Boundary Data for Local Boundaries

in General Relativity

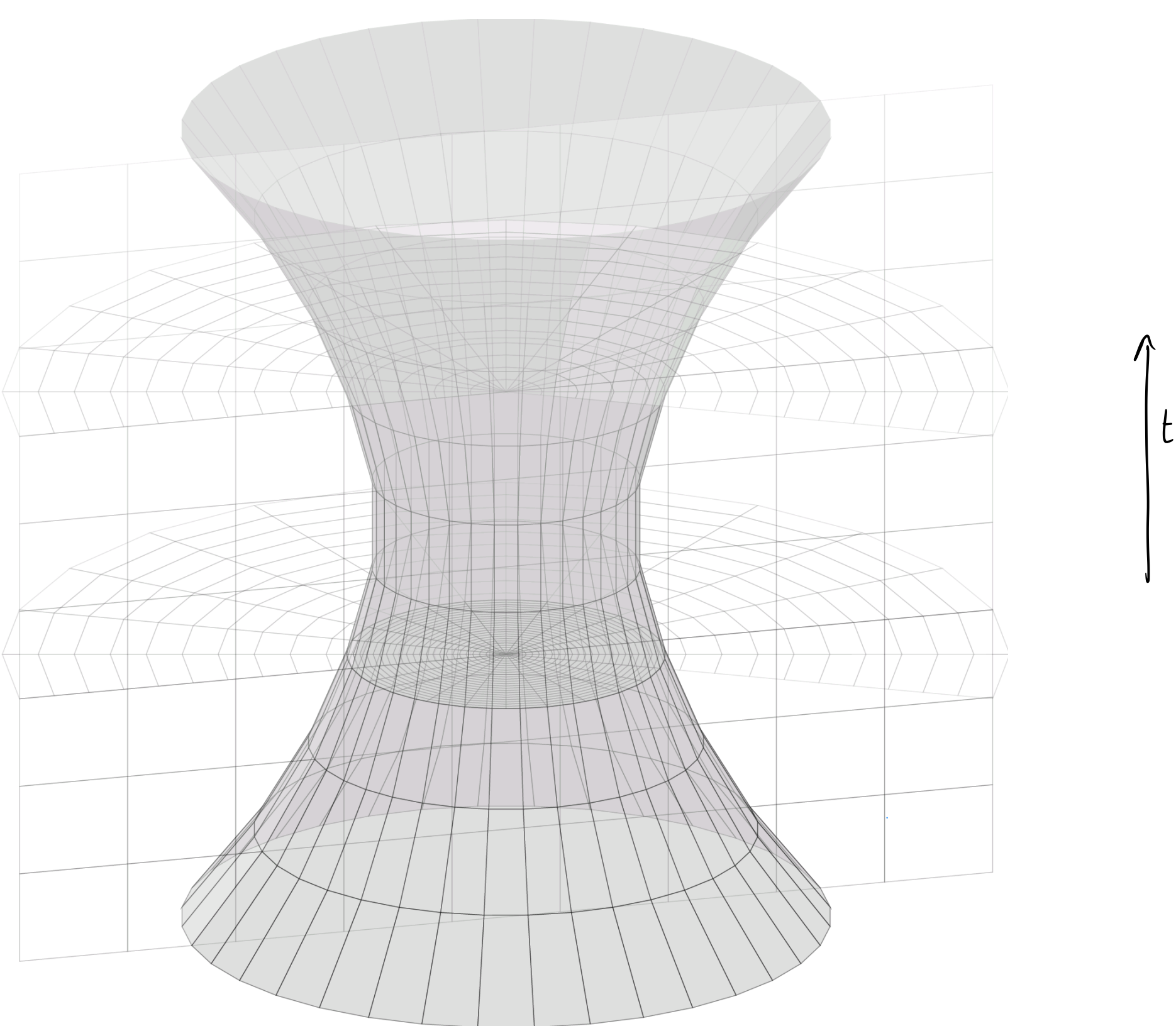
Antonia Seifert

work in progress with Luis Lehner and Fernando Abalos

Theory Canada, June 19, 2026

PERIMETER INSTITUTE

UNIVERSITY OF WATERLOO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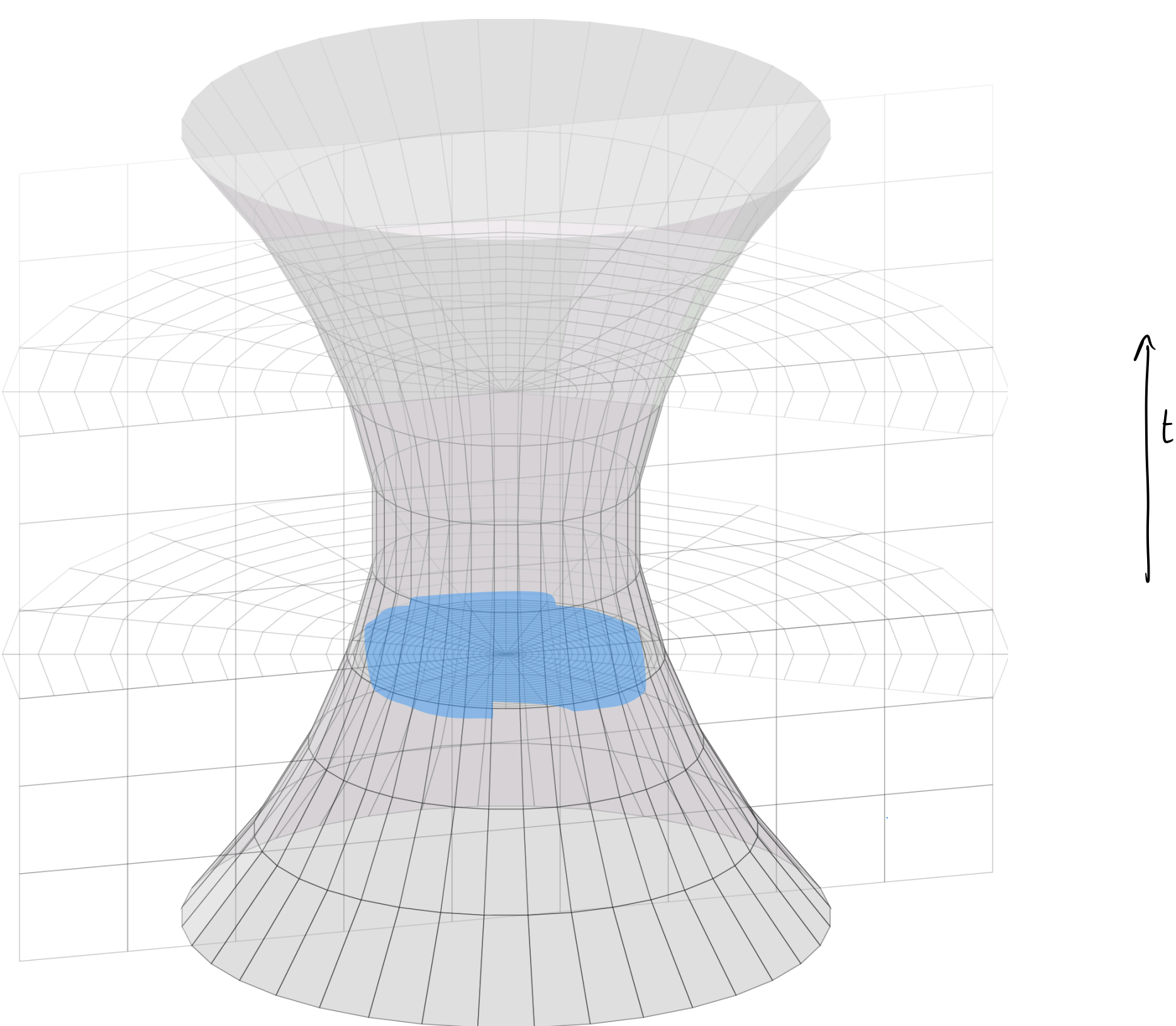
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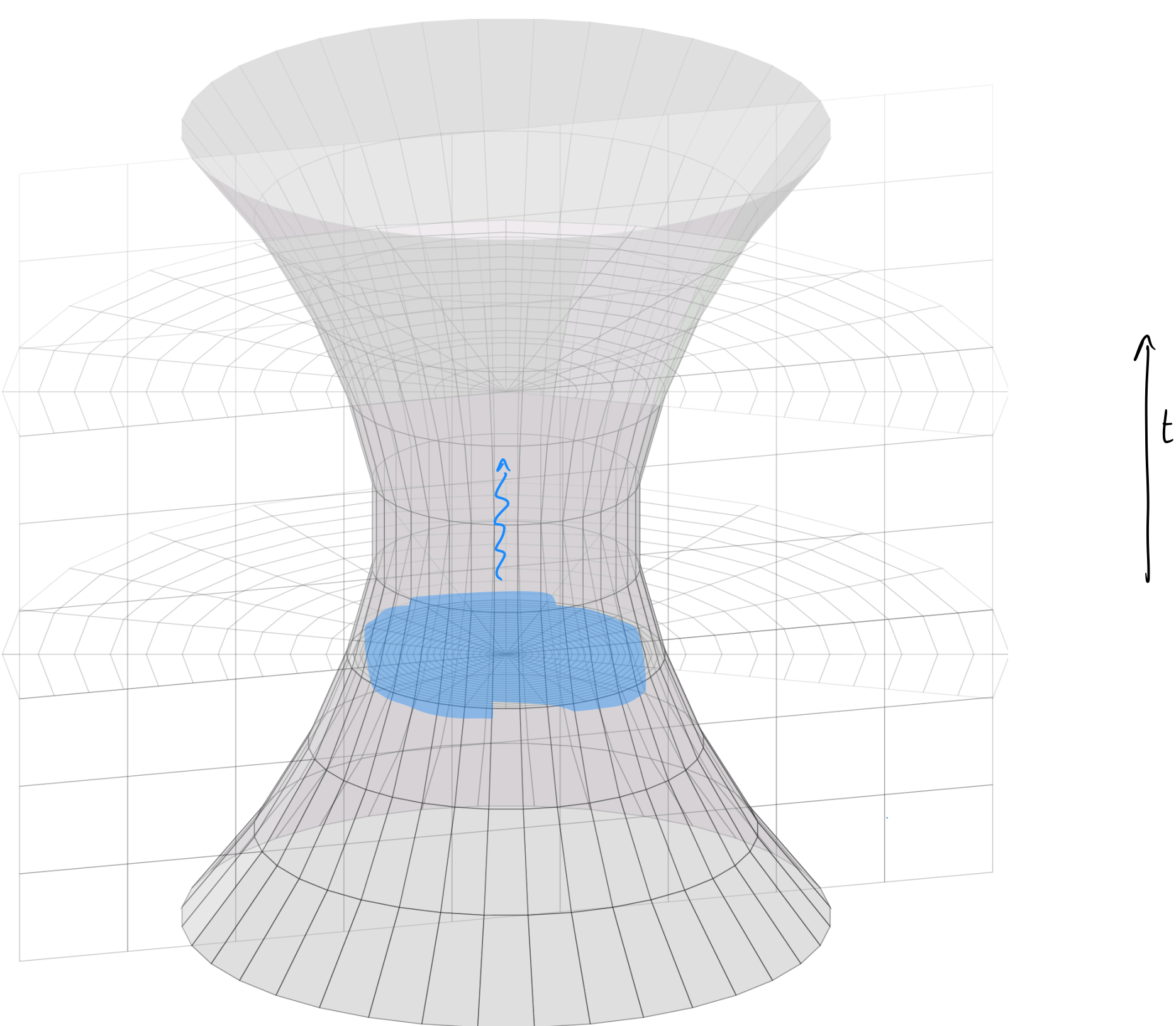
