Dynamics of nucleation in thermal phase transitions

in collaboration with
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based on

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Theory Canada, Montreal

June 19, 2026

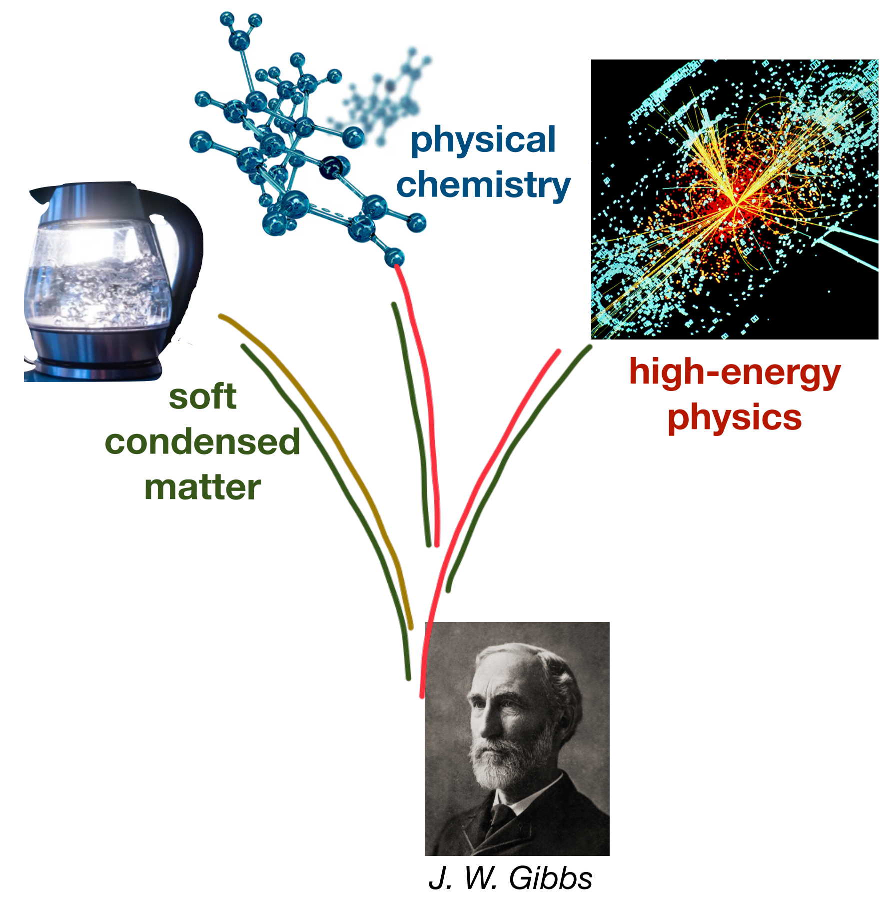
Thermal phase transitions

- plays a fundamental role in various branches of physics

  from quantum cosmology to quantum matter

- have been studied for ~150 years

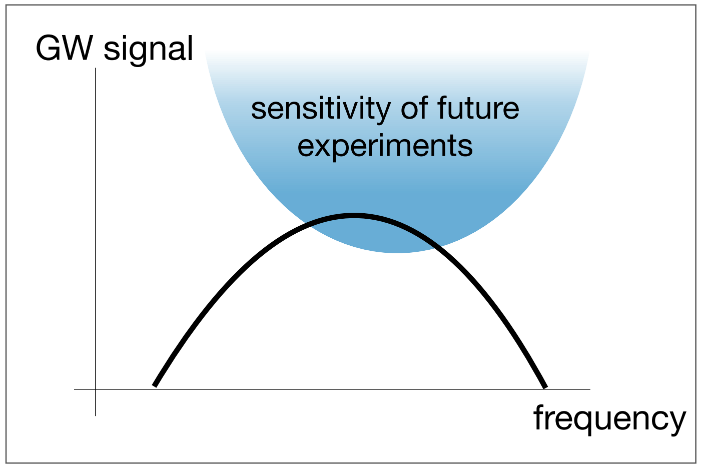
- many *dynamical* aspects are still poorly understood



Motivation

First order phase transitions in the early universe

Nucleating, propagating and colliding bubbles generate gravitational waves (GWs). This is a motivation to improve the existing and build new GW detectors.



