

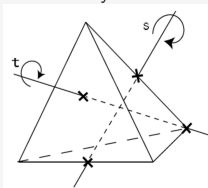
Neutrino oscillations in the context of discrete flavor models

Janusz Gluza
University of Silesia in Katowice

1st Annual MAPS Meeting (Indico)

*"Bridging Theory and Experiments:
Pushing the Boundaries of Neutrino Oscillation Precision Physics"*

Ruder Bošković Institute, Zagreb
1 July 2026





President Clinton's talk at MIT's 1998 Commencement



President William Jefferson Clinton—1998 MIT Commencement

Just yesterday in Japan, physicists announced a discovery that tiny neutrinos have mass. Now, that may not mean much to most Americans, but **it may change our most fundamental theories -- from the nature of the smallest subatomic particles to how the universe itself works, and indeed how it expands.**

.....

The larger issue is that these kinds of findings have **implications that are not limited to the laboratory.** They affect the whole of society -- not only our economy, but our very view of life, our understanding of our relations with others, and **our place in time.**

Source: Kajita, Neutrino'2022, <https://www.youtube.com/watch?v=9LheUWrXUHU>

N_ν

QED deconvolution

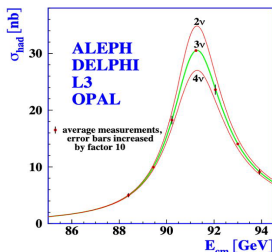


Reminder: Measuring N_ν at LEP

- The Z lineshape determination**

Phys. Rep. 427 (2006) 257

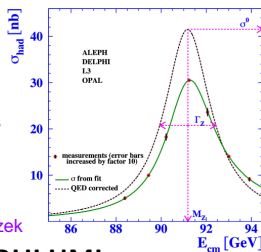
- Measure hadronic and leptonic cross sections (σ_{had} and σ_{lept}) as a function of E_{cm} ($\sqrt{s}$)



Fit to a Breit-Wigner shape
folded with QED ISR effects

Get m_Z , Γ_Z and σ^0
Get also $R_t^0 = \sigma_{\text{had}}^0 / \sigma_{\text{lept}}^0$

Staszek



KKMC, BHLUMI

- The peak cross section σ^0 is very sensitive to N_ν**

- The smaller the peak cross section, the larger the number of light neutrino active species



Reminder: Measuring N_ν at LEP

- What was done in practice to extract N_ν :

- Total Z decay width : $\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{\text{had}} + N_\nu \Gamma_{\nu\nu}$. $R_\ell^0 = \frac{\Gamma_{\text{had}}}{\Gamma_\ell}$
- Divide by Γ_ℓ : $\Gamma_Z / \Gamma_\ell = 3 + \delta_\tau + R_\ell^0 + N_\nu (\Gamma_{\nu\nu} / \Gamma_\ell)$
 - δ_τ is a small phase-space correction due to the finite τ mass
 - $(\Gamma_{\nu\nu} / \Gamma_\ell)$ rather immune to SM parameters (m_{top} , m_H , ...): taken from SM
 - Γ_Z / Γ_ℓ taken from Breit-Wigner peak expression $\sigma_{\text{had}}^0 = \frac{12\pi}{m_Z^2} \frac{\Gamma_{ee} \Gamma_{\text{had}}}{\Gamma_Z^2}$
- Solve for N_ν :

$$N_\nu \left(\frac{\Gamma_{\nu\nu}}{\Gamma_\ell} \right)_{\text{SM}} = \left(\frac{12\pi}{m_Z^2} \frac{R_\ell^0}{\sigma_{\text{had}}^0} \right)^{\frac{1}{2}} - R_\ell^0 - 3 - \delta_\tau \approx -2.263 \cdot 10^{-3}$$

SM prediction:

= 1.99125 ± 0.00083 in 2005

= 1.99060 ± 0.00021 in 2019

Dubovik, Freitas, Gluza, Riemann, Usovitch

Phys. Lett. B 783 (2018) 86

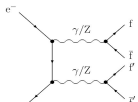
Measured **BHLUMI**

Phys. Rep. 427 (2006) 257

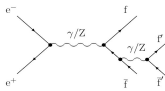
Light fermion pair production

Four-fermion final state (with at least one e^+e^- pair) may pass the event selection

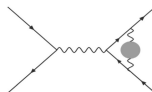
Conversion



Annihilation



Virtual correction at the same order
(interference with tree-level graph)



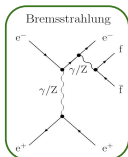
J. Gluza et al, arXiv:0807.4691
Phys. Rev. D 78 (2008) 085019

Specific four fermion
MC generators

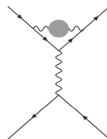
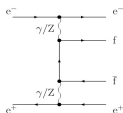
Positive correction

Negative correction

Dominant



Multiperipheral



Delicate cancelation w/o cuts, but **negative correction** when selection cuts are included
(smaller momentum and larger acoplanarity angle for the e^+e^- pair in the four-fermion final state)



Our result made its way to the Hall of Fame in 2023

Z HADRONIC POLE CROSS SECTION $41.4802 \pm 0.0325 \text{ nb}$

OUR EVALUATION is obtained using the fit procedure and correlations as determined by the LEP Electroweak Working Group (see the note "The Z boson" and ref. LEP-SLC 2006). Corrections as discussed in VOUTSINAS 2020 and JANOT 2020 are also included. This quantity is defined as

$$\sigma_h^0 = \frac{12\pi}{M_Z^2} \frac{\Gamma(e^+e^-) \Gamma(\text{hadrons})}{\Gamma_Z^2}$$

It is one of the parameters used in the Z lineshape fit.

VALUE [nb]	EVTS	DOCUMENT ID	TECN
41.4802 ± 0.0325		OUR EVALUATION	
41.4802 ± 0.0325		¹ JANOT 2020	
		² VOUTSINAS 2020	
		LEP-SLC 2006	
41.500 ± 0.037			
41.541 ± 0.037			
41.501 ± 0.055	4.10M	² ABBIDI 2001A	OPAL
41.578 ± 0.069	3.70M	ABREU 2000F	DUPH
41.535 ± 0.055	3.54M	ACCIAIARI 2000C	L3
41.559 ± 0.058	4.07M	⁴ BARATE 2000C	ALEP
42 ± 4	450	ABRAMS 1989B	MRK2

¹ JANOT 2020 applies a correction to LEP-SLC 2006 using an updated Bhhabha cross section calculation account for correlated luminosity bias as presented in VOUTSINAS 2020.

² VOUTSINAS 2020 applies a correction to LEP-SLC 2006 to account for correlated luminosity bias.

See <https://pdglive.lbl.gov/>

- [Number of neutrino types](#)
- [Z properties](#) (under Gauge and Higgs bosons)

Number of Light ν Types 2.996 ± 0.007
(already in the PDG in 2020)

VALUE	DOCUMENT ID	TECN
2.9963 ± 0.0074	¹ JANOT 2020	
2.9918 ± 0.0081	² VOUTSINAS 2020	
2.9840 ± 0.0082	³ LEP-SLC 2006	RVUE

• • We do not use the following data for averages, fits, limits, etc. • •

$E_{\text{CM}}^{\text{Z}} = 88 - 94 \text{ GeV}$

Z WIDTH $2.4955 \pm 0.0023 \text{ GeV}$

OUR EVALUATION is obtained using the fit procedure and correlations as determined by the LEP Electroweak Working Group (see the note "The Z boson" and ref. LEP-SLC 2006). Corrections as discussed in VOUTSINAS 2020 and JANOT 2020 are also included.

VALUE [GeV]	EVTS	DOCUMENT ID	TECN	COMMENT
2.4955 ± 0.0023		OUR EVALUATION		
2.4955 ± 0.0023		¹ JANOT 2020		
		² VOUTSINAS 2020		
		LEP-SLC 2006		
2.4955 ± 0.0023				
2.4952 ± 0.0023				

• • We do not use the following data for averages, fits, limits, etc. • •

$E_{\text{CM}}^{\text{Z}} = 88 - 94 \text{ GeV}$

$N_\nu = 2.9963 \pm 0.0074$ - *virtual trace of new physics*

Excellent example of connection between theory (NNLO plus BHLUMI) and experimental data analysis.

N_{eff} : LEP and Now

ALEPH, OPAL, L3, DELPHI, MARKII (SLC): $N_\nu = 3.12 \pm 0.19$

CERN, 13.10.1989, [Video](#) ($\sim 12,000$ Z decays)

[\[LEP, 2006\]](#) (~ 17 mln Z decays)

$$N_\nu = 2.9840 \pm 0.0082$$

Update: [\[P. Janot and S. Jadach, 2019\]](#) (only 1σ off from $N=3$)

$$N_\nu = 2.9963 \pm 0.0074$$

Theorem: [\[C. Jarlskog, 1990\]](#)

In the Standard Model with n left-handed lepton doublets and $N - n$ right-handed neutrinos, the effective number of neutrinos, N_ν , defined by

$$\Gamma(Z \rightarrow \nu' s) \equiv N_\nu \Gamma_0,$$

where Γ_0 is the standard width for one massless neutrino, satisfies

$$N_\nu \leq n.$$

$$\nu_\alpha^{(f)} = \underbrace{\left(V_{\text{osc}} \right)_{\alpha i} \nu_i^{(m)}}_{\text{SM part}} + \underbrace{\left(V_{lh} \right)_{\alpha j} \tilde{\nu}_j^{(m)}}_{\text{BSM part}}$$

Cosmology: $N_{\text{eff}} = 3.044$. J. Froustey, C. Pitrou, M. Volpe, [JCAP 12 \(2020\) 015](#),

J. Bennett, G. Buldgen, M. Drewes, Y. Wong, [JCAP 03 \(2020\) 003](#), [JCAP 03 \(2021\) A01](#)

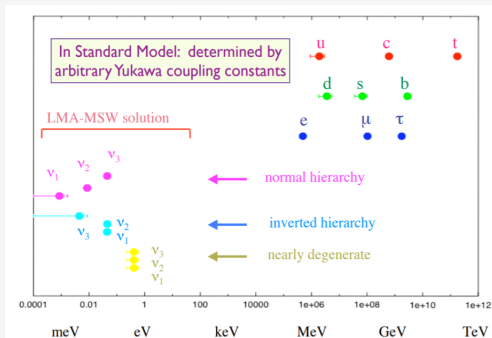
Neutrinos mix

Before oscillations era (PDG, 1996):

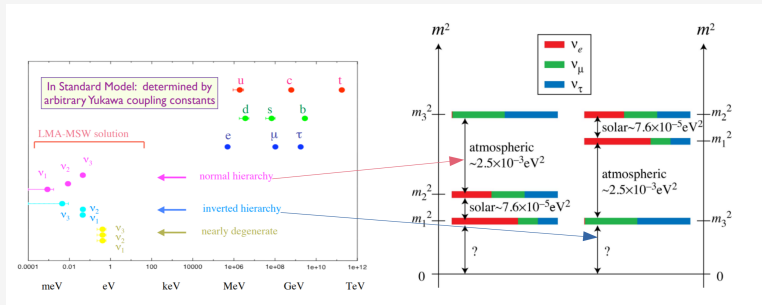
1. $m_{\nu_e} < 15 \text{ eV}$;
2. $m_{\nu_\mu} < 170 \text{ keV}$;
3. $m_{\nu_\tau} < 24 \text{ MeV}$;

$$U_{PMNS} \approx U_{TB} = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Quantum superposition.
$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{PMNS} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix},$$



Neutrino parameters and oscillations



L_e, L_μ, L_τ - lepton numbers broken!

