Static and axisymmetric black holes in improved GUP: theory and phenomenology

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Outline

1. Static and Rotating GUP Metrics
2. Shadow and Phenomenology

GUP

Generalized uncertainty principle (GUP): based on modification of algebra

$$\Delta q \Delta p \geq \frac{1}{2} \left|\langle [q,p] \rangle\right| \Rightarrow \text{modification of RHS} \Leftrightarrow \text{modification of LHS}$$

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Our remedy: Analytic extension of interior to exterior

Interior of BH in AB variables

Classical interior metric in AB variables:

$$ds^2 = -\frac{N\left(p_b, p_c\right)}{t^2}dt^2 + \frac{p_b^2}{L_0^2 p_c}dr^2 + p_c d\Omega^2$$

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Classical interior Hamiltonian (constraint) in AB variables

$$H = -\frac{N\left(p_b, p_c\right)}{2G\gamma^2}\left[\left(b^2 + \gamma^2\right)\frac{p_b}{\sqrt{p_c}} + 2bc\sqrt{p_c}\right]$$

with algebra

$$\{b, p_b\} = 2G\gamma, \qquad \{c, p_c\} = G\gamma$$

GUP-Modified Solution

1. GUP-modify the algebra

$$\{b, p_b\} = 2G\gamma\left(1 + \beta_b b^2\right), \qquad \{c, p_c\} = G\gamma\left(1 + \beta_c c^2\right)$$

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5. Correct classical and asymptotic limits?

Asymptotic Issues

Correct Classical limits:

$$\lim_{\beta_b,\beta_c\to 0} \mathsf{g}_{\mu\nu}^{\mathsf{GUP}} = \mathsf{g}_{\mu\nu}^{\mathsf{Schw}}$$

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Incorrect Kretschmann fall off

$$K_{\mathrm{GUP}}(r \to \infty) \propto \frac{1}{r^4} \text{ and not } \frac{1}{r^6}$$

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Similar issues in LQG BH/LQC

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Remedied by “Improved Schemes”:

Quantum parameter(s) $\Rightarrow$ momentum-dependent

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Remedied by "Improved Schemes":

Quantum parameter(s) ⇒ momentum-dependent

$$\beta \rightarrow \bar{\beta}\left(p\right)$$

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Apply improved scheme to GUP

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In our model

$$\beta_b \to \bar{\beta}_b = \frac{\beta_b L_0^4}{p_b^2}, \qquad \beta_c \to \bar{\beta}_c = \frac{\beta_c L_0^4}{p_c^2}$$

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Apply improved scheme to GUP

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which means

$$\{b, p_b\} = 2G\gamma \left(1 + \frac{\beta_b L_0^4}{p_b^2} b^2\right), \qquad \{c, p_c\} = G\gamma \left(1 + \frac{\beta_c L_0^4}{p_c^2} c^2\right)$$

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$$\dot{p}_b = \{p_b, H\}, \qquad \dot{p}_c = \{p_c, H\}$$

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4. Has correct limits:

$$\lim_{\beta_b, \beta_c \to 0} g_{\mu\nu}^{\text{GUP-Imp}} = g_{\mu\nu}^{\text{Schw}}$$

$$\lim_{r \to \infty} g_{\mu\nu}^{\text{GUP-Imp}} = \eta_{\mu\nu}$$

GUP-Modified Improved Metric

Result: first-ever GUP BH metric derived [Fragomeno, Gingrich, Hergott, Rastgoo, Vienneau, PRD 111 (2025) 2, 024048; Gingrich, Rastgoo, PRD 111 (2025) 10, 104017]

$$g_{00}^{\text{GUP-Imp}} = -\left(1+\frac{Q_b}{r^2}\right)\left(1+\frac{Q_c R_s^2}{4r^8}\right)^{-1/4}\left(1-\frac{R_s}{\sqrt{r^2+Q_b}}\right)$$

$$g_{11}^{\text{GUP-Imp}} = \left(1+\frac{Q_c R_s^2}{4r^8}\right)^{1/4}\left(1-\frac{R_s}{\sqrt{r^2+Q_b}}\right)^{-1}$$

$$g_{22}^{\text{GUP-Imp}} = r^2\left(1+\frac{Q_c R_s^2}{4r^8}\right)^{1/4}$$

with dimensionful quantum parameters

$$Q_b = -\,\text{sgn}(\beta_b)|\beta_b|\gamma^2 L_0^2, \qquad Q_c = -\,\text{sgn}(\beta_c)|\beta_c|\gamma^2 L_0^6$$

