Resonant boundary effects of graviton-photon conversion

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June 19th, 2026

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Bonjour tout le monde! C'est un plaisir d'être de retour à l'Université de Montréal, où j'ai fait mon baccalauréat et ma maîtrise. Je vais donner cette présentation en anglais, mais vous pouvez me poser des questions en français ou en anglais.

Hi everyone! It is a pleasure to be back here at Université de Montréal, where I did my undergraduate and masters' degree! I will now speak in English, but feel free to ask questions in French or in English.

Summary of the presentation

- Summary of the Gertsenshtein effect (graviton-photon coupling)
- How it is amplified by a resonant boundary effect

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The coupling between gravity and electromagnetism are described by the Einstein field equation and Maxwell equations in curved spacetime:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa^2 \left(F_{\mu\lambda} F_{\nu}{}^{\lambda} - \frac{1}{4} g_{\mu\nu} F_{\lambda\rho} F^{\lambda\rho} \right), \quad \nabla_{\nu} F_{\mu}{}^{\nu} = 0, \tag{1}$$

where $\kappa := \sqrt{8\pi G}$.

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Unless, one of those two photons is from a background field instead of a wave...

Summary of the Gertsenshtein effect (graviton-photon coupling)

The dynamics is controlled by a dimensionless parameter

$$\lambda := \frac{\kappa}{\sqrt{2}\,k}\sqrt{(B_y + E_z)^2 + (B_z - E_y)^2}\,, \tag{2}$$

where E_y, E_z, B_y and B_z are components of the background electromagnetic field (this assumes that the waves propagate along the positive x axis). It is almost always true that $\lambda \ll 1$, so we expand around λ.

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The background electromagnetic field splits the dispersion relation (which would be just $\omega - k = 0$ in a vacuum) into two (birefringence):

$$\omega - k = \mp\lambda k + k\, O(\lambda^2). \qquad (3)$$

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This is a **weak** effect; it is suppressed by λ.

How it is amplified by a resonant boundary effect

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Consider a vacuum region at $x < 0$, and an constant background electromagnetic field at $x > 0$. Hence, the background electromagnetic tensor is

$$\overline{F}_{\mu\nu} = \begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & B_z & -B_y \\ E_y & -B_z & 0 & B_x \\ E_z & B_y & -B_x & 0 \end{bmatrix} \Theta(x). \quad (4)$$

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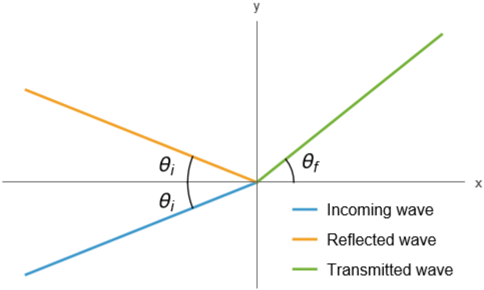
We send a pure gravitational wave from $x \to -\infty$ and making an angle θ_i with the positive direction of the x axis. Part of it is reflected back to $x \to -\infty$ and makes an angle θ_i in with the x axis in the other direction. Another part of it is transmitted towards $x \to +\infty$ and makes an angle θ_f. Without loss of generality, we consider take the $x - y$ plane to be the plane of the waves.



Conservation of energy and momentum tangential to the boundary imposes (Snell's law):

$$\begin{aligned} \sin\theta_f &= (1 \mp \lambda + O(\lambda^2))\sin\theta_i\,, \\ \cos\theta_f &= (1 \mp \lambda + O(\lambda^2))\sqrt{\cos^2\theta_i \pm 2\lambda + O(\lambda^2)}\,, \end{aligned} \tag{5}$$

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We can see that beyond the critical angle $\theta_c := \arccos\sqrt{2\lambda}$, half of the eigenstates (the ones with the lower signs) become evanescent on the other side of the boundary: $\sin\theta_f > 1$ and $\cos^2\theta_f < 0$. Therefore, **these eigenstates are totally reflected**.

We now want to calculate the various reflection coefficients, which describe what fraction of the incoming gravitational wave's energy is reflected; and among how much is reflected back as a gravitational wave, and how much is converted into electromagnetic radiation. There are also transmission coefficients, but these are not interesting to this specific setup (in which the background electromagnetic in $x > 0$ goes forever).