Generation of baryon asymmetry of the universe

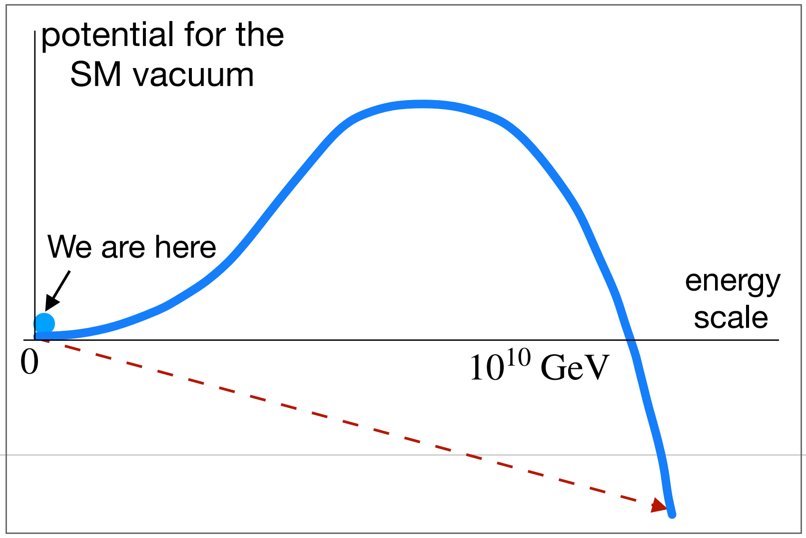
Expanding bubbles moving through cosmic plasma can generate asymmetry between particles and anti-particles.

Metastability of the Standard Model vacuum

According to the measured values of the parameters of the Standard Model (SM), our "fundamental" vacuum may itself be metastable.

In habitable parts of the present-day universe the decay probability is very small.

This may not be so in extreme environments (e.g. black holes) or earlier epochs (e.g. inflation).



Experimental tests of nucleation theory

Zenesini et al, Nature Physics 20, 558–563 (2024) — first experimental result using a cold atom system

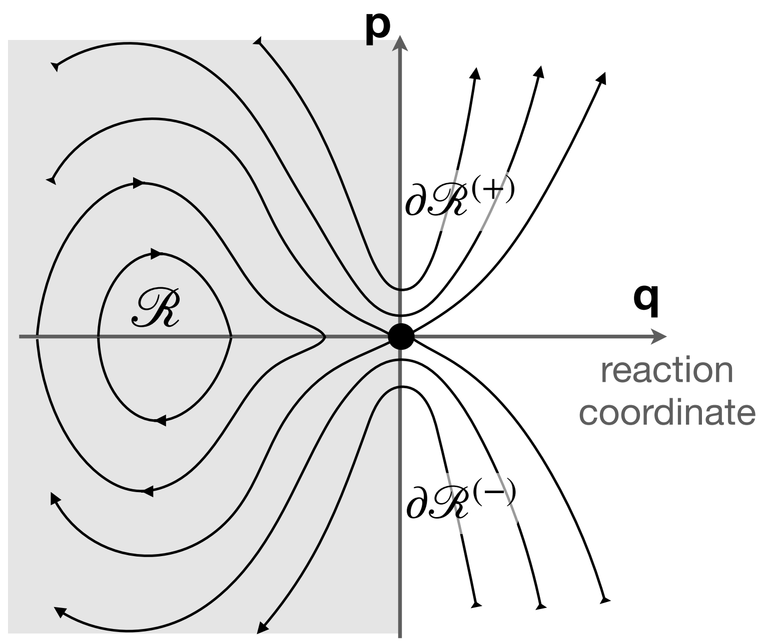
Classical TST rate

- The “equilibrium” rate can be defined through the phase-space probability flux:

$$\Gamma_{\rm TST} = \frac{1}{Z_{\mathcal{R}}} \int_{\partial\mathcal{R}} \mathrm{d}S\, \mathbf{n}\cdot\dot{\mathbf{z}}\, \theta(\mathbf{n}\cdot\dot{\mathbf{z}})\, \mathrm{e}^{-H(\mathbf{z})/T}$$

