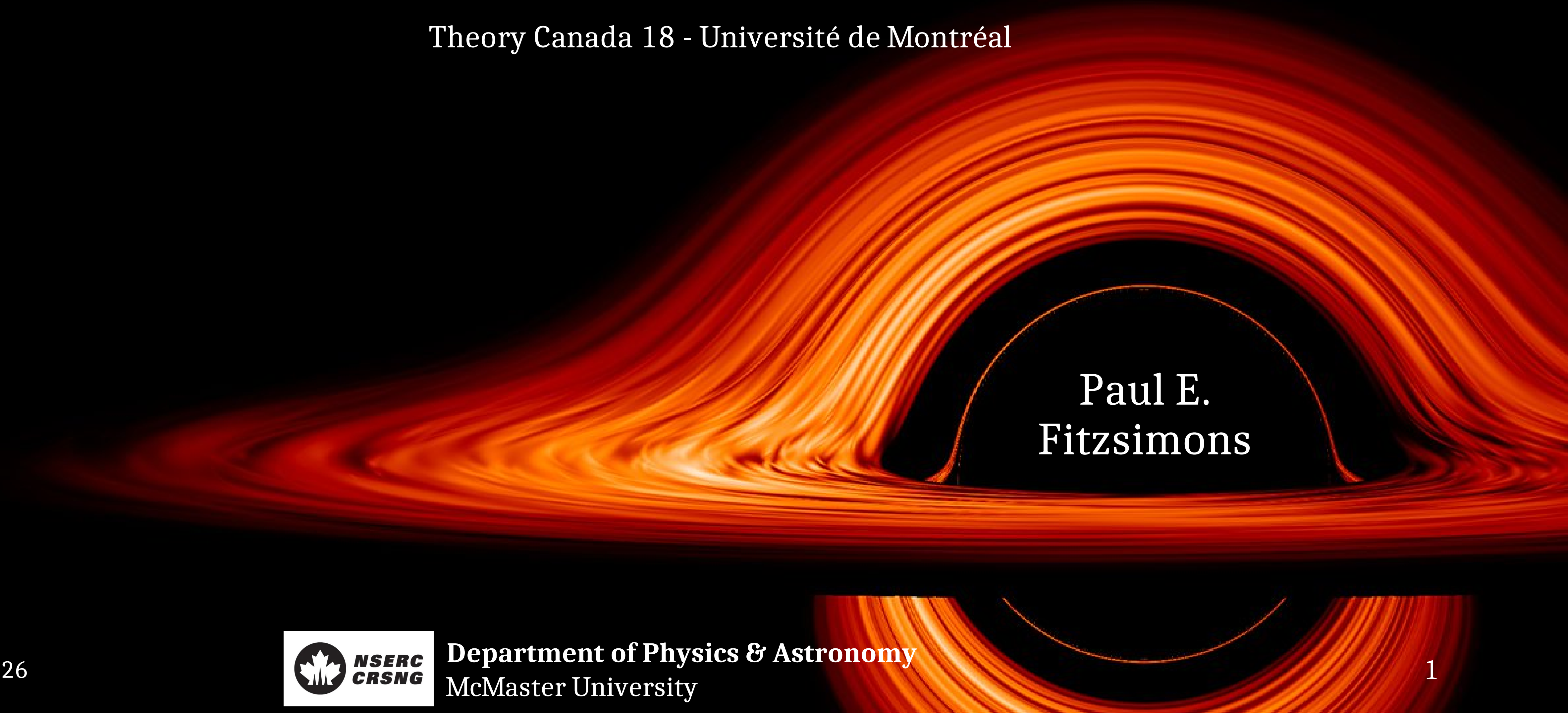
Near the event horizon of black holes

Theory Canada 18 - Université de Montréal



Paul E. Fitzsimons

Department of Physics & Astronomy
McMaster University

Introduction

- Classical formula for B-H entropy [1]

- Quantum-derived formula for 3D solution [2], [3]
  $\rightarrow$ foreshadows AdS/CFT correspondence

- Recent work focused on <u>extremal</u> black holes

What we know:

Well-defined geometry of extremal black holes

EBH entropy derived from Cardy Formula

Symmetry persists in low frequency waves in non-extreme black holes

Objective:

Study geometric structure of general BHs

Connect to holographic descriptions of spacetime

Find geometric origin of persisting symmetry

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Stationary Black hole review

- Spacetime metrics: $\mathrm{d}s^2 = g_{\alpha\beta}\mathrm{d}x^\alpha \mathrm{d}x^\beta$
- Uniqueness Theorem: Mass M, Angular Momentum J, Charge Q [3]

