, USA*

(Received 21 March 2024; accepted 23 May 2024; published 24 June 2024)

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The paper found:

→ SU(3) symmetry is excluded at > 5σ

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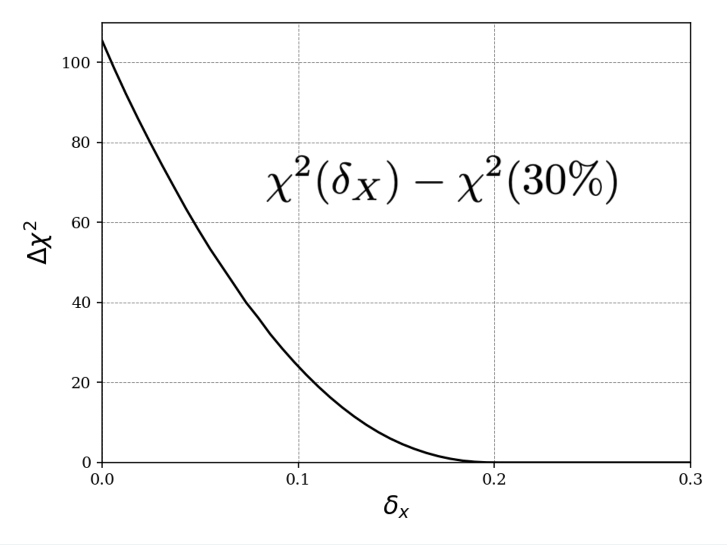

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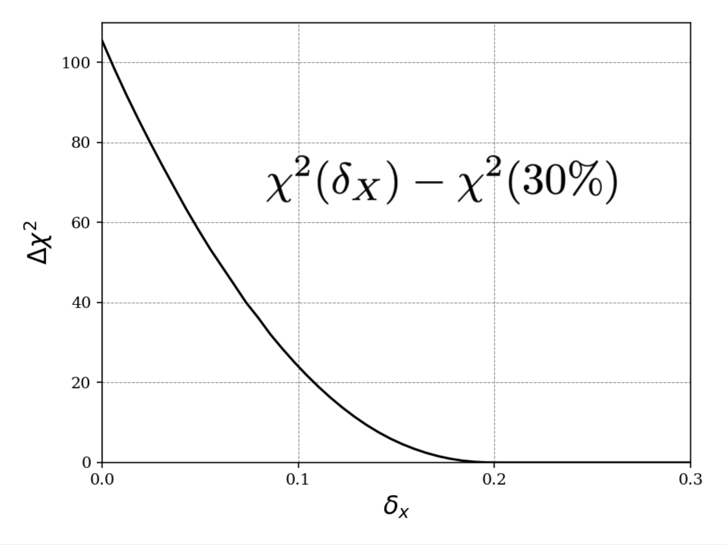
The paper found:

- SU(3) symmetry is excluded at $> 5\sigma$
- Breaking of 20% is enough to explain the data



But what if we expand to more observables by including $\bar{B} \to \bar{D}P$?

$\bar{B} \to \bar{D}P$

- $\Delta S = 0$: 8 decays
- $\Delta S = -1$: 8 decays

- We have then a total 16 decays: 5 branching ratios measured
- We have 7 parameters in total: $\tilde{T}, \tilde{C}, \tilde{E}, \tilde{A} \quad \delta_{\tilde{C}}, \delta_{\tilde{E}}, \delta_{\tilde{A}}$
- The SU(3) breaking parameters: $\tilde{T}_i, \tilde{C}_i, \tilde{E}_i, \tilde{A}_i, \delta_{\tilde{T}_i}, \delta_{\tilde{C}_i}, \delta_{\tilde{E}_i}, \delta_{\tilde{A}_i}$ (16 total)

!