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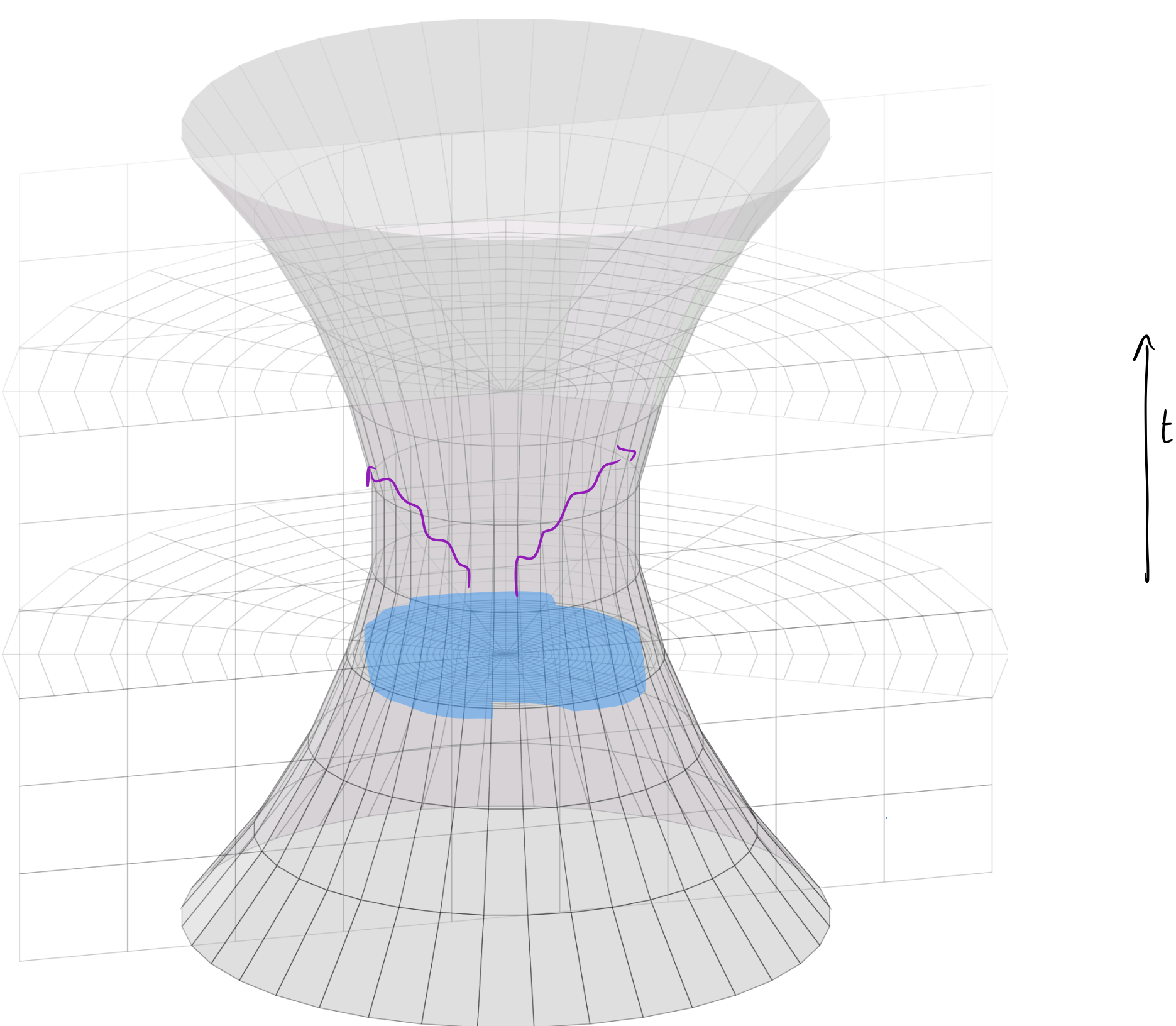
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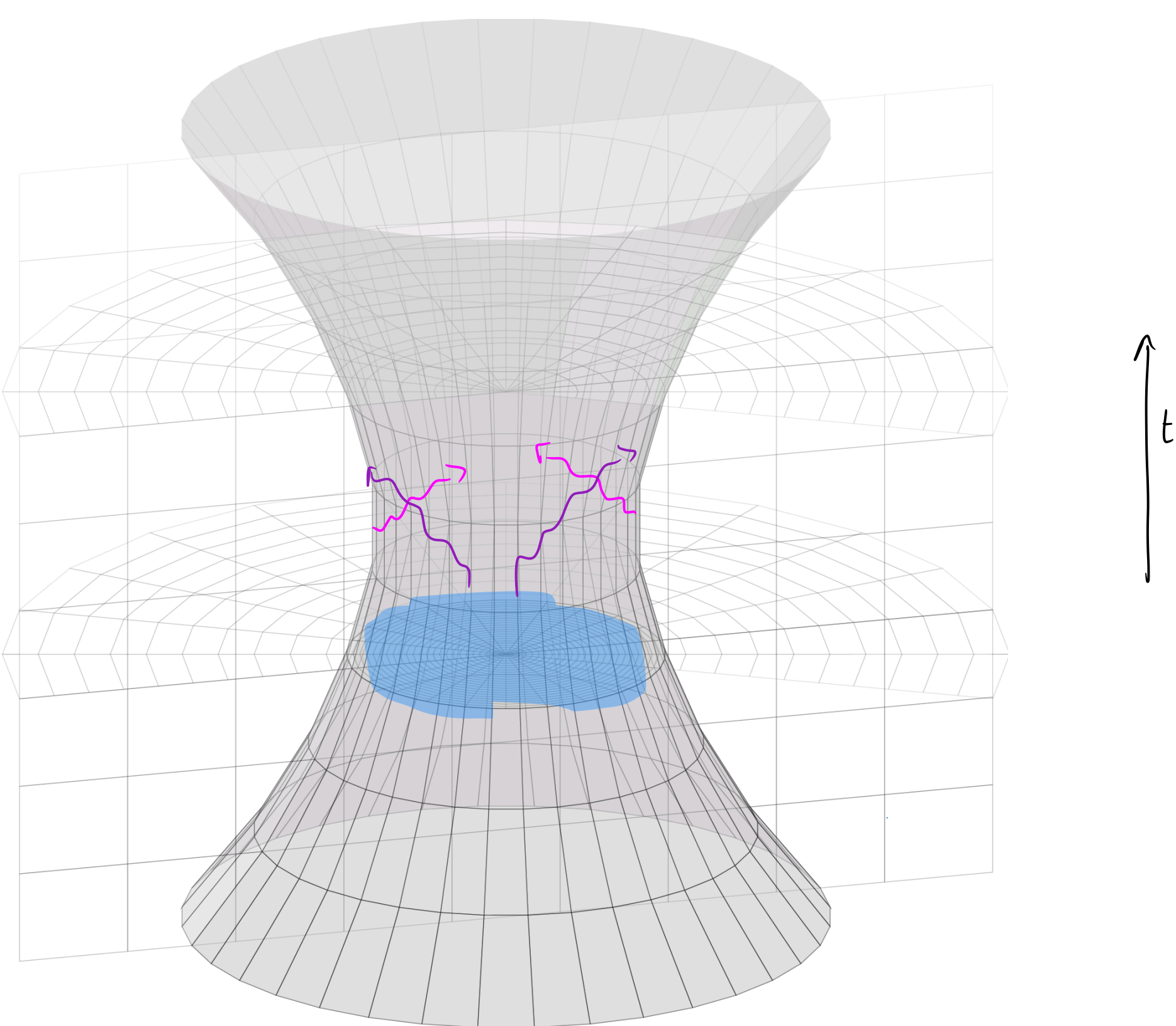
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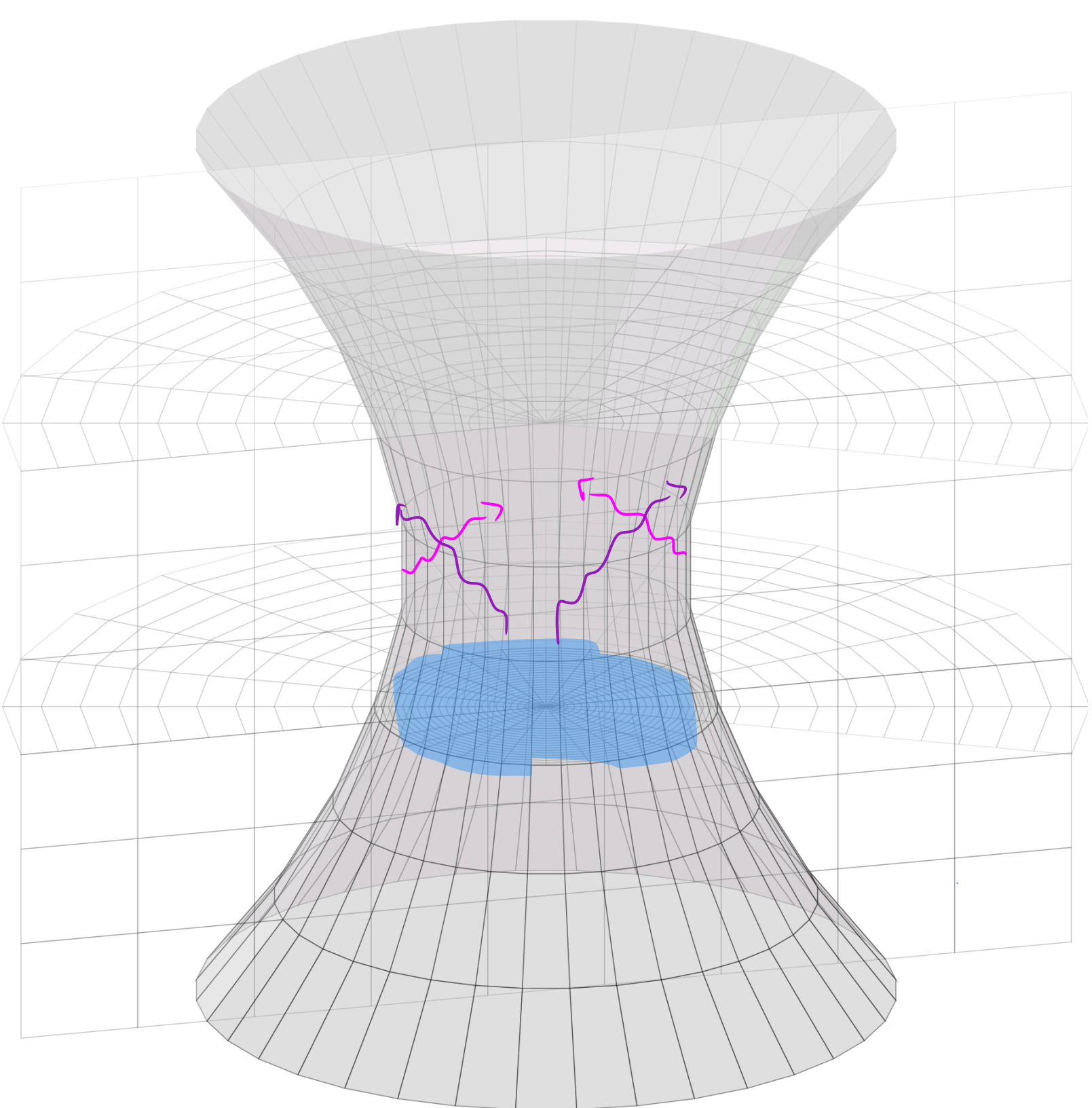
Initial
Boundary
Value
Problem
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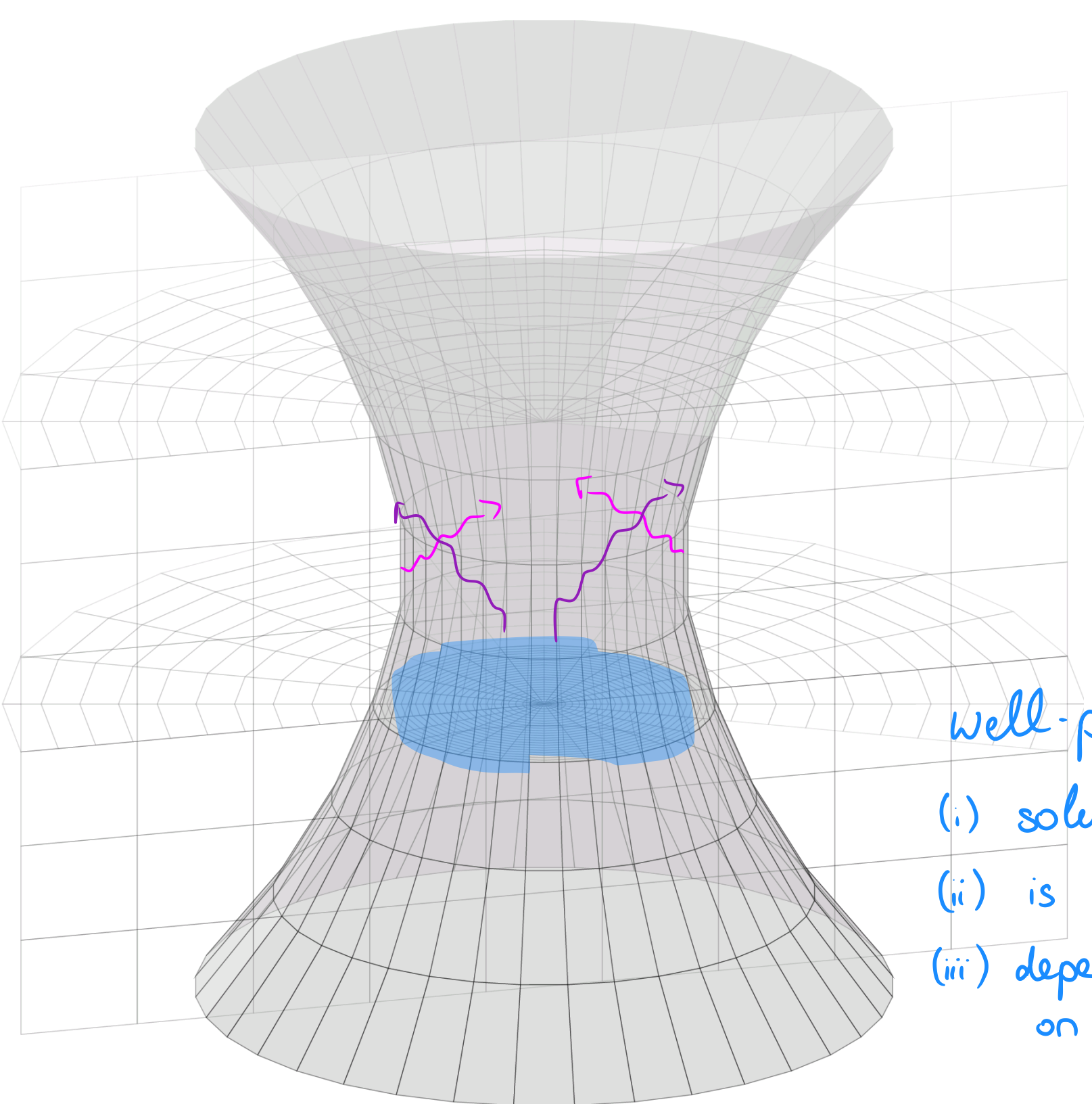