4 common types

- Schwarzschild: M
- Kerr: M, J
- Reissner-Nordström: M, Q
- Kerr-Newman: M, J, Q

$$\mathrm{d}s^2 = -\left(1 - \frac{2M}{r}\right)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{1 - \frac{2M}{r}} + r^2\mathrm{d}\Omega_2^2$$

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Symmetry

- Characterized by commutators $[\,\cdot\,,\,\cdot\,]$ and relevant dynamical operator

$[P\,,H]=0 \rightarrow$ translational symmetry $\quad [L\,,H]=0 \rightarrow$ rotational symmetry

- Isometry vs. Non-Geometric symmetry:
  - Isometries act on spacetime
  - Symmetries act on system

Geometric symmetries in GR

- Killing vectors:
  - Timelike $\longrightarrow$ time symmetry
  - Spacelike $\longrightarrow$ spatial symmetry
  - Null $\longrightarrow$ event horizon (if on hypersurface)

- Kerr: $\xi^\alpha = \partial_t$ is timelike (stationary) ; $\psi^\alpha = \partial_\phi$ is spacelike (axisymmetry) ; $\chi^\alpha = \partial_v + \Omega_H \partial_\phi$ is null when $r = r_H$

Black Holes:

Classical quantities

Event Horizon

Singularity on horizon

Symmetry:

Conservation after transformation

Isometries vs. non-geometric symmetries

Simplifies systems

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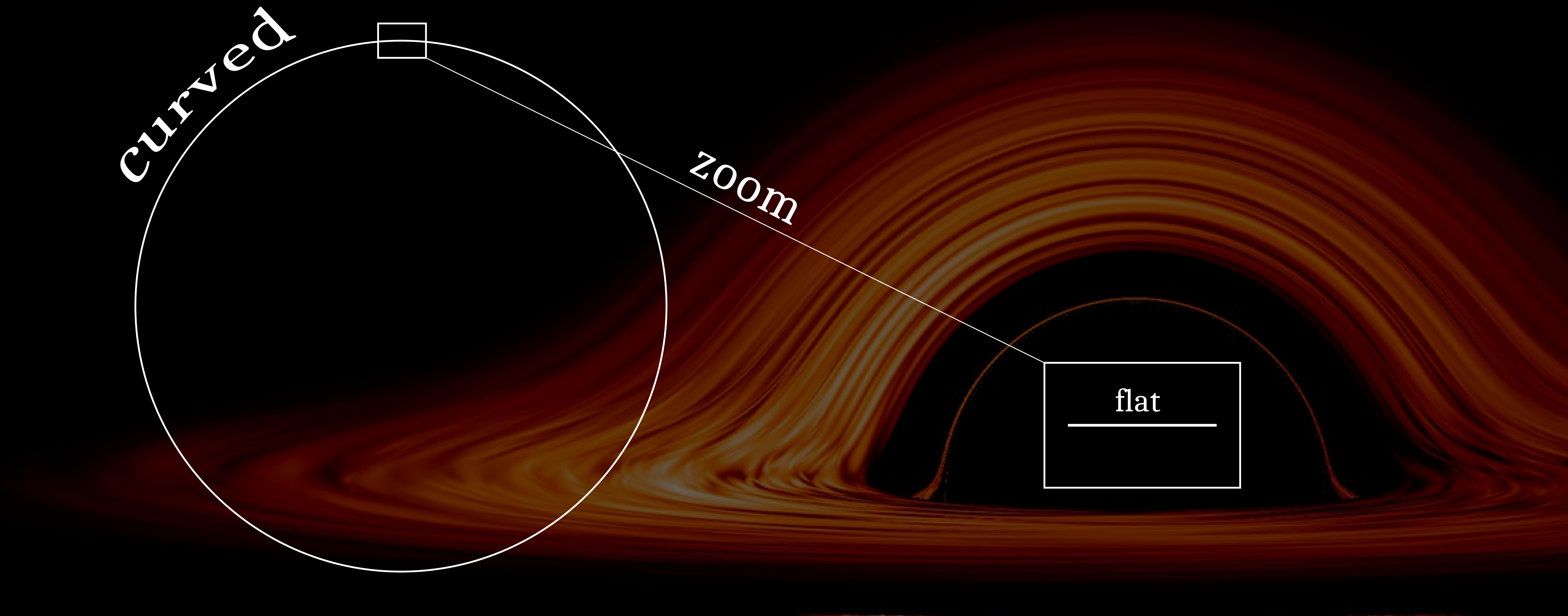
The right coordinate system
(Gaussian null coordinates)

- Applicable in all stationary solutions [4]
- Explicit derivation is generally difficult
- e.g. Eddington-Finkelstein coordinates:

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dv^2 + 2 dv dr + r^2 d\Omega_2^2$$

In principle



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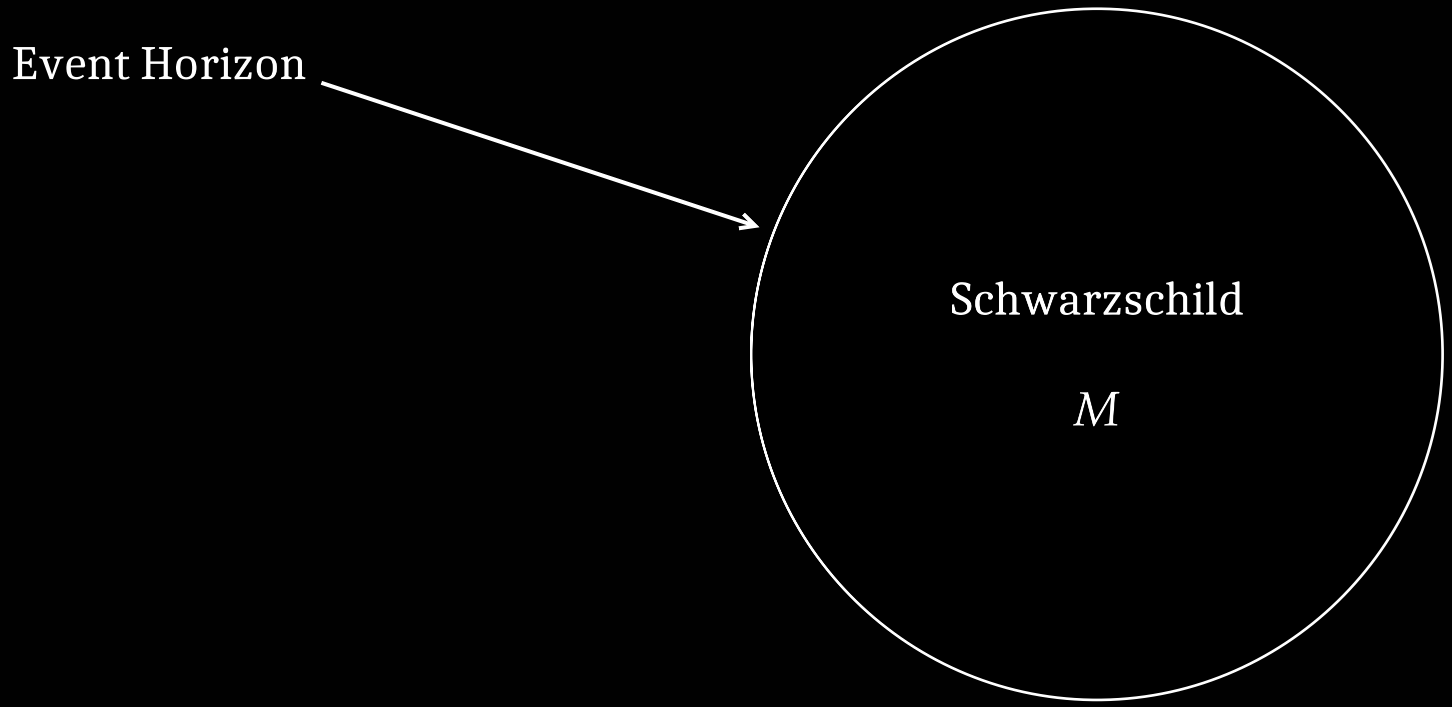
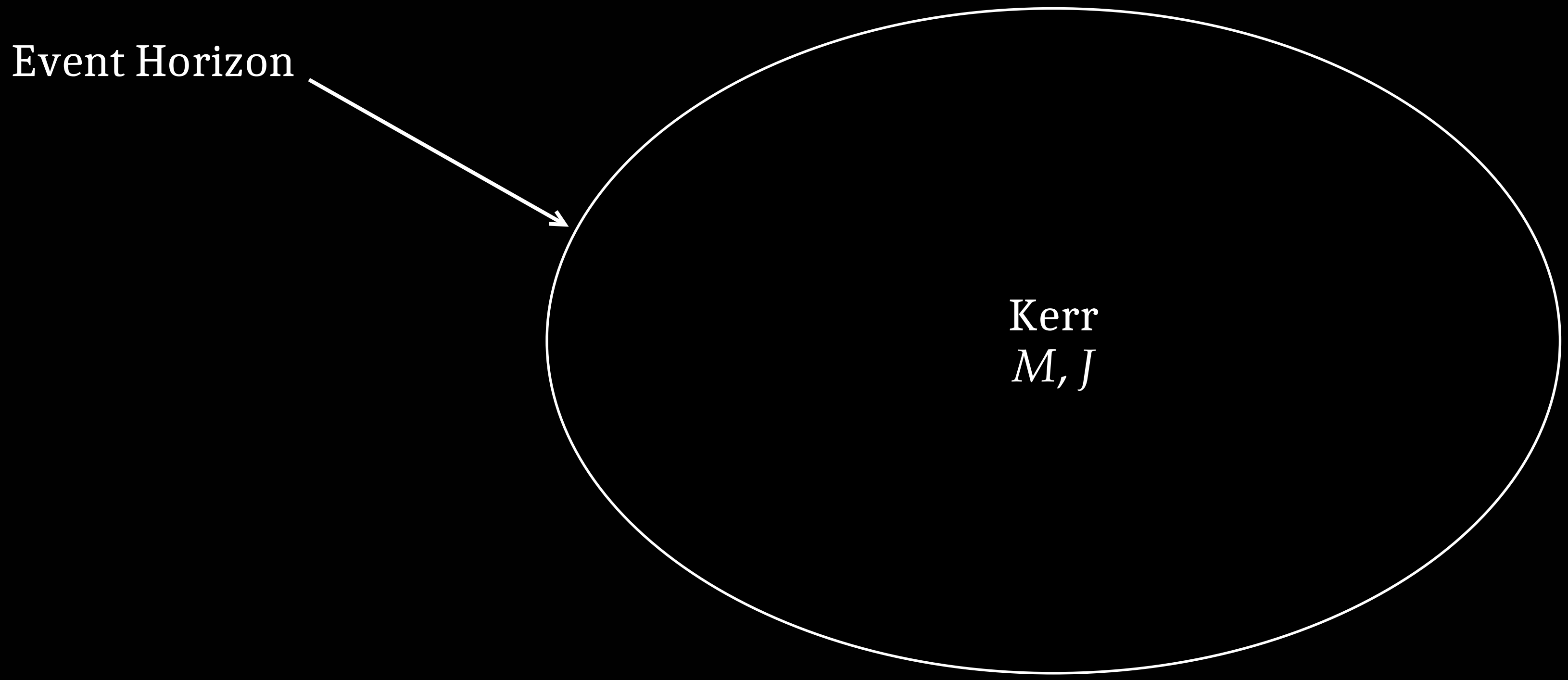
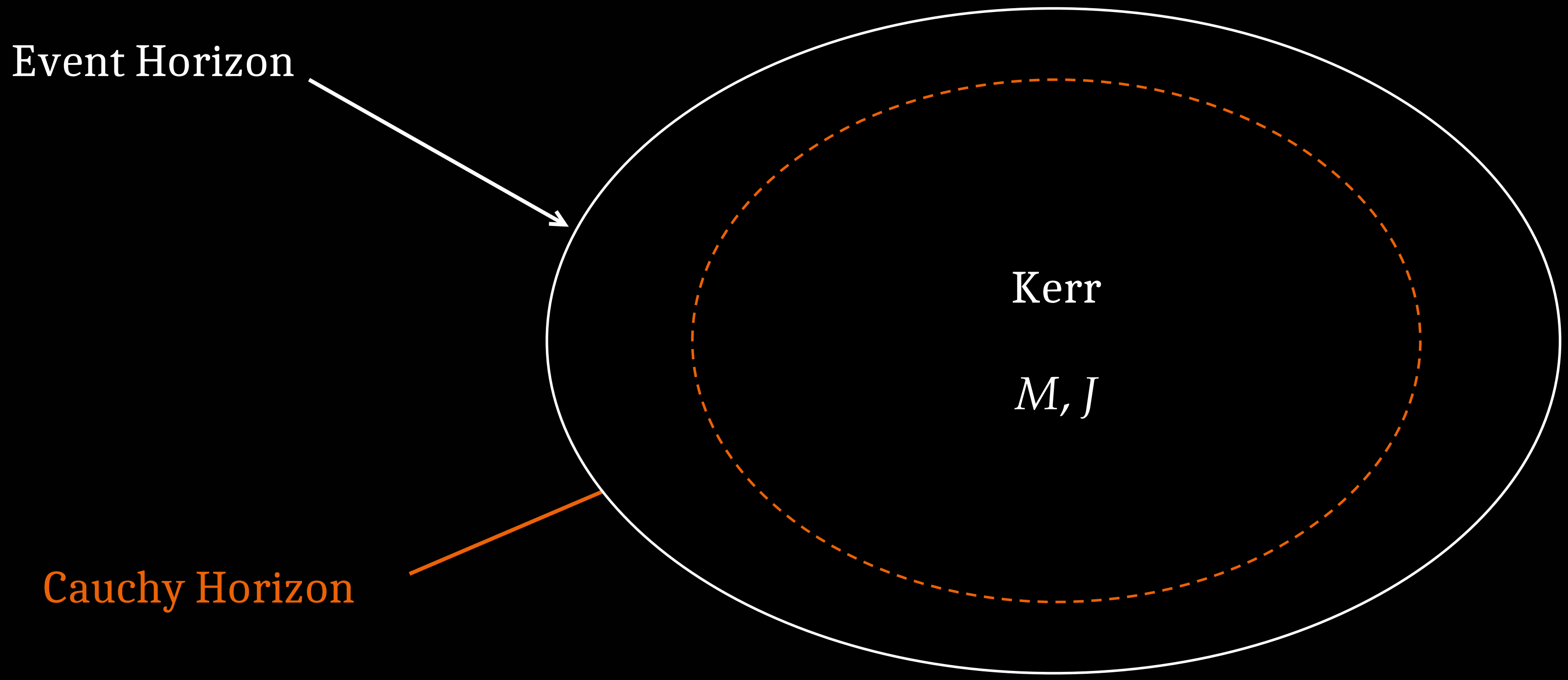
Schwarzschild
M