Present main Issues

- **Neutrino Mixing:** Flavor eigenstates and mass eigenstates are related, Pontecorvo-Maki-Nakagawa-Sakata parametrization

$$U = \begin{bmatrix} C_{12}C_{13} & S_{12}C_{13} & S_{13}e^{-i\delta} \\ -S_{12}C_{23} - C_{12}S_{13}S_{23}e^{i\delta} & C_{12}C_{23} - S_{12}S_{13}S_{23}e^{i\delta} & C_{13}S_{23} \\ S_{12}S_{23} - C_{12}S_{13}C_{23}e^{i\delta} & C_{12}S_{23} - S_{12}S_{13}C_{23}e^{i\delta} & C_{13}C_{23} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_{21}/2} & 0 \\ 0 & 0 & e^{i\alpha_{31}/2} \end{bmatrix}$$

here $C_{ij} = \cos \theta_{ij}$ and $S_{ij} = \sin \theta_{ij}$.

- **Neutrino mixing parameters:** the known and unknowns

Three mixing angles: θ_{12} , θ_{23} and θ_{13}

Dirac CP-violating phase: δ_{CP}

Two mass squared differences: $\Delta m_{\odot}^2 = m_2^2 - m_1^2$, $\Delta m_A^2 = |m_3^2 - m_1^2|$

6 parameters involved in neutrino oscillation, still ambiguity over 3 parameters

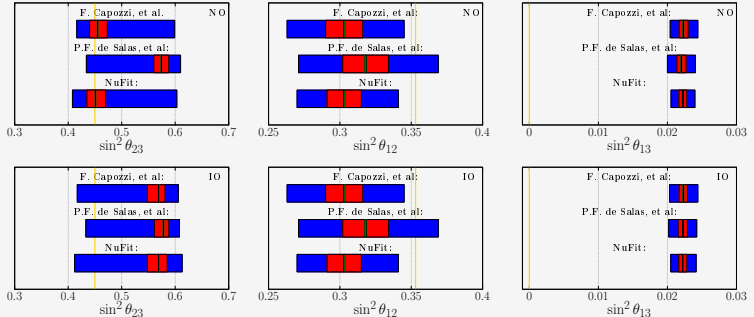
Two Majorana phase: α_{21} and α_{31} (Not sensitive to oscillation experiments)

Current main questions in neutrino oscillation physics:

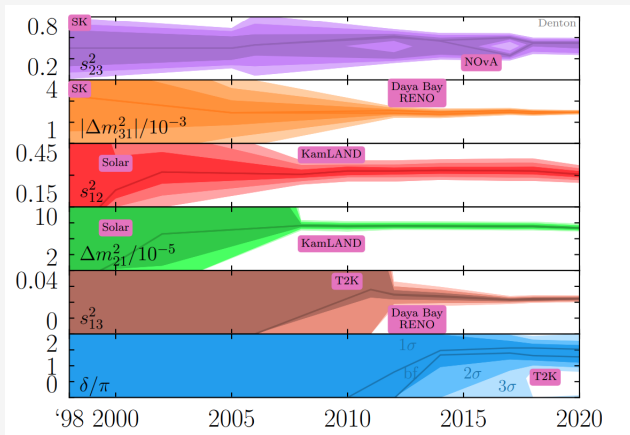
- (i) what is the mass ordering of the neutrinos (i.e sign of $|\Delta m_{32(1)}^2|$)
- (ii) what is the octant of θ_{23}
- (iii) is CP symmetry violated in the leptonic sector?

'Big' Data, Neutrino parameters at present, PPNP 138 (2024) 104126

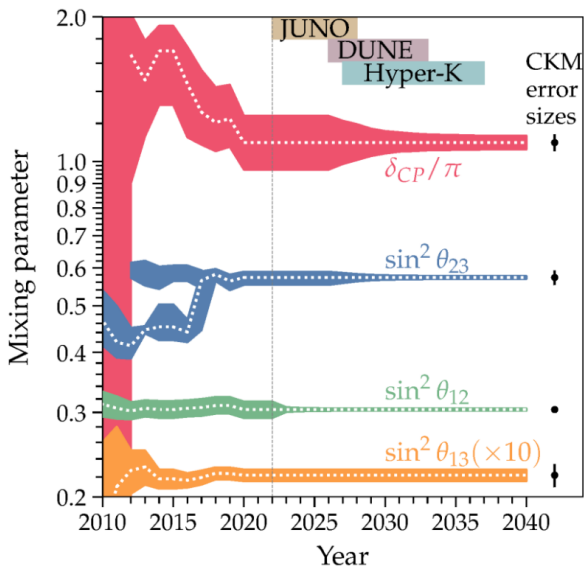
Parameter	Ordering	NuFit 5.2		de Salas et al.		Capozzi et al.	
		$\text{bf} \pm 1\sigma$	3σ range	$\text{bf} \pm 1\sigma$	3σ range	$\text{bf} \pm 1\sigma$	3σ range
$\sin^2 \theta_{12}/10^{-1}$	NO, IO	$3.03^{+0.12}_{-0.12}$	2.70 – 3.41	$3.18^{+0.16}_{-0.16}$	2.71 – 3.69	$3.03^{+0.13}_{-0.13}$	2.63 – 3.45
$\sin^2 \theta_{23}/10^{-1}$	NO	$4.51^{+0.19}_{-0.16}$	4.08 – 6.03	$5.74^{+0.14}_{-0.14}$	4.34 – 6.10	$4.55^{+0.18}_{-0.15}$	4.16 – 5.99
	IO	$5.69^{+0.16}_{-0.21}$	4.12 – 6.13	$5.78^{+0.10}_{-0.17}$	4.33 – 6.08	$5.69^{+0.12}_{-0.21}$	4.17 – 6.06
$\sin^2 \theta_{13}/10^{-2}$	NO	$2.225^{+0.056}_{-0.059}$	2.052 – 2.398	$2.200^{+0.069}_{-0.062}$	2.000 – 2.405	$2.23^{+0.07}_{-0.06}$	2.04 – 2.44
	IO	$2.223^{+0.058}_{-0.058}$	2.048 – 2.416	$2.225^{+0.064}_{-0.070}$	2.018 – 2.424	$2.23^{+0.06}_{-0.06}$	2.03 – 2.45
δ/π	NO	$1.29^{+0.20}_{-0.14}$	0.80 – 1.94	$1.08^{+0.13}_{-0.12}$	0.71 – 1.99	$1.24^{+0.18}_{-0.13}$	0.77 – 1.97
	IO	$1.53^{+0.12}_{-0.16}$	1.08 – 1.91	$1.58^{+0.15}_{-0.16}$	1.11 – 1.96	$1.52^{+0.15}_{-0.11}$	1.07 – 1.90
$\Delta m_{21}^2/10^{-5} \text{eV}^2$	NO, IO	$7.41^{+0.21}_{-0.20}$	6.82 – 8.03	$7.50^{+0.22}_{-0.20}$	6.94 – 8.14	$7.36^{+0.16}_{-0.15}$	6.93 – 7.93
$ \Delta m_{atm}^2 /10^{-3} \text{eV}^2$	NO	$2.507^{+0.026}_{-0.027}$	2.427 – 2.590	$2.55^{+0.02}_{-0.03}$	2.47 – 2.63	$2.485^{+0.023}_{-0.031}$	2.401 – 2.565
	IO	$2.486^{+0.028}_{-0.025}$	2.406 – 2.570	$2.45^{+0.02}_{-0.03}$	2.37 – 2.53	$2.455^{+0.030}_{-0.025}$	2.376 – 2.541



Neutrino parameters: Progress



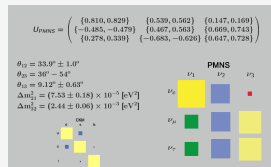
Neutrino parameters: Future



courtesy of Shirley Li

Neutrino Physics Enters Precision Era

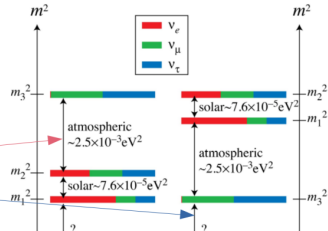
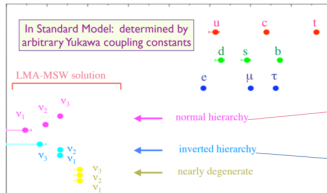
Super-K, Hyper-K, T2K, NOvA, Antares, KM3NeT, Juno, DUNE, SNO+, Daya Bay, Double Chooz, RENO, ...



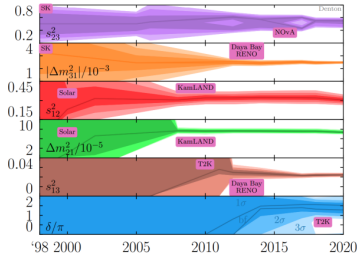
Conclusion: Neutrino Physics stepped in the precision era.

Till 2030: mass hierarchy (JUNO!), δ_{CP} (maybe), absolute masses, Majorana-Dirac, L. Wen, EPS2021.

Neutrino parameters and the known unknowns



We need a theory of flavor!



	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.6$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2 \theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.269 \rightarrow 0.343$	$0.304^{+0.012}_{-0.012}$	$0.269 \rightarrow 0.343$
θ_{12}°	$33.44^{+0.77}_{-0.74}$	$31.27 \rightarrow 35.86$	$33.45^{+0.77}_{-0.74}$	$31.27 \rightarrow 35.87$
$\sin^2 \theta_{23}$	$0.573^{+0.018}_{-0.023}$	$0.405 \rightarrow 0.620$	$0.578^{+0.017}_{-0.021}$	$0.410 \rightarrow 0.623$
θ_{23}°	$49.2^{+1.0}_{-1.3}$	$39.5 \rightarrow 52.0$	$49.5^{+1.0}_{-1.2}$	$39.8 \rightarrow 52.1$
$\sin^2 \theta_{13}$	$0.02220^{+0.00068}_{-0.00062}$	$0.02034 \rightarrow 0.02430$	$0.02238^{+0.00064}_{-0.00062}$	$0.02053 \rightarrow 0.02434$
θ_{13}°	$8.57^{+0.13}_{-0.12}$	$8.20 \rightarrow 8.97$	$8.60^{+0.12}_{-0.12}$	$8.24 \rightarrow 8.98$
δ_{CP}°	194^{+52}_{-25}	$105 \rightarrow 405$	287^{+27}_{-32}	$192 \rightarrow 361$
$\frac{\Delta m_{21}^2}{10^{-5} \text{eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \rightarrow 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \rightarrow 8.04$
$\frac{\Delta m_{3l}^2}{10^{-3} \text{eV}^2}$	$+2.515^{+0.028}_{-0.028}$	$+2.431 \rightarrow +2.599$	$-2.498^{+0.028}_{-0.029}$	$-2.584 \rightarrow -2.413$

The goal of many theoretical approaches is to reconcile oscillation parameters with DM, leptogenesis (matter-antimatter), total mass limits (cosmology), LFV.