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We consider the specific case in which the boundary physics (currents, energy-momentum tensor) does not interfere at all with the waves (i.e. total transparency, **this is not always true**). This is equivalent to having the waves and their first derivatives be continuous across the boundary:

$$\Delta h_{ab} = 0, \quad \Delta a_a = 0, \quad \Delta(\partial_x h_{ab}) = 0, \quad \Delta(\partial_x a_a) = 0, \tag{6}$$

where Latin indices (a, b, c, etc.) represent boundary coordinates.

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This is enough information to calculate the reflection coefficients.

How it is amplified by a resonant boundary effect

When $\theta_i < \theta_c$, the reflection and transmission coefficients are

$$R^{(g)} = \frac{\left(1 - \sqrt{1 - \frac{4\lambda^2}{\cos^4\theta_i}}\right)^2}{\left(1 + \sqrt{1 + \frac{2\lambda}{\cos^2\theta_i}}\right)^2 \left(1 + \sqrt{1 - \frac{2\lambda}{\cos^2\theta_i}}\right)^2},$$

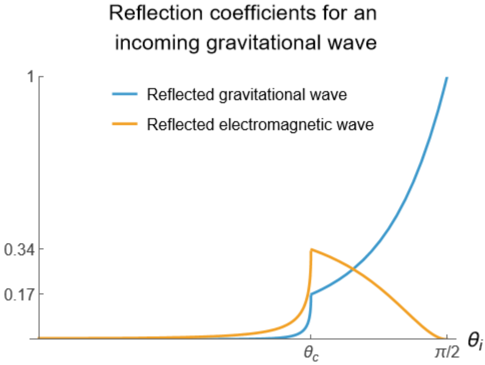
$$R^{(\gamma)} = \frac{\left(\sqrt{1 + \frac{2\lambda}{\cos^2\theta_i}} - \sqrt{1 - \frac{2\lambda}{\cos^2\theta_i}}\right)^2}{\left(1 + \sqrt{1 + \frac{2\lambda}{\cos^2\theta_i}}\right)^2 \left(1 + \sqrt{1 - \frac{2\lambda}{\cos^2\theta_i}}\right)^2}. \quad (7)$$

When $\theta_i > \theta_c$, the reflection and transmission coefficients are now

$$R^{(g)} = \frac{2\lambda}{\cos^2\theta_i \left(1 + \sqrt{1 + \frac{2\lambda}{\cos^2\theta_i}}\right)^2}, \quad R^{(\gamma)} = \frac{2}{\left(1 + \sqrt{1 + \frac{2\lambda}{\cos^2\theta_i}}\right)^2}. \quad (8)$$

How it is amplified by a resonant boundary effect

Graphically, this looks as follows.



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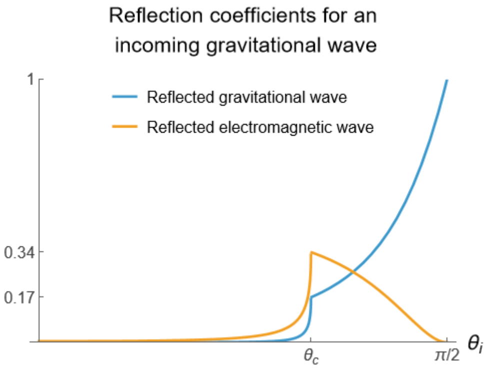

At the critical angle θ_c, $R^{(g)} = \dfrac{1}{3+2\sqrt{2}} \approx 17\%$ and $R^{(\gamma)} = \dfrac{2}{3+2\sqrt{2}} \approx 34\%$.
Approximately $34\,\%$ **of the gravitational wave's energy is reflected and converted into electromagnetic waves**.

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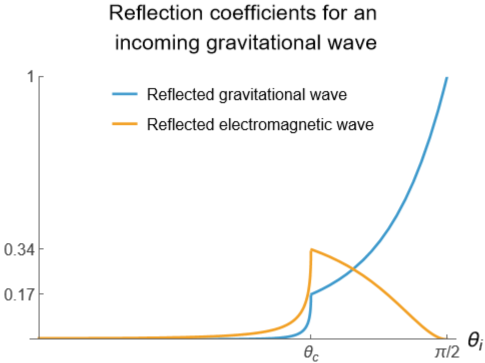

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Approximately 34 % **of the gravitational wave's energy is reflected and converted into electromagnetic waves**. However, the catch is that since $\lambda \ll 1$, θ_c is **really** close to 90 °. So we only have a significant conversion around a very narrow range of angles around the critical value. That makes it a **resonant boundary effect**.

Merci pour votre attention. Je vais répondre à vos questions.

Thank you for your attention. I will answer your questions.