$B^- \to \pi^0 D_s^-$

$\bar{B}^0 \to \pi^+ D_s^-$

The two decays are related by isospin → $\Delta \mathcal{B}r \sim 1\sigma$

Deviation independent of SU(3) breaking

GPP Groupe de Physique des Particules Université de Montréal
Université de Montréal

Combining the two channels

Global fit

- Now we include both channels: $\bar{B} \to DP$ & $\bar{B} \to \bar{D}P$
- We add CP-violation observables: $S, C, \mathcal{A}_{CP}$
- We have a total of 26 decays, 25 observables and 13 parameters
- The fit is constrained by the CKM angle $\gamma = 65.9^{\mathrm{o}}$ (World average)

$\bar{B} \to \bar{D}P$

$\tilde{T}, \tilde{C}, \tilde{E}, \tilde{A},$

$\delta_{\tilde{T}}, \delta_{\tilde{C}}, \delta_{\tilde{E}}, \delta_{\tilde{A}}$

$\bar{B} \to DP$

T, C, E

δ_C, δ_E

We now have 12 degrees of freedom

We perform a fit with no breaking in SU(3) ($\delta_X = 0\%$):

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We allow a breaking of 30% ⇒ $\chi^2 = 15$

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We find the lowest χ^2 at $\delta_X = 60\%$ ➡ $\chi^2 = 2.6$

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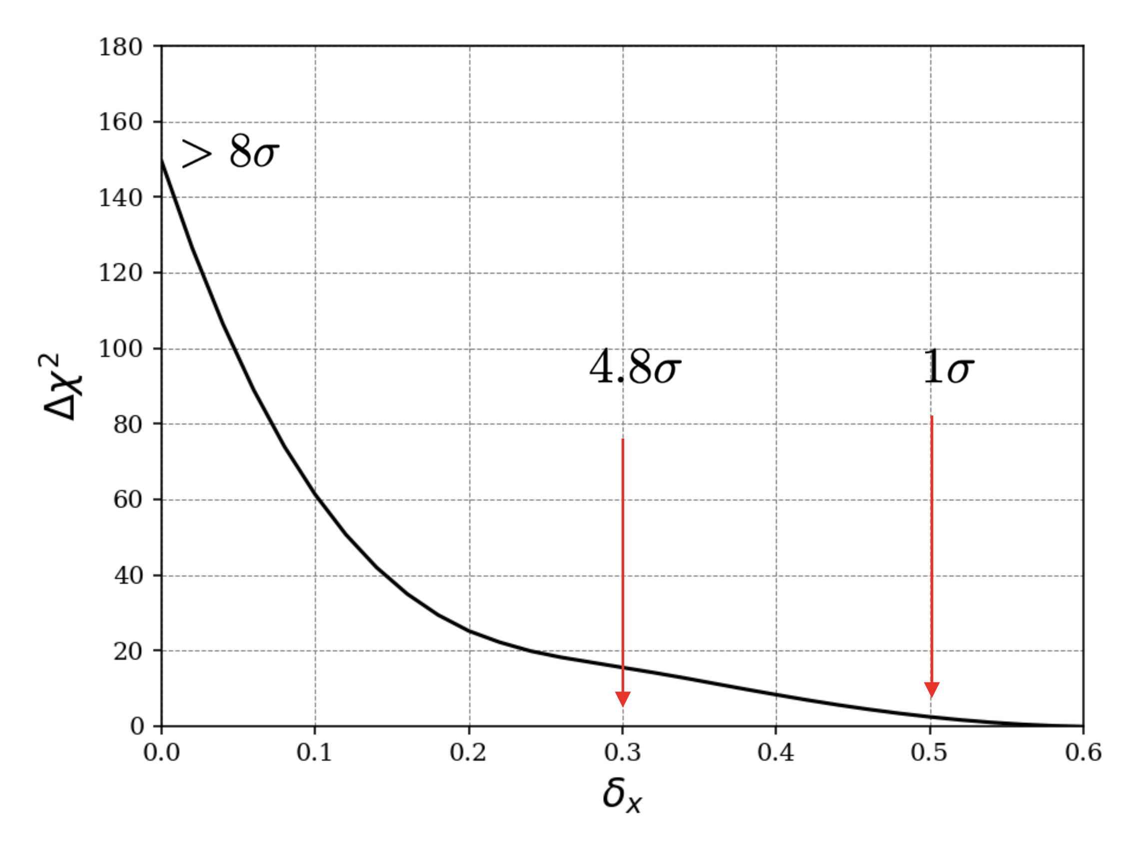
We allow a breaking of 30% → $\chi^2 = 15$