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Reality of the metric on $r \in [0, \infty)$ dictates

$$Q_b > 0 \Rightarrow \text{sgn}(\beta_b) = -1, \qquad Q_c > 0 \Rightarrow \text{sgn}(\beta_c) = -1$$

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$$g_{22}^{\text{GUP-Imp}} = r^2\left(1 + \frac{Q_c R_s^2}{4r^8}\right)^{1/4}$$

Q_b: controls the exterior/near horizon quantum effects

Q_c: controls deep interior/high curvature quantum effects

Rotating GUP BH Metric

Newman-Janis algorithm $\Rightarrow$ rotating metric

$$g_{\bar{0}\bar{0}} = -\frac{\rho^2}{\Sigma^2}\left(\Sigma - \mathit{\Lambda}\right), \quad g_{\bar{0}\bar{3}} = -\frac{a\sin^2(\theta)\,\rho^2}{\Sigma^2}\mathit{\Lambda}, \qquad g_{\bar{1}\bar{1}} = \frac{\rho^2}{\Delta},$$

$$g_{\bar{2}\bar{2}} = \rho^2, \qquad g_{\bar{3}\bar{3}} = \frac{\rho^2\sin^2(\theta)}{\Sigma^2}\left[\Sigma^2 + a^2\sin^2(\theta)\left(\Sigma + \mathit{\Lambda}\right)\right],$$

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where

$$\rho^2 = g_{22} + a^2\cos^2(\theta) = r^2\left(1 + \frac{Q_c R_s^2}{4r^8}\right)^{\frac{1}{4}} + a^2\cos^2(\theta),$$

$$\Delta = \frac{g_{22}}{g_{11}} + a^2 = r^2\left(1 - \frac{R_s}{\sqrt{r^2 + Q_b}}\right) + a^2,$$

$$\Sigma = \frac{g_{22}}{\sqrt{-g_{00}g_{11}}} + a^2\cos^2(\theta) = \left(\rho^2 - a^2\cos^2(\theta)\right)\left(1 + \frac{Q_b}{r^2}\right)^{-\frac{1}{2}} + a^2\cos^2(\theta),$$

$$\Lambda = \Sigma - \Delta + a^2\sin^2(\theta).$$

Limits

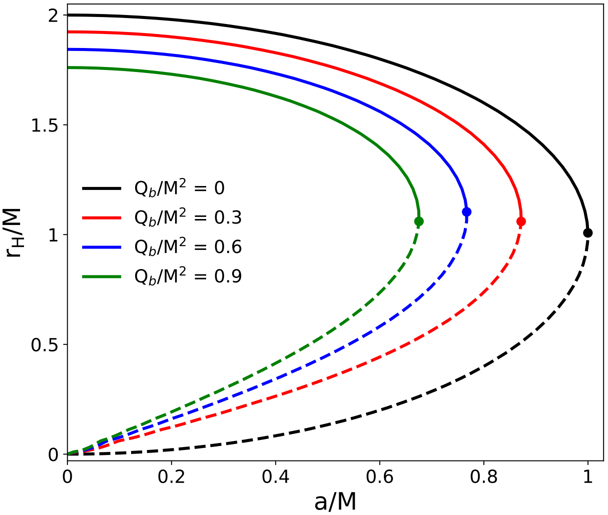
Correct limits:

$$\text{Static limit: } \lim_{a\to 0} g_{\bar{\mu}\bar{\nu}} = g_{\mu\nu}^{\text{GUP-Imp}}$$

$$\text{Classical limit: } \lim_{Q_b\to 0, Q_c\to 0} g_{\bar{\mu}\bar{\nu}} = g_{\mu\nu}^{\text{Kerr}}$$

$$\text{Asymptotic expansion: } g_{\bar{\mu}\bar{\nu}}\big|_{Q_b\to 0, Q_c\to 0, r\to\infty} = g_{\mu\nu}^{\text{Kerr}}\big|_{r\to\infty}$$

