Marginally-outer trapped surfaces in spacetimes with cosmological constant

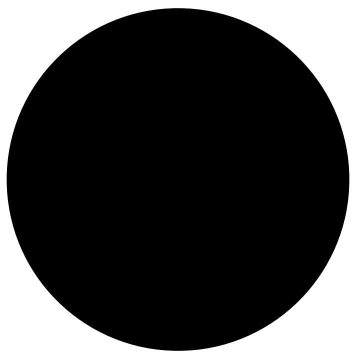
at Theory Canada 18

June 20, 2026

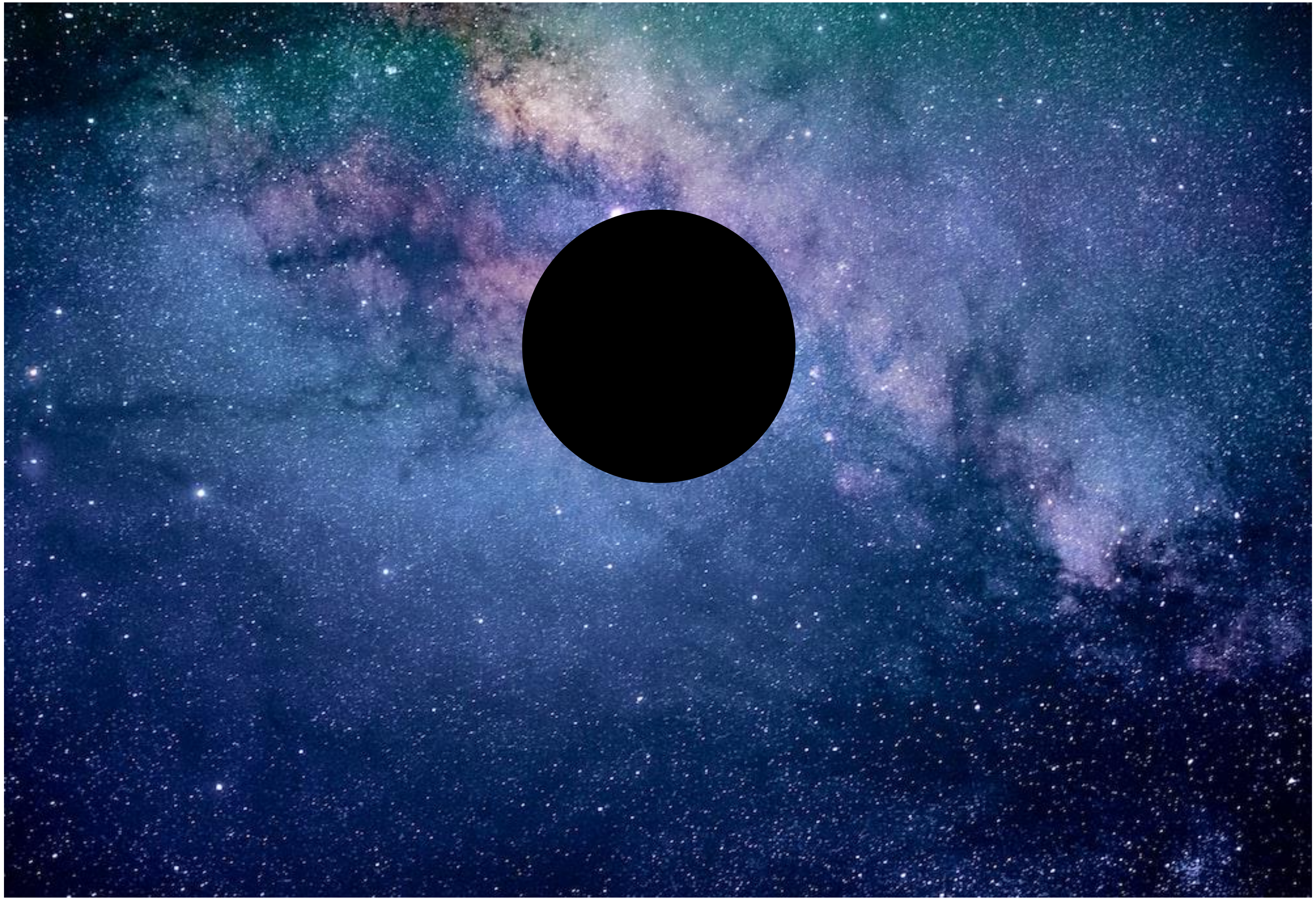
Billy Sievers (McMaster U)

Based on:
2111.09373
2210.15685
2312.00769
2607.xxxxxx

Black Hole

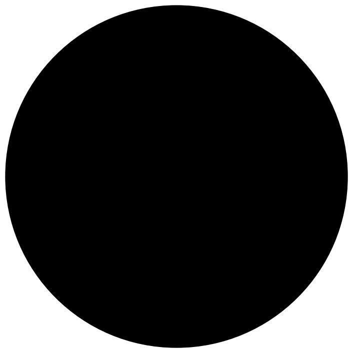


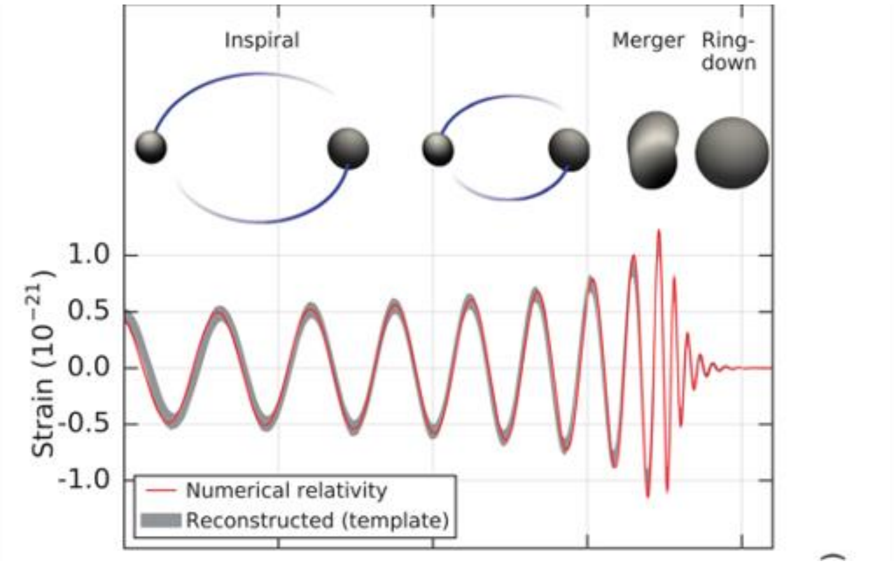
Black Hole



Event Horizon Telescope Collaboration, 2019

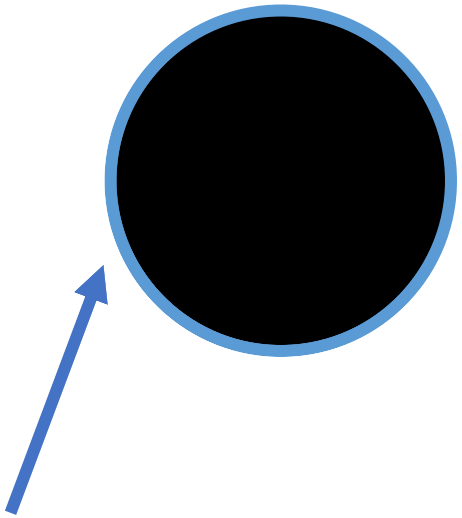
Black Holes





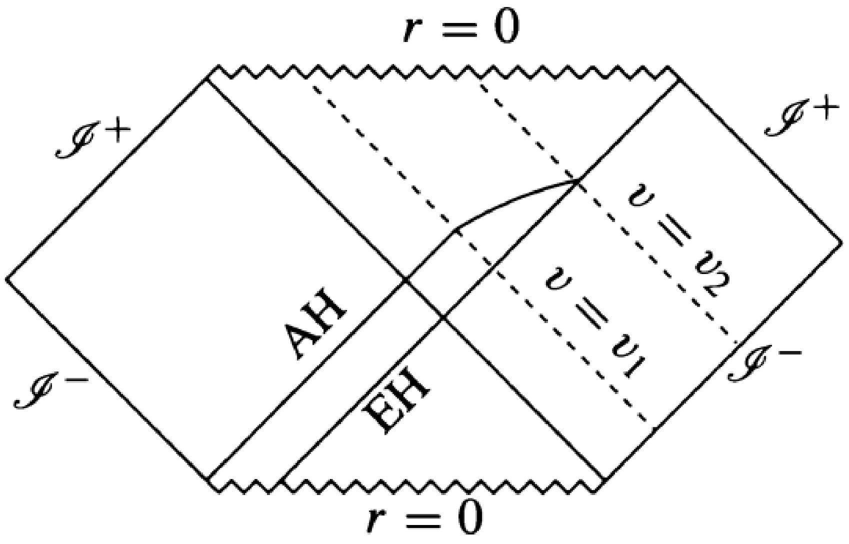
LIGO-VIRGO Collaboration, 2016

Black Hole Boundary



Event Horizon (?)

Apparent Horizon (?)



