Uses and abuses of dimension-zero scalar fields

Jim Cline, McGill University

with Anamaria Hell, IPMU, Tokyo

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Dimension-zero scalar field?

"I thought scalars are supposed to have mass dimension 1."

We will discuss an exotic scalar with 4th-order action

$$S = -\int d^4x\sqrt{-g}\,\varphi\Delta_4\varphi\,,$$

where

$$\Delta_4 = \Box^2 + 2R^{\mu\nu}\nabla_\mu\nabla_\nu - \frac{2}{3}R\Box + \frac{1}{3}(\nabla^\mu R)\nabla_\mu$$

is known as the Paneitz operator. It is the unique conformally invariant 4th-order differential operator, analogous to

$$\Delta_2 = \Box + \tfrac{1}{6}R$$

at second order.

The mass dimension of 4th-order scalar is obviously zero.

History of dim-0 scalars

1982: The 4th-order scalars were discovered by Fradkin and Tseytlin, in context of conformal supergravity theories, which have remarkable properties and were widely studied. φ is a scalar graviton mode in those theories.

1983: Independently discovered by Paneitz, who died the same year (his conference proceeding cited by 648, never published).

1984: Riegert used dim-0 scalar as an auxiliary field, producing nonlocal action whose variation is the trace anomaly.

2021: Boyle and Turok invoke dim-0 scalars to cancel cosmological constant and Weyl anomaly of Standard Model (arXiv: 2110.06258).

2023: Boyle and Turok invoke dim-0 scalars to explain cosmological perturbations without inflation (arXiv: 2302.00344).

We do not agree with the last two. In addition the theory has a an obvious problem . . .

The problem of ghosts

In QFT, we are taught that higher-derivative theories should only be treated as low-energy effective theories.

Putting p^4 in the propagator leads to extra pole with wrong-sign residue: a ghost. Rewrite action as 2nd-order using auxiliary field σ:

$$
\begin{aligned}
S &= \int d^4x\sqrt{-g}\Big[\partial_\mu\sigma\partial^\mu\varphi R^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \tfrac{1}{3}R(\partial\varphi)^2 - \tfrac{1}{2}\sigma^2\Big] \\
&= \int d^4x\sqrt{-g}\left[\frac{1}{2}\begin{pmatrix}\partial_\mu\varphi\\ \partial_\mu\sigma\end{pmatrix}^T\begin{pmatrix}R^{\mu\nu}-\frac{1}{3}Rg^{\mu\nu} & g^{\mu\nu}\\ g^{\mu\nu} & 0\end{pmatrix}\begin{pmatrix}\partial_\mu\varphi\\ \partial_\mu\sigma\end{pmatrix} - \frac{1}{2}\sigma^2\right].
\end{aligned}
$$

Kinetic mixing matrix goes to $\left(\begin{smallmatrix}0 & 1\\ 1 & 0\end{smallmatrix}\right)$ in flat-space limit. It has eigenvalues ± 1: one normal scalar plus one ghost.

One can show that the Hamiltonian

$$\mathcal{H} = \dot\varphi\,\dot\sigma + \vec\nabla\varphi\cdot\vec\nabla\sigma + \tfrac{1}{2}\sigma^2$$

is unbounded from below, demonstrating the ghost.

There is no gauge symmetry

Boyle and Turok claim the ghost problem is cured by a "gauge symmetry"

$$\varphi \to \varphi + \alpha(x) \quad \text{where} \quad \Box\alpha = 0$$

But this is no gauge symmetry in the usual sense, e.g., observables like $T_{\mu\nu}$ do not remain invariant under such transformations;

$$T_{\mu\nu} = \partial_\mu\varphi\,\partial_\nu\sigma + \partial_\mu\sigma\,\partial_\nu\varphi - \eta_{\mu\nu}\left(\frac{\sigma^2}{2} + \partial_\gamma\varphi\partial^\gamma\sigma\right)$$

$T_{\mu\nu}$ is only invariant if the additional restriction $\sigma = 0$ is imposed, which renders the theory trivial.

Their reference to Bogoliubov *et al.* QFT textbook on this point is confused by not recognizing the extra restriction.

Rivelles (2003) showed the higher derivative theory can be made trivial by adding FP ghost to realize BRST symmetry—but this is similar to above restriction.