Horizons



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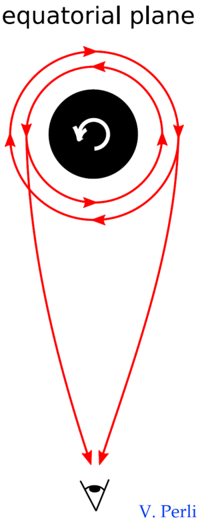
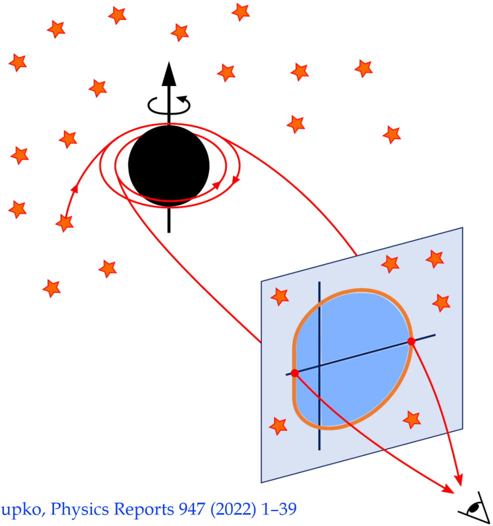
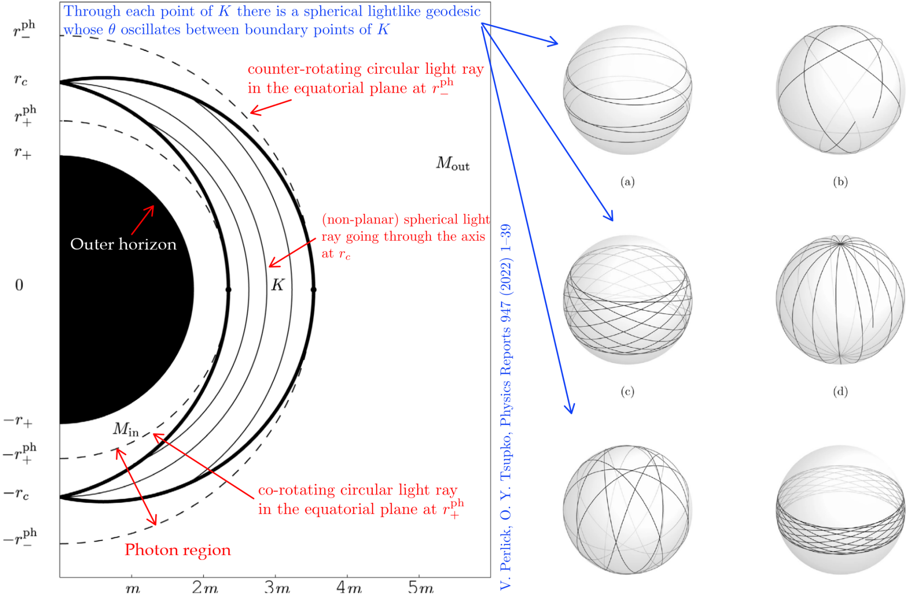
Shadow - Axisymmetric - Planar





V. Perlick, O. Y. Tsupko, Physics Reports 947 (2022) 1–39

Shadow - Axisymmetric



Through each point of K there is a spherical lightlike geodesic whose θ oscillates between boundary points of K