Penrose diagram of Vaidya spacetime

(A Relativist's Toolkit, E Poisson)

Binary Black Hole Merger

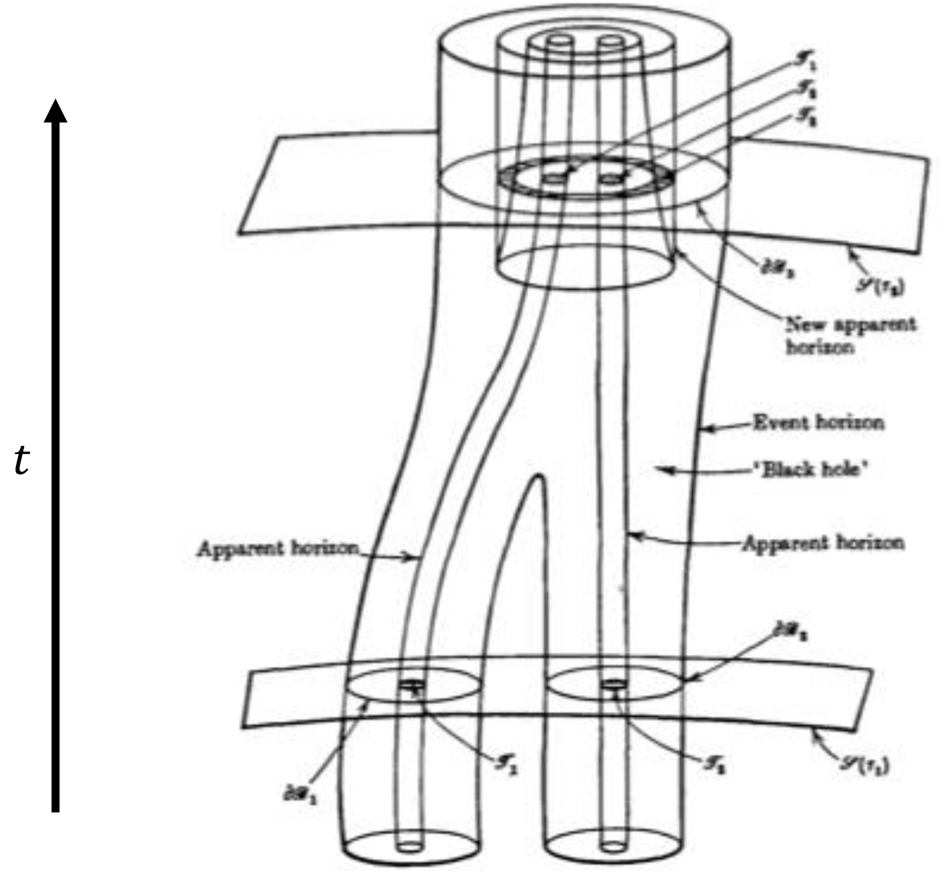


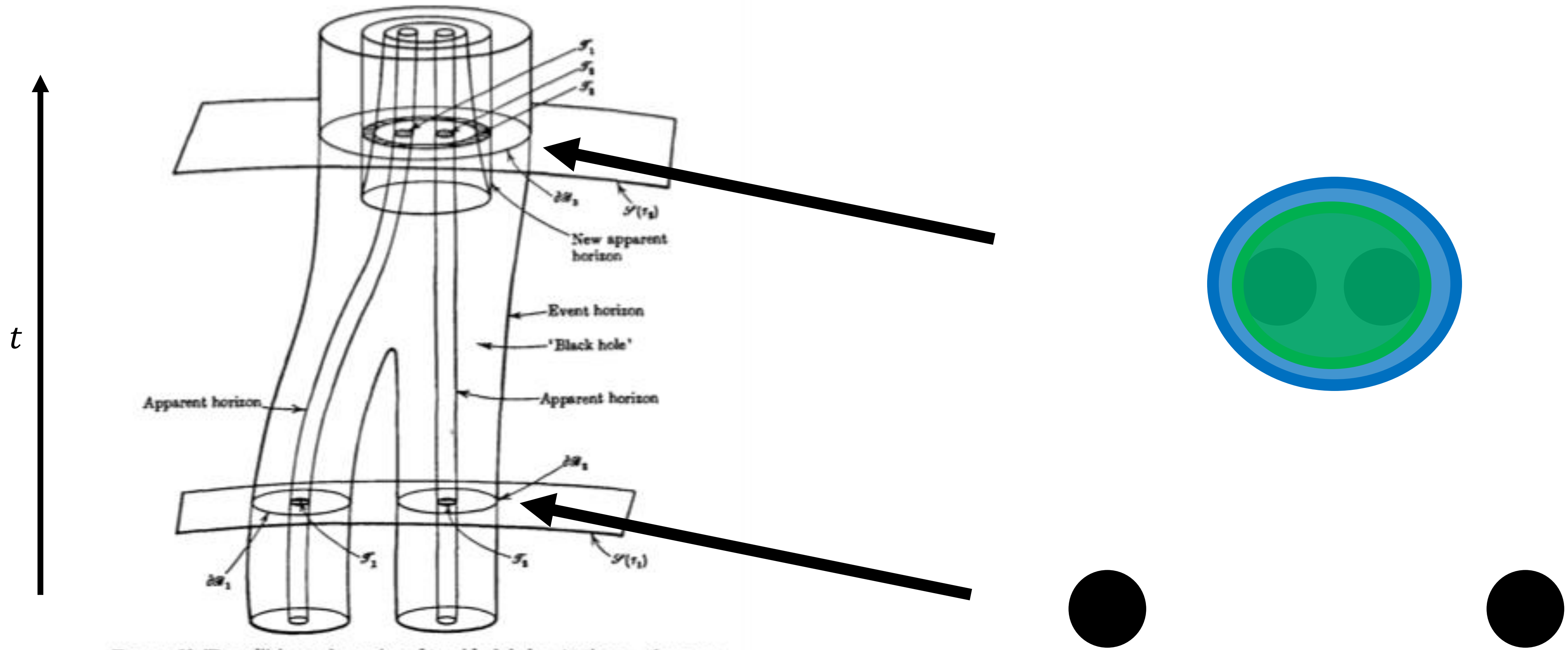
FIGURE 60. The collision and merging of two black holes. At time τ_1, there are apparent horizons $\partial\mathscr{T}_1$, $\partial\mathscr{T}_2$ inside the event horizons $\partial\mathscr{B}_1$, $\partial\mathscr{B}_2$ respectively. By time τ_2, the event horizons have merged to form a single event horizon; a third apparent horizon has now formed surrounding both the previous apparent horizons.