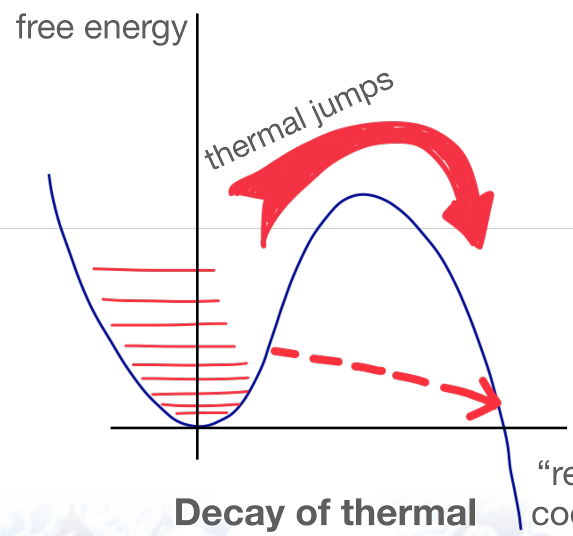
In the saddle-point approximation:

$$\Gamma_{\rm TST} \overset{\text{1-loop}}{=} \frac{|\omega_-|}{\pi T}\cdot \mathrm{Im}\, F$$



Phase-space trajectories and the metastable region $\mathscr{R}$

- But this is not how the nucleation really happens!



Decay of thermal metastable state

Start from $\rho(\mathbf{z}; t=0) = \frac{1}{Z_{\mathcal{R}}}\mathrm{e}^{-H(\mathbf{z})/T}\theta(\mathbf{z}\in\mathcal{R})$ and wait…

At *late times* one expects $\Gamma = \mathscr{A}_{\rm dyn.}\Gamma_{\rm TST}$ – *stationary flux* regime

Methods to compute $\mathscr{A}_{\rm dyn.}$ have developed, but puzzles remain.

Langer 69; Affleck 81; Arnold, McLerran 87; Hirvonen 25
Moore, Rummukainen, Tranberg 01

Puzzles of thermal decays

- How universal the factorization into statistical and dynamical parts is?
  Does it hold beyond 1 loop?

- How the choice of the transition state does not matter?
  The TST rate depends on the TS.

- How to account for non-perturbative dynamical effects?
  The role of *oscillons* in vacuum decay?

- One-particle mechanics vs field theory

Example from simulations

Pirvu, Johnson, Sibiryakov 23; Pirvu, AS, Sibiryakov 24

- Take scalar field theory in (1+1) dimensions, with unstable potential, evolve it on a classical lattice, *measure* the survival probability.

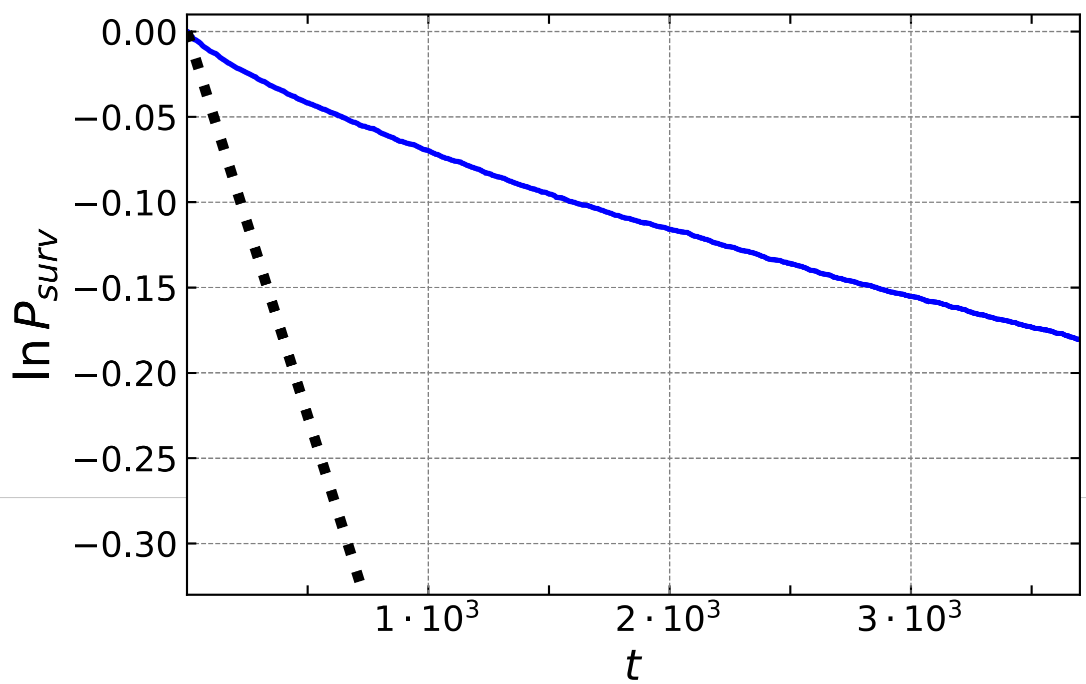
$$\ln P_{surv}(t) = \text{const} - \Gamma L \cdot t$$