counter-rotating circular light ray in the equatorial plane at r_-^{ph}

(non-planar) spherical light ray going through the axis at r_c

co-rotating circular light ray in the equatorial plane at r_+^{ph}

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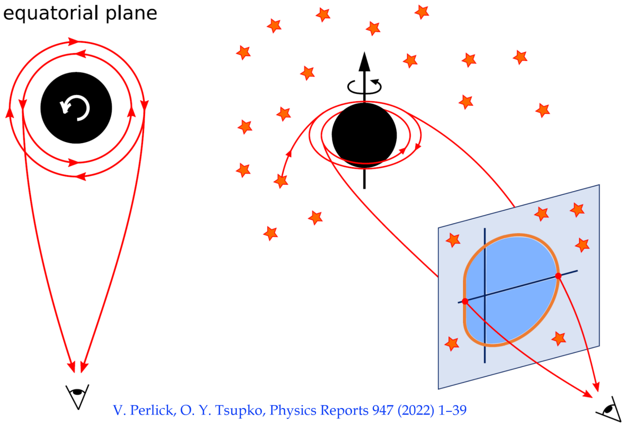
Shadow Boundary



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Boundary of the shadow:

Governed by two impact parameters:

- $\xi = \frac{L}{E}$, photon's perpendicular distance from the rotation axis as seen by a distant observer
- $\eta = \frac{\mathcal{K}}{E}$, measures motion out of the equatorial plane

Shadow

Apparent shape of the shadow

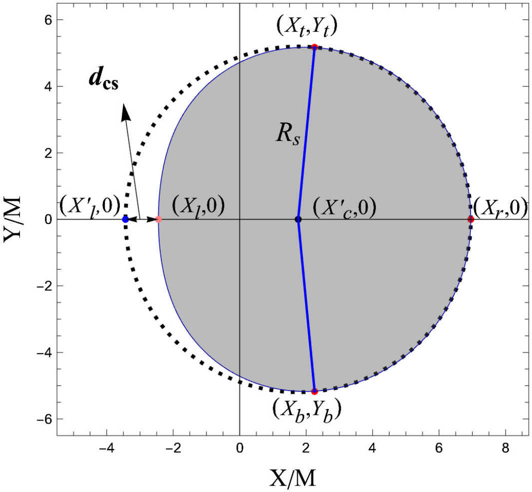
- measured by a distant observer
- located at asymptotically flat infinity
- inclined at an angle θ_0

described by the celestial coordinates

$$\alpha\left(r_{\mathrm{ph}}\right) = -\xi\left(r_{\mathrm{ph}}\right)\csc\left(\theta_0\right),$$

$$\beta\left(r_{\mathrm{ph}}\right) = \pm\sqrt{\eta\left(r_{\mathrm{ph}}\right) + a^2\cos^2\left(\theta_0\right) - \xi^2\cot^2\left(\theta_0\right)}.$$

Shadow of GUP Rotating Metric



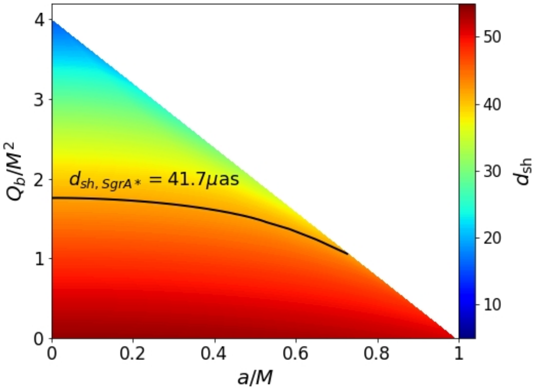
- D: distance of observer to BH,
- Shadow area A

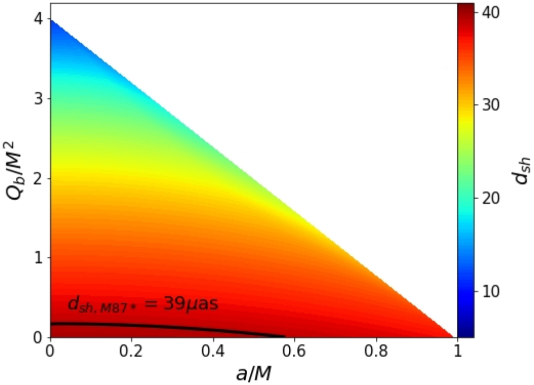
$$A = 2\int_{r_{ph}^-}^{r_{ph}^+} \beta\left(r_{ph}\right)\frac{d\alpha\left(r_{ph}\right)}{dr_{ph}}dr_{ph}$$