“Pair of pants” diagram

The Large Scale Structure of space-time,

Hawking & Ellis 1973

Binary Black Hole Merger



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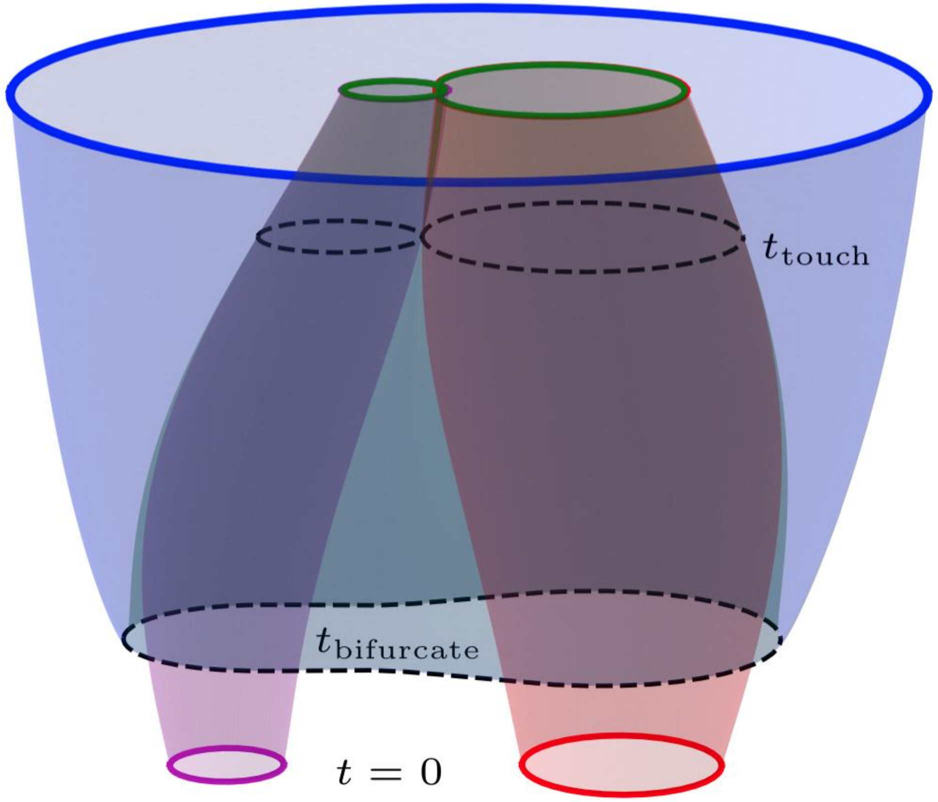
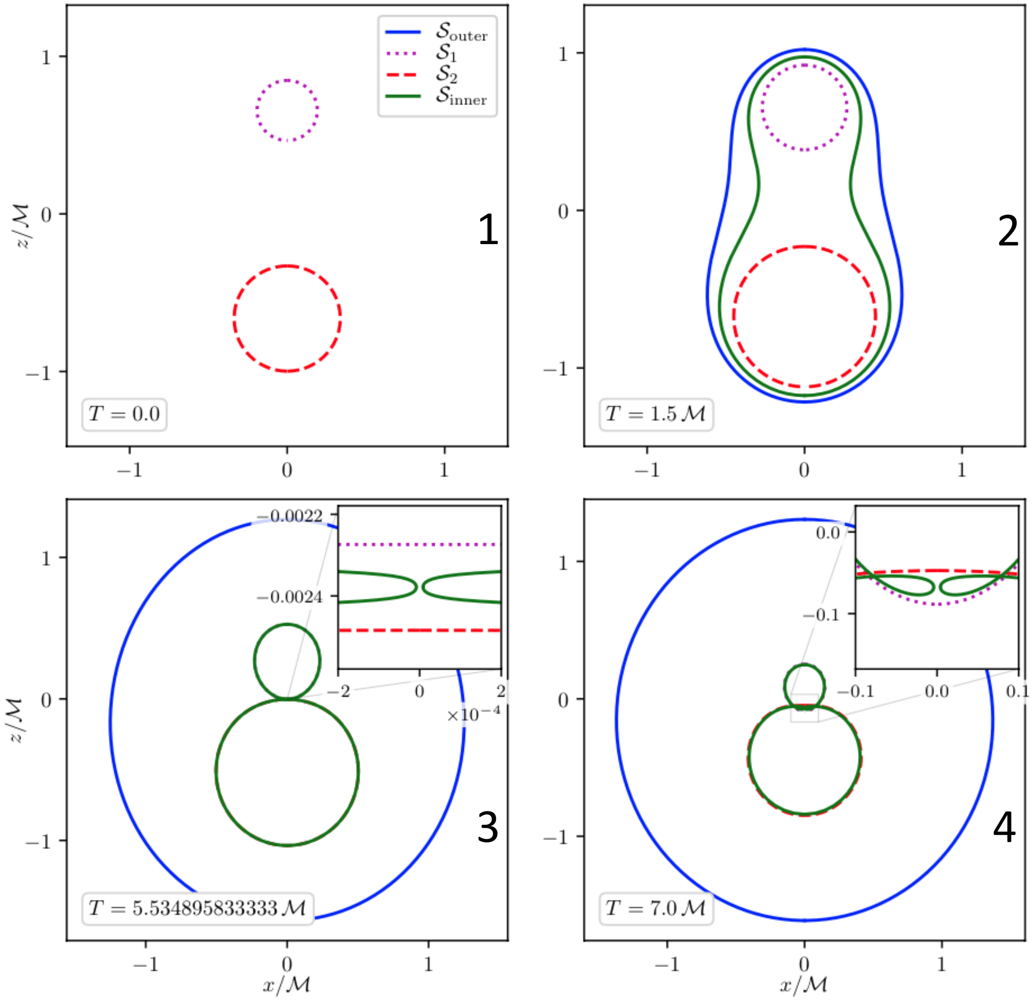
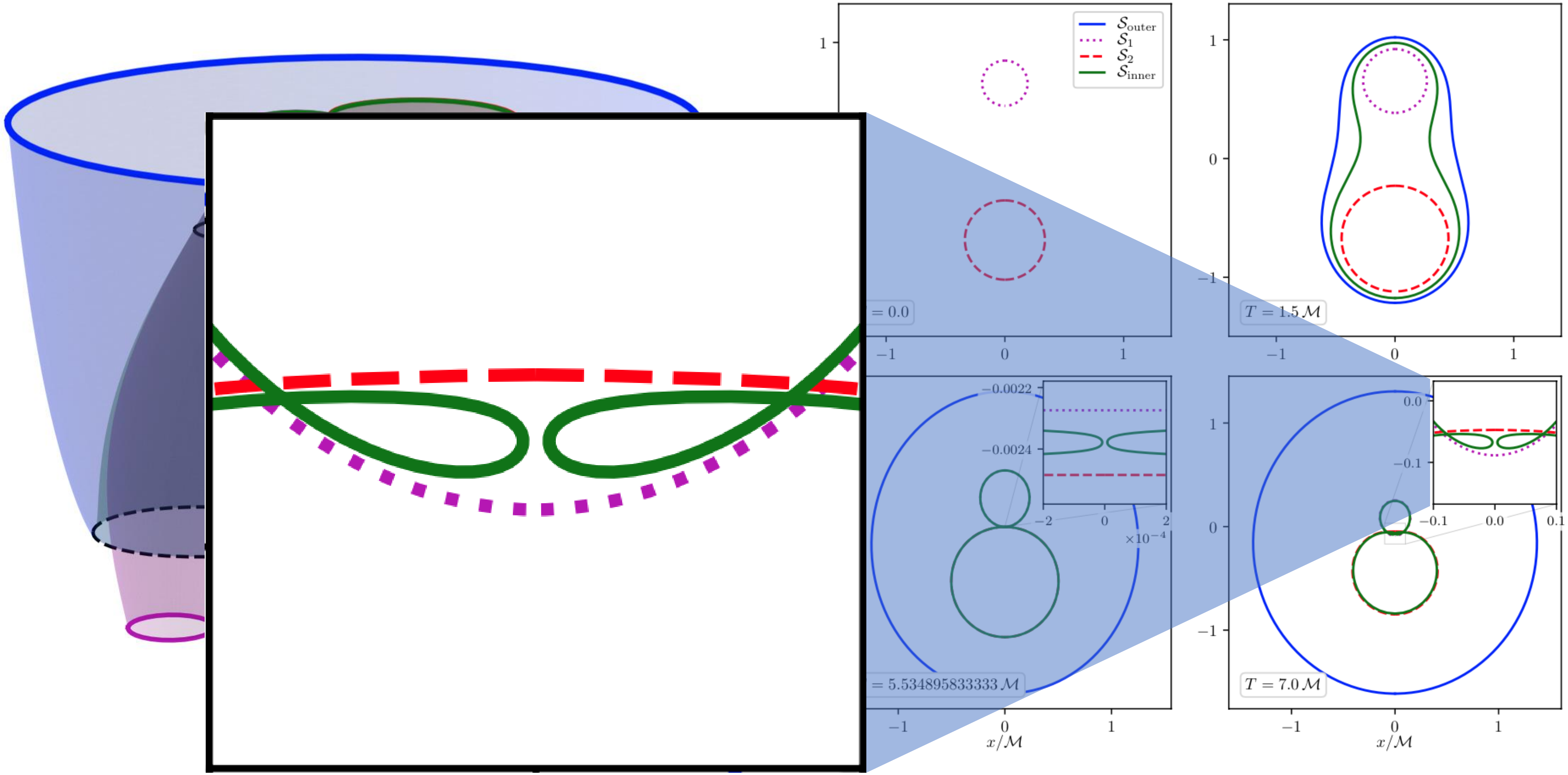
Binary Black Hole Merger