- ▶ "Phenomenology of the flavor symmetric scoto-seesaw model with dark matter and TM_1 mixing", Joy Ganguly, JG, Biswajit Karmakar, Satyabrata Mahapatra, Phys.Rev.D 110 (2024) 3, 035012, [2311.15997 \[hep-ph\]](#)
- ▶ "Common origin of Θ_{13} and dark matter within the flavor symmetric scoto-seesaw framework", Joy Ganguly, JG, Biswajit Karmakar JHEP 11 (2022) 074, [2209.08610 \[hep-ph\]](#)
- ▶ In nutshell: "Phenomenology of Flavor Symmetries: An Example", JG, Biswajit Karmakar Acta Phys.Polon.Supp. 18 (2025) 6, 6-A20, [pdf](#)

General Framework

Anarchy

- ▶ Neutrino mixing anarchy is the hypothesis that the leptonic mixing matrix can be described as the result of a random draw from an unbiased distribution of unitary 3×3 matrices.
- ▶ Random analysis **without imposing prior theories or symmetries** on the mass and mixing matrices.
- ▶ This hypothesis **does not make any correlation among the neutrino masses and mixing parameters**

de Gouvea, Haba, Hall, Murayama : 9911341, 0009174, 1204.1249

Texture

- ▶ More specific studies with imposed mass or mixing textures for which models with underlying symmetries can be sought.
- ▶ It's an intermediate approach
- ▶ Some texture zeros of neutrino mass matrices can be eliminated.

Alejandro Ibarra, Graham Ross: Phys.Lett.B 2003

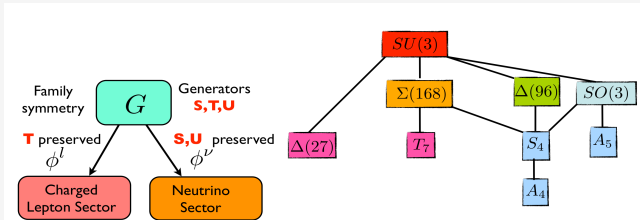
Symmetry

- ▶ Theoretical studies where some **explicit symmetries at the Yukawa Lagrangian** level are assumed and corresponding **extended particle sector is defined**.
- ▶ The symmetry-based approach to explain the non-trivial mixing in the lepton sector known as **family symmetry or horizontal symmetry**

Reviews: Tanimoto *et.al.* 1003.3552, Altarelli, Feruglio 1002.0211, King 1301.1340

General Framework: Symmetry based approach

- ▶ Fundamental symmetry in the lepton sector can easily explain the origin of neutrino mixing which is considerably different from quark mixing.
- ▶ Incidentally, both **Abelian or non-Abelian family symmetries** have potential to shade light on the Yukawa couplings.
- ▶ The Abelian symmetries (such as Froggatt-Nielsen symmetry) only points towards a **hierarchical** structure of the Yukawa **charged lepton** couplings.
- ▶ Non-Abelian symmetries are more equipped to explain the **non-hierarchical** structures of the observed **neutrino** mixing as observed by the oscillation experiments.



S. F. King 1301.1340

$$G \rightarrow G_e, G_\nu \text{ typically, } G_e = Z_3 \text{ and } G_\nu = Z_2 \times Z_2.$$

MIXING

Flavor symmetries, why?

$$U_{PMNS} = \begin{pmatrix} C_{12}C_{13} & S_{12}C_{13} & S_{13}e^{-i\delta} \\ -S_{12}C_{23} - C_{12}S_{13}S_{23}e^{i\delta} & C_{12}C_{23} - S_{12}S_{13}S_{23}e^{i\delta} & C_{13}S_{23} \\ S_{12}S_{23} - C_{12}S_{13}C_{23}e^{i\delta} & -C_{12}S_{23} - S_{12}S_{13}C_{23}e^{i\delta} & C_{13}C_{23} \end{pmatrix}$$

↓

(Prior to 2012)

$$s_{23} = 1/\sqrt{2} \ (\theta_{23} = 45^\circ) \text{ and } \theta_{13} = 0$$

↓

$$U_0 = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

$$\begin{array}{llll} \theta_{12} = 45^\circ (s_{12} = 1/\sqrt{2}) & \theta_{12} = 35.26^\circ (s_{12} = 1/\sqrt{3}) & \theta_{12} = 31.7^\circ & \theta_{12} = 30^\circ (s_{12} = 1/2) \\ \text{Bimaximal Mixing} & \text{Tribimaximal Mixing} & \text{Golden Ratio Mixing} & \text{Hexagonal Mixing} \end{array}$$

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{\sqrt{2}}{3} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{\varphi}{\sqrt{2+\varphi}} & \frac{1}{\sqrt{2+\varphi}} & 0 \\ \frac{-1}{\sqrt{4+2\varphi}} & \frac{\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{4+2\varphi}} & \frac{-\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{\sqrt{3}}{4} & \frac{1}{2} & 0 \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Fukugita, Tanimoto, Yanagida PRD98; Harrison Perkins, Scott PLB02; Dutta, Ramond NPB03; Rodejohann et. al. EPJC10

$$(\text{GR: } \tan \theta_{12} = 1/\phi \text{ where } \phi = (1 + \sqrt{5})/2)$$

Flavor symmetries, why?

Simple example: $\mu - \tau$ permutation symmetry and TBM

$$m_\nu = U_0^\star \text{diag}(m_1, m_2, m_3) U_0^\dagger,$$

such a mixing matrices can easily diagonalize a $\mu - \tau$ symmetric (transformations $\nu_e \rightarrow \nu_e$, $\nu_\mu \rightarrow \nu_\tau$, $\nu_\tau \rightarrow \nu_\mu$ under which the neutrino mass term remains unchanged) neutrino mass matrix of the form

$$m_\nu = \begin{pmatrix} A & B & B \\ B & C & D \\ B & D & C \end{pmatrix},$$

With $A + B = C + D$ this matrix yields tribimaximal mixing pattern where $s_{12} = 1/\sqrt{3}$ i.e., $\theta_{12} = 35.26^\circ$

• Compatible vs Observed Mixing Matrix :

$$U_{\text{TBM}} \simeq \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$U_{\text{PMNS}} \simeq \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \epsilon \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} (?) \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} (?) \end{pmatrix}$$

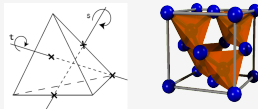
```
in[688]:= Sin[8.6*Pi / 180]
          Sin[8.6*Pi / 180] ^ 2
Out[688]= 0.149535
Out[689]= 0.0223608
```

Another example: A_4 Discrete Symmetry

A_4 is the alternating group of degree 4, which is the group of even permutations of four elements. It has 12 elements and can be seen as a subgroup of the symmetric group S_4 , which includes all permutations of four elements. Geometrically, it is a symmetry group of a tetrahedron.

A_4 Symmetry in Diamond Lattice Structure

- ▶ The diamond lattice structure is a specific arrangement of carbon atoms in a crystalline form, exhibiting remarkable symmetry and unique physical properties. One interesting aspect of this structure is its connection to the mathematical concept of A_4 symmetry, which can be understood through group theory.
- ▶ The diamond lattice is a variation of the face-centered cubic (FCC) structure. Each carbon atom in a diamond lattice is tetrahedrally coordinated, bonded to four other carbon atoms

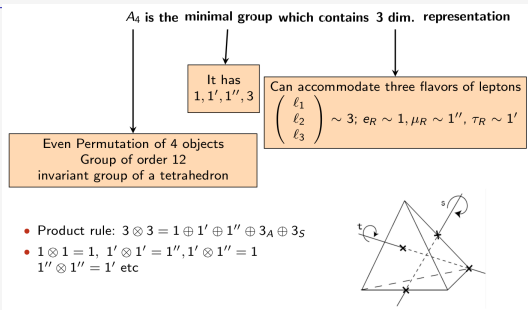


A_4 Symmetry as a tetrahedron and the diamond lattice

A_4 Symmetry in Neutrino Physics

A_4 can be considered to be a symmetry in the neutrino sector. This symmetry in the Lagrangian can explain the observed neutrino mixing.

Constructions with A_4 , step 1: Yukawa Lagrangian



- SM scalars (flavons) introduced (with wished "flavor" VEV directions), e.g.:
 $\langle \phi_S \rangle = v_S (1, 1, 1)^T$ $\langle \xi \rangle = v_\xi$, $\langle \phi_T \rangle = v_T (1, 0, 0)^T$

$$\begin{aligned} \mathcal{L}_l &= \frac{y_e}{\Lambda} (\bar{L} \phi_T)_1 H e_R + \frac{y_\mu}{\Lambda} (\bar{L} \phi_T)_{1'} H \mu_R + \frac{y_\tau}{\Lambda} (\bar{L} \phi_T)_{1''} H \tau_R + h.c. \\ &= \frac{y_e}{\Lambda} (\bar{L}_1 \phi_{T_1} + \bar{L}_2 \phi_{T_3} + \bar{L}_3 \phi_{T_2}) H e_R + \frac{y_\mu}{\Lambda} (\bar{L}_3 \phi_{T_3} + \bar{L}_1 \phi_{T_2} + \bar{L}_2 \phi_{T_1}) H \mu_R + \dots \end{aligned}$$

$$1 \rightarrow (S = 1, T = 1), 1' \rightarrow (S = 1, T = \omega), 1'' \rightarrow (S = 1, T = \omega^2)$$

$$1 \sim x_1 y_1 + x_2 y_3 + x_3 y_2, 1' \sim x_3 y_3 + x_1 y_2 + x_2 y_1, \dots$$

Non-zero θ_{13}

	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.6$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2 \theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.269 \rightarrow 0.343$	$0.304^{+0.012}_{-0.012}$	$0.269 \rightarrow 0.343$
$\theta_{12}/^\circ$	$33.44^{+0.77}_{-0.74}$	$31.27 \rightarrow 35.86$	$33.45^{+0.77}_{-0.74}$	$31.27 \rightarrow 35.87$
$\sin^2 \theta_{23}$	$0.573^{+0.018}_{-0.023}$	$0.405 \rightarrow 0.620$	$0.578^{+0.017}_{-0.021}$	$0.410 \rightarrow 0.623$
$\theta_{23}/^\circ$	$49.2^{+1.0}_{-1.3}$	$39.5 \rightarrow 52.0$	$49.5^{+1.0}_{-1.2}$	$39.8 \rightarrow 52.1$
$\sin^2 \theta_{13}$	$0.02220^{+0.00068}_{-0.00062}$	$0.02034 \rightarrow 0.02430$	$0.02238^{+0.00064}_{-0.00062}$	$0.02053 \rightarrow 0.02434$
$\theta_{13}/^\circ$	$8.57^{+0.13}_{-0.12}$	$8.20 \rightarrow 8.97$	$8.60^{+0.12}_{-0.12}$	$8.24 \rightarrow 8.98$
$\delta_{CP}/^\circ$	194^{+52}_{-25}	$105 \rightarrow 405$	287^{+27}_{-32}	$192 \rightarrow 361$
$\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \rightarrow 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \rightarrow 8.04$
$\frac{\Delta m_{3\ell}^2}{10^{-3} \text{ eV}^2}$	$+2.515^{+0.028}_{-0.028}$	$+2.431 \rightarrow +2.599$	$-2.498^{+0.028}_{-0.029}$	$-2.584 \rightarrow -2.413$

Bimaximal Mixing

Tribimaximal Mixing

Golden Ratio Mixing

Hexagonal Mixing

$$U_0 = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{\varphi}{\sqrt{2+\varphi}} & \frac{-\varphi}{\sqrt{2+\varphi}} & 0 \\ \frac{-1}{\sqrt{4+2\varphi}} & \frac{\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{4+2\varphi}} & \frac{-\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \sqrt{\frac{3}{4}} & \frac{1}{2} & 0 \\ -\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Decadents of fixed pattern mixing schemes

Non-zero θ_{13} : Dependents of tribimaximal mixing

$$U_{TBM} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad U_{PMNS} \simeq \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \epsilon \simeq 0.15 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} (?) \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} (?) \end{pmatrix}$$



$$|U_{TM1}| = \begin{pmatrix} \frac{2}{\sqrt{6}} & * & * \\ \frac{1}{\sqrt{6}} & * & * \\ \frac{1}{\sqrt{6}} & * & * \end{pmatrix}$$

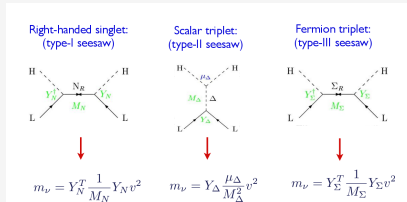
$$|U_{TM2}| = \begin{pmatrix} * & \frac{1}{\sqrt{3}} & * \\ * & \frac{1}{\sqrt{3}} & * \\ * & \frac{1}{\sqrt{3}} & * \end{pmatrix},$$

On top of the mixing issue:

MASS

Neutrino Mass Generation

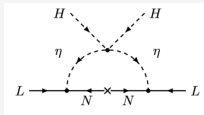
Seesaw frameworks



• Type-I Seesaw, Type-II Seesaw, Type-III Seesaw, ...