Event Horizon
Schwarzschild
M


Event Horizon
Kerr
M, J



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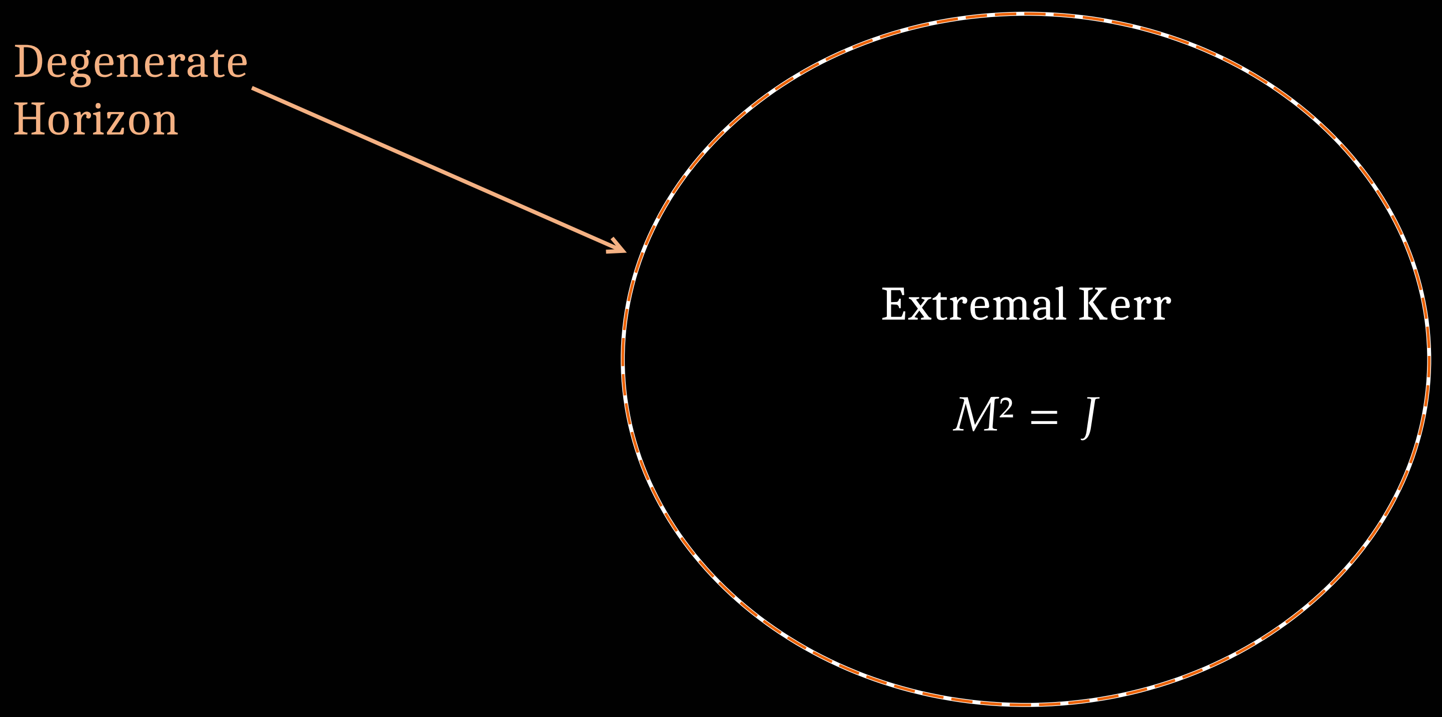
Degenerate
Horizon
Extremal Kerr
M² = J