- Angular shadow diameter

$$d_{sh} = \frac{2}{D}\sqrt{\frac{A}{\pi}},$$

Comparison to EHT Measurements: Sgr A* vs. M87*





Summary

- 1st static and NJ-rotated GUP BH metric derived since the beginning of GUP
- Two quantum parameters $Q_b,\ Q_c$
- LQG tricks used (improved scheme, interior-to-exterior)
- Comparing theoretical shadow with EHT data allows putting bounds on $Q_b,\ Q_c$
- Limits the spin parameter to $\frac{a}{M} < 1$ for naked singularities
- Considering systematics, etc., limits M87* spin to $\frac{a}{M} < 0.6$

Limits and Singularity Resolution

Singularity is resolved

- Radius of 2-spheres

$$\sqrt{g_{22}^{\text{GUP-Imp}}} = \left(r^8 + \frac{Q_c R_s^2}{4}\right)^{\frac{1}{8}}, \qquad \sqrt{g_{22}^{\text{GUP-Imp}}}\Big|_{r=0} = \left(\frac{Q_c R_s^2}{4}\right)^{\frac{1}{8}}$$

- Kretschmann

$$K = \frac{\cdots}{\sqrt{r^8 + \frac{1}{4} Q_c R_s^2}} + \frac{\cdots}{\left(r^2 + Q_b\right)^5 \left(r^8 + \frac{1}{4} Q_c R_s^2\right)^{9/2}}, \qquad \lim_{r \to 0^+} K = \frac{8}{R_s \sqrt{Q_c}}$$

- Null expansion scalar

$$\begin{aligned} \theta_+ \left(r > r_H\right) &> 0, \quad \theta_+ \left(r \leq r_H\right) < 0, \\ \theta_- \left(r > r_H\right) &< 0, \quad \theta_- \left(r \leq r_H\right) < 0, \\ \theta_\pm \left(r = 0\right) &= 0 \end{aligned}$$

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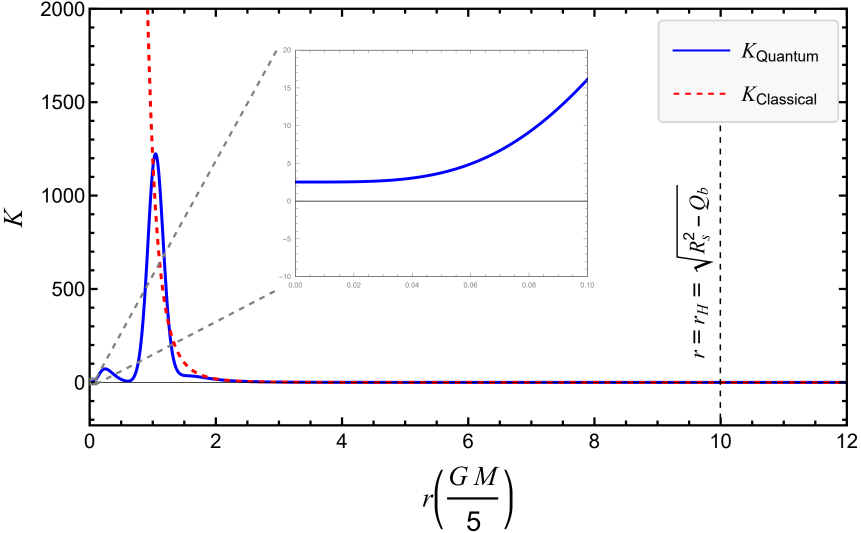