Initial Boundary Value Problem

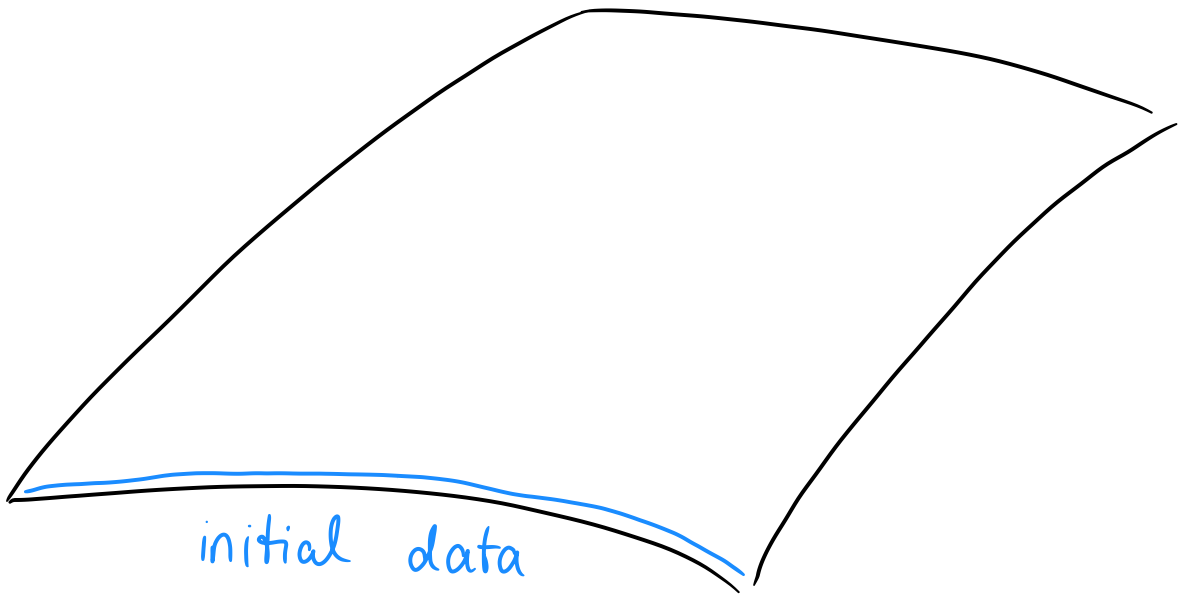
well-posed iff

(i) solution exists

(ii) is unique

(iii) depends continuously on initial data

Initial Boundary Value Problem



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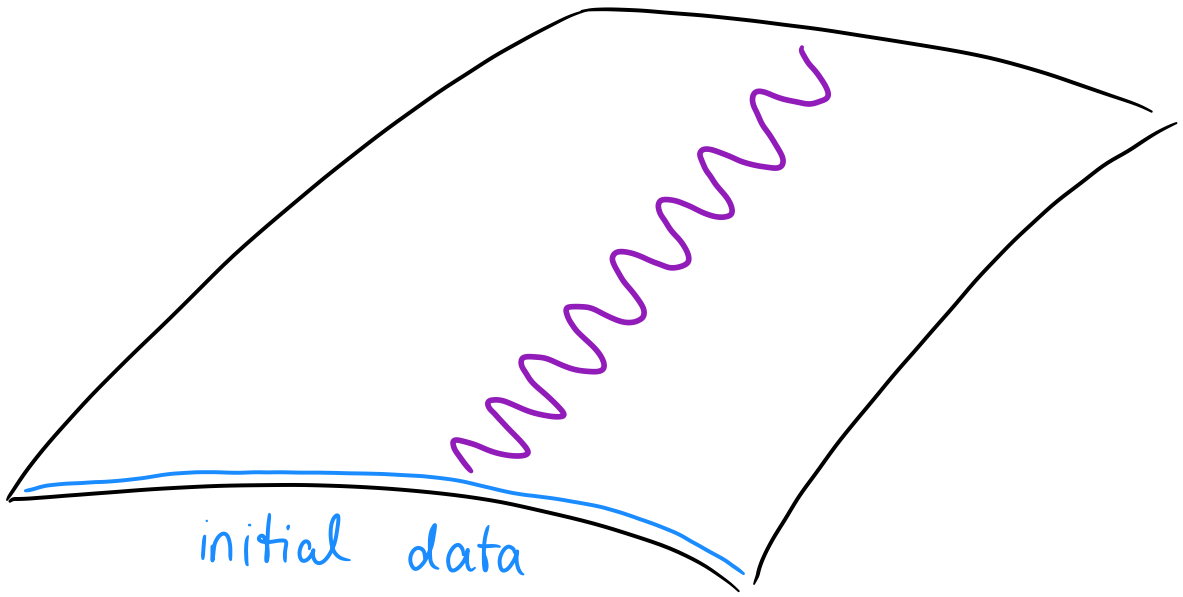

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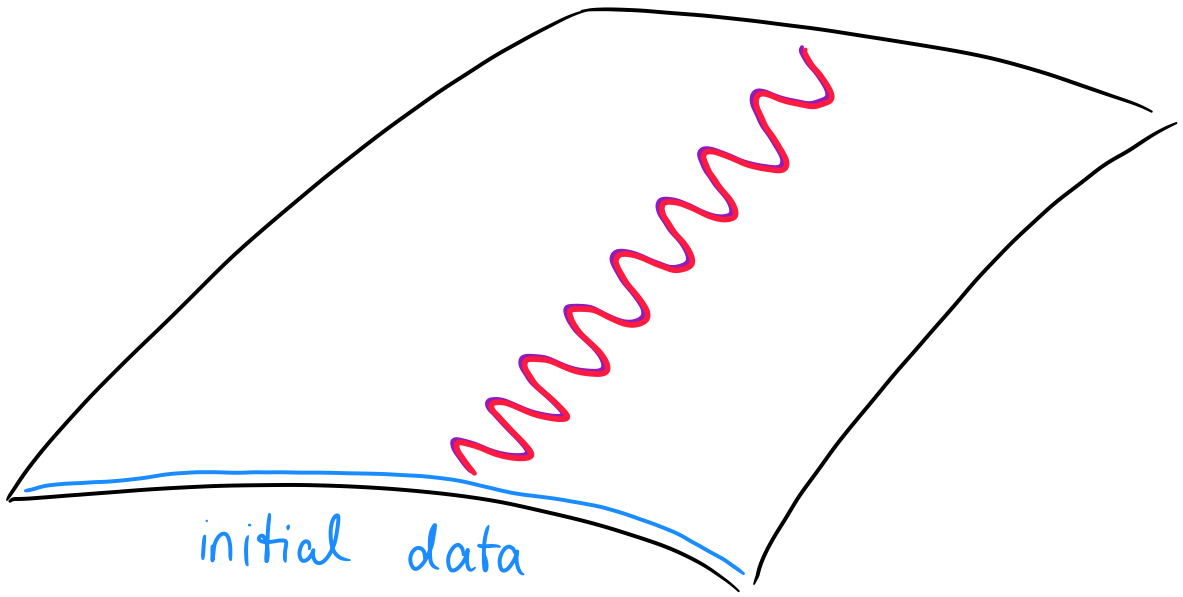

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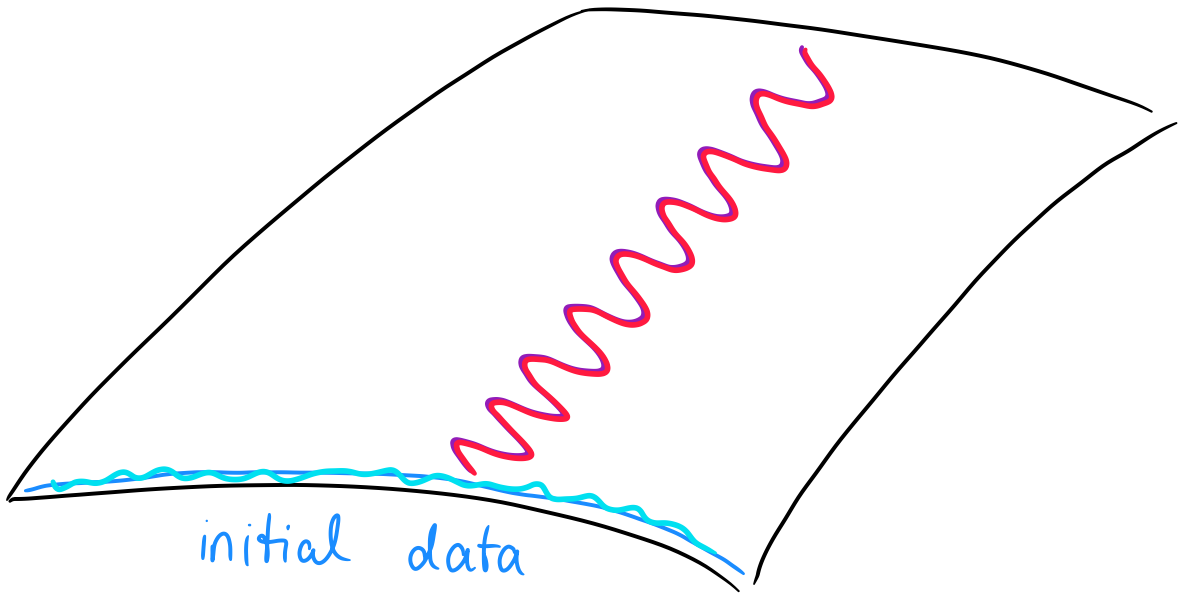

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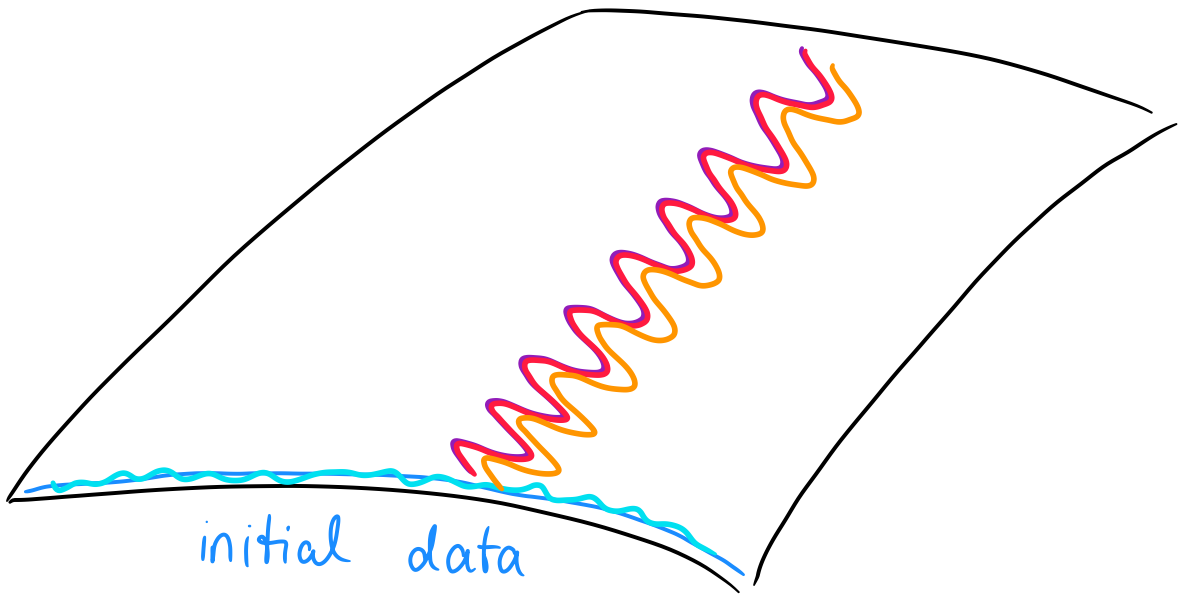