Extremal Black Holes

Extremal black holes:

- Maximal spin/charge for given mass
- Zero surface gravity
- Degenerate horizon

Exact derivations of near-horizon metrics for many solutions

Extremal BH & symmetry

Symmetry enhancement theorem [5]:

Extremal BH with $R \times U(1)^{n-3}$ isometry in Einstein gravity coupled with matter field $\lambda \leq 0$ $\rightarrow$ field equations solutions have $SO(2,1) \times U(1)^{n-3}$ isometry

$$ds^2 = \Gamma(x)(-r^2 dv^2 + 2dvdr) + d\theta^2 + \gamma_{ij}(dx^i + k^i r dv)(dx^j + k^j r dv)$$

AdS$_2$ spacetime

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Gaussian null coordinates:

Widely applicable

Zooms on the horizon

Useful to study horizon geometry

Extremal Black Hole:

Degenerate horizons

Highly symmetric

CFT entropy = B-E entropy

Non-Extremal symmetries

- Extension of isometry outside extremality?
- Conjectured as the "Kerr/CFT Correspondence" [6]
- Connection established with non-geometric symmetries [7]

Wave equation in Kerr solution

- $SO(2,1)$ symmetry when in the right regime
  - Found with low frequency + approximations
- Three vectors commute with wave operator
  - matches $SO(2,1)$ structure
- Not symmetries of spacetime!

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Example

- Non-extremal Kerr (GNCs to first order in r):

$$ds^2 = \Gamma(\theta)[\text{ Rindler }] + g_{\phi\phi}\left(d\phi + r dv(2a\Gamma\kappa + 2ar_+)\right)^2 + \Gamma(\theta)d\theta^2$$

$$\Gamma(\theta) = r_+^2 + a^2\cos^2(\theta)$$

- Extremal Kerr (exact):

$$ds^2 = \frac{(1+\cos^2\theta)}{2}[\text{ AdS}_2\text{ }] + \frac{2a^2\sin^2\theta}{1+\cos^2\theta}\left(d\phi + \frac{r dv}{2a^2}\right)^2$$

(similar results for Kerr-Newman and black ring)

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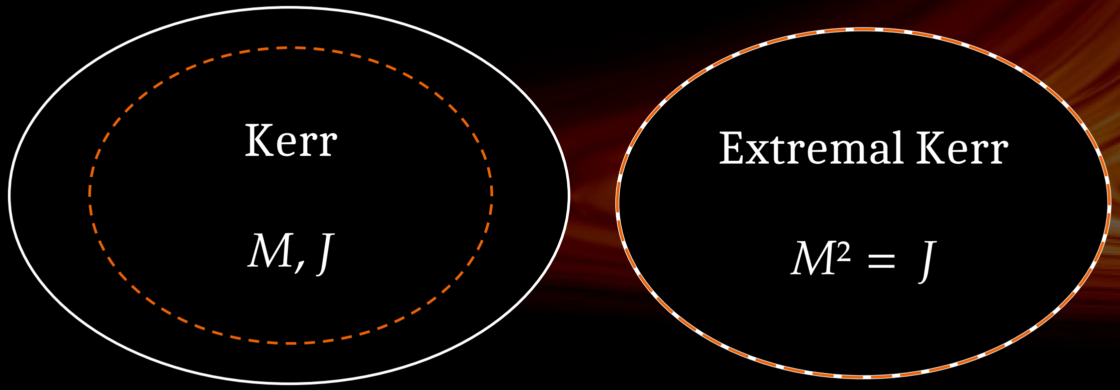
Current work

- Solving wave equation using approximate GNCs
  - $\rightarrow$ looking for geometric origin of symmetry
- Rindler structure is recovered for black ring solution
  - $\rightarrow$ Is this a general result for n-dimensions? To investigate

Questions

Symmetry enhancement?
Extremal BH?
Gaussian null coordinates?

$$\mathrm{d}s^2 = \Gamma(\theta)[\ \text{Rindler}\ ] + g_{\phi\phi}(\mathrm{d}\phi + r\mathrm{d}v(2a\Gamma\kappa + 2ar_+))^2 + \Gamma(\theta)\mathrm{d}\theta^2$$



References

[1] - D. Bekenstein, Lett. Nuovo Cim. 4 (1972), 737-740

[2] - Banados, M. Henneaux, C. Teitelboim and J. Zanelli, Phys. Rev. D 48 (1993), 1506-1525 [erratum: Phys. Rev. D 88 (2013), 069902]