Minkowski 77; Gellman, Ramond, Slansky 80; Glashow, Yanagida 79; Mohapatra, Senjanovic 80; Lazarides, Shafi; Schechter, Valle 81; Schechter, Valle 80; Mohapatra, Senjanovic 81; Lazarides, Shafi, Wetterich 81; Mohapatra Valle 86; Foot, Lew, He, Joshi 89; Ma 98; Bajc, Senjanovic 07....

Radiative neutrino mass



• Radiative models

started in 80s: Zee 80, Cheng, Li 80; Zee 86; Babu 88; Babu, Ma, Valle, 02; Ma 06;

• For a review of radiative models: Cai, Herrero-Garcia, Schmidt, Vicente, Volkas 17;

Hybrid Scenarios??

Flavor Symmetry and Hybrid Mass Mechanisms: Why?

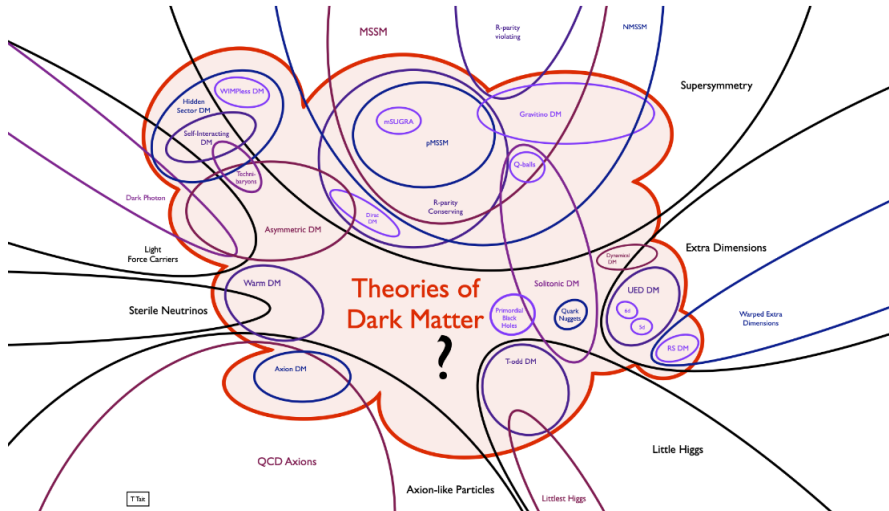
- Ratio of solar to atmospheric mass difference :

$$r = \frac{\Delta m_{\text{SOL}}^2}{\Delta m_{\text{ATM}}^2} \simeq \frac{7.41 \times 10^{-5} \text{ eV}^2}{2.51 \times 10^{-3} \text{ eV}^2} \simeq 3 \times 10^{-2}$$

- Two different mass scales that might originate from **entirely separate mechanisms !**

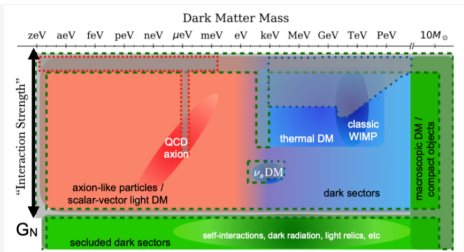
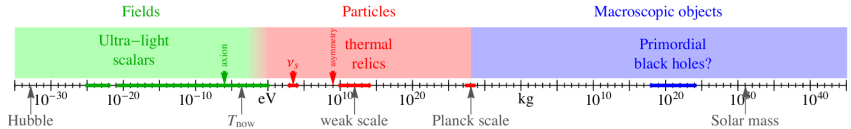
- Minimal scoto-seesaw scenario: **Rojas, Srivastava, Valle 1807.11447**
- Minimal **Scoto Seesaw** scenario: **Rojas, Srivastava, Valle 1807.11447**
Greek word 'skótos' → 'darkness'
- Seesaw Analog : Sequential dominance: **Antusch, King 0405272**
- Scoto-seesaw scenarios : **Accommodates DM candidates**
- Flavor symmetric scoto-seesaw / discrete dark matter scenarios:
 - Observed neutrino mixing**
 - Hierarchy of neutrino masses**
 - Accommodates DM candidates**
 - Low Energy Signatures**

BSM - terra incognita: DM & RHN, ...



BSM - terra incognita: DM & RHN, ...

- $10^{-22} \text{ eV} \lesssim m_\chi \lesssim 1 \text{ eV}$: “Ultralight” Dark Matter
- $1 \text{ eV} \lesssim m_\chi \lesssim 1 \text{ GeV}$: “Light” particle Dark Matter
- $1 \text{ GeV} \lesssim m_\chi \lesssim 100 \text{ TeV}$: “Heavy” particle Dark Matter
- $m_\chi \gtrsim 100 \text{ TeV}$: “Ultra-Heavy” Dark Matter (UHDM)



Flavor Symmetry and Hybrid Mass Mechanisms: Why?

- Minimal **Scoto** **Seesaw** scenario:

Rojas, Srivastava, Valle 1807.11447

$$\frac{\Delta m_{\text{SOL}}^2}{\Delta m_{\text{ATM}}^2} \sim \left(\frac{1}{32\pi^2} \right)^2 \lambda_5^2 \left(\frac{M_N M_f}{M_f^2 - m_{\eta}^{(R)2}} \right)^2 \left(\frac{\mathbb{Y}_{(f)}^2}{\mathbb{Y}_{(N)}^2} \right)^2$$

$$\mathcal{L} = -Y_N^k \bar{L}^k i\sigma_2 H^* N_R + \frac{1}{2} M_R \bar{N}_R^c N_R + Y_f^k \bar{L}^k i\sigma_2 \eta^* f + \frac{1}{2} M_f \bar{f}^c f$$

The number of right-handed neutrinos added to the SM is not fixed as they do not carry any anomaly

Schechter, Valle 1980

- The total neutrino mass reads:

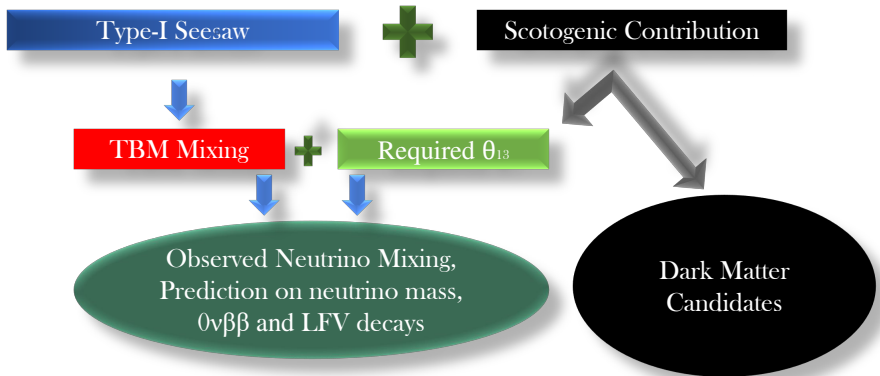
$$M_\nu^{ij} = -\frac{v^2}{M_N} Y_N^i Y_N^j + \mathcal{F}(M_{\eta_R}, M_{\eta_I}, M_f) M_f Y_f^i Y_f^j.$$

where

$$\mathcal{F}(M_{\eta_R}, M_{\eta_I}, M_f) = \frac{1}{32\pi^2} \left[\frac{M_{\eta_R}^2 \log(M_f^2/M_{\eta_R}^2)}{M_f^2 - M_{\eta_R}^2} - \frac{M_{\eta_I}^2 \log(M_f^2/M_{\eta_I}^2)}{M_f^2 - M_{\eta_I}^2} \right],$$

where M_{η_R} and M_{η_I} are the masses of the neutral component of η .

Standard Model with A_4 discrete flavor symmetry



TM₂ mixing

$$U_{TBM} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad U_{PMNS} \simeq \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \epsilon \simeq 0.15 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} (?) \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} (?) \end{pmatrix}$$



$$|U_{TM_1}| = \begin{pmatrix} \frac{2}{\sqrt{6}} & * & * \\ \frac{1}{\sqrt{6}} & * & * \\ \frac{1}{\sqrt{6}} & * & * \end{pmatrix}$$

$$|U_{TM_2}| = \begin{pmatrix} * & \frac{1}{\sqrt{3}} & * \\ * & \frac{1}{\sqrt{3}} & * \\ * & \frac{1}{\sqrt{3}} & * \end{pmatrix},$$

Step 2, choice of neutrino mixing, Flavor symmetric scoto-seesaw: TM_2 mixing

Type-I Seesaw contribution: Z_8 : Barreiros, Joaquim, Srivastava, Valle 2012.05189, E.g. A_4 : 2209.08610

$$\mathcal{L}_{\text{TREE}} = \frac{y_{N1}}{\Lambda} (\bar{L} \phi_s) \tilde{H} N_{R1} + \frac{y_{N2}}{\Lambda} (\bar{L} \phi_a) \tilde{H} N_{R2} + \frac{1}{2} M_{N1} \bar{N}_{R1}^c N_{R1} + \frac{1}{2} M_{N2} \bar{N}_{R2}^c N_{R2} + h.c.,$$

• L , ϕ_a and $\phi_s \rightarrow A_4$ triplets; H , N_{R1} , $N_{R2} \rightarrow A_4$ singlets