Cosmological backgrounds

We found two exact cosmological solutions to EOMs:

$$
\begin{aligned}
a(t) &\sim t^{2/3}, \quad \varphi \sim t \\
a(t) &\sim e^{Ht}, \quad \varphi \sim a^{-1} \text{ or } a^{-3}
\end{aligned}
$$

We linearized the fluctuations of metric and scalar around these backgrounds to find single scalar fluctuation with kinetic term

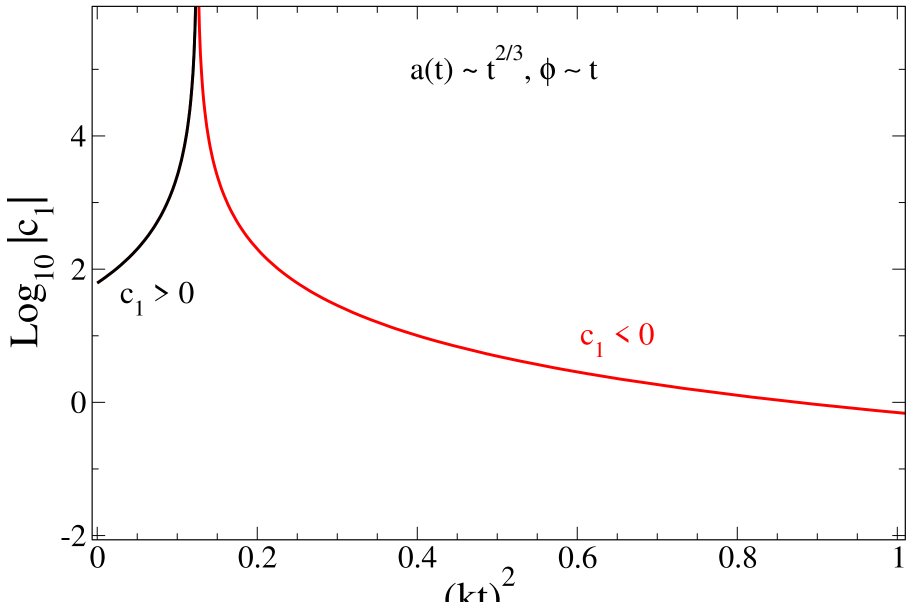
$$\tilde{\mathcal{L}}_{Sk} = c_1(t,k^2)\, |\dot{\tilde{\sigma}}_{\vec{k}}|^2 - c_2(t,k^2)\, k^2\, |\tilde{\sigma}_{\vec{k}}|^2$$

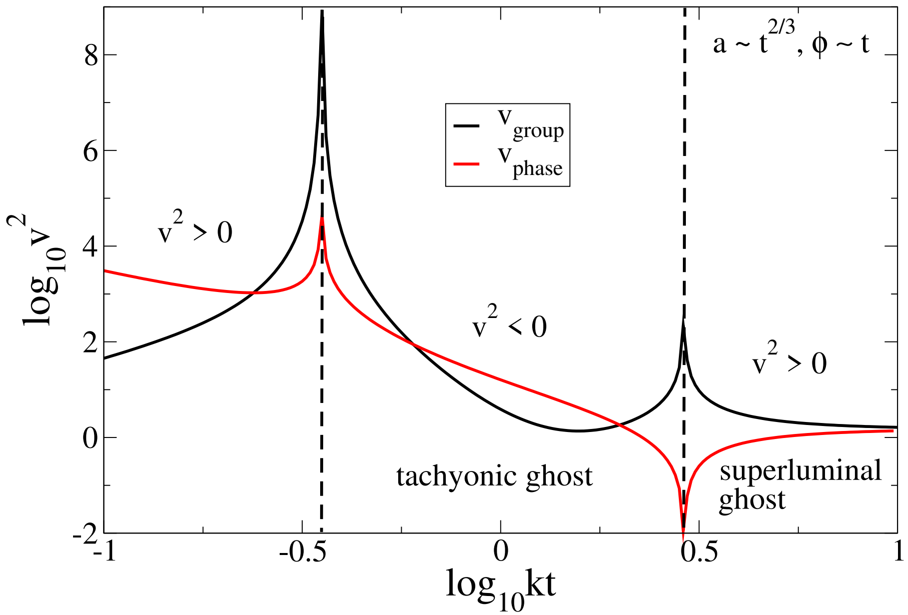
Sign of c_1 determines whether ghost, and propagation speed is $v_p = \sqrt{c_2/c_1}$. Imaginary v_p means tachyon.