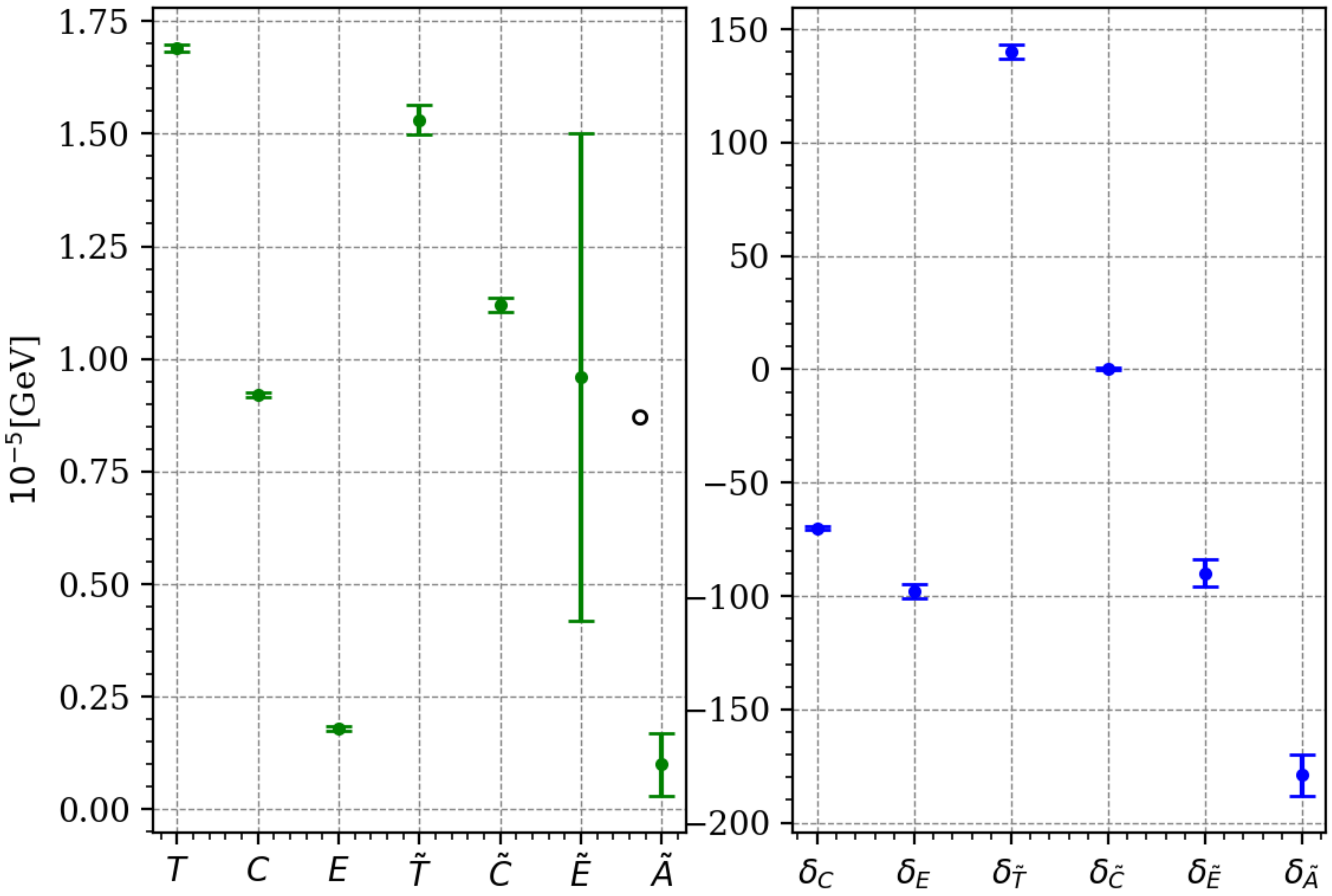
We find the lowest χ^2 at $\delta_X = 60\%$ → $\chi^2 = 2.6$

We run a profile of $\Delta\chi^2(\delta_X)$ relative to $\delta_X = 60\%$

$$\Delta\chi^2 = \chi^2(\delta_X) - \chi^2(60\%)$$



- We have good precision for diagrams: $T, C, E, \tilde{T}, \tilde{C}$
- Ratios of the diagrams relative to T are higher than the expected hierarchy: $|E/T|, |A/T| \sim 0.05 \quad |C/T| \sim 0.2$