• A_4 multiplication rules: If we have two triplets (a_1, a_2, a_3) and (b_1, b_2, b_3) , their products are given by
 $\Rightarrow 3 \otimes 3 = 1 + 1' + 1'' + 3_A + 3_S$

$$\begin{aligned} 1 &\sim a_1 b_1 + a_2 b_3 + a_3 b_2, 1' \sim a_3 b_3 + a_1 b_2 + a_2 b_1, 1'' \sim a_2 b_2 + a_3 b_1 + a_1 b_3, \\ 3_S &\sim \begin{bmatrix} 2a_1 b_1 - a_2 b_3 - a_3 b_2 \\ 2a_3 b_3 - a_1 b_2 - a_2 b_1 \\ 2a_2 b_2 - a_1 b_3 - a_3 b_1 \end{bmatrix}, 3_A \sim \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_1 b_2 - a_2 b_1 \\ a_3 b_1 - a_1 b_3 \end{bmatrix}. \end{aligned}$$

• Flavan fields get VEVs along $\langle \phi_s \rangle = (0, v_s, -v_s)$, $\langle \phi_a \rangle = (v_a, v_a, v_a)$

$$\begin{aligned} \frac{y_{N1}}{\Lambda} (\bar{L} \phi_s)_1 \tilde{H} N_{R1} &= \frac{y_{N1}}{\Lambda} (\bar{L}_1 \phi_{s1} + \bar{L}_2 \phi_{s3} + \bar{L}_3 \phi_{s2})_1 \tilde{H} N_{R1} = \frac{y_{N1}}{\Lambda} (0 - \bar{L}_2 v_s + \bar{L}_3 v_s)_1 \tilde{H} N_{R1} \\ \frac{y_{N2}}{\Lambda} (\bar{L} \phi_a)_1 \tilde{H} N_{R2} &= \frac{y_{N2}}{\Lambda} (\bar{L}_1 \phi_{a1} + \bar{L}_2 \phi_{a3} + \bar{L}_3 \phi_{a2})_1 \tilde{H} N_{R2} = \frac{y_{N2}}{\Lambda} (\bar{L}_1 v_a + \bar{L}_2 v_a + \bar{L}_3 v_a)_1 \tilde{H} N_{R2} \end{aligned}$$

• Dirac neutrino mass matrix :

$$M_D = \frac{v}{\Lambda} \begin{pmatrix} 0 & y_{N2} v_a \\ -y_{N1} v_s & y_{N2} v_a \\ y_{N1} v_s & y_{N2} v_a \end{pmatrix} = v Y_N, \quad M_R = \begin{pmatrix} M_{N1} & 0 \\ 0 & M_{N2} \end{pmatrix}.$$

Step 2, choice of neutrino mixing, Flavor symmetric scoto-seesaw: TM_2 mixing

Scotogenic contribution:

For more see 2209.08610

$$\begin{aligned}\mathcal{L}_{\text{LOOP}} &= \frac{y_s}{\Lambda^2} (\bar{L} \phi_s) \xi i \sigma_2 \eta^* f + \frac{1}{2} M_f \bar{f}^c f + h.c., \\ (M_\nu)_{\text{LOOP}} &= \mathcal{F}(m_{\eta_R}, m_{\eta_I}, M_f) M_f Y_f^i Y_f^j, \\ Y_F &= (Y_F^e, Y_F^\mu, Y_F^\tau)^T = (y_s \frac{v_s}{\Lambda} \frac{v_\xi}{\Lambda}, 0, -y_s \frac{v_s}{\Lambda} \frac{v_\xi}{\Lambda})^T.\end{aligned}$$

Therefore, the corresponding mass matrix takes the form

$$(M_\nu)_{\text{LOOP}} = C \begin{pmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{pmatrix}, \quad C = \mathcal{F}(m_{\eta_R}, m_{\eta_I}, M_f) y_s^2 \frac{v_s^2 v_\xi^2}{\Lambda^4}.$$

Here $\mathcal{F}(m_{\eta_R}, m_{\eta_I}, M_f)$ is the loop function

Step 2, choice of neutrino mixing, Flavor symmetric scoto-seesaw: TM_2 mixing

► Effective neutrino mass matrix:

$$\begin{aligned} M_\nu &= -M_D M_R^{-1} M_D^T + (M_\nu)_{\text{LOOP}} \\ &= (M_\nu)_{\text{TREE}} + (M_\nu)_{\text{LOOP}} \\ &= \begin{pmatrix} -B+C & -B & -B-C \\ -B & -A-B & A-B \\ -B-C & A-B & -A-B+C \end{pmatrix}, A = \frac{v^2 v_s^2 y_{N_1}^2}{\Lambda^2 M_{N_1}}, B = \frac{v^2 v_a^2 y_{N_2}^2}{\Lambda^2 M_{N_2}}. \end{aligned}$$

► After rotation by TBM matrix:

$$\begin{aligned} M'_\nu &= U_{TB}^T M_\nu U_{TB} \\ &= \frac{1}{2} \begin{pmatrix} 3C & 0 & -\sqrt{3}C \\ 0 & -6B & 0 \\ -\sqrt{3}C & 0 & -4A+C \end{pmatrix}. \end{aligned}$$

Flavor symmetric scoto-seesaw : TM_2 mixing

■ Effective neutrino mixing matrix (TM_2 mixing):

$$U_\nu = U_{TB} U_{13} = \begin{pmatrix} \sqrt{\frac{2}{3}} \cos \theta & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}} e^{i\phi} \sin \theta \\ -\frac{\cos \theta}{\sqrt{6}} + \frac{e^{i\phi} \sin \theta}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -\frac{\cos \theta}{\sqrt{2}} - \frac{e^{i\phi} \sin \theta}{\sqrt{6}} \\ -\frac{\cos \theta}{\sqrt{6}} - \frac{e^{i\phi} \sin \theta}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{\cos \theta}{\sqrt{2}} - \frac{e^{i\phi} \sin \theta}{\sqrt{6}} \end{pmatrix} U_m.$$

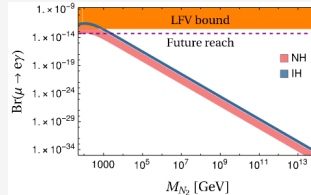
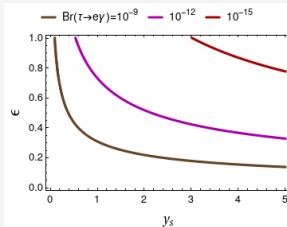
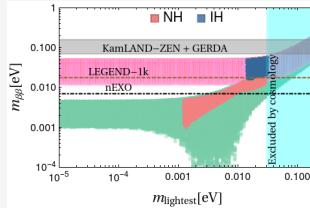
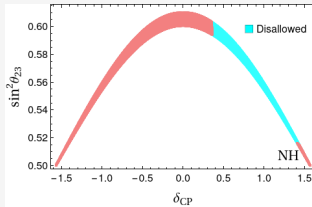
■ Corelations, comparing with U_{PMNS}

$$\sin \theta_{13} e^{-i\delta_{CP}} = \sqrt{\frac{2}{3}} e^{-i\phi} \sin \theta, \quad \tan^2 \theta_{12} = \frac{1}{2 - 3 \sin^2 \theta_{13}},$$
$$\tan^2 \theta_{23} = \frac{\left(1 + \frac{\sin \theta_{13} \cos \phi}{\sqrt{2 - 3 \sin^2 \theta_{13}}}\right)^2 + \frac{\sin^2 \theta_{13} \sin^2 \phi}{(2 - 3 \sin^2 \theta_{13})}}{\left(1 - \frac{\sin \theta_{13} \cos \phi}{\sqrt{2 - 3 \sin^2 \theta_{13}}}\right)^2 + \frac{\sin^2 \theta_{13} \sin^2 \phi}{(2 - 3 \sin^2 \theta_{13})}}.$$

For more see 2209.08610

Flavor symmetric scoto-seesaw : TM_2 mixing

■ Predictions:



TM₁ mixing

$$U_{TBM} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad U_{PMNS} \simeq \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \epsilon \simeq 0.15 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} (?) \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} (?) \end{pmatrix}$$



$$|U_{TM_1}| = \begin{pmatrix} \frac{2}{\sqrt{6}} & * & * \\ \frac{1}{\sqrt{6}} & * & * \\ \frac{1}{\sqrt{6}} & * & * \end{pmatrix}$$

$$|U_{TM_2}| = \begin{pmatrix} * & \frac{1}{\sqrt{3}} & * \\ * & \frac{1}{\sqrt{3}} & * \\ * & \frac{1}{\sqrt{3}} & * \end{pmatrix},$$

Flavor symmetric scoto-seesaw : TM_1 mixing

- Contributions to the neutrino mass:

2311.15997

$$\begin{aligned}\mathcal{L} &= \frac{y_N}{\Lambda} (\bar{L} \phi_s) \bar{H} N_R + \frac{1}{2} M_N \bar{N}_R^c N_R + \frac{y_s}{\Lambda^2} (\bar{L} \phi_a) \xi i \sigma_2 \eta^* f + \frac{1}{2} M_f \bar{f}^c f + \text{h.c.}, \\ M_\nu &= -\frac{v^2}{M_N} Y_N^i Y_N^j + \mathcal{F}(m_{\eta_R}, m_{\eta_I}, M_f) M_f Y_f^i Y_f^j\end{aligned}$$

$$Y_N = (Y_N^e, Y_N^\mu, Y_N^\tau)^T = (0, y_N \frac{v_s}{\Lambda}, -y_N \frac{v_s}{\Lambda})^T, ; Y_F = (Y_F^e, Y_F^\mu, Y_F^\tau)^T = (y_s \frac{v_\xi}{\Lambda} \frac{v_a}{\Lambda}, y_s \frac{v_\xi}{\Lambda} \frac{2v_a}{\Lambda}, 0)^T$$

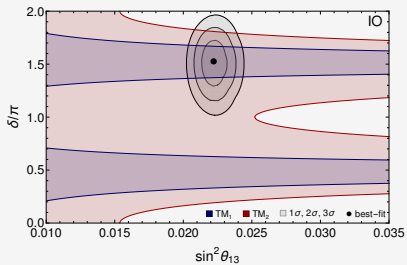
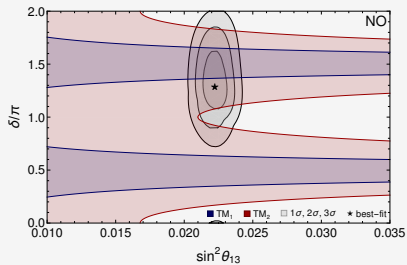
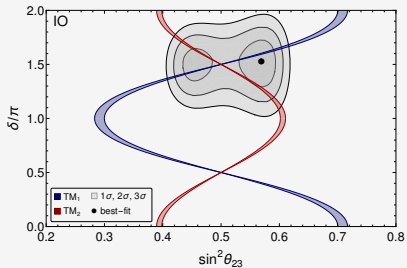
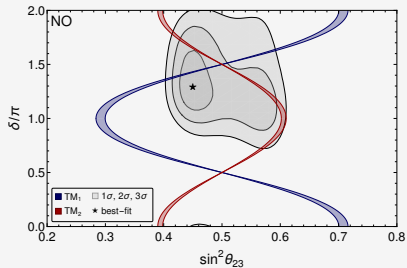
- Light neutrino mass matrix :

$$M_\nu = \begin{pmatrix} b & 2b & 0 \\ 2b & -a + 4b & a \\ 0 & a & -a \end{pmatrix}, a = y_N^2 \frac{v^2}{M_N} \frac{v_s^2}{\Lambda^2}, b = y_s^2 \frac{v_\xi^2}{\Lambda^2} \frac{v_a^2}{\Lambda^2} \mathcal{F}(m_{\eta_R}, m_{\eta_I}, M_f) M_f = \kappa^2 \mathcal{F}(m_{\eta_R}, m_{\eta_I})$$