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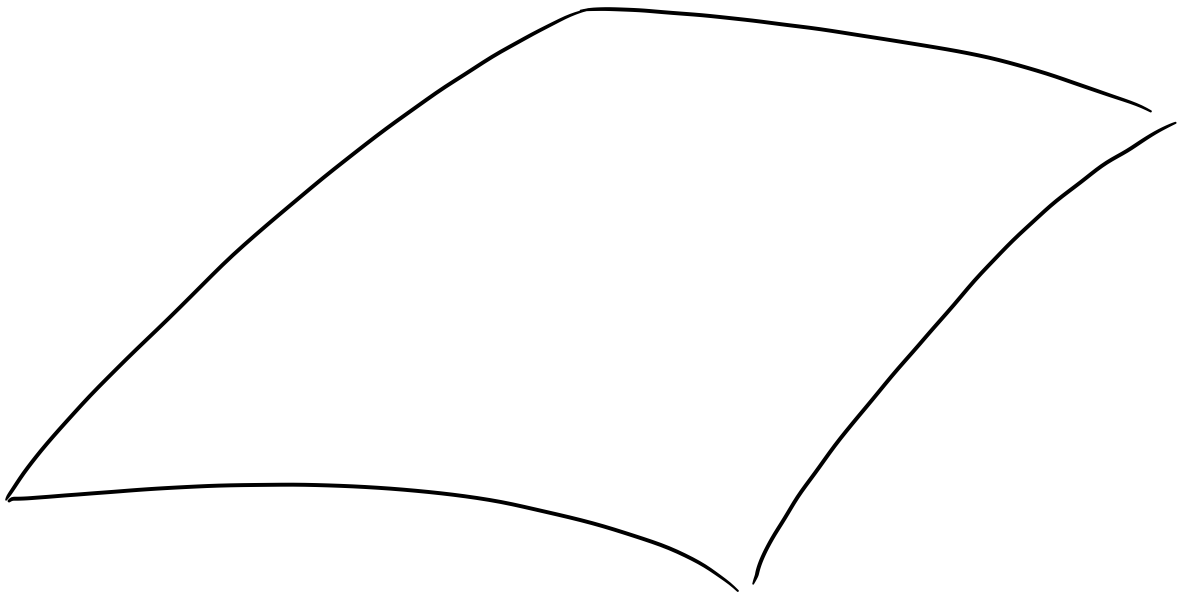
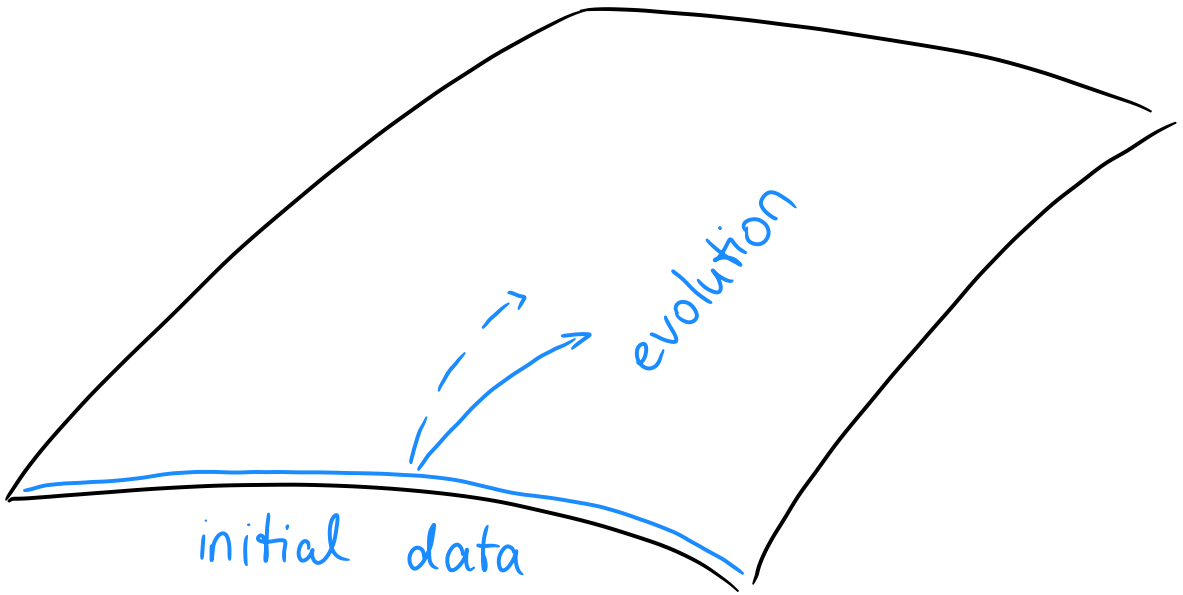
4 dimensional GR $\Rightarrow$ 10 equations



4 dimensional GR $\Rightarrow$ 10 equations

gauge choice to fix evolution direction n $\hookrightarrow$ 4 gauge eqs



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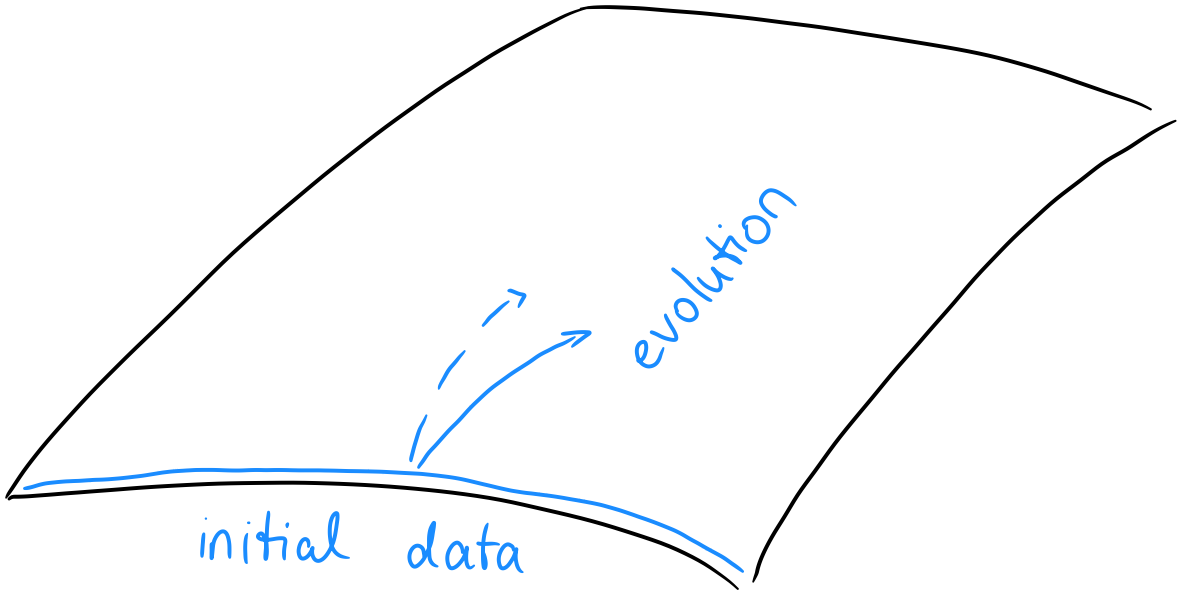

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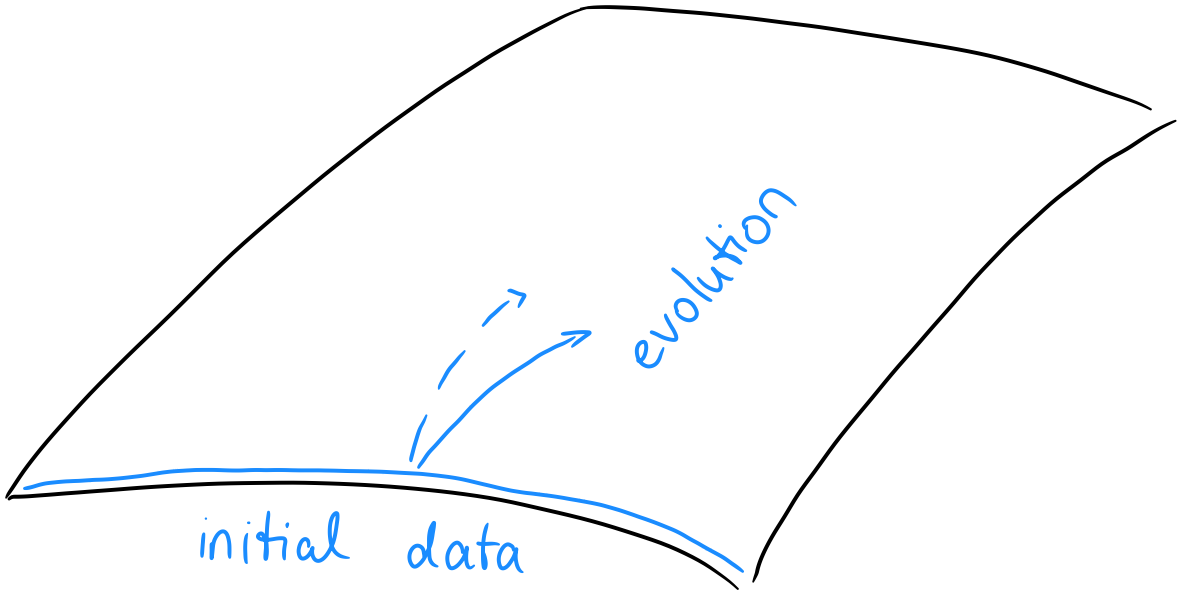
2 remaining eqs



4 dimensional GR

gauge choice to fix evolution direction n

satisfy projection of Einstein eqs along n

$\Rightarrow$ 10 equations

↳ 4 gauge eqs

↳ 4 constraint eqs

2 remaining eqs

need boundary data for 6 eqs

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wave-like equations

$$\left[-\partial_t^2 + \omega^2 \partial_x^2\right] \varphi = 0$$

4 dimensional GR $\Rightarrow$ 10 equations

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need boundary data for 6 eqs

Wave-like equations $\rightarrow$ 1 outgoing and 1 incoming mode per eq.

$$\left[-\partial_t^2 + \omega^2 \partial_x^2\right] \varphi = 0$$

$$\varphi = \lambda e^{i(\omega t \pm x)}$$

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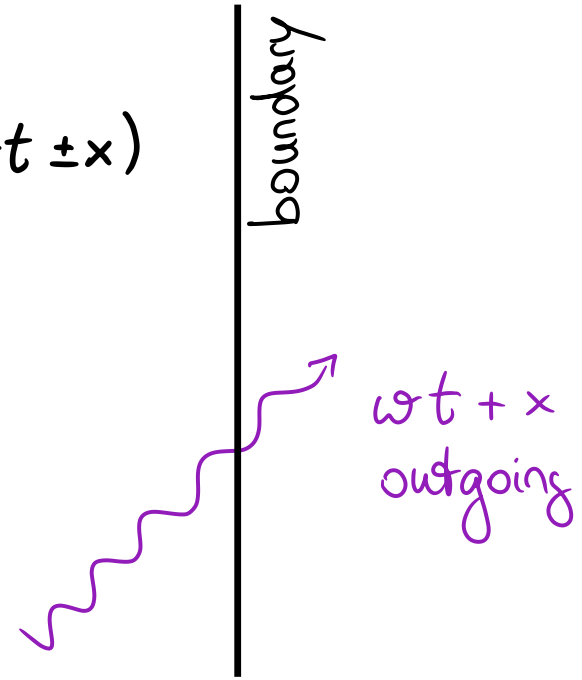

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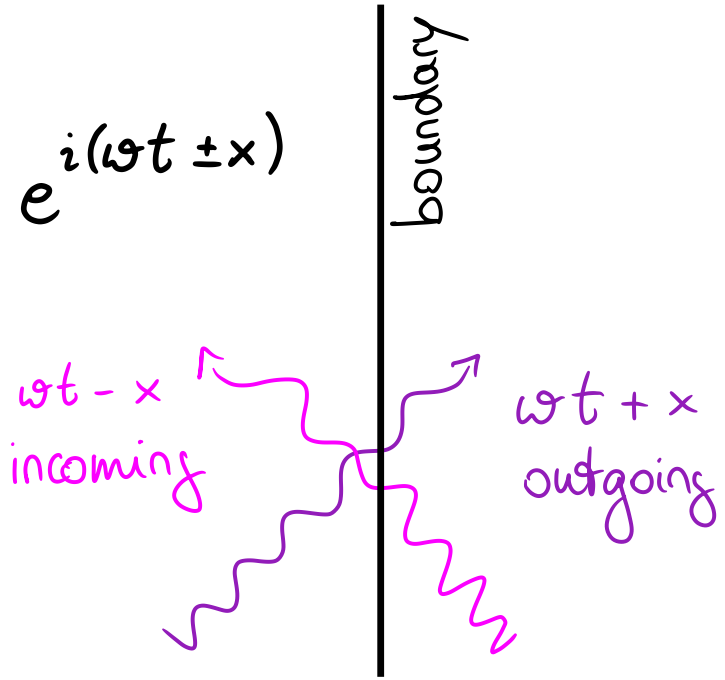

4 dimensional GR $\Rightarrow$ 10 equations

gauge choice to fix evolution direction n $\hookrightarrow$ 4 gauge eqs

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2 remaining eqs

need boundary data for 6 eqs

need to give boundary data for 6 incoming modes

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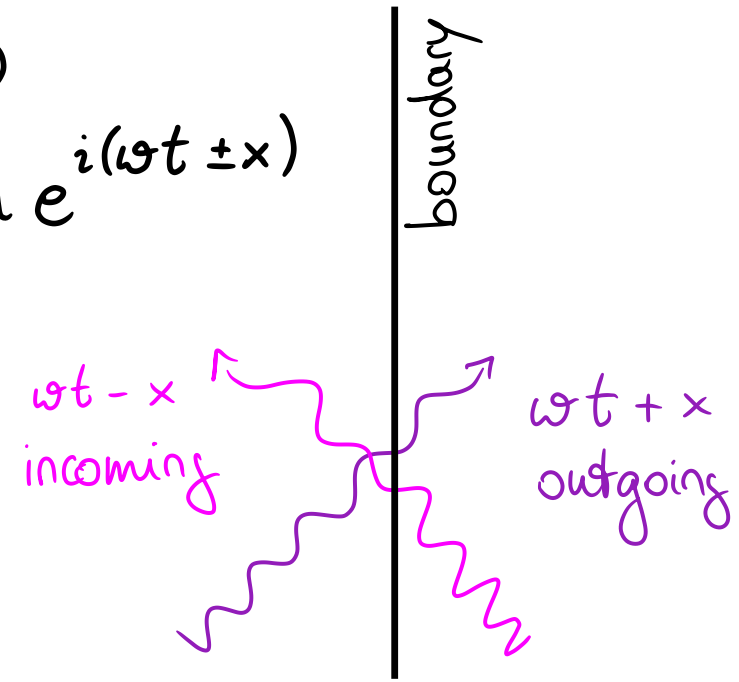

10 equations

↳ 4 gauge eqs

↳ 4 constraint eqs

~~4~~ 2 remaining eqs

→ 6 incoming modes

↳ 4 incoming gauge modes

↳ 2 incoming physical modes

10 equations

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Our formalism: Background metric + perturbation

⇒ Which modes in which components of the perturbation?