Best fit values with SU(3) breaking

$$|\frac{C}{T}| = 0.55 \pm 0.01$$

$$|\frac{E}{T}| = 0.10 \pm 0.01$$

$$|\frac{\tilde{C}}{\tilde{T}}| = 0.73 \pm 0.02$$

$$|\frac{\tilde{E}}{\tilde{T}}| = 0.63 \pm 0.35$$

Takeaways & conclusions

- SU(3)-flavour symmetry breaking of 30% is not sufficient to explain the data when combining all channels.
- A 30% breaking model has a 4.8σ deviation relative to the 60% breaking model.
- The hierarchy between the diagrams is two times higher than what was expected.
- Having good precision for T & C diagrams we can predict the branching ratios of some unmeasured decays:

$$\mathcal{B}r(\overline{B^0_s} \to K^+ D^-) \qquad \mathcal{B}r(\overline{B^0} \to \bar{K}^0 \bar{D}^0) \qquad \mathcal{B}r(\overline{B^0_s} \to K^0 \bar{D}^0)$$

Backup slides

- A more rigorous way is to multiply the initial and final states, the Hamiltonian by a mass difference spurion and find the new RMEs.

- The mass spurion operator is given by:

$$Q_M \equiv -2s\bar{s} + d\bar{d} + u\bar{u}$$

The new contributions are:

$$\epsilon^1 = \langle f|\mathcal{H} \otimes Q_M|B\rangle + \langle Q_M \otimes f|\mathcal{H}|B\rangle + \langle f|\mathcal{H}|Q_M \otimes B\rangle$$

Thus obtaining new RMEs:

$\bar{B} \to DP$

$$O^1_{\overline{15}} \equiv \langle \overline{\mathbf{15}}||\,\mathbf{8_1}\,||\overline{\mathbf{3}}\rangle,\ O^1_{\overline{6}} \equiv \langle \mathbf{6}||\,\mathbf{8_1}\,||\overline{\mathbf{3}}\rangle,\ O^1_{\overline{3}} \equiv \langle \overline{\mathbf{3}}||\,\mathbf{8_1}\,||\overline{\mathbf{3}}\rangle,$$

$$O^2_{\overline{15}} \equiv \langle \overline{\mathbf{15}}||\,\mathbf{8_2}\,||\overline{3}\rangle,\ O^2_{\overline{6}} \equiv \langle \mathbf{6}||\,\mathbf{8_2}\,||\overline{\mathbf{3}}\rangle,\ O^2_{\overline{3}} \equiv \langle \overline{\mathbf{3}}||\,\mathbf{8_2}\,||\overline{\mathbf{3}}\rangle,$$

$$W_{\overline{15}} \equiv \langle \overline{\mathbf{15}}||\,\overline{\mathbf{10}}\,||\overline{\mathbf{3}}\rangle,\ Z_{\overline{15}} \equiv \langle \overline{\mathbf{15}}||\,\mathbf{27}\,||\overline{\mathbf{3}}\rangle,\ V_6 \equiv \langle \mathbf{6}||\,\mathbf{15}\,||\overline{\mathbf{3}}\rangle.$$

Refs : Michael Gronau, Phys. Rev. D **91**, 076007 (2015)

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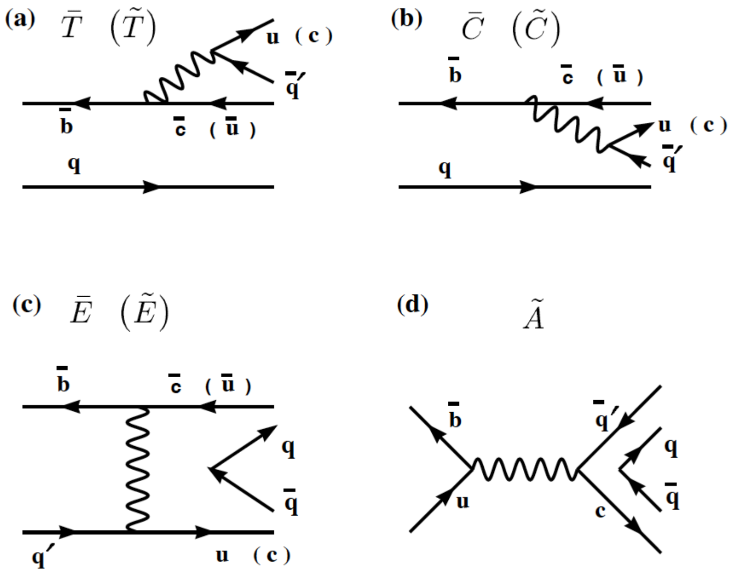
Diagram-RME relations