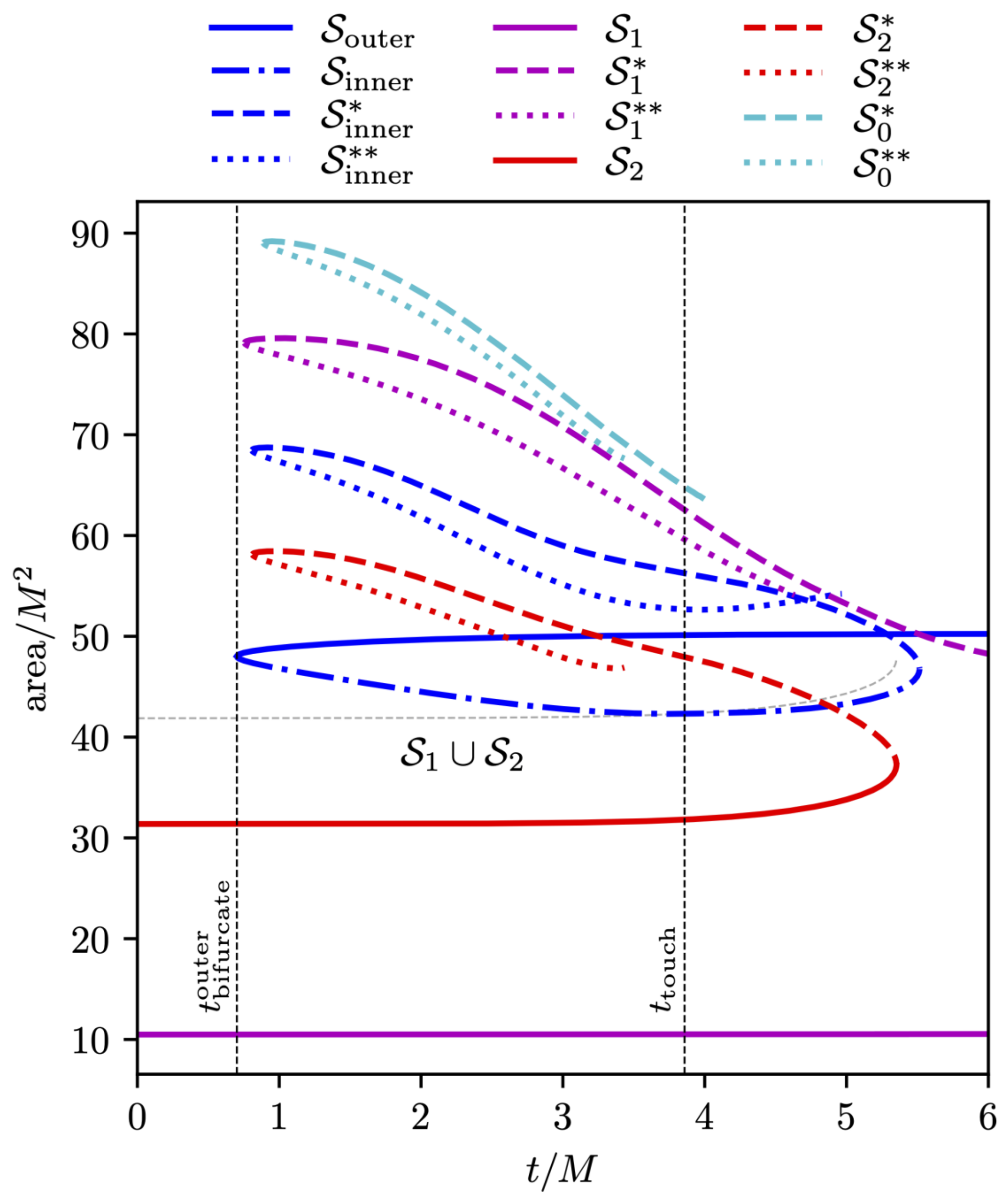


Pook-Kolb, Birnholtz, Krishnan, Schnetter, PRL 2019

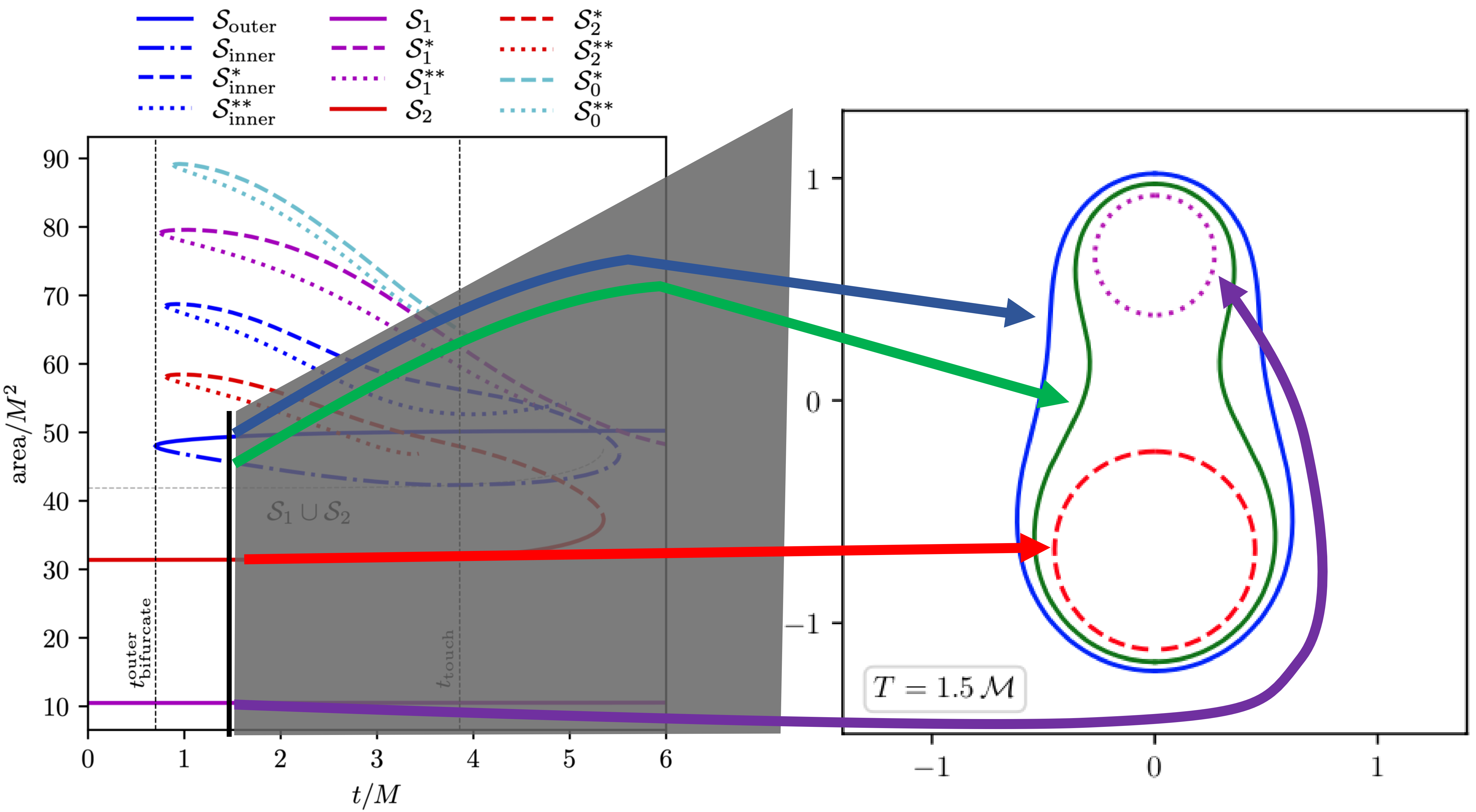
Binary Black Hole Merger



Pook-Kolb, Birnholtz, Krishnan, Schnetter, PRL 2019



Pook-Kolb, Booth, Hennigar, 2021, PRD104, 084084



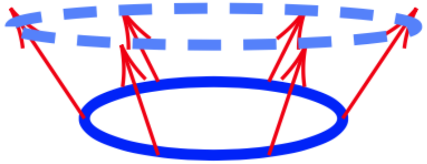
Pook-Kolb, Booth, Hennigar, 2021, PRD104, 084084

Marginally Outer-Trapped Surface (MOTS)

ℓ^{α} : outward null normal ; q_{AB} : Riemannian geometry on $\mathcal{S}$

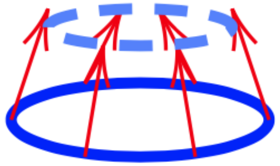
$$\Theta_{(\ell)} = q^{\alpha\beta}\nabla_{\alpha}\ell_{\beta}$$: outward null expansion

- Untrapped Surface :



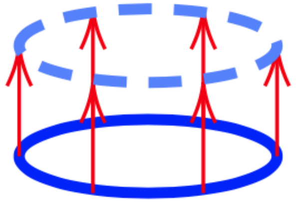
$\Theta_{(\ell)} > 0$

- Outer-Trapped Surface :



$\Theta_{(\ell)} < 0$

- **Marginally Outer-Trapped Surface**:



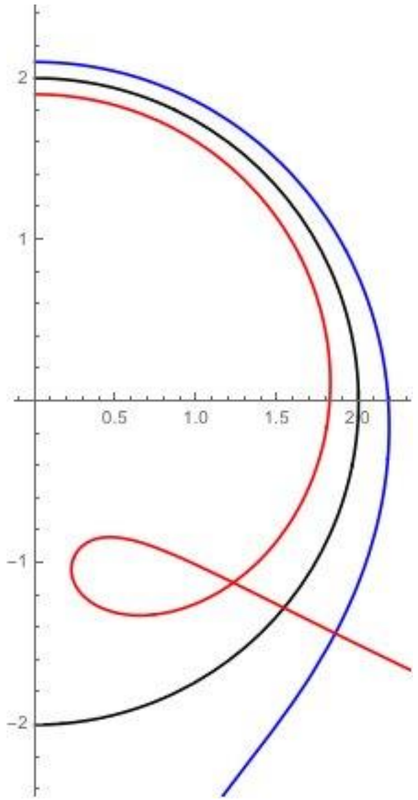
$\Theta_{(\ell)} = 0$

Marginally Outer-Trapped Surface (MOTS)

Equations for axisymmetric MOTS in Schwarzschild
In PG coordinates on the const.-Φ orbit space

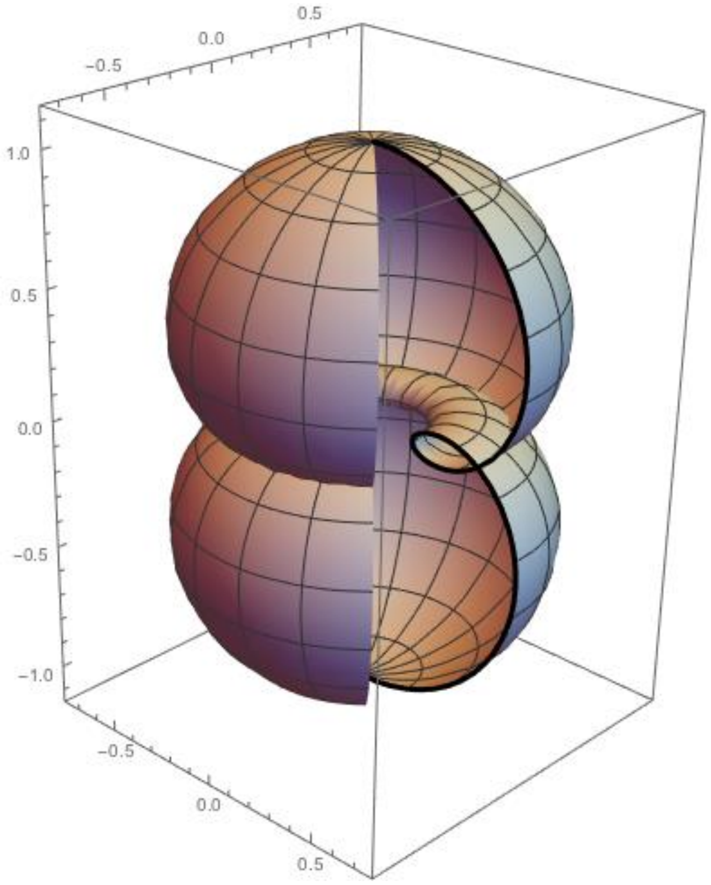
$$\ddot{R} = R\dot{\Theta}\left(\dot{\Theta} - \kappa\right) \quad \text{for} \quad \kappa = -\dot{\Theta} + \frac{\cot\Theta}{R}\dot{R}$$