Kretschmann Scalar comparison, with $G = 1$, $M = 5$, $Q_b = 0.1 = Q_c$

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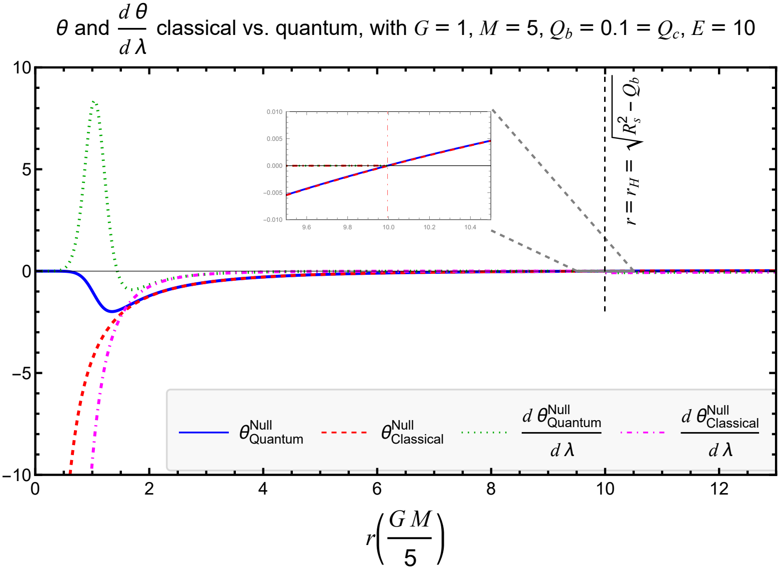

Horizon

There is a single horizon $g^{11}(r_H) = 0$ or $g_{00}(r_H) = 0$ located at

$$r_H = R_s \sqrt{1 - \frac{Q_b}{R_s^2}} = R_s - \frac{1}{2}\frac{Q_b}{R_s} + \mathcal{O}\left(\frac{Q_b^2}{R_s^3}\right)$$

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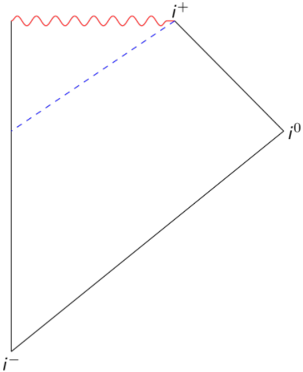
Based on the value of M and Q_b

$$\begin{cases} R_s^2 > Q_b \Rightarrow M > \frac{\sqrt{Q_b}}{2G}, & \text{Black hole} \\ R_s^2 = Q_b \Rightarrow M = \frac{\sqrt{Q_b}}{2G}, & \text{Min mass, remnant} \\ R_s^2 < Q_b \Rightarrow M < \frac{\sqrt{Q_b}}{2G}, & \text{No horizon, wormhole} \end{cases}$$

Penrose Diagram



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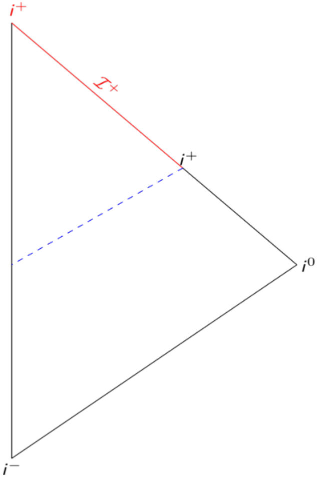

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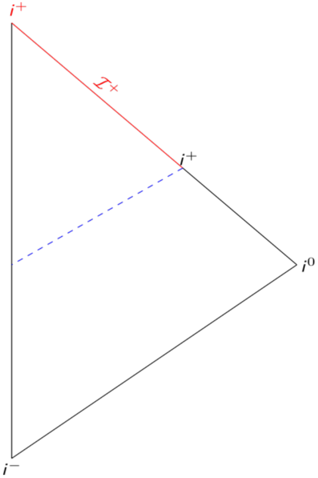

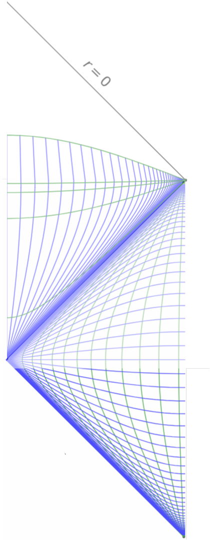

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3) Find the Newman-Penrose null tetrads

$$g^{(\mathrm{EF})\tilde{\mu}\tilde{\nu}} = -l^{\tilde{\mu}}n^{\tilde{\nu}} - l^{\tilde{\nu}}n^{\tilde{\mu}} + m^{\tilde{\mu}}\bar{m}^{\tilde{\nu}} + m^{\tilde{\nu}}\bar{m}^{\tilde{\mu}}$$