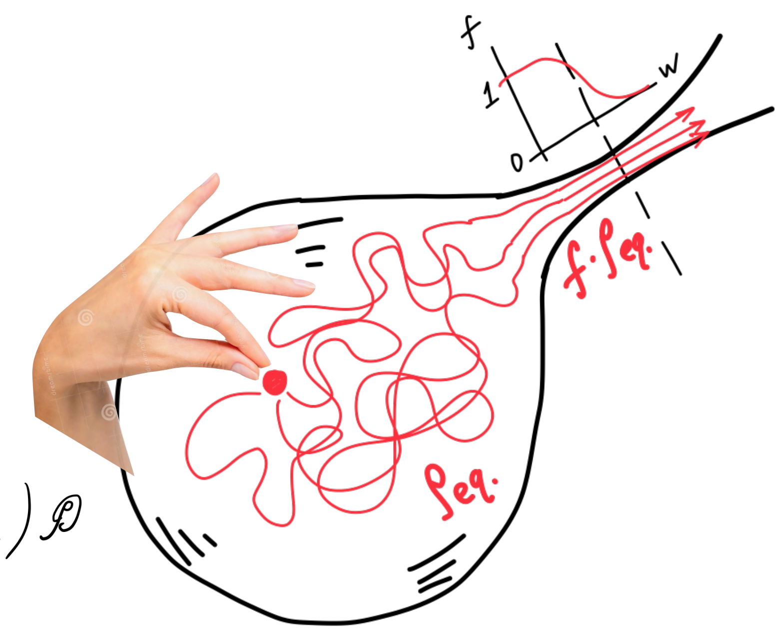
↑ thermal decay rate

$$\frac{\partial \mathcal{P}}{\partial t} = -\frac{\partial \mathcal{J}_I}{\partial z_I}$$

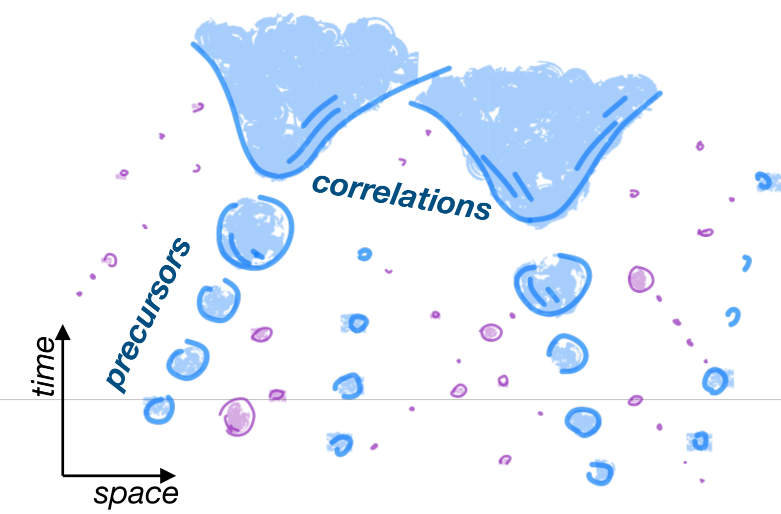
$$\mathcal{J}_I = -\mathcal{M}_{IJ}\left(\frac{\partial \mathcal{H}}{\partial z_J} + T\frac{\partial}{\partial z_J}\right)\mathcal{P}$$

$$\Gamma = \int_{w=0} dz\, \mathcal{J}_\perp$$





Stationary flux regime



Real-time dynamics of critical bubble nucleation

Here we evolve massive scalar field with negative quartic coupling:

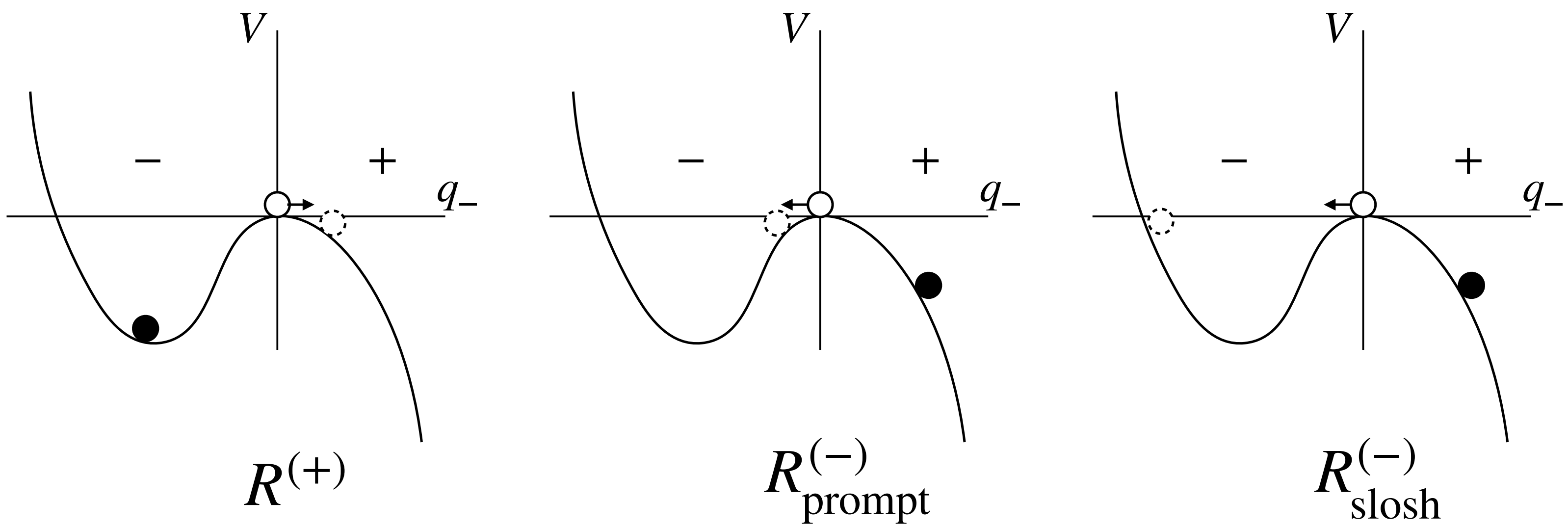
$$\begin{cases} \dot\phi = \pi \\ \dot\pi = \Delta\phi - m^2\phi + \lambda\phi^3 \end{cases}$$

Dynamical decay rate

- In the stationary flux regime $\Gamma = \Gamma_{\rm TST} \cdot (1 - R)$

Gould, Hirvonen, AS, Sibiryakov *(in preparation)*

$R = R^{(+)} + R^{(-)}$ — turnaround (recrossing) probability



This is similar to the reactive-flux method in chemistry

Chandler 77
P. Hanggi, P. Talkner and M. Borkovec 90

and to the MRT method in HEP

Moore, Rummukainen 00
Moore, Rummukainen, Tranberg 01

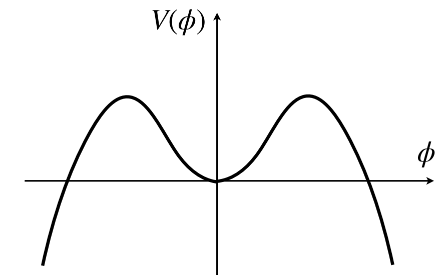
- This expression is exact (non-perturbative).
- $R_{\rm pert.}$ can be systematically calculated. But $R_{\rm slosh}$ can give big contribution.
- Practically, choose $q_- = 0$ as a dividing surface,

sample $\rho^{(\pm)}(\mathbf{q}, \mathbf{p}; t = 0) = \mathcal{N} \delta(q_-) \theta(\pm \dot{q}_-) |\dot{q}_-| \mathrm{e}^{-H(\mathbf{q},\mathbf{p})/T}$