[3] – Strominger, A. "Black Hole Entropy from Near-Horizon Microstates"; JHEP 9802:009,1998

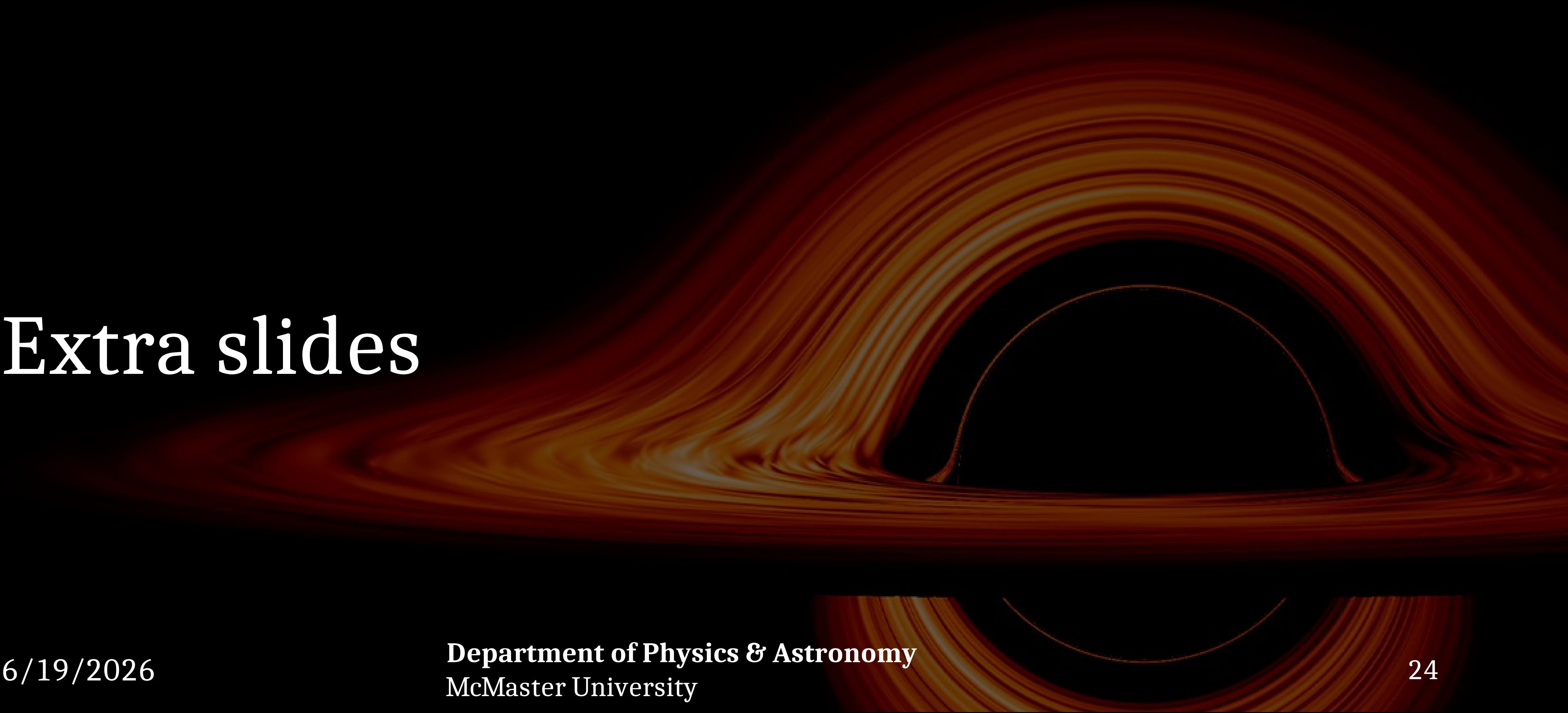
[4] - Web of Stories. Listeners: Ken Ford. Recorded December 1996; published 24 January 2008

[5] - Kunduri, Hari K.; Lucietti, James (2009). "A classification of near-horizon geometries of extremal vacuum black holes". Journal of Mathematical Physics. 50 (8): 082502

[6] - Compère, Geoffrey (2012-10-22). "The Kerr/CFT Correspondence and its Extensions". Living Reviews in Relativity. 15 (1) 11. Springer Science and Business Media LLC

[7] – Castro, A.; Maloney, A.;, Strominger, A. (2010). “Hidden Conformal Symmetry of the Kerr Black Hole” . Phys.Rev.D 82 (2010), 024008

Extra slides



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- Higher and lower dimensions
  - n-D Myers-Perry black holes, spherical topology S^{n-2} *(Myers, Perry, 1985)*
  - n-D black ring, toric topology $S^1 \times S^{n-3}$ *(Emparan, Reall, 2001)*
  - 3D BTZ black hole, negatively curved spacetime *(Bañados–Teitelboim–Zanelli, 1992)*