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$$g^{(\mathrm{EF})\tilde{\mu}\tilde{\nu}} = -l^{\tilde{\mu}}n^{\tilde{\nu}} - l^{\tilde{\nu}}n^{\tilde{\mu}} + m^{\tilde{\mu}}\bar{m}^{\tilde{\nu}} + m^{\tilde{\nu}}\bar{m}^{\tilde{\mu}}$$

where

$$\begin{aligned} g^{(\mathrm{EF})}_{\tilde{\mu}\tilde{\nu}} l^{\tilde{\mu}} l^{\tilde{\nu}} = g^{(\mathrm{EF})}_{\tilde{\mu}\tilde{\nu}} n^{\tilde{\mu}} n^{\tilde{\nu}} = g^{(\mathrm{EF})}_{\tilde{\mu}\tilde{\nu}} m^{\tilde{\mu}} m^{\tilde{\nu}} = g^{(\mathrm{EF})}_{\tilde{\mu}\tilde{\nu}} l^{\tilde{\mu}} m^{\tilde{\nu}} = g^{(\mathrm{EF})}_{\tilde{\mu}\tilde{\nu}} n^{\tilde{\mu}} m^{\tilde{\nu}} &= 0 \\ g^{(\mathrm{EF})}_{\tilde{\mu}\tilde{\nu}} l^{\tilde{\mu}} n^{\tilde{\nu}} &= -1 \\ g^{(\mathrm{EF})}_{\tilde{\mu}\tilde{\nu}} m^{\tilde{\mu}} \bar{m}^{\tilde{\nu}} &= 1 \end{aligned}$$

Modified Newman-Janis Algorithm

4) Unlike traditional NJ, do not assume $\sqrt{2\mathrm{g}_{\tilde{2}\tilde{2}}}$ (and hence *r*) is complex

Modified Newman-Janis Algorithm

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5)Make a transformation (introduce rotation deformation)

$$\begin{aligned} r' &= r + ia\cos(\theta)\,, & \theta' &= \theta \\ u' &= u - ia\cos(\theta)\,, & \phi' &= \phi \end{aligned}$$

under which

$$l^{\tilde{\mu}} \to l^{\bar{\mu}},\; n^{\tilde{\mu}} \to n^{\bar{\mu}},\; m^{\tilde{\mu}} \to m^{\bar{\mu}}$$

and

$$g_{\mu\nu} \to \breve{g}_{\mu\nu}$$

and later we demand $\lim_{a\to 0} \breve{g}_{\mu\nu} = g_{\mu\nu}$

Modified Newman-Janis Algorithm

6) Find $g^{(\mathsf{EF})\mu'\nu'} = -l^{\mu'} n^{\nu'} - l^{\nu'} n^{\mu'} + m^{\mu'} \bar{m}^{\nu'} + m^{\nu'} \bar{m}^{\mu'}$ and then find $g^{(\mathsf{EF})}_{\mu'\nu'}$

Modified Newman-Janis Algorithm

6) Find $g^{(\mathrm{EF})\mu'\nu'} = -l^{\mu'}n^{\nu'} - l^{\nu'}n^{\mu'} + m^{\mu'}\bar{m}^{\nu'} + m^{\nu'}\bar{m}^{\mu'}$ and then find $g^{(\mathrm{EF})}_{\mu'\nu'}$

7) Make a Boyer-Lindquist transformation

$$\begin{aligned} du' &= d\bar{t} + \lambda\left(\bar{r}\right) d\bar{r} \\ d\phi' &= d\bar{\phi} + \chi\left(\bar{r}\right) d\bar{r} \end{aligned}$$

and demand $g_{\bar{t}\bar{r}} = 0 = g_{\bar{r}\bar{\phi}}$, λ and χ are only functions or $\bar{r}$, and $\lim_{a\to 0} \breve{g}_{\mu\nu} = g_{\mu\nu}$