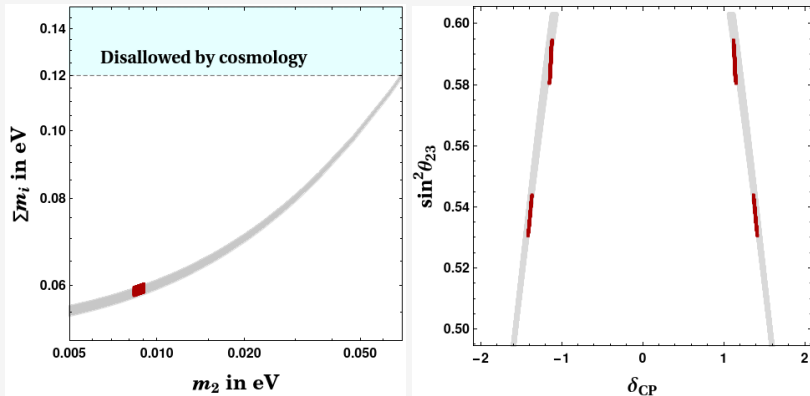
- Diagonalizing matrix : TM_1 mixing

$$U_\nu = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{\cos \theta}{\sqrt{3}} & \frac{e^{-i\psi} \sin \theta}{\sqrt{3}} \\ -\frac{1}{\sqrt{6}} & \frac{\cos \theta}{\sqrt{3}} + \frac{e^{i\psi} \sin \theta}{\sqrt{2}} & -\frac{\cos \theta}{\sqrt{2}} + \frac{e^{-i\psi} \sin \theta}{\sqrt{3}} \\ -\frac{1}{\sqrt{6}} & \frac{\cos \theta}{\sqrt{3}} - \frac{e^{i\psi} \sin \theta}{\sqrt{2}} & \frac{\cos \theta}{\sqrt{2}} + \frac{e^{-i\psi} \sin \theta}{\sqrt{3}} \end{pmatrix} U_m$$

TM_1 vs TM_2 , 2310.20681 (PPNP)



Flavor symmetric scoto-seesaw : TM_1 mixing



m_2 (meV)	m_3 (meV)	$\sum m_i$ (meV)	$m_{\beta\beta}$ (meV)
8.3 – 9.0	49.7 – 51.3	58.0 – 60.3	1.61 – 3.85

Flavor symmetric scoto-seesaw : TM_1 mixing

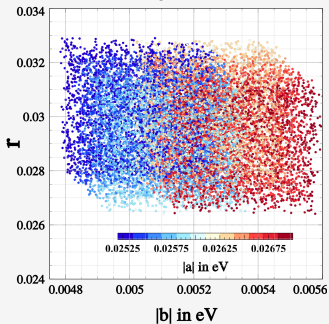
- Mass eigenvalues:

2311.15997

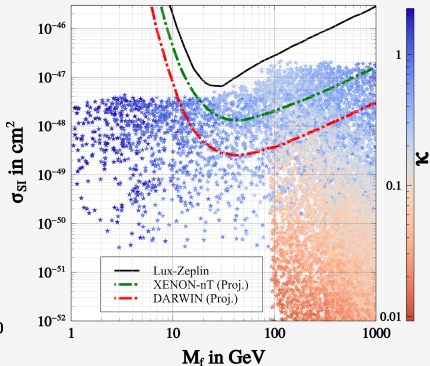
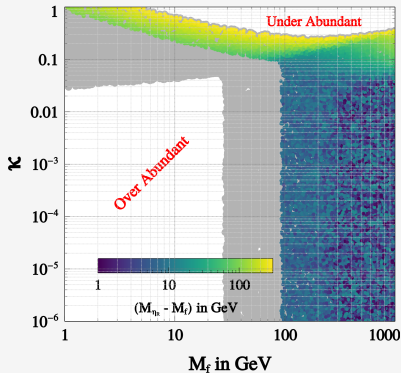
$$\begin{aligned}\widetilde{m}_1 &= 0, \\ \widetilde{m}_2 &= \frac{1}{2} \left(-2a + 5b - \sqrt{4a^2 + 4ab + 25b^2} \right), \\ \widetilde{m}_3 &= \frac{1}{2} \left(-2a + 5b + \sqrt{4a^2 + 4ab + 25b^2} \right).\end{aligned}$$

- Ratio of the solar to atmospheric mass-squared differences:

$$r \sim \frac{m_2^2}{m_3^2}$$



FSS₁ phenomenology: Dark Matter

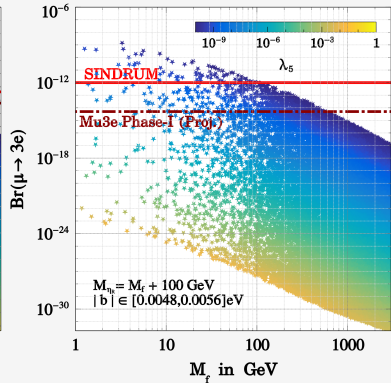
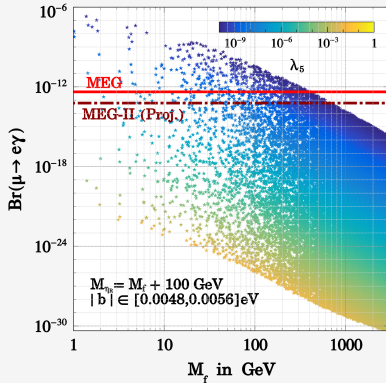


- Neutrino mixing dependence of dark matter phenomenology :

$$\Rightarrow \kappa^2 = \frac{|b|}{\mathcal{F}(M_{\eta_R}, M_{\eta_I}, M_f) M_f}$$

FSS₁ phenomenology: Lepton Flavor Violation

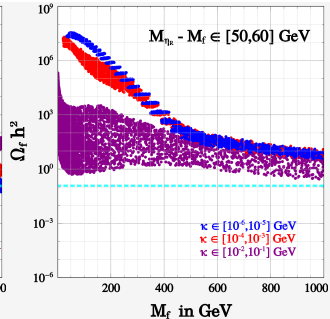
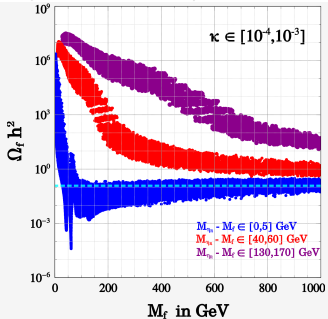
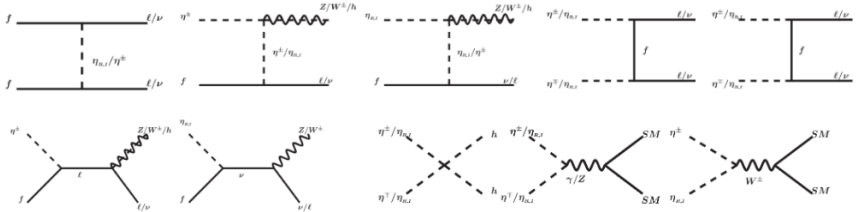
Decay Modes	Scotogenic contribution	Seesaw Contribution	Remarks
$\mu \rightarrow e \gamma$	✓	55	$Y_N^e = 0$
$\tau \rightarrow e \gamma$	55	55	$Y_F^e = 0, Y_N^e = 0$
$\tau \rightarrow \mu \gamma$	55	✓	$Y_F^e = 0$
$\mu \rightarrow 3e$	✓	55	$Y_N^e = 0$
$\tau \rightarrow 3e$	55	55	$Y_F^e = 0, Y_N^e = 0$
$\tau \rightarrow 3\mu$	55	✓	$Y_F^e = 0$

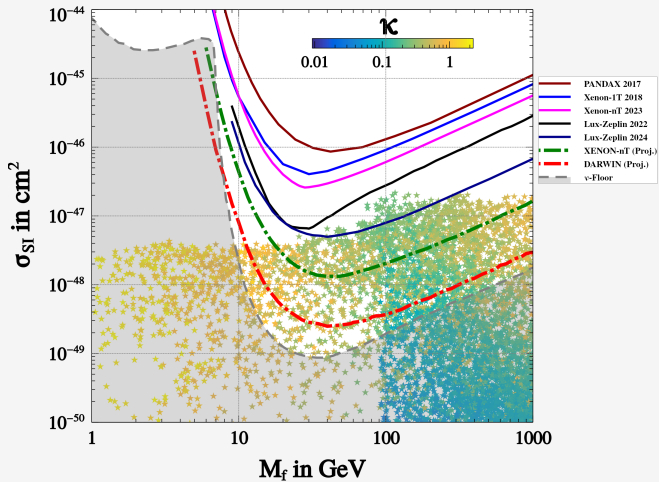


λ_5 is the coupling for the interaction $(H^\dagger \eta)(H^\dagger \eta)$

FSS phenomenology: Dark Matter

- 2 viable DM candidates $\Rightarrow$ the **lightest neutral scalar** (Mandal, Srivastava, Valle, 2104.13401)
 $\Rightarrow$ the **singlet fermion** (Ganguly, Gluza, Karmakar, Mahapatra 2311.15997).





$$\Rightarrow \kappa^2 = \frac{|b|}{\mathcal{F}(M_{\eta_R}, M_{\eta_I}, M_f) M_f}$$

- ▶ Link: [Hitting the neutrino floor?](#)
- ▶ Coherent elastic neutrino nucleus scattering, or $\text{CE}\nu\text{NS}$ (pronounced “sevens”), link: [\$\text{CE}\nu\text{NS}\$](#)

Neutrinos Rising from the Floor

June 29, 2021 • *Physics* 14, 96

A neutrino background that could confound dark matter searches is now becoming an opportunity for probing new physics.