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THIS YIELDS A GENERAL FORM OF THE METRIC PERTURBATION

metric perturbation

$$\hookrightarrow = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}$$

↑ evolution direction

↑ boundary direction

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⟹ find 5 degrees of freedom in boundary metric

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$$\hat{=} \begin{pmatrix} & \\ & \boxed{3\times 3 \text{ boundary metric}} \end{pmatrix}$$

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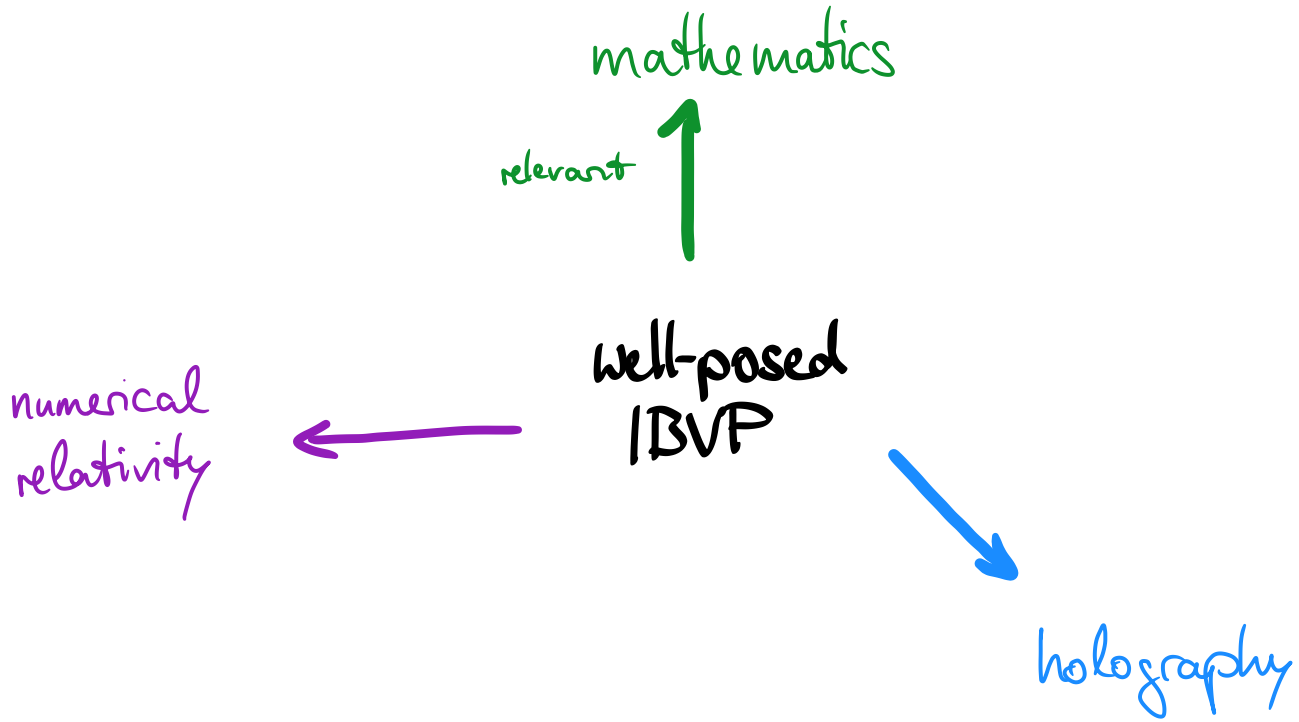
metric perturbation

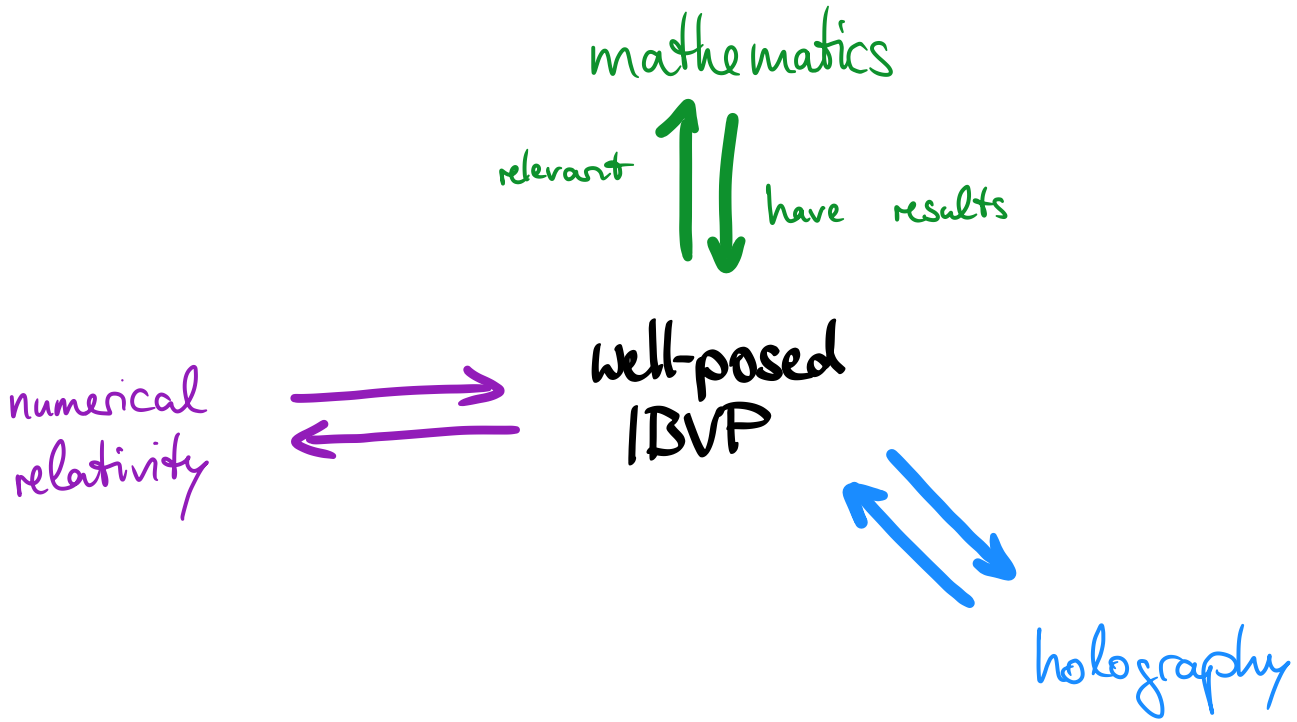
↳ = (3×3 boundary metric)

↑ evolution direction

↑ boundary direction

→ STUDY CHOICES OF BOUNDARY DATA


mathematics
relevant
well-posed IBVP
numerical relativity
holography


mathematics
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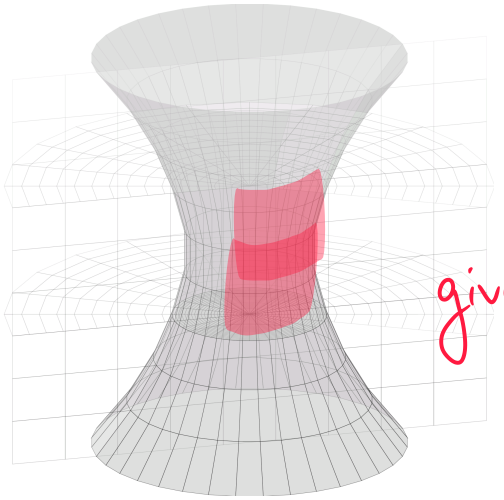
mathematics

relevant ⇅ have results

numerical relativity ⇄ well-posed IBVP

well-posed IBVP ⇅ holography

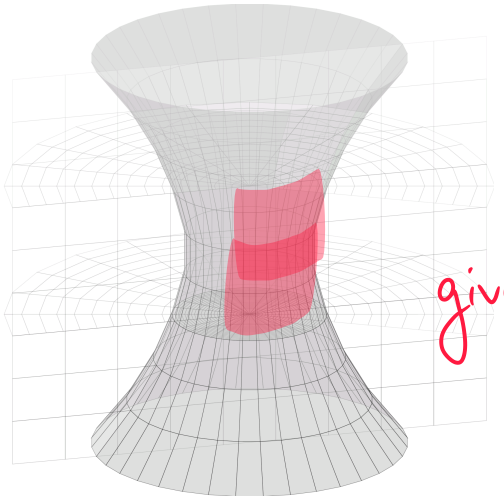


Dirichlet data : give boundary metric

mathematics

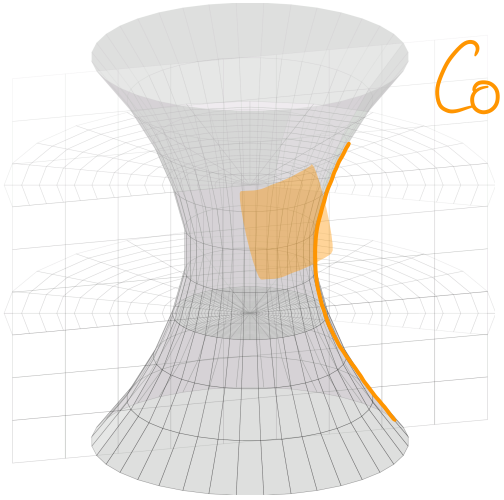
relevant ⇅ have results

numerical relativity ⇄ well-posed IBVP ⇄ holography



Dirichlet data :

give boundary metric



Conformal- mean curvature data

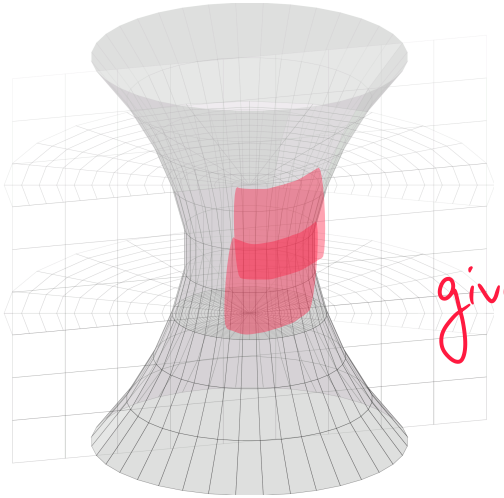
give boundary metric without scale

(↪ conformal class)

+ trace of extrinsic curvature

$\Longrightarrow$ find 5 degrees of freedom in boundary metric

$\longrightarrow$ one extrinsic gauge degree of freedom



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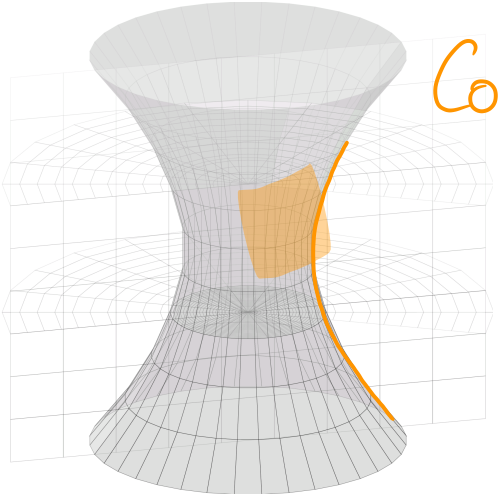

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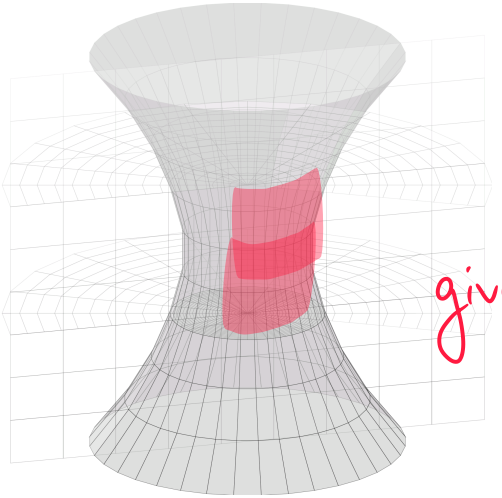