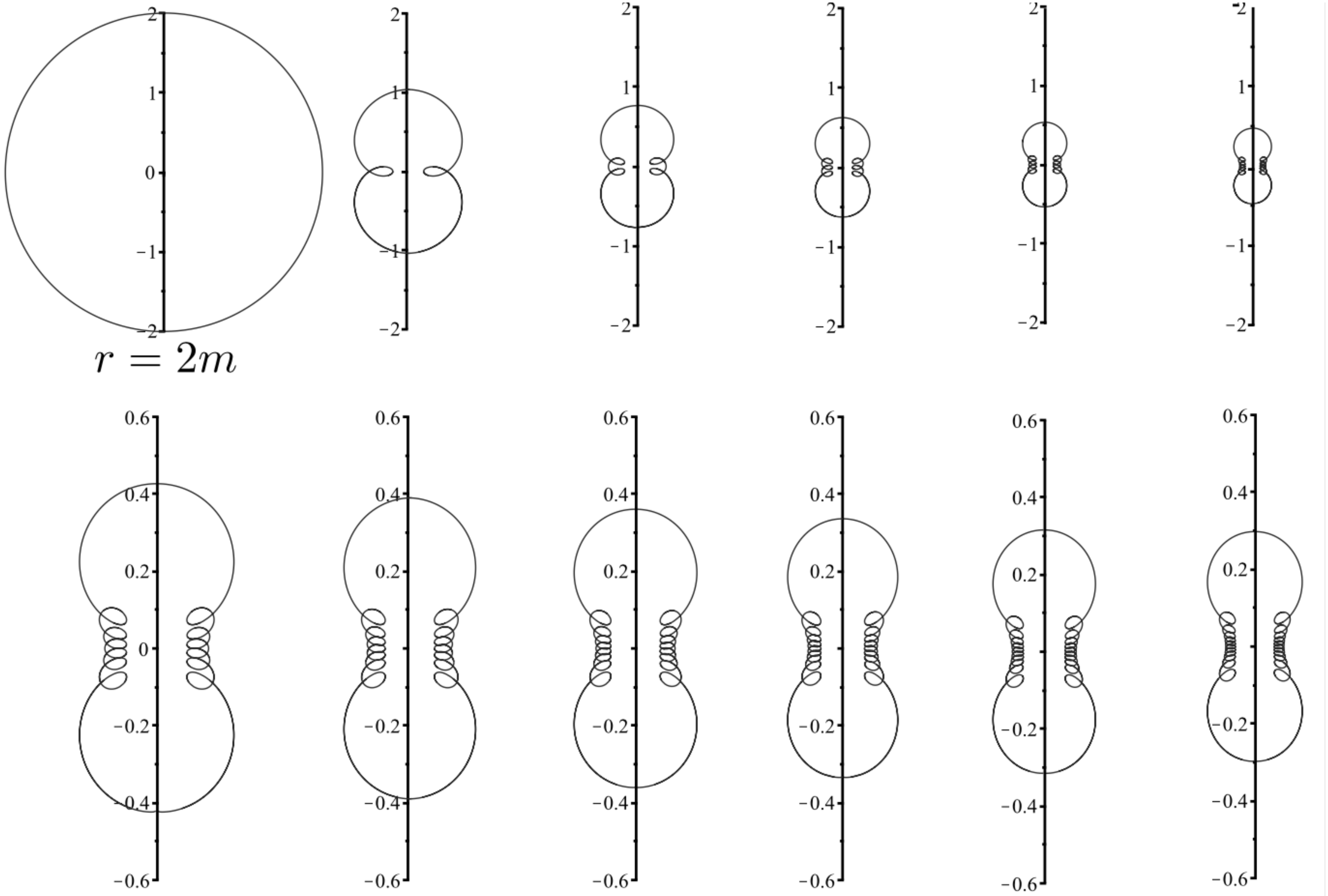
$$\ddot{\Theta} = -\frac{\dot{R}}{R}\left(2\dot{\Theta} - \kappa\right) \qquad \pm\sqrt{\frac{m}{2R^3}}\left(3R^2\dot{\Theta}^2 + 1\right)$$



Booth, Hennigar, Mondal, 2020, PRD102, 044031

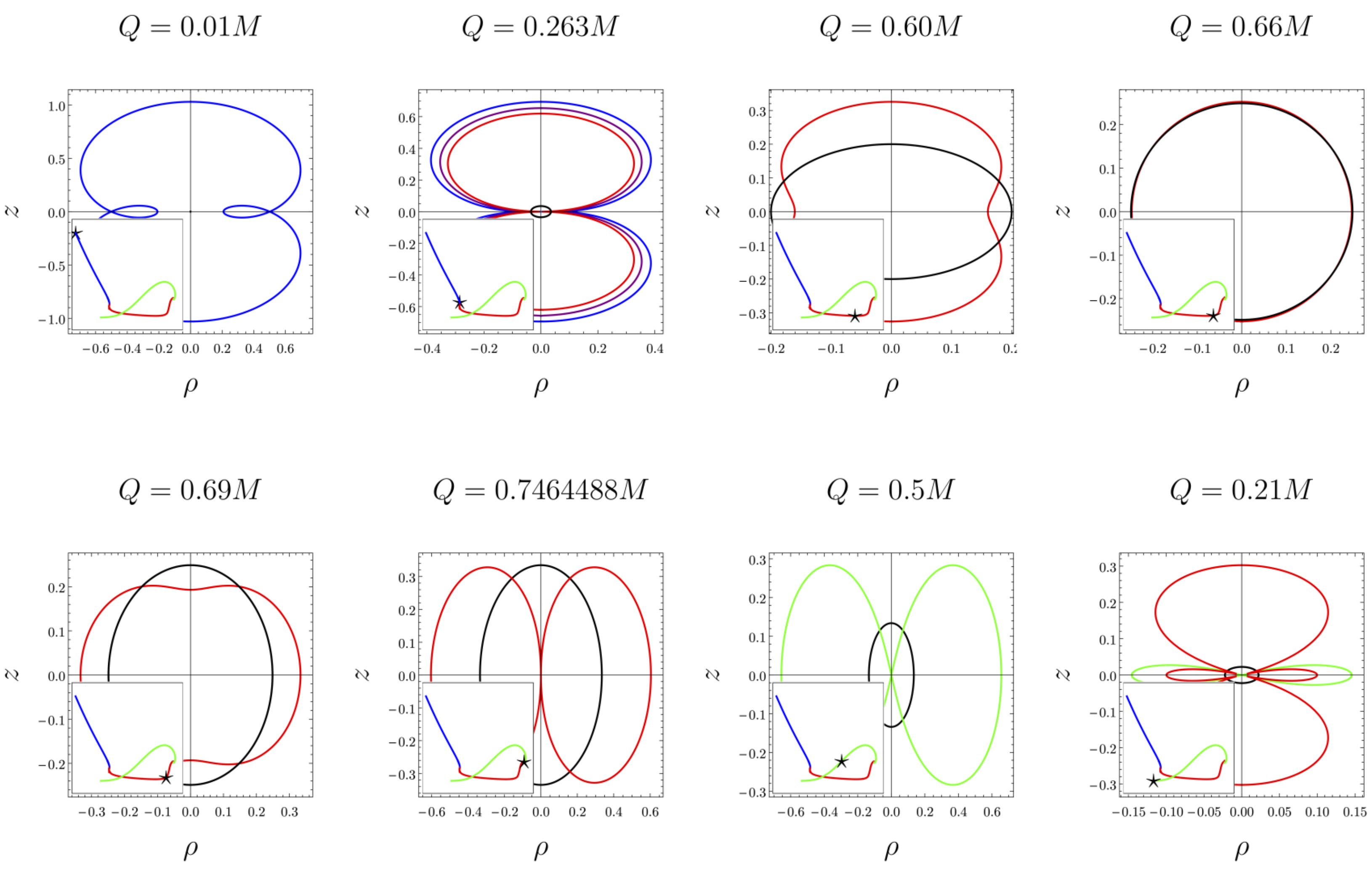
Schwarzschild Interior Self-Intersections