$$O_{\bar{3}} \equiv \langle \bar{3} || 8 || \bar{3} \rangle$$

$$O_{\bar{15}} \equiv \langle \bar{15} || 8 || \bar{3} \rangle$$

$$O_6 \equiv \langle 6 || 8 || \bar{3} \rangle$$

$$S_{15} \equiv \langle 15 || 6 || \bar{3} \rangle$$

$$S_3 \equiv \langle 3 || 6 || \bar{3} \rangle$$

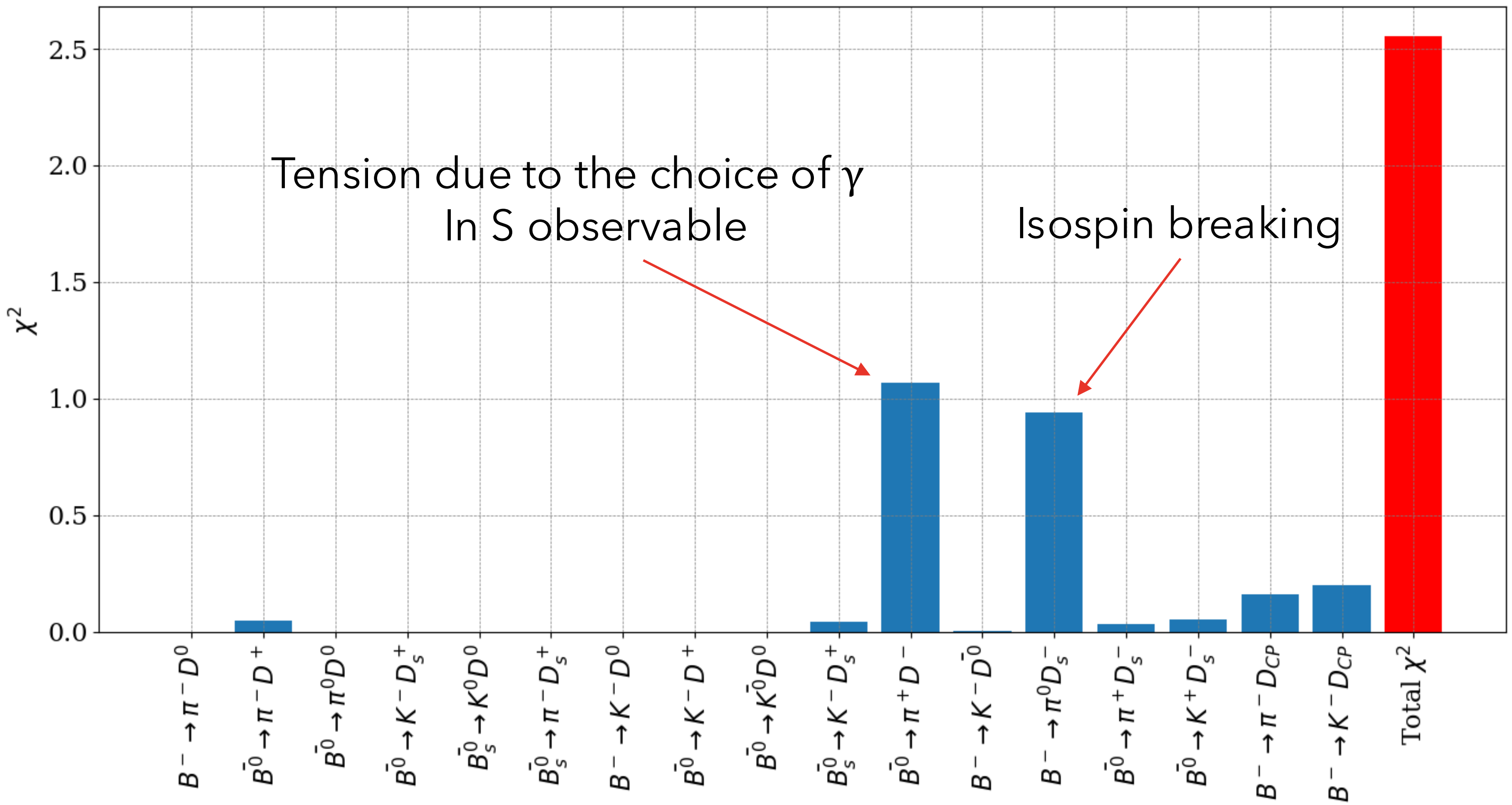
$$T_3 \equiv \langle 3 || \bar{3} || \bar{3} \rangle$$