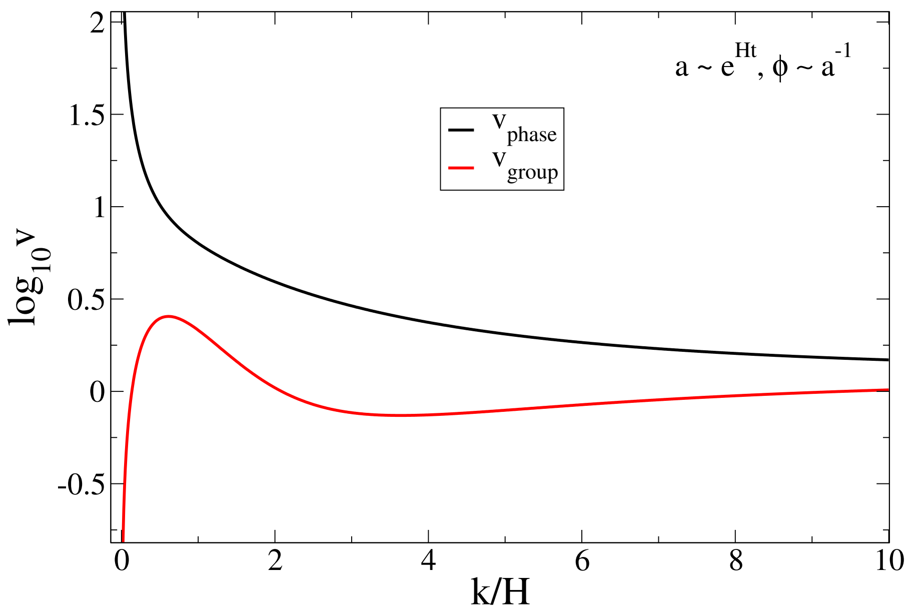
Depending on wavenumber k, we find ghosts, tachyons, and superluminal propagation...

Pathological perturbations

Examples of ghosts, tachyonic ghosts, and superluminal excitations







Top: c_1 and velocities for "matter-dominated" background cosmology.

Left: fluctuations are always ghosts in inflationary background, and approach $v = \sqrt{3/2}$ as $k \to \infty$.

Cosmological constant Λ

Boyle and Turok hypothesize 36 dim-0 scalar fields. This leads to a remarkable coincidence: simultaneous cancellation of Weyl anomaly and cosmological constant in Standard Model + 3 RH νs.

We point out caveats to this argument.

1. Assumes Higgs boson is composite state of SM fermions, not independent degree of freedom. Huh???

2. Cancellation of Λ is only exact if all particles massless. In presence of particle masses,

$$\rho_\Lambda \sim 0 \times M_p^4 + O(m^2 M_p^2)$$

Still too big by 91 orders of magnitude.

3. Who cares about the Weyl anomaly?

Unitarity violation

Choice of $\pm i\epsilon$ in ghost propagator determines whether it carries negative energy or negative probability.

For Λ cancellation, Boyle and Turok implicitly assume $E > 0$, negative probability.

Unitarity (optical theorem) is violated. Do field redefinition

$$\varphi = \frac{1}{2\,M}(\eta + \psi), \quad \sigma = M(\eta - \psi)\,.$$

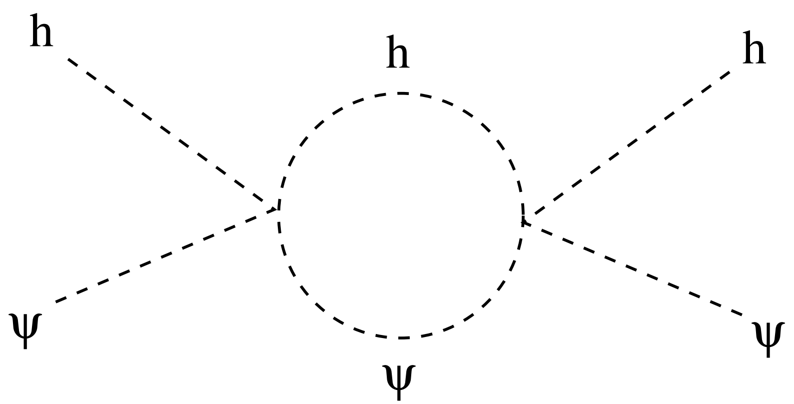
M is arbitrary parameter to give correct dimensions to canonically normalized fields η, ψ; must drop out of observable amplitudes. Lagrangian becomes