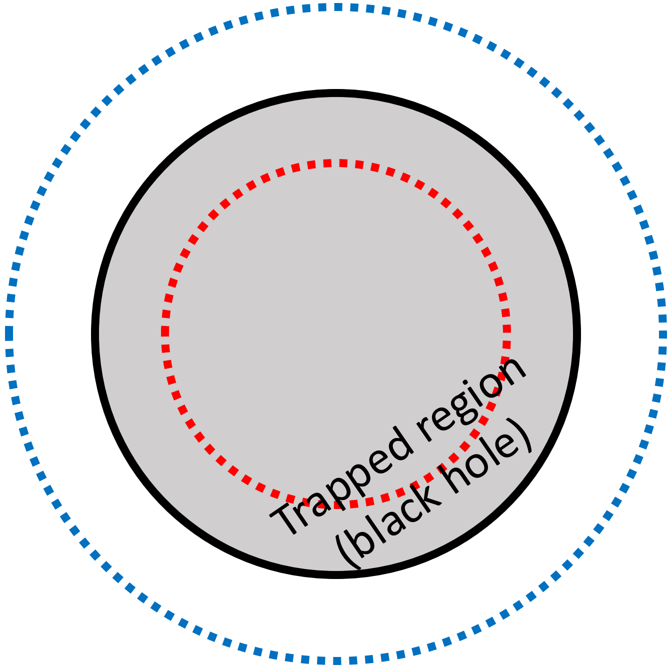
Booth, Hennigar, Mondal, 2020

Reissner-Nordström spacetime



MOTS as Black Hole Boundaries

- Outer-most MOTS is the apparent horizon



$\Theta_{(\ell)} = 0$
$\Theta_{(\ell)} < 0$
$\Theta_{(\ell)} > 0$

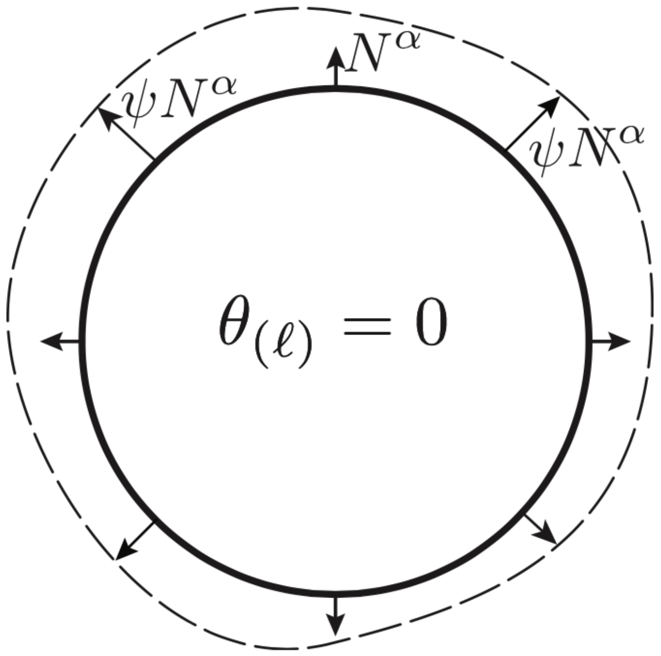
Ivan Booth, CJP, 2005

Stability Operator

- Consider a MOTS perturbed some distance-function ψ

$$L_\Sigma \psi = -\Delta\psi + \left(\frac{1}{2}\mathcal{R} - 2||\sigma_{(\ell)}||^2 - 2G_{++} - G_{+-}\right)\psi$$

2D surface Laplacian | 2D Ricci Scalar | Null shear (squared) | Matter contributions



- Consider the eigenvalue spectrum of L_Σ:

$$L_\Sigma \psi_i = \lambda_i \psi_i$$

Take smallest to be <u>principal eigenvalue</u> λ_0,

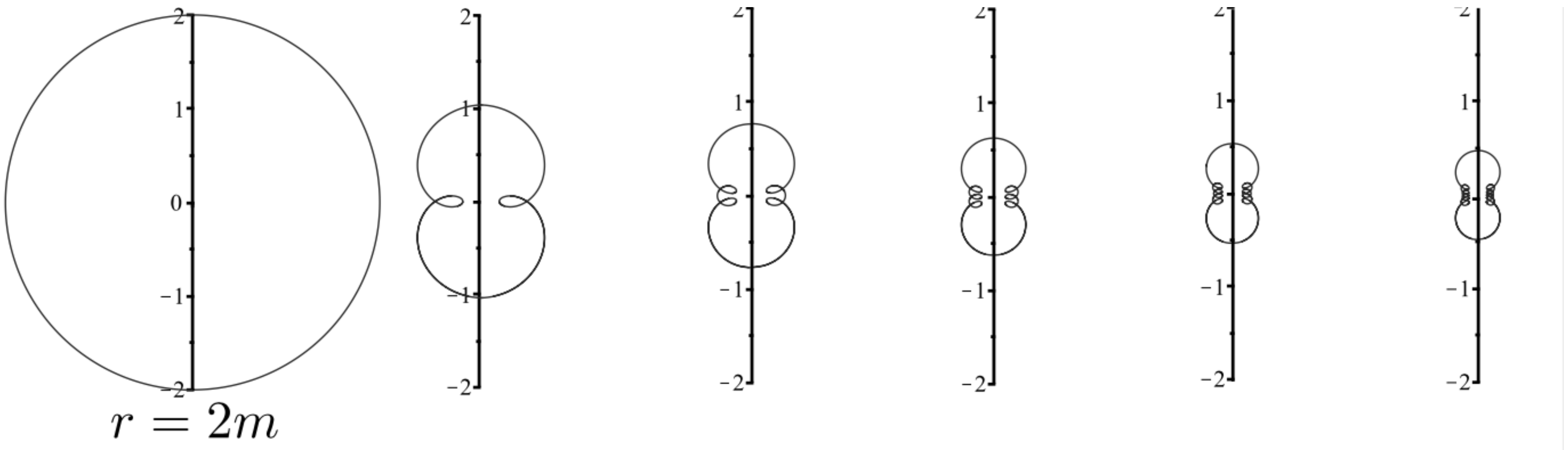
- ➢ annihilation/bifurcation when $\lambda_0 = 0$
- ➢ MOTS has *barrier* property if $\lambda_0 > 0$

Andersson, Mars, Simon, PRL95, 111102

Stability Operator

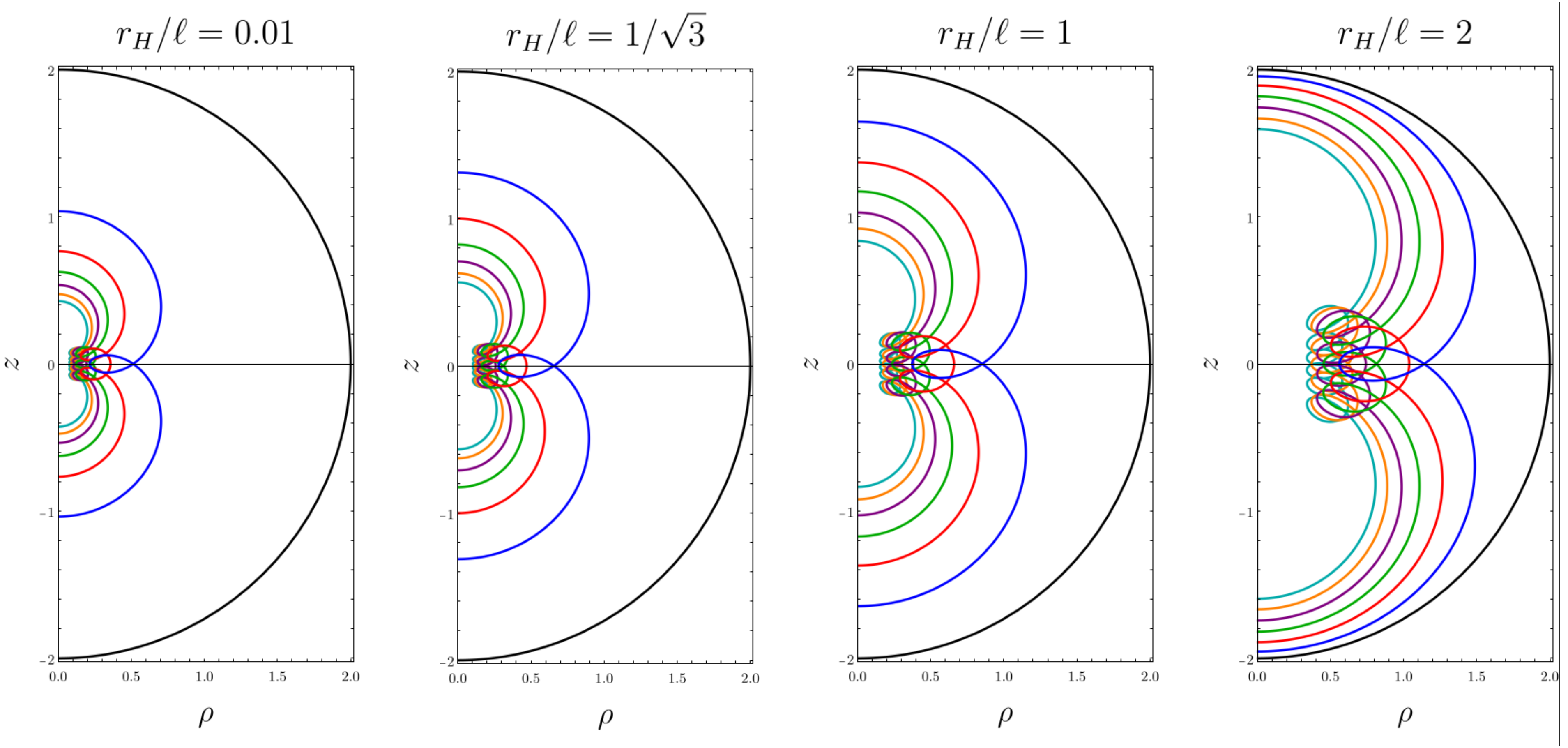
Number of negative eigenvalues observed thus far:

0, 1 | 2, 3 | 4, 5 | 6, 7 | 8, 9 | 10, 11

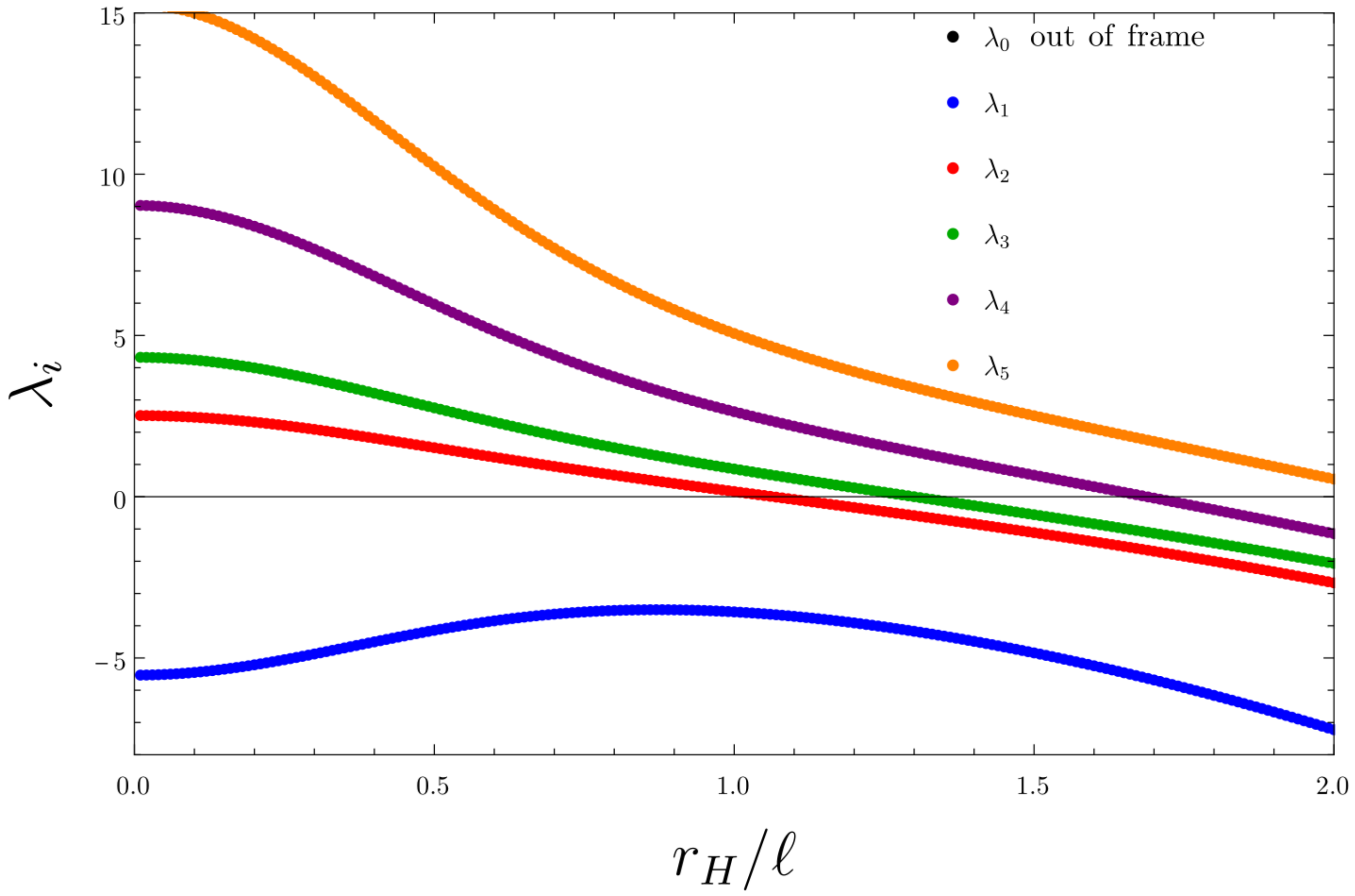


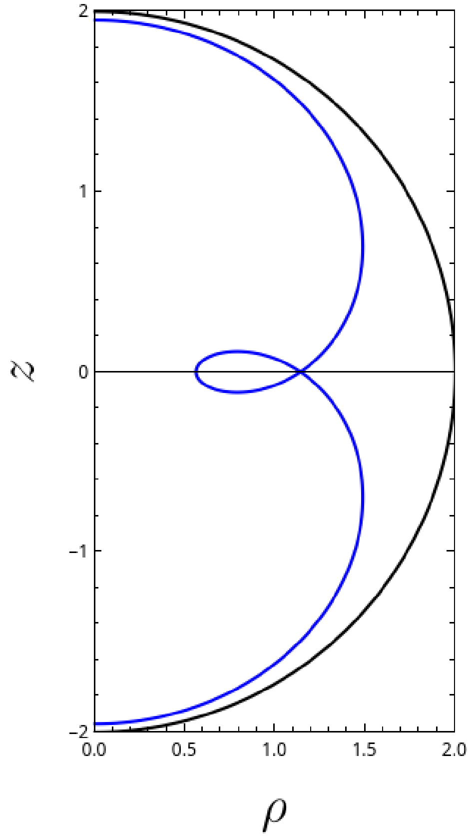
MOTSs in Schwarzschild-AdS

$$f(r) = 1 - \frac{\mu}{r} + \frac{r^2}{\ell^2}$$



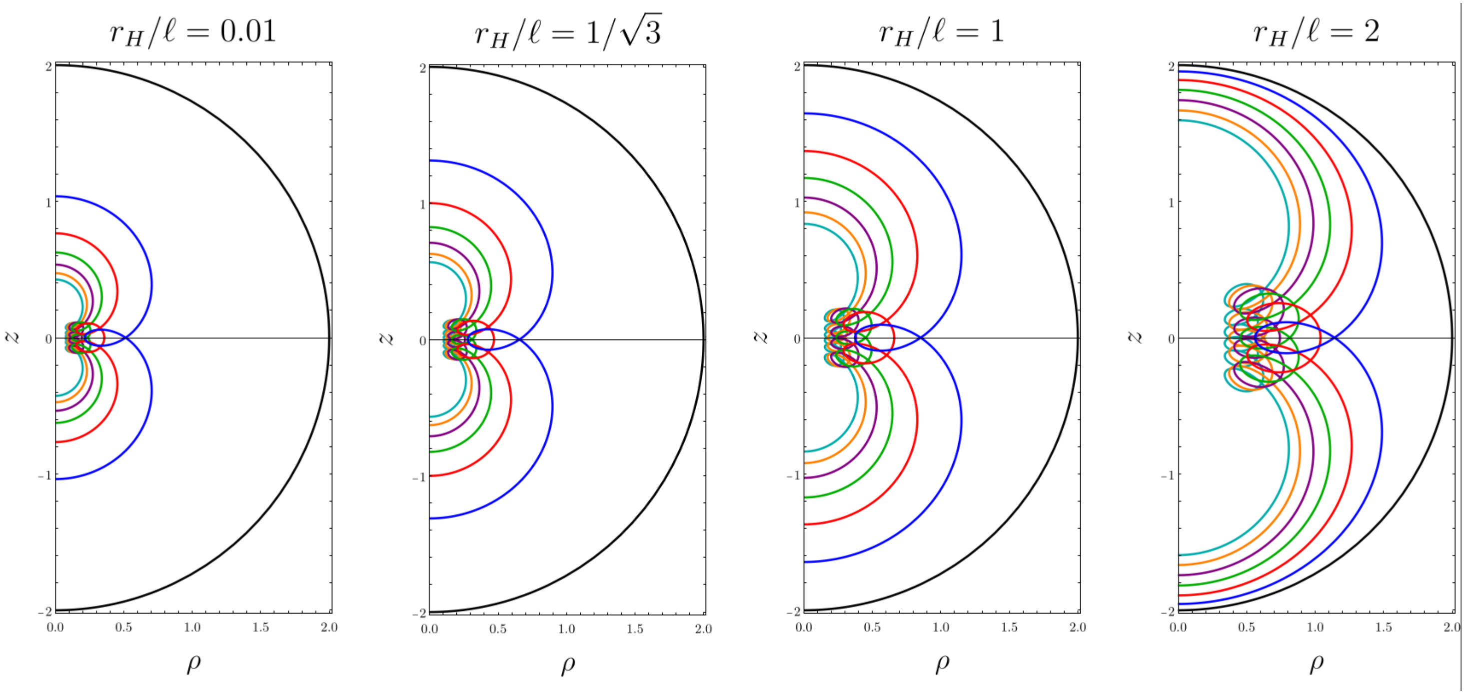
MOTSs in Schwarzschild-AdS



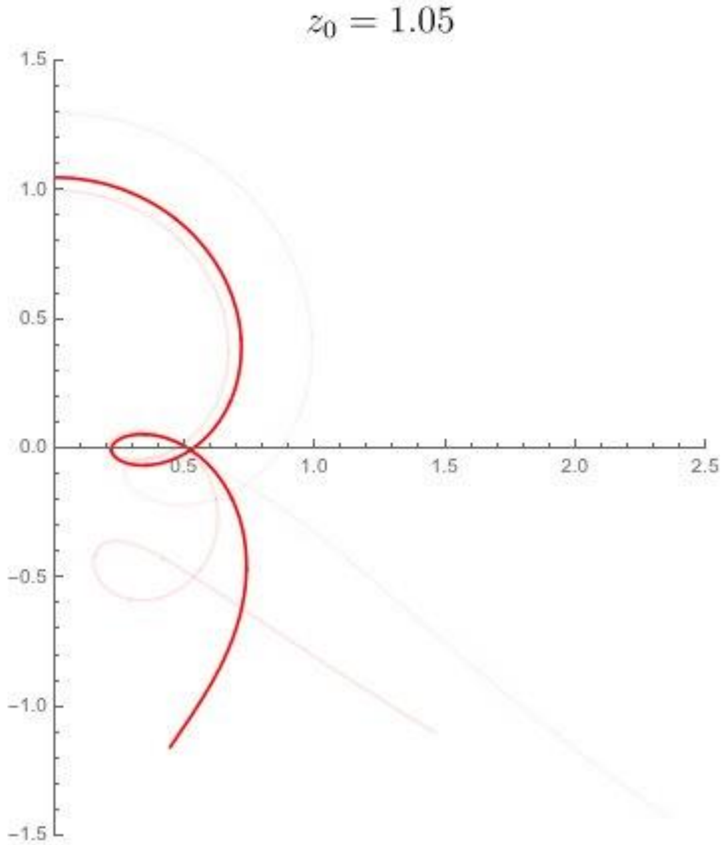


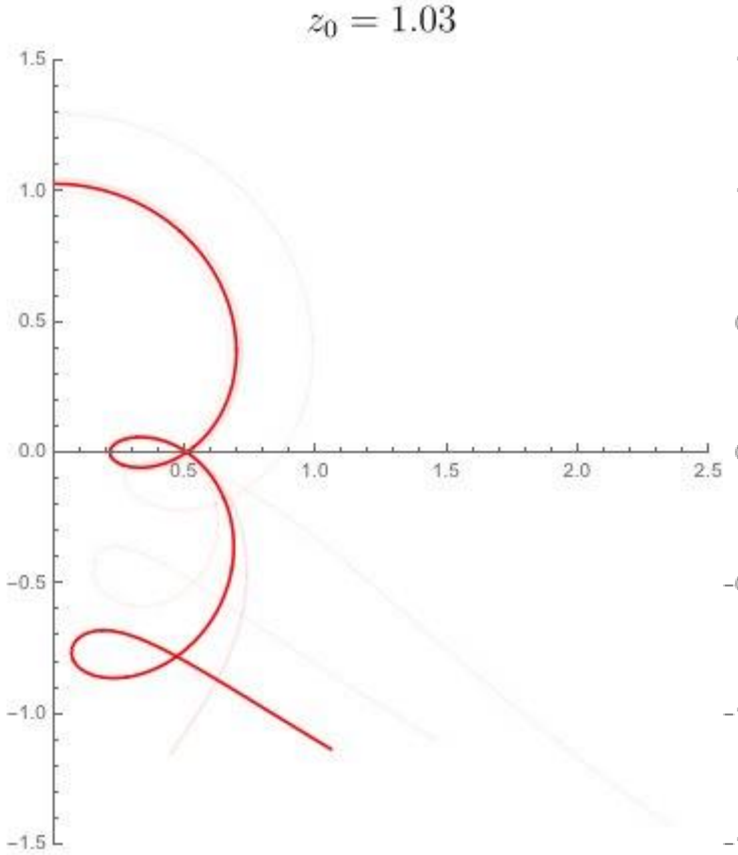
Summary

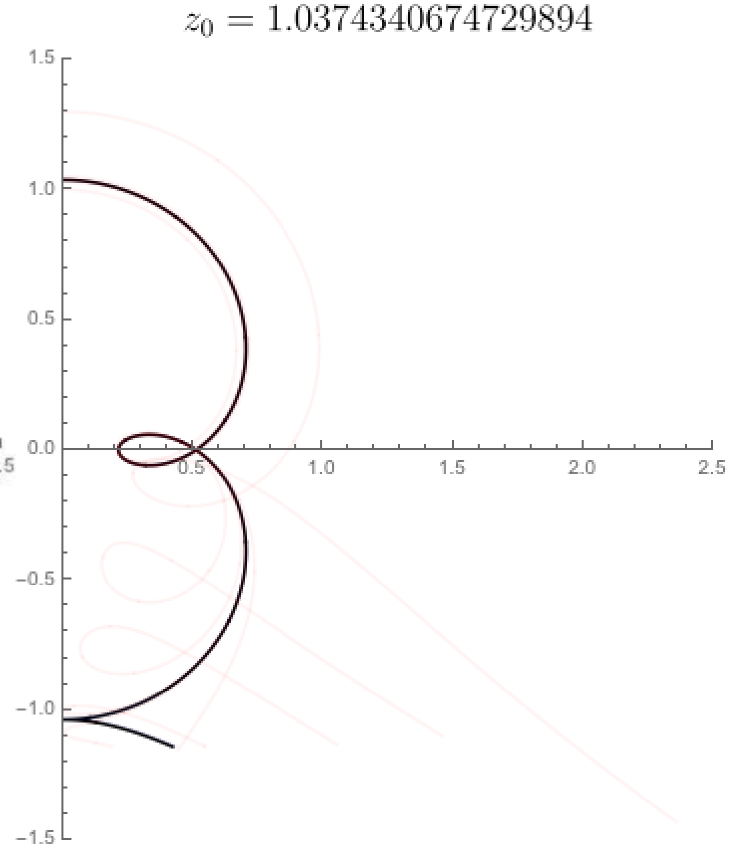
- Self-intersecting MOTSs are found generically in static, stationary black hole interiors and probes the geometry of the spacetime
- Asymptotically AdS behaviour introduces unexpected stability spectrum