XENON Collaboration

The XENON1T detector shown from below. The bottom array of

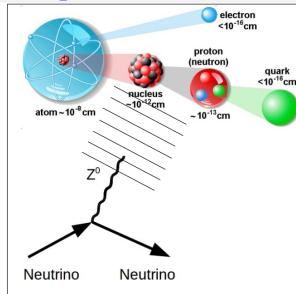
- ▶ Neutrino geology [Neutrino Geology](#)
- ▶ Coherent elastic neutrino nucleus scattering, or $\text{CE}\nu\text{NS}$ (pronounced “sevens”), link: [CE \$\nu\$ NS](#)

$\text{CE}\nu\text{NS}$: The simple Picture

1. ν -wavelength big enough
→ Z scattering @whole nucleus
2. small momentum transfer

→ coherent scattering

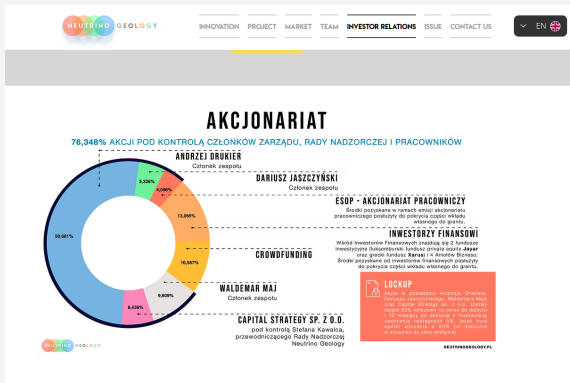
- Coherence length $\sim 1/E$
 - E_ν below $\mathcal{O}(50)$ MeV
 - lower for full coherence
- cross section $\sim (E_\nu)^2$
 - overcompensate:
 - very high ν flux
 - coherence $\sim N^2$



E.g. Germanium:

$N \simeq 40\text{-}44 \rightarrow N^2 \simeq 1600\text{-}1900 \rightarrow$ conventional detector mass 10t $\leftrightarrow$ few kg

NCBiR grant and crowdfunding (2024): shares



"If possible I would short-sell this stock :-)"

$$2D + 2D + 2D \rightarrow 2^1H + 4He + e^- + \nu_e + 21.63 \text{ MeV}$$

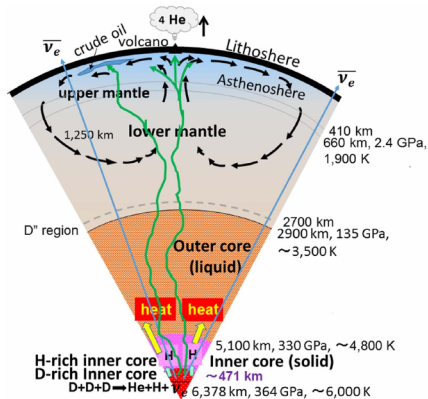


Figure 2. Earth's cross-section showing the crust, upper- and lower-mantle, and outer- and inner-cores. The inner core comprises an H-rich core and a D-rich core. A substantial amount of heat is generated by nuclear dynamic fusion of deuterons squeezed in highly compressed hexagonal close-packed (hcp) Fe-rich crystal lattice near the inner core centre. The H and He atoms and the anti-neutrino $\bar{\nu}_e$ that are produced are incorporated as Fe-H based alloys in the H-rich inner core, are released from Earth's interior to the universe, and pass through Earth, respectively.

"Possible generation of heat from nuclear fusion in Earth's inner core"

<https://www.nature.com/articles/srep37740>

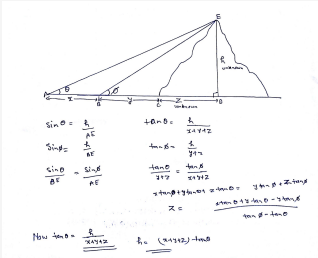
Why weak?

- ▶ The Sun emits $2 \cdot 10^{38}$ neutrinos per seconde!
- ▶ On Earth, about 40 billion neutrinos passes through 1 cm^2 in a second.
- ▶ A detector containing 1000 tons of heavy water is observing 30 cases per day
- ▶ To catch a single neutrino with 100% certainty you would need a layer of lead about 5 light years thick
- ▶ Assume a man's mass of 80 kg and a density of $\rho = 1 \text{ kg/dm}^3$ in the course of a person's lifetime "registers" one neutrino
- ▶ Since cross sections are very small (10^{-48} m^2): large detectors
- ▶ Natural radiation in the Earth (Unknown): 20,000 GW (about 20,000 nuclear reactors!) which translates into about by 6 million neutrinos per seconde per cm^2 .

"Weak interactions move mountains, move continents?" Can it be possible?

How much work to uplift Mount Everest?

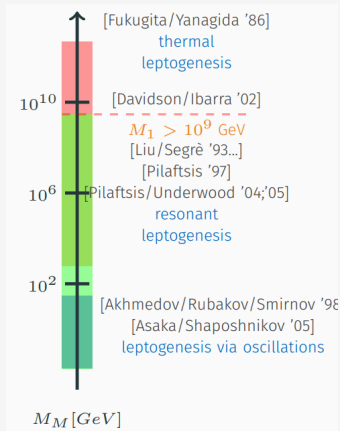




OK, power suitable for melting rocks and tectonic movements

Summary - Flavor symmetric scoto-seesaw FSS_1 and FSS_2 mixing

- ▶ FSS_1 (TM_1 mixing)
 - One copy of N_R, f and η .
 - non-trivial A_4 flavons, gauge singlets
 - Symmetry : $A_4 \times Z_4 \times Z_3 \times Z_2$
 - stable DM
 - Leptogenesis
- ▶ FSS_2 (TM_2 mixing)
 - 2 RHNs,
 - DM,
 - Leptogenesis



Meaning of Z_N symmetries

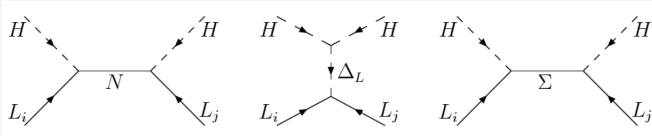
TABLE I. Field content and transformations under the symmetries of the FSS₁ model. Product rules of the A_4 singlets and triplets are given in the Appendix. The flavon fields mentioned in the second block are essential to implement the A_4 symmetry, and $\omega (= e^{2i\pi/3})$ is the cube root of unity.

Fields	e_R, μ_R, τ_R	L_α	H	N_R	f	η	ϕ_s	ϕ_a	ϕ_T	ξ
A_4	1, $1'', 1'$	3	1	1	1	1	3	3	3	$1''$
Z_4	-1	i	1	1	1	1	i	i	$-i$	1
Z_3	1	ω	ω	1	1	1	ω^2	ω	1	1
Z_2	-1	1	1	1	-1	-1	1	-1	-1	-1

Fields	e_R, μ_R, τ_R	L_α	H	N_R	f	η	ϕ_s	ϕ_a	ϕ_T	ξ
A_4	1, $1'', 1'$	3	1	1	1	1	3	3	3	$1''$
Z_4	i	i	1	1	1	1	i	i	1	1
Z_3	ω^2	ω	1	ω^2	1	1	ω^2	ω	ω^2	1
Z_2	1	1	1	1	-1	-1	1	-1	1	-1

- ▶ $Z_N \Rightarrow$ Auxiliary Symmetries
- ▶ Here $Z_N = Z_4 \otimes Z_3 \otimes Z_2$
- ▶ Z_N Symmetries also dictates the flavor structure of the Yukawa couplings, allows necessary terms, forbids unwanted terms
- ▶ Wrong Z_N symmetries $\Rightarrow$ invalid models
- ▶ Example: Majorana mass ($\bar{N}_R^c N_R$) terms are not allowed in the (wrong) 2nd table due to inappropriate Z_3 symmetry.
- ▶ Another advantage : breaking of $G_f = A_4 \otimes Z_N \Rightarrow$ remnant Z_2 symmetry ensures stability of dark matter as well as correct neutrino mixing.

Low scale CP and leptogenesis from RHNs sector

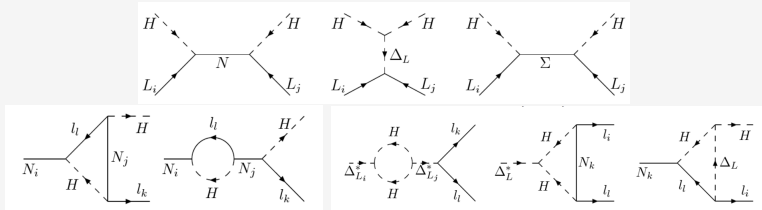


	Type of seesaw model		
	Type-I	Type-II	Type-III
Seesaw states	N	$\Delta_L = \begin{pmatrix} \delta^+/\sqrt{2} & \delta^{++} \\ \delta^0 & -\delta^+/\sqrt{2} \end{pmatrix}$	$\Sigma = \begin{pmatrix} \Sigma^0/\sqrt{2} & \Sigma^+ \\ \Sigma^- & -\Sigma^0/\sqrt{2} \end{pmatrix}$
Kin. term	$i\bar{N}\not{\partial}N$	$\text{Tr}[(D_\mu \Delta_L)^\dagger (D^\mu \Delta_L)]$	$\text{Tr}[\bar{\Sigma}i\not{\partial}\Sigma]$
Mass term	$-\frac{1}{2}\text{Tr}[\bar{N}m_N N^c + \bar{N}^c m_N^* N]$	$-m_\Delta^2 \text{Tr}[\Delta_L^\dagger \Delta_L]$	$-\frac{1}{2}\text{Tr}[\bar{\Sigma}m_\Sigma \Sigma^c + \bar{\Sigma}^c m_\Sigma^* \Sigma]$
Interactions	$-\tilde{\phi}^\dagger \bar{N} Y_N L - \bar{L} Y_N^\dagger N \tilde{\phi}$	$-L^T Y_\Delta C i\tau_2 \Delta_L L + \mu \tilde{H}^T i\tau_2 \Delta_L \tilde{H}$	$-\tilde{\phi}^\dagger \bar{\Sigma} \sqrt{2} Y_\Sigma L - \bar{L} \sqrt{2} Y_\Sigma^\dagger \Sigma \tilde{\phi}$
ν masses	$\mathcal{M}_\nu^N = -\frac{v^2}{2} Y_N^T \frac{1}{m_N} Y_N$	$\mathcal{M}_\nu^\Delta = 2Y_\Delta v_{\Delta_L} = Y_\Delta \mu^* \frac{v^2}{m_\Delta}$	$\mathcal{M}_\nu^\Sigma = -\frac{v^2}{2} Y_\Sigma^T \frac{1}{m_\Sigma} Y_\Sigma$
CP asym.	$\varepsilon_N \equiv \frac{\Gamma(N \rightarrow LH) - \Gamma(N \rightarrow \bar{L}\bar{H})}{\Gamma(N \rightarrow LH) + \Gamma(N \rightarrow \bar{L}\bar{H})}$	$\varepsilon_\Delta \equiv 2 \frac{\Gamma(\Delta_L \rightarrow LL) - \Gamma(\Delta_L \rightarrow \bar{L}\bar{L})}{\Gamma_\Delta + \Gamma_{\bar{\Delta}}}$	$\varepsilon_\Sigma \equiv \frac{\Gamma(\Sigma \rightarrow LH) - \Gamma(\bar{\Sigma} \rightarrow \bar{L}\bar{H})}{\Gamma(\Sigma \rightarrow LH) + \Gamma(\bar{\Sigma} \rightarrow \bar{L}\bar{H})}$

Leptogenesis: beyond the minimal type I seesaw scenario, Thomas Hambye, [1212.2888](#)

Low scale CP and leptogenesis from RHNs sector

Minimal setup: > 1 RHNs needed (or Δ_L), complex couplings.



$$\begin{aligned}
 \varepsilon_N &= -\frac{3}{32\pi^2} \frac{m_N^3}{\Gamma_N v^4} \text{Im}[(\mathcal{M}_\nu^N)_{\beta\alpha} (\mathcal{M}_\nu^H)_{\alpha\beta}^\dagger] \quad \longrightarrow \text{induced by decaying } N \text{ and "H" in loop} \\
 \varepsilon_\Delta &= -\frac{1}{16\pi^2} \frac{m_\Delta^3}{\Gamma_\Delta v^4} \text{Im}[(\mathcal{M}_\nu^\Delta)_{\beta\alpha} (\mathcal{M}_\nu^H)_{\alpha\beta}^\dagger] \quad \longrightarrow \text{by decaying } \Delta_L \text{ and heavy "H" in loop} \\
 \varepsilon_\Sigma &= -\frac{1}{32\pi^2} \frac{M_\Sigma^3}{\Gamma_\Sigma v^4} \text{Im}[(\mathcal{M}_\nu^\Sigma)_{\beta\alpha} (\mathcal{M}_\nu^H)_{\alpha\beta}^\dagger] \quad \longrightarrow \text{by decaying } \Sigma \text{ and heavy "H" in loop}
 \end{aligned}$$