$$T_{\bar{6}} \equiv \langle \bar{6} || \bar{3} || \bar{3} \rangle$$

$$O_r = \sum_i a_i P_i$$

RMEs

Diagrams


Tension due to the choice of γ
In S observable
Isospin breaking
χ^2
2.5
2.0
1.5
1.0
0.5
0.0
$B^- \to \pi^- D^0$
$\bar{B}^0 \to \pi^- D^+$
$\bar{B}^0 \to \pi^0 D^0$
$\bar{B}^0 \to K^- D_s^+$
$\bar{B}_s^0 \to K^0 D^0$
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$B^- \to K^- D^0$
$\bar{B}^0 \to K^- D^+$
$\bar{B}^0 \to K^0 \bar{D}^0$
$\bar{B}_s^0 \to K^- D_s^+$
$\bar{B}^0 \to \pi^+ D^-$
$B^- \to K^- \bar{D}^0$
$B^- \to \pi^0 D_s^-$
$\bar{B}^0 \to \pi^+ D_s^-$
$\bar{B}^0 \to K^+ D_s^-$
$B^- \to \pi^- D_{CP}$
$B^- \to K^- D_{CP}$
Total χ^2

Observables

Branching ratios

Decay modes	Value	Diagram contributions
$(\Delta S = 0) \;\propto V_{cb}V_{ud}^* = \mathcal{O}(\lambda^2)$		
$\mathcal{B}(B^- \to \pi^- D^0)$	$(4.61 \pm 0.10).\ 10^{-3}$	T, C
$\mathcal{B}(\bar{B}^0 \to \pi^- D^+)$	$(2.51 \pm 0.08).\ 10^{-3}$	T, E
$\mathcal{B}(\bar{B}^0 \to \pi^0 D^0)$	$(2.67 \pm 0.09).\ 10^{-4}$	C, E
$\mathcal{B}(\bar{B}^0 \to K^- D_s^+)$	$(2.7 \pm 0.5).\ 10^{-5}$	E
$\mathcal{B}(\bar{B}_s^0 \to K^0 D^0)$	$(4.3 \pm 0.9).\ 10^{-4}$	C
$\mathcal{B}(\bar{B}_s^0 \to \pi^- D_s^+)$	$(2.98 \pm 0.14).\ 10^{-3}$	T
$(\Delta S = 1) \;\propto V_{cb}V_{us}^* = \mathcal{O}(\lambda^3)$		
$\mathcal{B}(B^- \to K^- D^0)$	$(3.64 \pm 0.15).\ 10^{-4}$	T, C
$\mathcal{B}(\bar{B}^0 \to K^- D^+)$	$(2.05 \pm 0.08).\ 10^{-4}$	T
$\mathcal{B}(\bar{B}^0 \to \bar{K}^0 D^0)$	$(5.5 \pm 0.4) \times 10^{-5}$	C
$\mathcal{B}(\bar{B}_s^0 \to \pi^- D^+)$	n.a.	E
$\mathcal{B}(\bar{B}_s^0 \to \pi^0 D^0)$	n.a.	E
$\mathcal{B}(\bar{B}_s^0 \to K^- D_s^+)$	$(1.94 \pm 0.21).\ 10^{-4}$	T, E