Extremal Black Holes

Reissner-Nordström $M = Q$:

$$\mathrm{d}s^2 = Q^2[-r^2\mathrm{d}v^2 + 2\mathrm{d}v\mathrm{d}r + \mathrm{d}\Omega_2^2]$$ has NH geometry $\mathrm{AdS}_2 \times S^2$

Kerr $M = \sqrt{J}$:

$$\mathrm{d}s^2 = \frac{(1+\cos^2\theta)}{2}\left[-\frac{r^2\mathrm{d}v^2}{2a^2} + 2\mathrm{d}v\mathrm{d}r + a^2\mathrm{d}\theta^2\right] + \frac{2a^2\sin^2\theta}{1+\cos^2\theta}\left(\mathrm{d}\phi + \frac{r\mathrm{d}v}{2a^2}\right)^2$$

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Gaussian null coordinates

- In neighbourhood of EBH degenerate horizons: $x^\alpha = (v, r, x^i)$

$$ds^2 = 2\mathrm{d}v\left(\mathrm{d}r + rh_i(r,x)\mathrm{d}x^i + \frac{1}{2}r^2 F(r,x)\mathrm{d}v\right) + \gamma_{ij}(r,x)\mathrm{d}x^i\mathrm{d}x^j$$

Purpose: get close to the event horizon $v \to \frac{v}{\epsilon}, r \to \epsilon r, \epsilon \to 0$

$$ds^2 = 2\mathrm{d}v\left(\mathrm{d}r + rh_i(x)\mathrm{d}x^i + \frac{1}{2}r^2 F(x)\mathrm{d}v\right) + \gamma_{ij}(x)\mathrm{d}x^i\mathrm{d}x^j$$

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Process

- Define geodesics orthogonal to horizon hypersurface:

$$X^\alpha(\rho) \approx X^\alpha|_{\rho=0} + \rho \frac{\mathrm{d}X^\alpha}{\mathrm{d}\rho}|_{\rho=0} + \frac{\rho^2}{2}\frac{\mathrm{d}^2X^\alpha}{\mathrm{d}\rho^2}|_{\rho=0}$$

$$X^\alpha|_H = [v, r_0, \theta, \phi] \quad \frac{\mathrm{d}X^\alpha}{\mathrm{d}\rho}|_H = n^\alpha \quad n^\beta \nabla_\beta n^\alpha = 0 \rightarrow \frac{\mathrm{d}^2X^\alpha}{\mathrm{d}\rho^2}|_H = -\Gamma^\alpha_{\beta\gamma} n^\beta n^\gamma$$

- Produce coordinate chart based off X^α

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Kerr derivation

- Non-extremal Kerr (GNC to first order):

$$2\kappa r \mathrm{d}v^2 + 2\mathrm{d}v\mathrm{d}r + \frac{4a^2 \sin(\theta)\cos(\theta)}{\Sigma_+} r\mathrm{d}v\mathrm{d}\theta + \Sigma_+^2 \mathrm{d}\theta^2$$

$$+\,2\left(\frac{2(r_+^2 + a^2)\kappa}{\Sigma_+} a \sin^2(\theta) + \frac{2ar_+(r_+^2 + a^2)}{\Sigma_+^2} \sin^2(\theta)\right) r\mathrm{d}v\mathrm{d}\phi + \frac{(r_+^2 + a^2)^2}{\Sigma_+} \sin^2(\theta)\mathrm{d}\phi^2$$

$$r \to \Gamma(\theta) r \xrightarrow{\textit{yields}} 2\mathrm{d}v\mathrm{d}r \to 2\Gamma \mathrm{d}v\mathrm{d}r + 2r\Gamma' \mathrm{d}\theta\mathrm{d}v$$

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Wave equation symmetry

- Conformal coordinates:

$$w^+ = \sqrt{\frac{r-r_+}{r-r_-}}e^{2\pi T_R \phi} \quad w^- = \sqrt{\frac{r-r_+}{r-r_-}}e^{2\pi T_L \phi - \frac{t}{2M}} \quad y = \sqrt{\frac{r-r_+}{r-r_-}}e^{\pi (T_R + T_L)\phi - \frac{t}{4M}}$$

$$H_1^\pm = i\partial_\pm \quad H_0^\pm = i\left(w^\pm \partial_\pm + \frac{1}{2} y \partial_y\right) \quad H_{-1}^\pm = i\left(w^{\pm 2}\partial_\pm + \frac{1}{2} w^\pm y \partial_y - y^2 \partial_\mp\right)$$

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