Footprints of CP from the heavy sector at low scale

- Presence of heavy RHN $\Rightarrow$ Majorana phases (aka high energy phases)
- They can be connected with low energy Dirac CP phase δ_{CP} (aka low energy phase)

G. Chauhan et al.

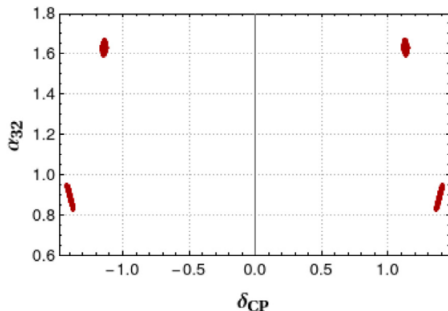


Fig. 3.6. Correlation between the Dirac CP phase δ_{CP} and the Majorana phase

$$U_\nu = U_{TB} U_{13} U_m, \quad U_m = \text{diag}(1, 1, e^{i\alpha_{32}/2})$$

Yet another approaches to
flavor symmetries

Flavor symmetry and neutrino mass: consequences

- Generalised mass sum rules:

$$A_1 \tilde{m}_1^p e^{i\chi_1} + A_2 \tilde{m}_2^p e^{i\chi_2} + A_3 \tilde{m}_3^p e^{i\chi_3} = 0$$

where $p \neq 0$, $\chi_1 \in [0, 2\pi]$, $A_i > 0$

King, Marle, Stuart 1307.2901

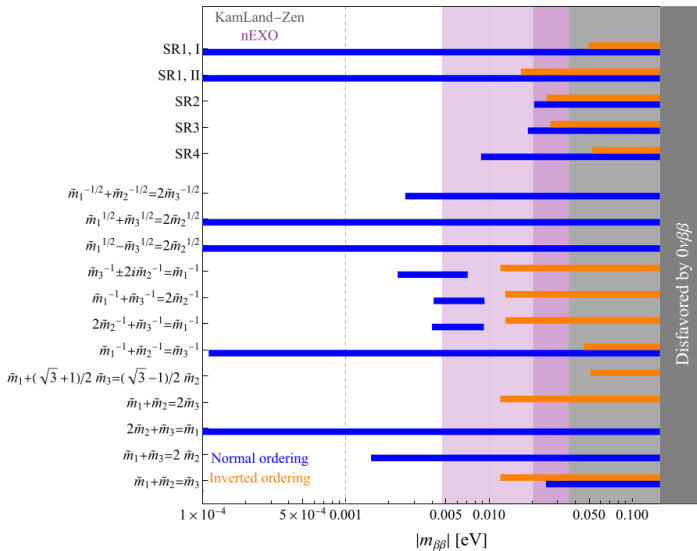
- Simplified Sum Rules obtained from various flavor models:

Sum Rule	Group	Seesaw Type
$\tilde{m}_1 + \tilde{m}_2 = \tilde{m}_3$	$A_4; S_4; A_5$	Weinberg
$\tilde{m}_1 + \tilde{m}_2 = \tilde{m}_3$	$\Delta(54); S_4$	Type II
$\tilde{m}_1 + 2\tilde{m}_2 = \tilde{m}_3$	S_4	Type II
$2\tilde{m}_2 + \tilde{m}_3 = \tilde{m}_1$	A_4	Weinberg
$2\tilde{m}_2 + \tilde{m}_3 = \tilde{m}_1$	$S_4; T_7^f; T_7$	Type II
$2\tilde{m}_1 + \tilde{m}_2 = 2\tilde{m}_3$	A_4	Dirac
$\tilde{m}_1 + \tilde{m}_2 = 2\tilde{m}_3$	S_4	Type II
$\tilde{m}_1 + \tilde{m}_2 = 2\tilde{m}_3$	$L_e - L_\mu - L_\tau$	Type II
$\tilde{m}_1 + \frac{\sqrt{3}+1}{2} \tilde{m}_3 = \frac{\sqrt{3}-1}{2} \tilde{m}_2$	A_5'	Weinberg
$\tilde{m}_1^{-1} + \tilde{m}_2^{-1} = \tilde{m}_3^{-1}$	$A_4; S_4; A_5$	Type I
$\tilde{m}_1^{-1} + \tilde{m}_2^{-1} = \tilde{m}_3^{-1}$	S_4	Type III
$2\tilde{m}_2^{-1} + \tilde{m}_3^{-1} = \tilde{m}_1^{-1}$	$A_4; T'$	Type I
$\tilde{m}_1^{-1} + \tilde{m}_3^{-1} = 2\tilde{m}_2^{-1}$	$A_4; T'$	Type I
$\tilde{m}_3^{-1} \pm 2i\tilde{m}_2^{-1} = \tilde{m}_1^{-1}$	$\Delta(96)$	Type I
$\tilde{m}_1^{1/2} - \tilde{m}_3^{1/2} = 2\tilde{m}_2^{1/2}$	A_4	Type I
$\tilde{m}_1^{1/2} + \tilde{m}_3^{1/2} = 2\tilde{m}_2^{1/2}$	A_4	Scotogenic
$\tilde{m}_1^{-1/2} + \tilde{m}_2^{-1/2} = 2\tilde{m}_3^{-1/2}$	S_4	Inverse

Flavor Symmetry and future $0\nu\beta\beta$ Experiments

G. Chauhan et al.

Progress in Particle and Nuclear Physics 138 (2024) 104126



Modular Symmetry

Possible Origin:

Superstring theory on certain compactifications may lead to Modular groups. In fact, torus compactification leads to Modular symmetry, which includes S_3, A_4, S_4, A_5 as its congruence subgroup.

Use of Modular Symmetry:

- ▶ It has been showed that neutrino mass might be of modular form ([F. Feruglio, \[arXiv:1706.08749 \[hep-ph\]\]](#)), introducing modular invariance approach to the lepton sector.
- ▶ The primary advantage is that the flavon fields might not be needed and the Yukawa couplings are written as modular forms, functions of only one complex parameter

$$\tau \rightarrow \tau' = \gamma\tau = \frac{a\tau + b}{c\tau + d},$$

where $a, b, c, d \in \mathbb{Z}$ and $ad - bc = 1, \text{Im}(\tau) > 0$.

- ▶ T. Kobayashi, K. Tanaka, T. H. Tatsuishi 1803.10391, J. T. Penedo, S. T. Petcov 1806.11040, F. J. de Anda, S. F. King, E. Perdomo 1812.05620, Wang, Zhou 2102.04358

Rich phenomenology : Yet to be explored

Backup

Other frontiers: High energy
(colliders) and cosmology

FSS phenomenology: Collider studies

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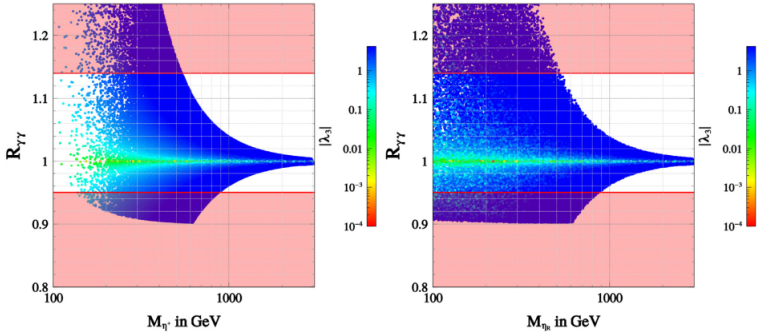


Fig. 4.5. Predictions for the diphoton decay of the Higgs boson within the FSS model [526]. $R_{\gamma\gamma}$ defined in (4.16) is plotted against $m_{\eta'}$ (left) and m_{η_s} (right). The white region is the current experimental allowed range measured by ATLAS [617]. Masses $M_{\eta'}$, M_{η_s} and the parameter λ_3 are defined in Section 5.2 where more phenomenological aspects of the FSS model in the context of cosmology will be discussed.

$$R_{\gamma\gamma} = \frac{[\sigma(gg \rightarrow H) \times \text{Br}(H \rightarrow \gamma\gamma)]_{\text{Model}}}{[\sigma(gg \rightarrow H) \times \text{Br}(H \rightarrow \gamma\gamma)]_{\text{SM}}} = \frac{\Gamma_{\text{Total}}^{\text{SM}} \times \Gamma(H \rightarrow \gamma\gamma)_{\text{Model}}}{\Gamma_{\text{Total}}^{\text{Model}} \times \Gamma(H \rightarrow \gamma\gamma)_{\text{SM}}}.$$

$$R_{\gamma\gamma} = 1.04^{+0.10}_{-0.09} \quad \text{ATLAS139 fb}^{-1}$$

Domain wall problem, GW with A_4, S_4 and gauged U(1)

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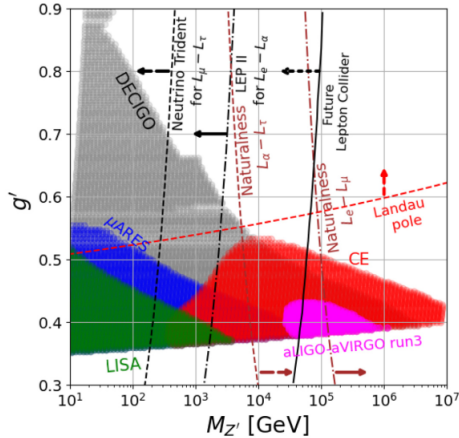
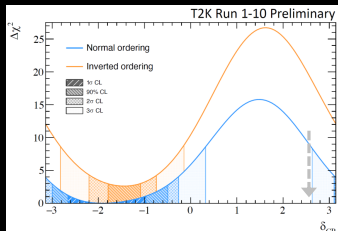


Fig. 5.10. Sensitivity of the future GW experiments in the mass-coupling plane of scale-invariant $U(1)_{L_e-L_\mu}$ models. The current constraint from non-observation of stochastic GW background in aLIGO-aVIRGO run 3 is shown in purple. The laboratory constraints from LEP-2 and neutrino trident experiments are shown for comparison, along with the future lepton collider sensitivities. The naturalness and perturbativity (Landau pole) constraints are also shown.

Source: Figure taken from Ref. [713].

The CP Phase



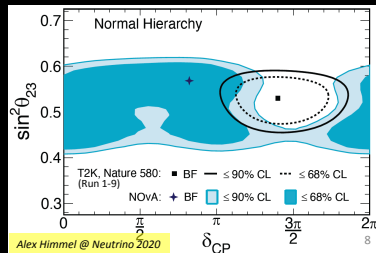
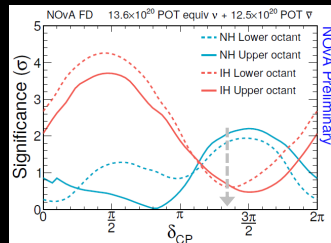
T2K

- $\delta = -\pi/2$ favored
- Large range of values of δ_{CP} around $+\pi/2$ are excluded at 99.7%

NOvA

- Best-fit $\delta = 0.82\pi$
- Exclude **IH** $\delta = \pi/2$ at $>3\sigma$
- Disfavor **NH** $\delta = 3\pi/2$ at $\sim 2\sigma$

Clear tension exists
NOvA + T2K joint analysis is underway



Alex Himmel @ Neutrino 2020