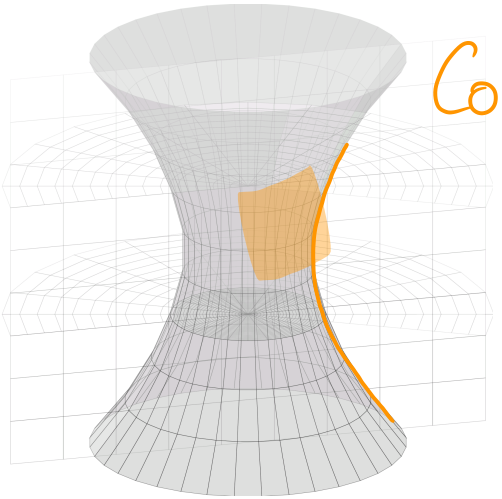
Dirichlet data :

give boundary metric

ill-posed :

over-constraining the boundary metric (no solution exists in general)

extrinsic gauge degree of freedom missing (non-uniqueness)



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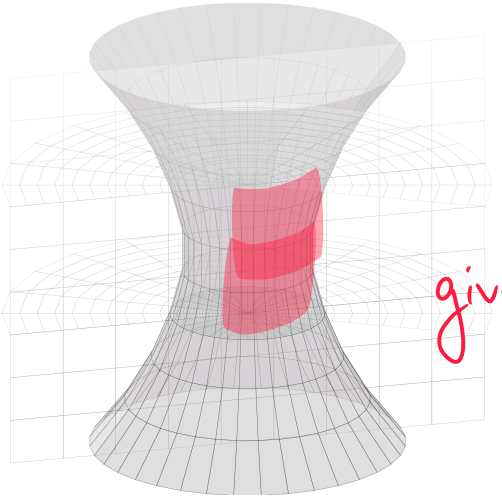

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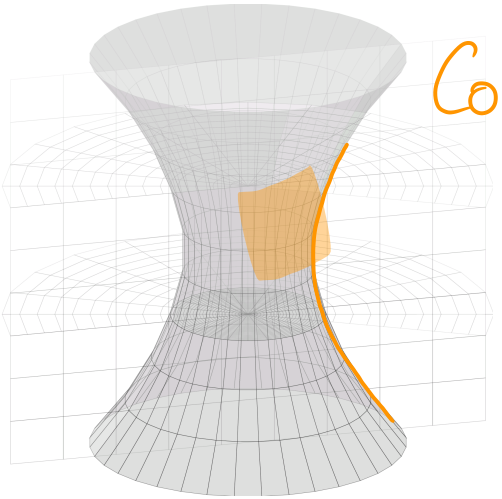

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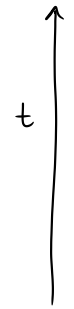
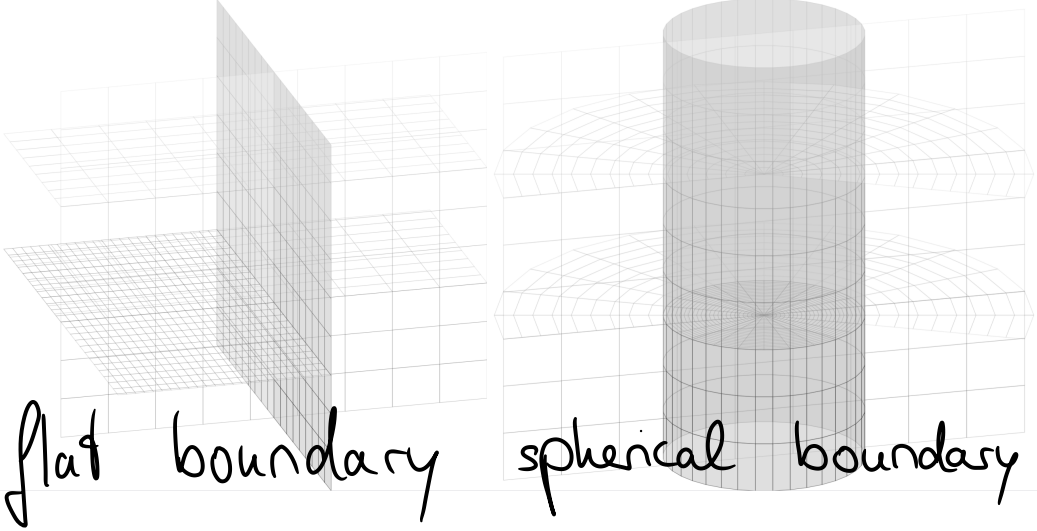
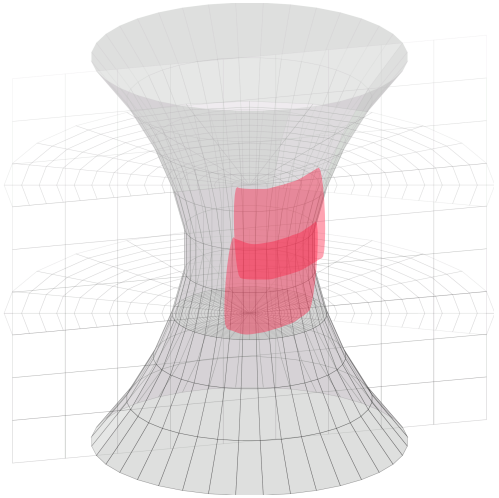
Well-posed



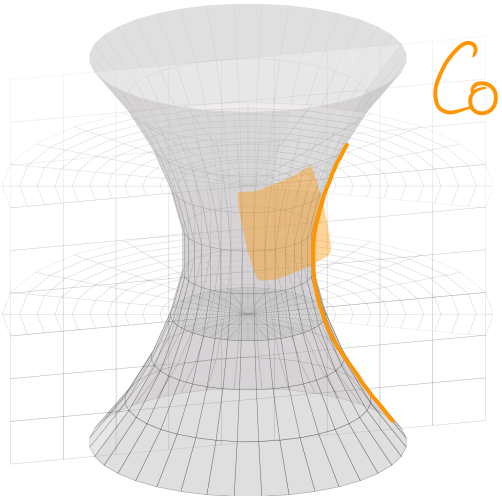


flat boundary spherical boundary



Dirichlet data

ill-posed



Conformal- mean curvature data

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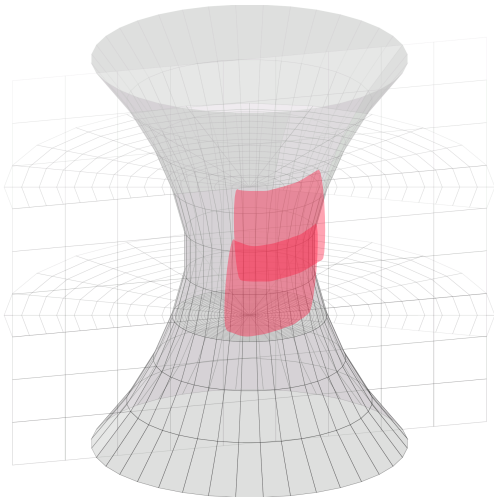

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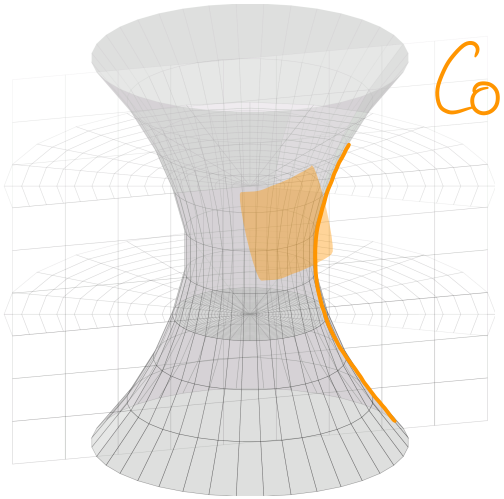

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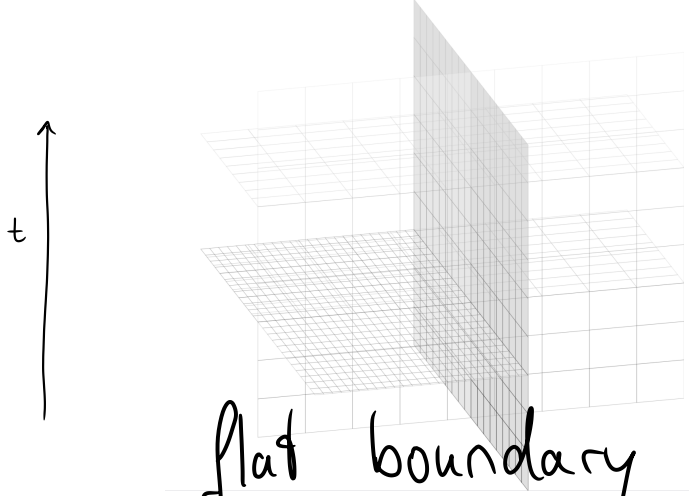
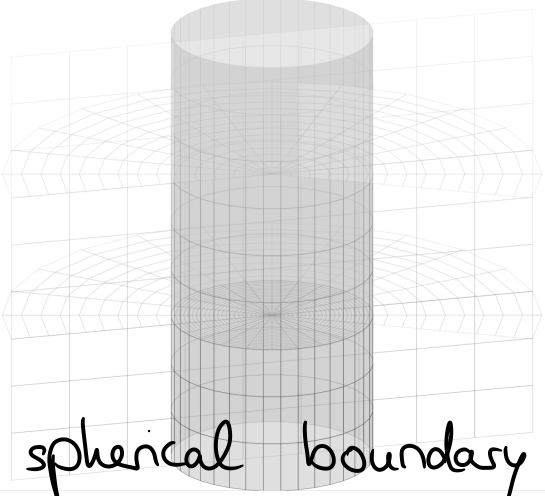
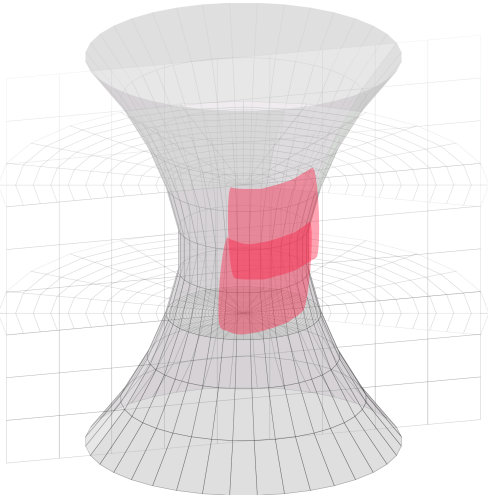




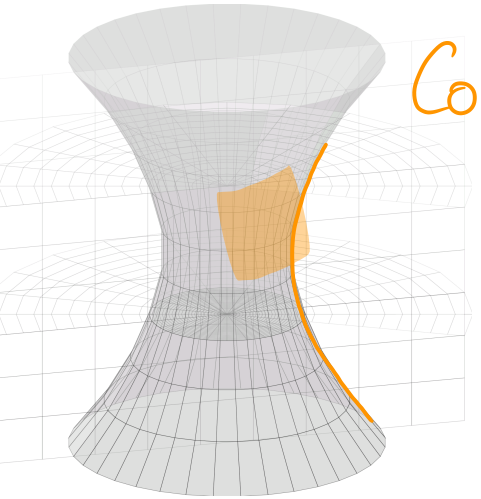
flat boundary	spherical boundary
ill-posed	ill-posed
well-posed	well-posed

> high-frequency modes do not discern whether a boundary is flat or spherical



Dirichlet data

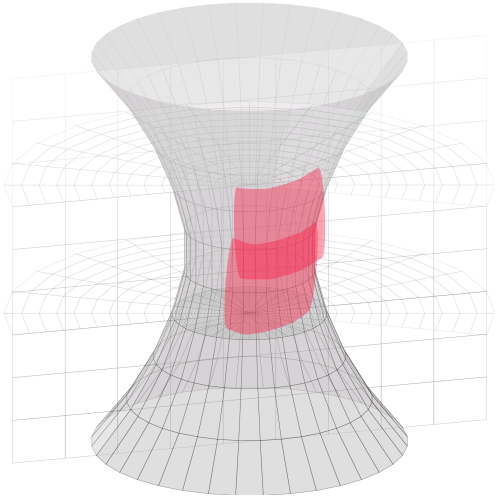
ill-posed



Conformal - mean curvature data

well-posed

flat boundary	spherical boundary
ill-posed	ill-posed
ill-posed Anninos, Galante, Maneerat 2023	maybe well-posed Anninos, Galante, Maneerat 2023
well-posed	well-posed
well-posed An & Anderson 2025 well-posed Anninos, Galante, Maneerat 2023	ill-posed Anninos, Galante, Maneerat 2023 Liu, Santos, Wiseman 2024 Liu, Reall, Santos, Wiseman 2025