Decay modes	Value	Diagram contributions
$\sim V_{ub}V_{cd}^* = \mathcal{O}(\lambda^4)$		
$\mathcal{B}(B^- \to \pi^- \bar{D}^0)$	n.a	C, A
$\mathcal{B}(B^- \to \pi^0 D^-)$	n.a.	T, A
$\mathcal{B}(B^- \to K^0 D_s^-)$	$< 3.5 \times 10^{-6}$	A
$\mathcal{B}(\bar{B}^0 \to \pi^+ D^-)$	$(7.3 \pm 1.2) \times 10^{-7}$	T, E
$\mathcal{B}(\bar{B}^0 \to \pi^0 \bar{D}^0)$	n.a.	C, E
$\mathcal{B}(\bar{B}^0 \to K^+ D_s^-)$	n.a.	E
$\mathcal{B}(\bar{B}^0{}_s \to K^+ D^-)$	n.a.	T
$\mathcal{B}(\bar{B}^0{}_s \to K^0 \bar{D}^0)$	n.a.	C
$\sim V_{ub}V_{cs}^* = \mathcal{O}(\lambda^3)$		
$\mathcal{B}(B^- \to K^- \bar{D}^0)$	$(3.60 \pm 0.24) \times 10^{-6}$	C, A
$\mathcal{B}(B^- \to \bar{K}^0 D^-)$	$< 2 \times 10^{-6}$	A
$\mathcal{B}(B^- \to \pi^0 D_s^-)$	$(1.6 \pm 0.5) \times 10^{-5}$	T
$\mathcal{B}(\bar{B}^0 \to \bar{K}^0 \bar{D}^0)$	n.a.	C
$\mathcal{B}(\bar{B}^0 \to \pi^+ D_s^-)$	$(2.03 \pm 0.18) \times 10^{-5}$	T
$\mathcal{B}(\bar{B}_s^0 \to K^+ D_s^-)$	$(2.6 \pm 1.2) \times 10^{-5}$	T, E
$\mathcal{B}(\bar{B}_s^0 \to \pi^+ D^-)$	n.a.	E
$\mathcal{B}(\bar{B}_s^0 \to \pi^0 \bar{D}^0)$	n.a.	E

CP-asymmetries

Decay modes	$C,\ \mathcal{A}_{CP}$	S
$\bar{B} \to DP$		
$(B^- \to \pi^- D^0)$	$\mathcal{A}_{CP} = -0.003 \pm 0.005$	
$(B^- \to \pi^- D_+^{CP})$	$\mathcal{A}_{CP} = -0.008 \pm 0.003$	
$(B^- \to \pi^- D_-^{CP})$	$\mathcal{A}_{CP} = 0.017 \pm 0.026$	
$(\bar{B}^0 \to \pi^- D^+)$	$C = -1$	$S = 0.038 \pm 0.021$
$(\bar{B}^0 \to \pi^0 D^0)$	$\mathcal{A}_{CP} = 0.004 \pm 0.024$	
$(B^- \to K^- D^0)$	$\mathcal{A}_{CP} = -0.017 \pm 0.005$	
$(B^- \to K^- D_+^{CP})$	$\mathcal{A}_{CP} = 0.136 \pm 0.009$	
$(B^- \to K^- D_-^{CP})$	$\mathcal{A}_{CP} = -0.14 \pm 0.05$	
$(\bar{B}_s^0 \to K^- D_s^+)$	$C = -0.73 \pm 0.15$	$S = -0.49 \pm 0.21$
$\bar{B} \to \bar{D}P$		
$(\bar{B}^0 \to \pi^+ D^-)$	$C = 1$	$S = 0.058 \pm 0.023$
$(\bar{B}_s^0 \to K^+ D_s^-)$	$C = 0.73 \pm 0.15$	$S = -0.52 \pm 0.21$

$\Delta\chi^2$ profile

$$\lambda = \frac{L(\delta_x; \theta)}{L(\hat{\delta_X}; \hat{\theta})}$$

Log-likelihood ratio test; θ are the nuisance parameters

$$-2\log(\lambda) = \Delta\chi^2$$

Wilks' theorem shows that this quantity behave like a χ^2 distribution with d.o.f. = 1