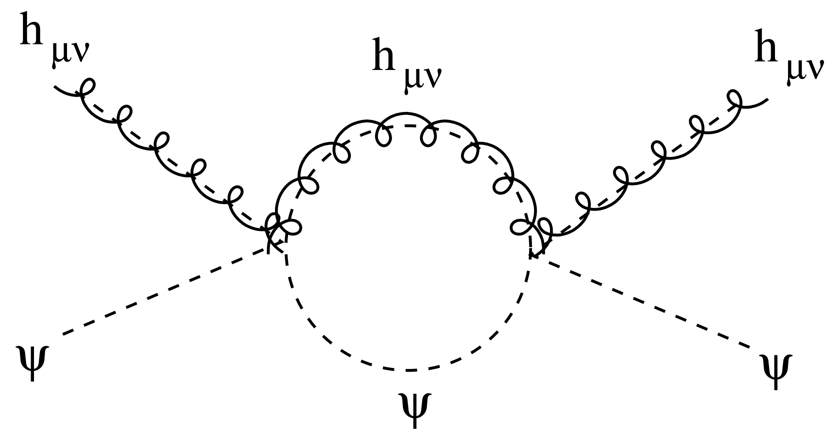
$$\mathcal{L} = \tfrac{1}{2}\Big((\partial\eta)^2 - (\partial\psi)^2 - M^2(\eta - \psi)^2\Big)\,.$$

Then η is normal, ψ is ghost.

Unitarity violation

Toy model: suppose ψ couples to Higgs boson. Then optical theorem is explicitly violated by diagram on left:





Internal ψ propagator has the wrong sign.

In reality, ψ need not couple to Higgs, but must couple to gravity. Diagram on right violates unitarity for same reason.

Primordial density perturbations?

Boyle and Turok note that φ has scale-invariant vacuum fluctuations

$$\langle\varphi^2\rangle = \int \frac{d^4k}{(2\pi)^4}\frac{i}{k^4} \sim \int \frac{d^3k}{(2\pi)^3}\frac{1}{k^3}$$

Could they replace the inflaton? They claim trace anomaly induces coupling of φ_j to energy densities ρ_i of SM particles,

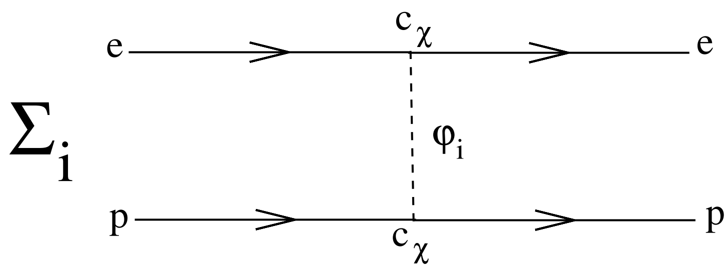
$$\mathcal{L} \sim \sum_{ij} c_\chi \rho_i \varphi_j \, ,$$

dominated by QCD contribution, $c_\chi \sim \alpha_s^2/(36\pi^2)$. Then thermal SM plasma sources φ_j fluctuations, claimed to agree with observed CMB fluctuations.

Whether or not you believe all that, the coupling of φ_j to SM particles is constrained . . .

New confining force

If such coupling exists, there is a new strong force between SM particles,



$$\sim \quad 36\, c_\chi^2\, m_e\, m_p\, (\bar{u}_e u_e)(\bar{u}_p u_p)\, \frac{1}{k^4}$$

Corresponding interaction potential comes from Fourier transforming amplitude,

$$V(\vec{x}) \sim 36\, c_\chi^2 m_e m_p |\vec{x}|\,.$$

At $x \sim a_0$ (Bohr radius), this is many times stronger than Coulomb force. Atomic structure is destroyed.

Conclusions

Dim-0 scalars are present in conformal supergravity. It was hoped a solution to the ghost problem would eventually be found, but none has so far.

Dim-0 scalars *do not* solve the cosmological constant problem, nor do they consistently replace inflationary fluctuations.

Dim-0 scalars *do* give rise to ghosts, tachyons, superluminal excitations, unitarity violation, and unacceptable new confining forces if coupled to standard model particles.

☺ THE END ☺