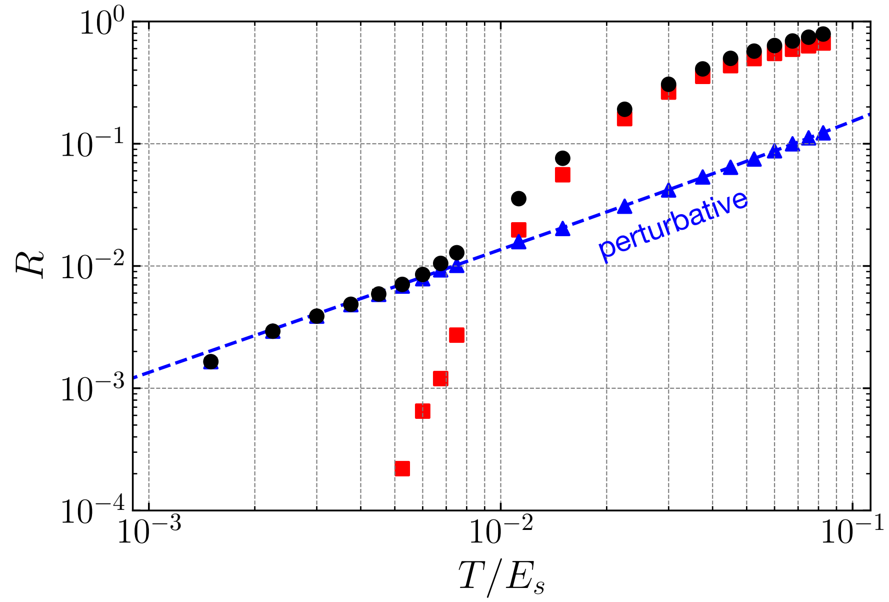
then $R^{(\pm)} = \lim_{t_{\rm dyn.} \ll t \ll t_{\rm dec.}} \int \mathrm{d}\mathbf{q} \mathrm{d}\mathbf{p}\, \rho^{(\pm)}(\mathbf{q}, \mathbf{p}; t) \theta(\mp q_-)$

Nonperturbative recrossings are important

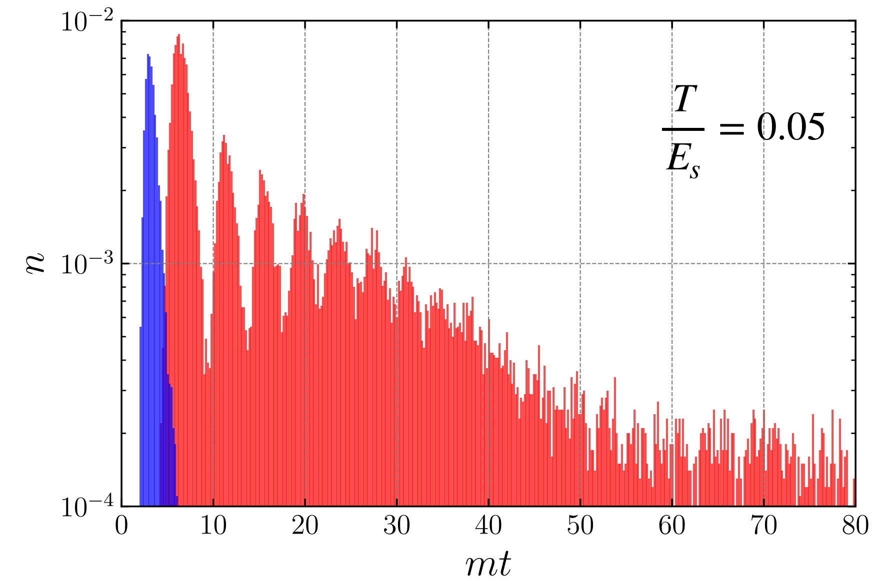
- Consider the theory $S = \int \mathrm{d}t\,\mathrm{d}x \left(\frac{(\partial_\mu \phi)^2}{2} - V(\phi) \right)$ $\quad V_1(\phi) = \frac{m^2\phi^2}{2} - \frac{\lambda\phi^4}{4}$