Dirichlet data

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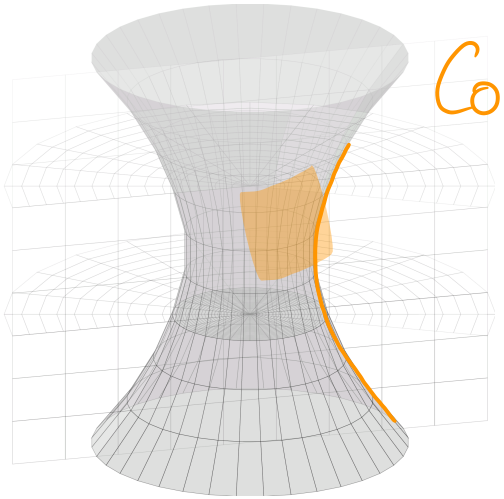
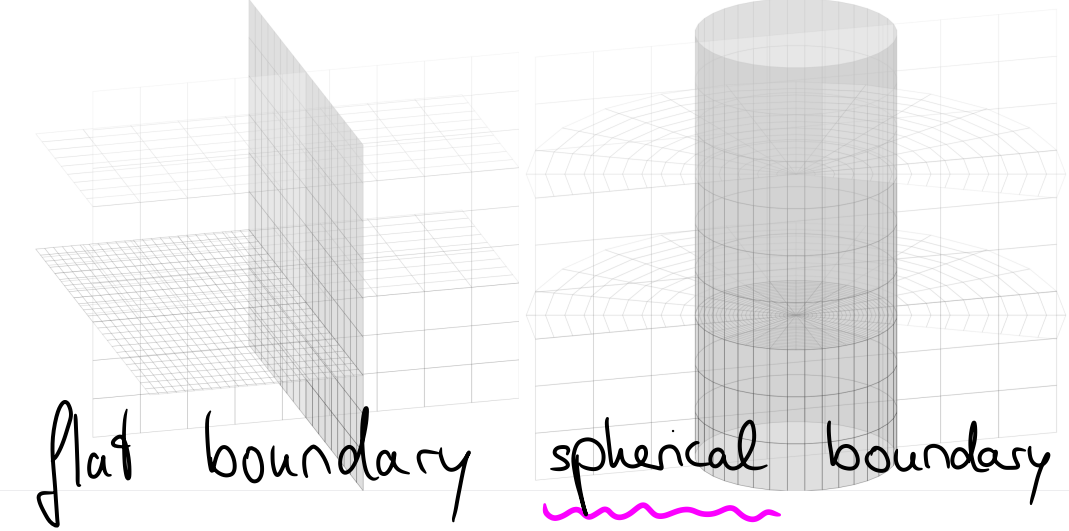


Conformal-mean curvature data

well-posed



flat boundary

spherical boundary

ill-posed

ill-posed

Annios, Galante, Maneerat 2023

ill-posed

maybe well-posed

Annios, Galante, Maneerat 2023

well-posed

well-posed

An & Anderson 2025

well-posed

Annios, Galante, Maneerat 2023

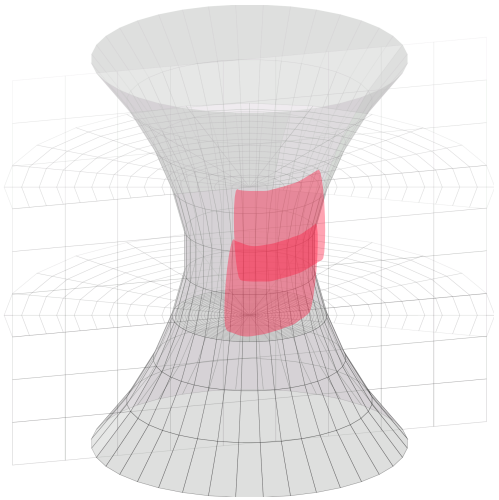
well-posed

ill-posed

Annios, Galante, Maneerat 2023

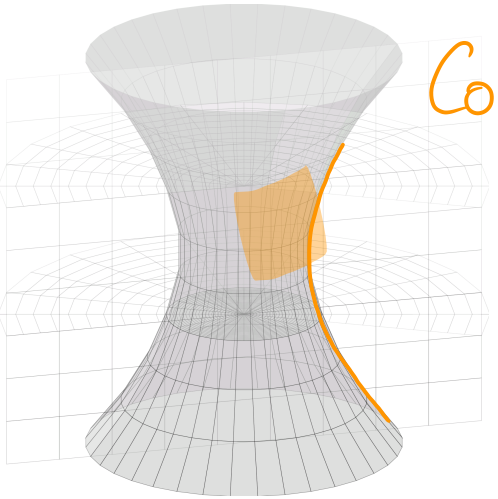
Liu, Santos, Wiseman 2024

Liu, Reall, Santos, Wiseman 2025



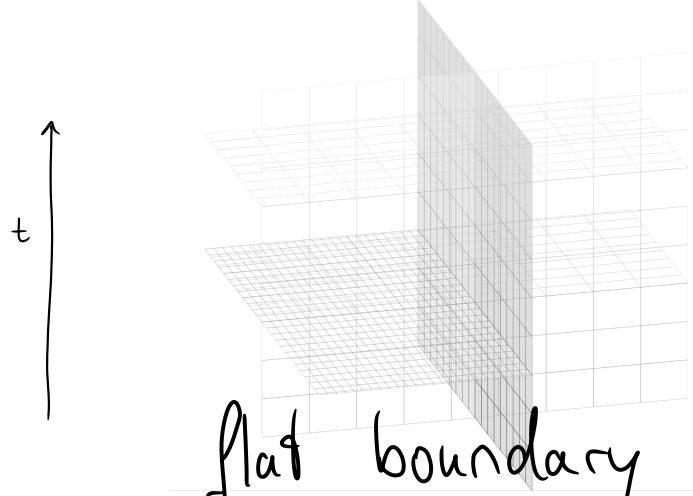
Dirichlet data

ill-posed



Conformal - mean curvature data

well-posed



flat boundary

harmonic gauge

ill-posed

ill-posed

Anninos, Galante, Maneerat 2023

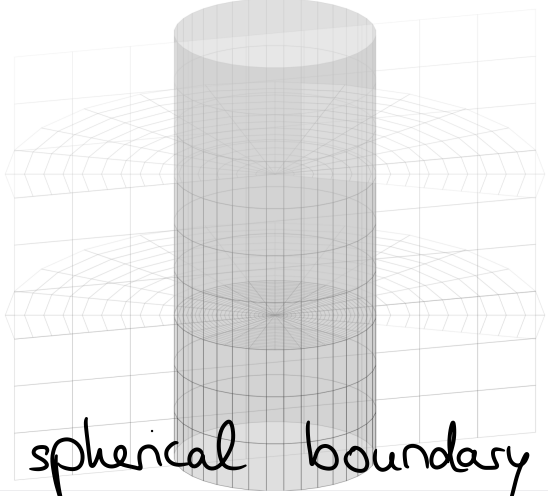
well-posed

Well-posed

An & Anderson 2025

Well-posed

Anninos, Galante, Maneerat 2023



spherical boundary

various gauges

ill-posed

maybe well-posed

Anninos, Galante, Maneerat 2023

well-posed

ill-posed

Anninos, Galante, Maneerat 2023

Liu, Santos, Wiseman 2024

Liu, Reall, Santos, Wiseman 2025

boundary data for 6 incoming modes

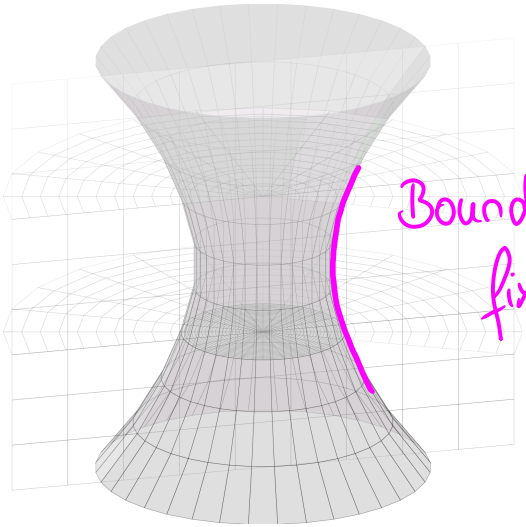
↳ 4 incoming gauge modes

↳ 2 incoming physical modes

boundary data for 6 incoming modes

↳ 4 incoming gauge modes

↳ 2 incoming physical modes

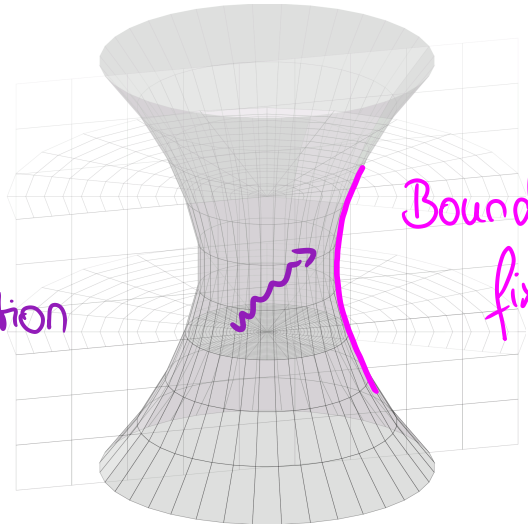


Boundary data fixes gauge modes at the boundary

boundary data for 6 incoming modes

↳ 4 incoming gauge modes

↳ 2 incoming physical modes



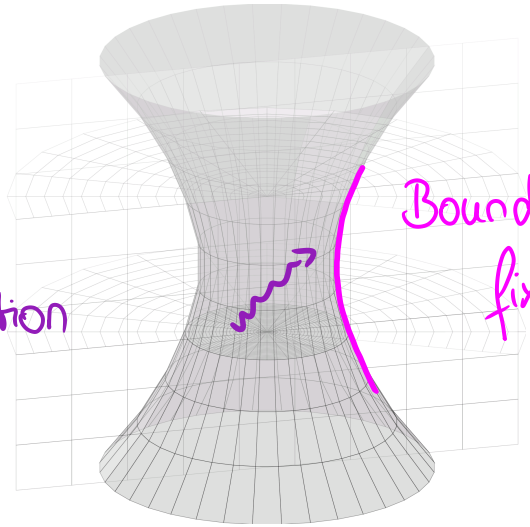
Boundary data fixes gauge modes at the boundary

pre-defined evolution of gauge modes in the bulk

boundary data for 6 incoming modes

↳ 4 incoming gauge modes

↳ 2 incoming physical modes



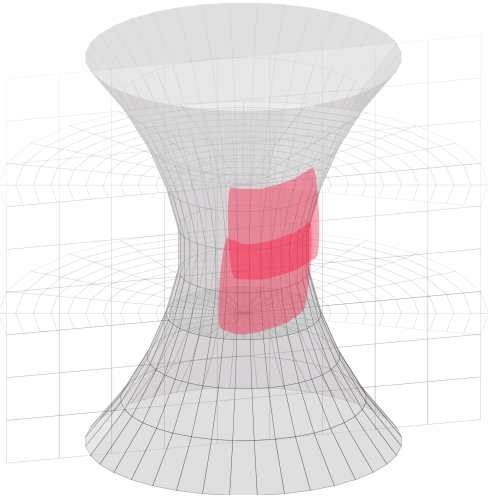
Boundary data fixes gauge modes at the boundary

pre-defined evolution of gauge modes in the bulk

↳ our formalism includes a gauge choice

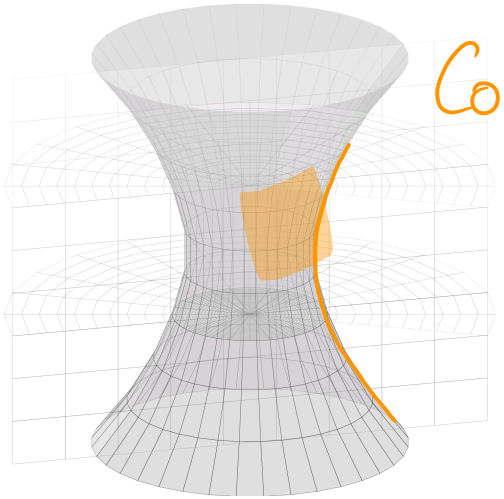
↳ well-posedness of the IBVP depends on the gauge choice