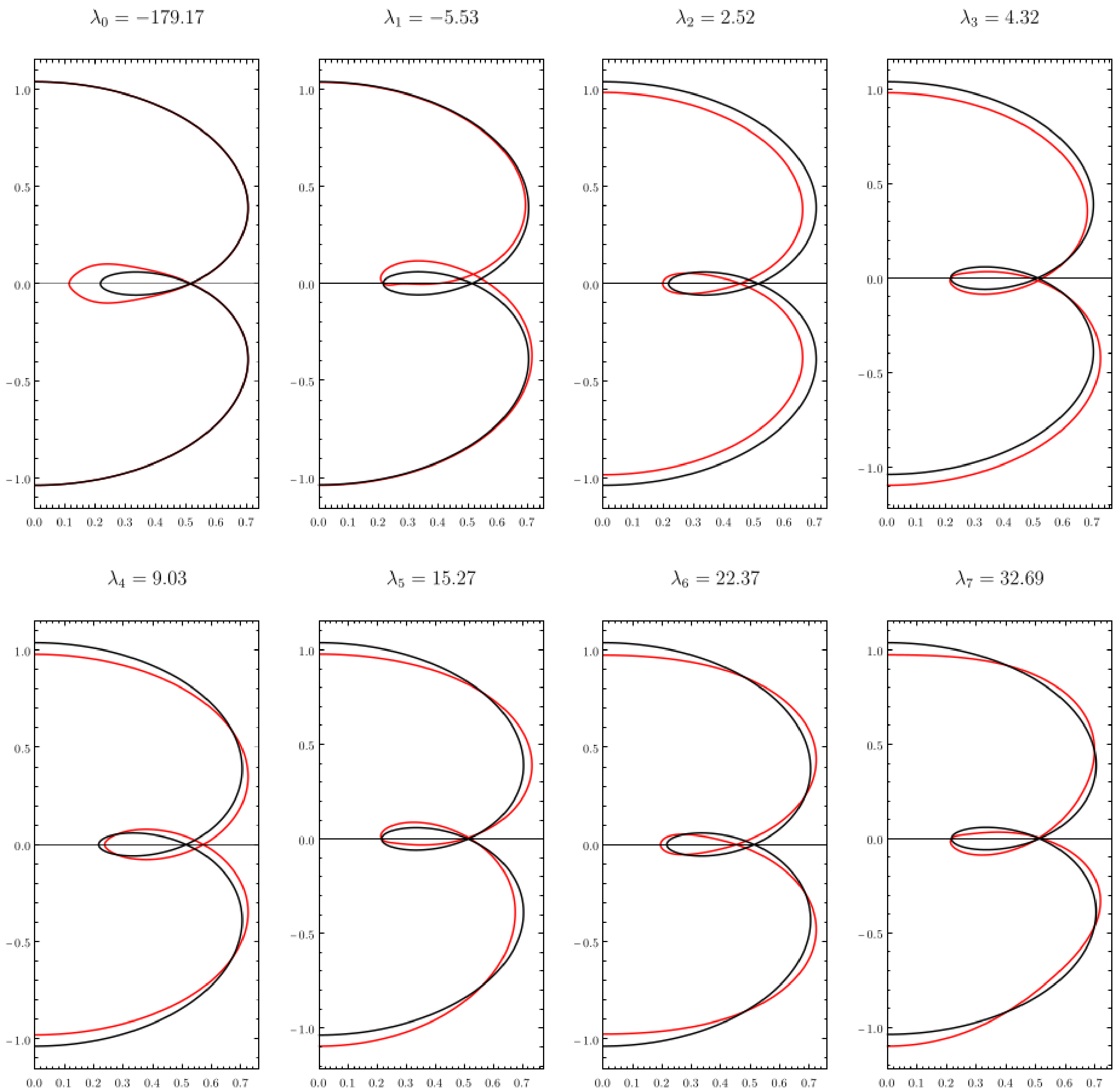


Extra Slides


$z_0 = 1.05$
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
0.5
1.0
1.5
2.0
2.5


$z_0 = 1.03$
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
0.5
1.0
1.5
2.0
2.5


$z_0 = 1.0374340674729894$
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
0.5
1.0
1.5
2.0
2.5


$\lambda_0 = -179.17$
$\lambda_1 = -5.53$
$\lambda_2 = 2.52$
$\lambda_3 = 4.32$
$\lambda_4 = 9.03$
$\lambda_5 = 15.27$
$\lambda_6 = 22.37$
$\lambda_7 = 32.69$

Gauss-Bonnet MOTS ($\alpha = \frac{1}{60} M^2$)