Dynamical factor *R* from simulations

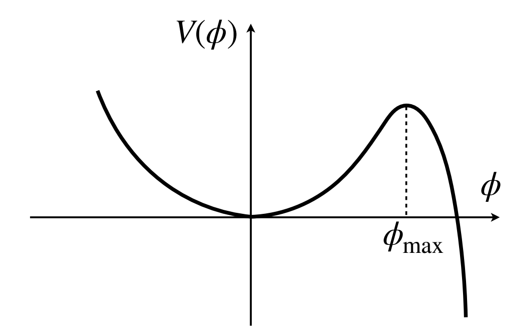


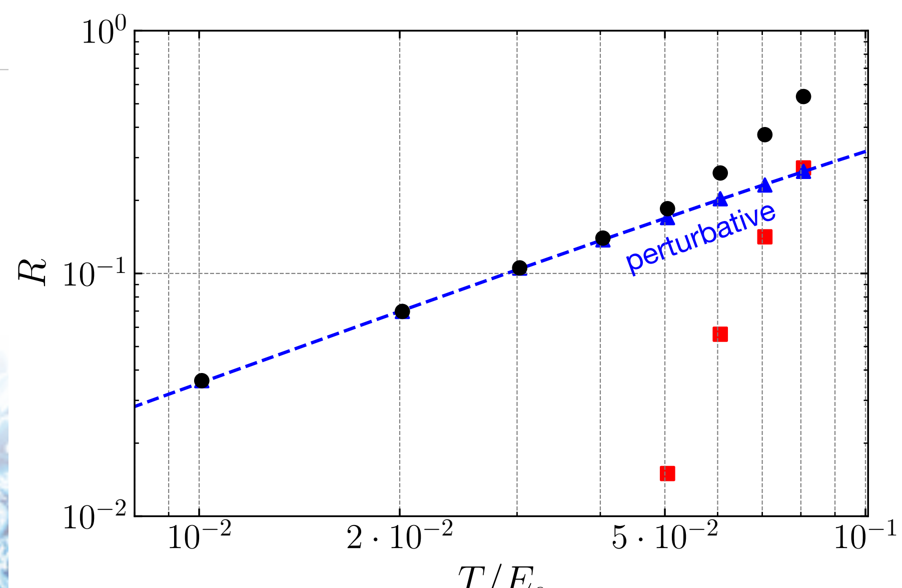
Recrossings as a function of time

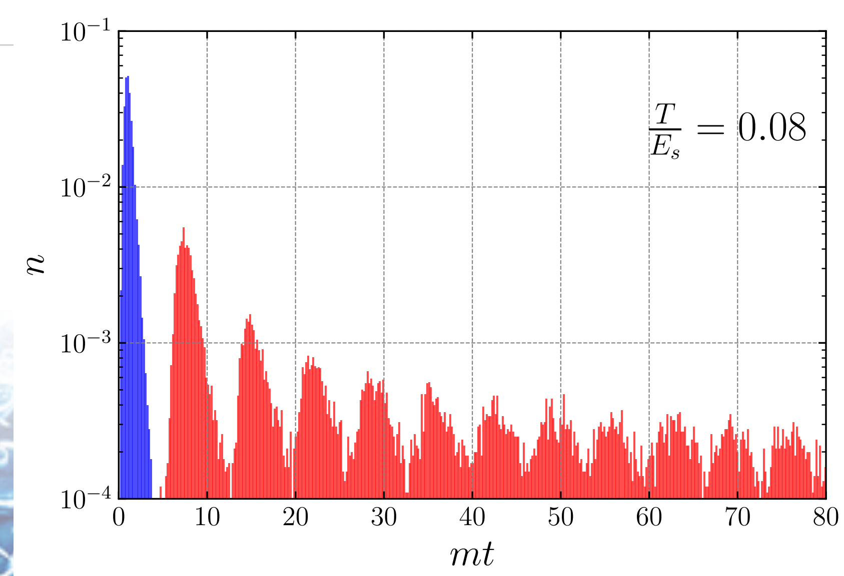


← they dominate recrossings even at $T/E_s \sim 10^{-2}$

- Take a theory with no oscillons: $V_2(\phi) = \frac{m^2\phi^2}{2} - \varkappa\, \mathrm{e}^{\phi}$