↳ it is not enough to fix the gauge data at the boundary



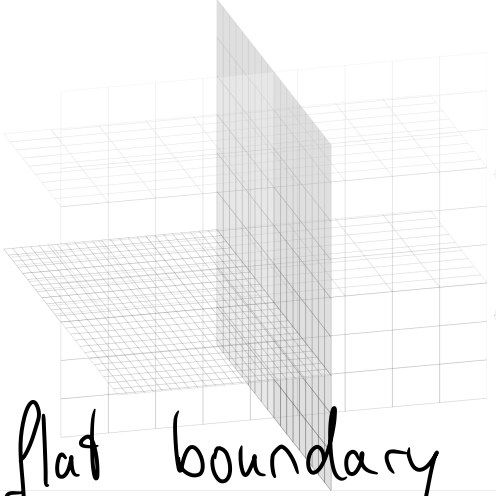
Dirichlet data

ill-posed



Conformal - mean curvature data

well-posed



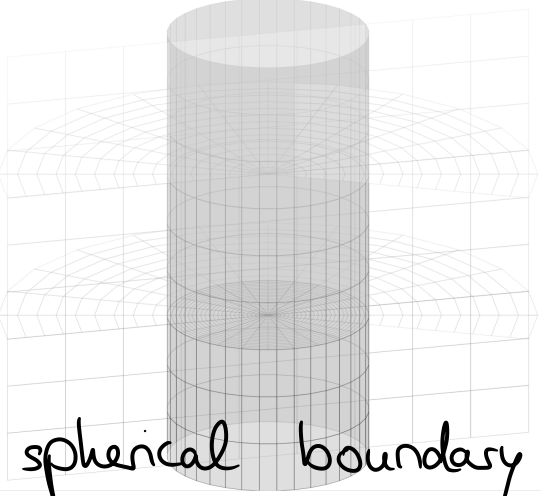
flat boundary

harmonic gauge

ill-posed

ill-posed

Anninos, Galante, Maneerat 2023



spherical boundary

various gauges

well-posed

well-posed

An & Anderson 2025

well-posed

Anninos, Galante, Maneerat 2023

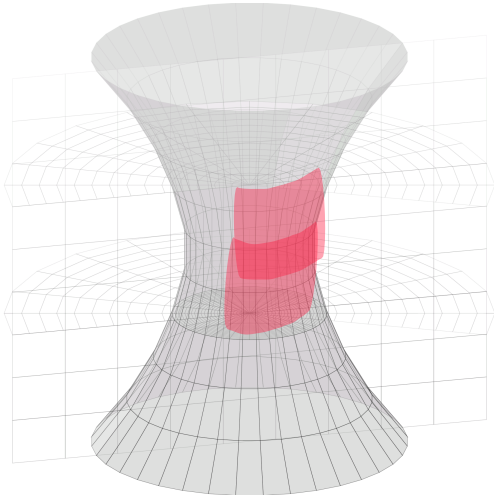
harmonic gauge

$$\Box x^{\alpha} = 0$$

or similar

↳ flat boundaries

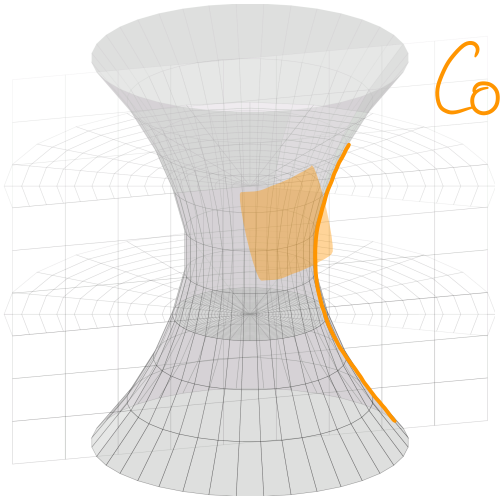
↳ our formalism



Dirichlet data

ill-posed

ill-posed



Conformal- mean curvature data

well-posed

well-posed

harmonic gauge

$\Box x^{\alpha} = 0$

or similar

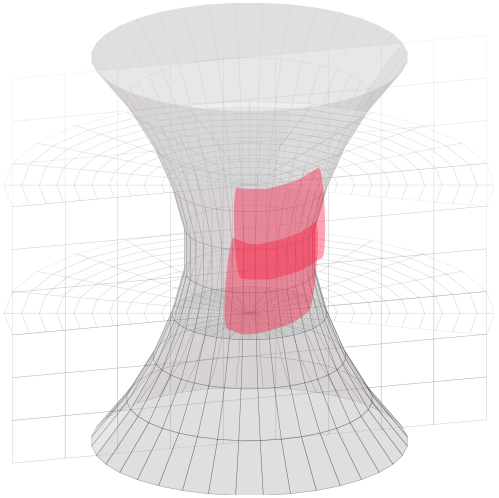
↳ flat boundaries

↳ our formalism

no gauge

conformal gauge

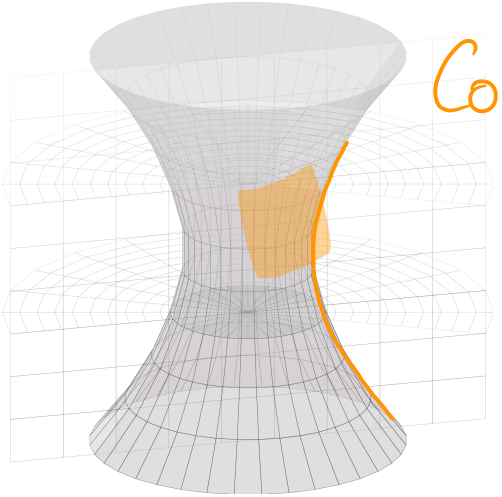
comes metric $\propto$ background



Dirichlet data

ill-posed

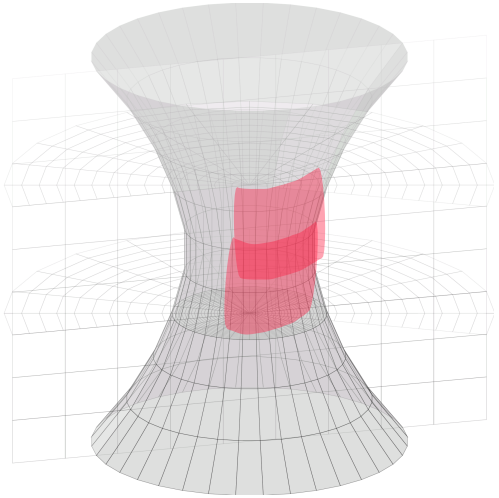
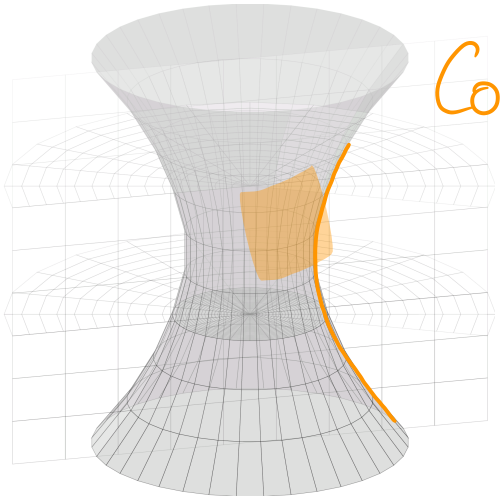
ill-posed

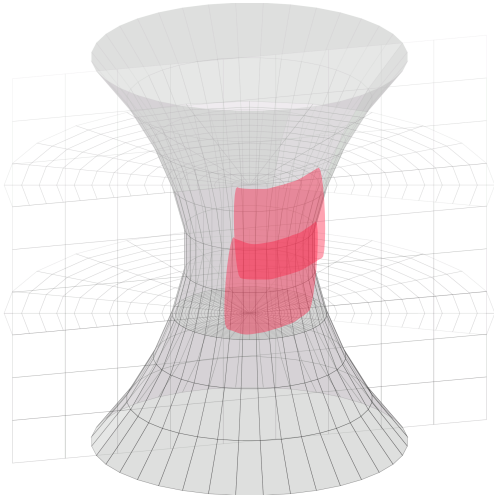
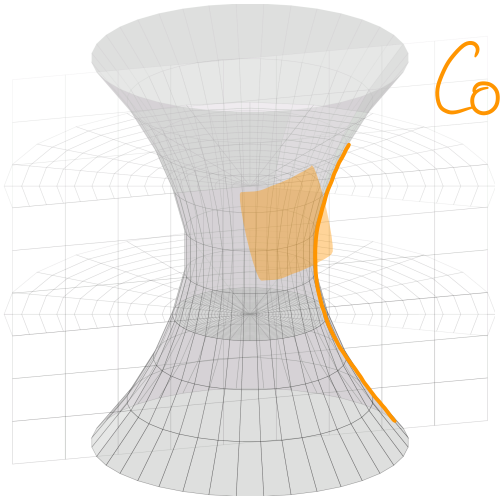


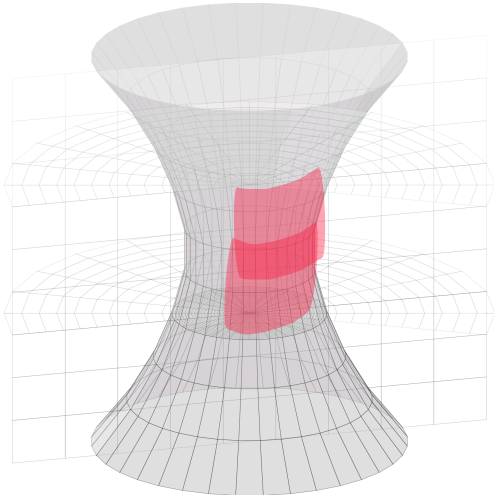
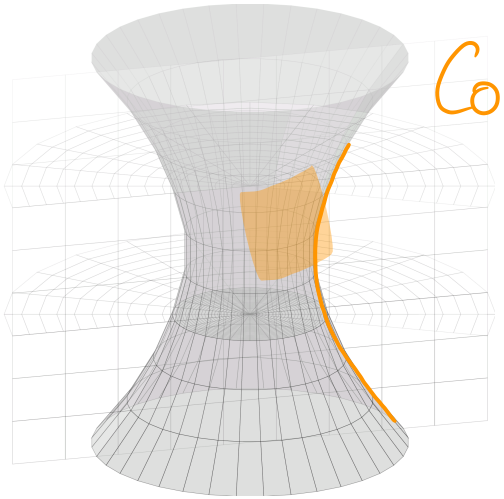
Conformal- mean curvature data

Well-posed

well-posed

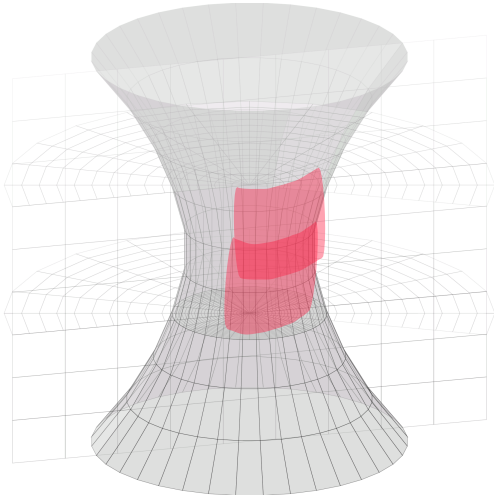
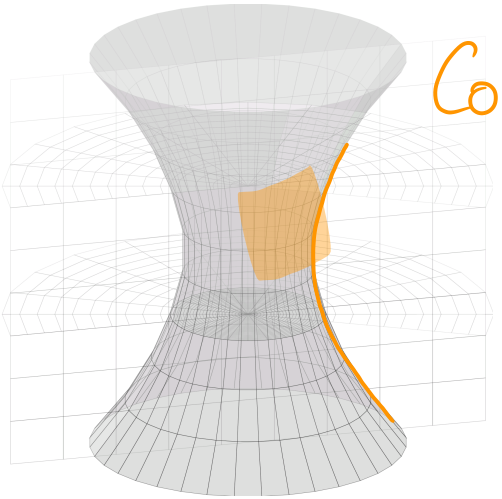
	harmonic gauge $\Box x^\alpha = 0$ or similar ↳ flat boundaries ↳ our formalism	no gauge	conformal gauge ↳ comes metric ∝ background
 Dirichlet data ill-posed	ill-posed	ill-posed	
 Conformal - mean curvature data Well-posed	well-posed	ill-posed	

	harmonic gauge $\Box x^\alpha = 0$ or similar ↳ flat boundaries ↳ our formalism	no gauge	conformal gauge corner metric $\propto$ background
 Dirichlet data ill-posed	ill-posed	ill-posed	well-posed* *work in progress
 Conformal- mean curvature data Well-posed	well-posed	ill-posed	ill-posed

	harmonic gauge $\Box x^\alpha = 0$ or similar ↳ flat boundaries ↳ our formalism	no gauge	conformal gauge corner metric $\propto$ background
 Dirichlet data ill-posed	ill-posed ill-posed Anninos, Galante, Maneerat 2023	ill-posed	well-posed* *work in progress
 Conformal-mean curvature data Well-posed	well-posed Well-posed An & Anderson 2025 Well-posed Anninos, Galante, Maneerat 2023	ill-posed ill-posed Liu, Santos, Wiseman 2024 Liu, Reall, Santos, Wiseman 2025	ill-posed ill-posed Anninos, Galante, Maneerat 2023

THE WELL-POSEDNESS DEPENDS ON THE GAUGE.

THE BOUNDARY DATA IS NOT ENOUGH TO FIX THE GAUGE.

	harmonic gauge $\Box x^\alpha = 0$ or similar ↳ flat boundaries ↳ our formalism	no gauge	conformal gauge corner metric $\propto$ background
 Dirichlet data ill-posed	ill-posed ill-posed Anninos, Galante, Maneerat 2023	ill-posed	well-posed* *work in progress
 Conformal- mean curvature data Well-posed	well-posed Well-posed An & Anderson 2025 Well-posed Anninos, Galante, Maneerat 2023	ill-posed ill-posed Liu, Santos, Wiseman 2024 Liu, Reall, Santos, Wiseman 2025	ill-posed ill-posed Anninos, Galante, Maneerat 2023

THE WELL-POSEDNESS DEPENDS ON THE GAUGE.

THE BOUNDARY DATA IS NOT ENOUGH TO FIX THE GAUGE.

THANK YOU!

QUESTIONS?