Comparison with direct decays

Pirvu, AS, Sibiryakov 24

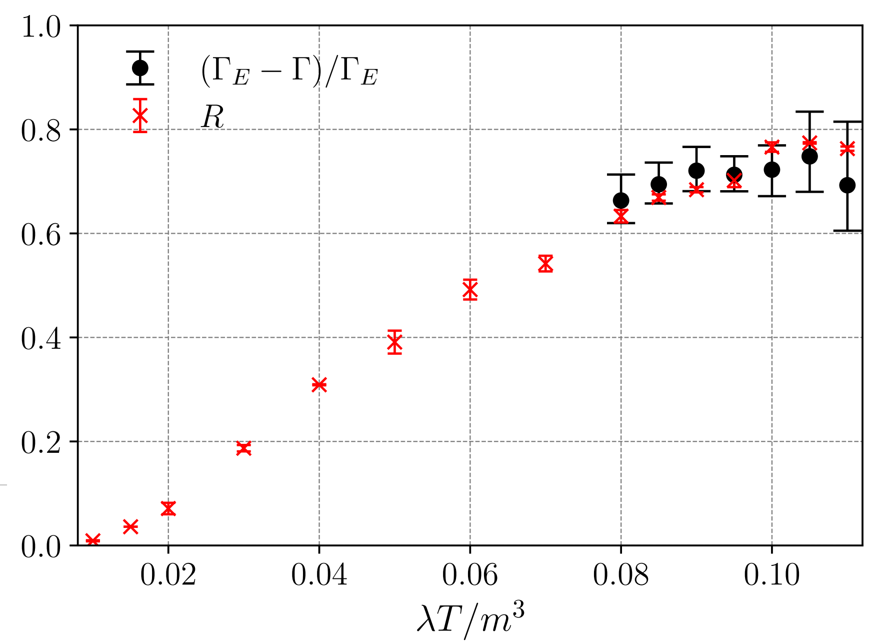
Gould, Hirvonen, AS, Sibiryakov *(in preparation)*

- We can directly check the relation $\Gamma = \Gamma_{\rm TST} \cdot (1 - R)$

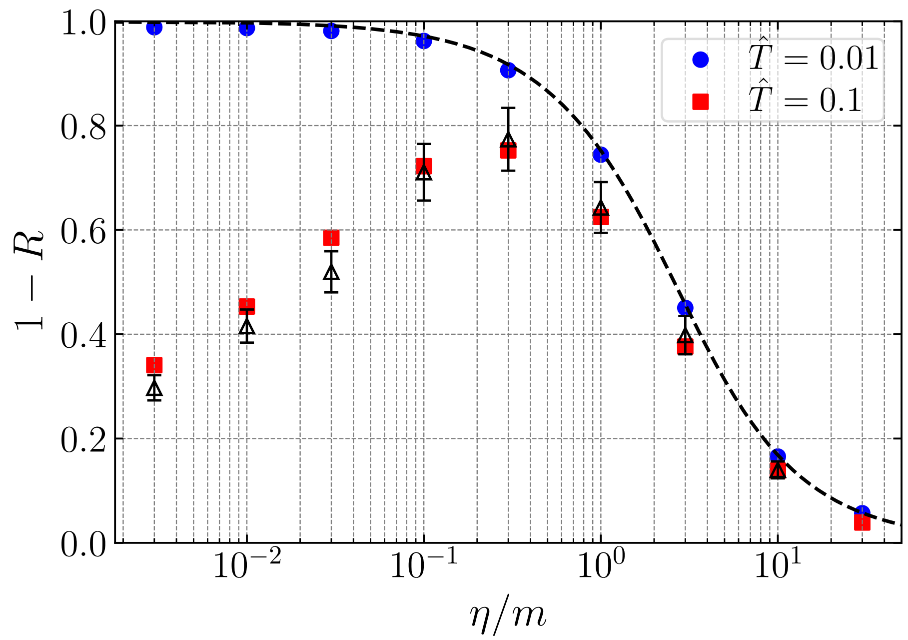
  by comparing recrossings with direct decays, when feasible.

  Take again the (1+1)-dim. theory with unstable quartic potential.

Recrossings vs decays



Recrossings with dissipation



$$\begin{cases} \dot{\phi}_i = \pi_i \\ \dot{\pi}_i = (\Delta\phi)_i - m^2\phi_i + \lambda\phi_i^3 - \eta\pi_i + \sigma\xi_i \end{cases}$$

$$\sigma^2 = 2\eta T/a$$

Discussion

- Application to cosmological phase transitions
- Application to *quantum simulators*
- Revisiting Coleman's tunnelling
- Other N^3 effects