

SUPERSCALING ACROSS MODEL, TARGET AND FLUX

Using the universality of the superscaling variable ψ' and L_1 and low
neutrino energy limitation.

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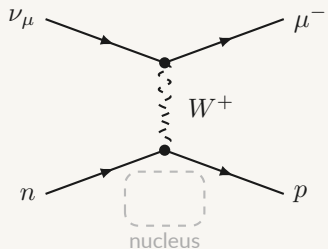
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SECTION 1

Introduction

CCQE NEUTRINO-NUCLEUS INTERACTION



Two reconstruction strategies

- ⊙ **RE:** uses measured E_p + constant S_{RE}
- ⊙ **LQE-TAR:** muon only, target-at-rest assumption

Energy conservation

$$E_\nu = E_\mu + E_p - M_n + S$$

What is measured

- ⊙ $\vec{p}_\mu$ — always accessible
- ⊙ E_p — near detector only
- ⊙ S (removal energy) — **not directly measurable**

SECTION 2

What are L_1 , L_2 and ΔE_p ?

■ Removal-energy reconstruction (hadronic)

$$S^{\text{RE}} = \overline{E_\nu - E_\mu - E_p + M_n} \Rightarrow E_\nu^{\text{RE}} = S^{\text{RE}} + E_\mu + E_p - M_n \quad (1)$$

■ Target-at-rest assumption

In the lepton-only quasielastic reconstruction, the initial nucleon is assumed to be at rest and the outgoing nucleon energy is inferred from the muon kinematics only:

$$q = \sqrt{p_{T\mu}^2 + \left(E_\nu^{\text{LQE}} - p_{z\mu}\right)^2}, \quad E_p^{\text{LQE}} = \sqrt{M_p^2 + q^2}.$$

Equivalently,

$$E_p^{\text{LQE}} = E_\nu^{\text{LQE}} - E_\mu + M_n - \bar{S}_{\text{LQE}}.$$

■ Muon-only neutrino-energy estimator

$$E_\nu^{\text{LQE}}(\bar{S}_{\text{LQE}}) = \frac{m_\mu^2 + 2E_\mu(\bar{S}_{\text{LQE}} - M_n) + (\bar{S}_{\text{LQE}} - M_n)^2 - M_p^2}{2(E_\mu + \bar{S}_{\text{LQE}} - M_n - p_\mu \cos \theta_\mu)}.$$

SUPERSCALING IN INCLUSIVE ELECTRON-NUCLEUS SCATTERING

I First-kind scaling (impulse approximation)

$$\frac{d^2\sigma_A}{d\Omega_\ell dE_\ell} \simeq \sigma_{\ell N}(\omega, |\vec{q}|) f(\psi')$$

$f(\psi')$ is **universal**: all kinematics enter only through the single dimensionless variable ψ' .

I Superscaling variable ψ' (Donnelly & Sick 1999)

$$\psi' = \chi_p \frac{\tilde{\lambda}' - \tau'}{\sqrt{(1 + \tilde{\lambda}')\tau' + \kappa' \sqrt{\tau'(1 + \tau')}}}$$

$$\kappa' = \frac{|\vec{q}|}{2M_n}, \quad \tilde{\lambda}' = \frac{\omega - \bar{S}_{\text{LQE}} + \Delta M}{2M_n}$$

$$\tau' = \kappa'^2 - \lambda'^2, \quad \chi_p = \frac{1}{\sqrt{\sqrt{1 + \eta_F^2} - 1}}, \quad \eta_F = \frac{k_F}{M_n}$$

I From (e, e') to ν -nucleus

In (e, e') : ω and $|\vec{q}|$ are **fully leptonic**, so ψ' is known event by event.

In ν scattering: E_ν is **unknown** $\Rightarrow \psi'$ must be *reconstructed*.

I Expected universality

Universality established in (e, e') should carry over to ν -nucleus scattering across targets and flux spectra.

$\Rightarrow$ Common framework to compare models and experiments.

TAYLOR EXPANSION OF ψ' AND MUON-ONLY COEFFICIENTS

Expansion variable

The superscaling variable is expanded around the LQE target-at-rest point, where

$$\Delta E_\nu \equiv E_\nu - E_\nu^{\text{LQE}} = 0, \quad \psi' = 0.$$

Taylor expansion

Around this point,

$$\psi'(\Delta E_\nu) = \chi_p \left[L_1(\vec{p}_\mu) \Delta E_\nu + L_2(\vec{p}_\mu) (\Delta E_\nu)^2 + L_3(\vec{p}_\mu) (\Delta E_\nu)^3 + \dots \right].$$

The Taylor coefficients are defined as

$$L_n(\vec{p}_\mu) \equiv \frac{1}{n!} \left. \frac{d^n(\psi'/\chi_p)}{d(\Delta E_\nu)^n} \right|_{\Delta E_\nu=0}.$$

RECONSTRUCTED ΔE_ν AND FACTORIZATION OF ψ'

Reconstructed energy bias

Replacing the unknown E_ν by the removal-energy estimator gives

$$\Delta E_{\nu,\text{RE}} \equiv E_\nu^{\text{RE}} - E_\nu^{\text{LQE}} = \underbrace{E_p - E_p^{\text{LQE}}}_{\Delta E_p} + (S_{\text{RE}} - \bar{S}_{\text{LQE}}).$$

Leading-order reconstructed superscaling variable

At first order in the Taylor expansion around $\Delta E_\nu = 0$,

$$\psi'_{\text{reco}} \simeq \chi_p \underbrace{L_1(\vec{p}_\mu)}_{\text{muon-only}} \left[\underbrace{\Delta E_p}_{\text{hadronic}} + \underbrace{S_{\text{RE}} - \bar{S}_{\text{LQE}}}_{\text{offset}} \right].$$

$$L_1 = \frac{\sqrt{2}}{M_N} \frac{\left(\widetilde{M}_N + p_{z\mu} - E_\mu\right)^2}{\sqrt{\left[p_{T\mu}^2 + \left(M_p - \widetilde{M}_N - p_{z\mu} + E_\mu\right)^2\right] \left[p_{T\mu}^2 + \left(M_p + \widetilde{M}_N + p_{z\mu} - E_\mu\right)^2\right]}}$$

WHY SHOULD L_1 BE APPROXIMATELY UNIVERSAL?

■ Starting point: leading-order factorization

At leading order, the reconstructed superscaling variable can be written as

$$\psi'_{\text{reco}} \simeq \chi_p L_1(\vec{p}_\mu) [\Delta E_p + S_{\text{RE}} - \bar{S}_{\text{LQE}}] .$$

■ ΔE_p as the nuclear-energy residual

$$\Delta E_p \equiv E_p - E_p^{\text{LQE}}$$

measures how much the measured outgoing nucleon energy differs from the target-at-rest quasielastic expectation.

■ Universality expectation

Since ψ' is constructed to organize the nuclear response into an approximately universal scaling variable, and since ΔE_p captures the corresponding hadronic nuclear residual, we expect the muon-only coefficient L_1 to inherit the same approximate universality.

LOW- $p_{T\mu}$ DIVERGENCE OF THE L_1 COEFFICIENT — ANALYTIC DERIVATION

■ Step-by-step derivation

- ❶ Definition of L_1 in terms of the LQE nucleon momentum:

$$L_1 = \frac{1}{\sqrt{2} \sqrt{(E_p^{\text{LQE}})^2 - M_p^2}}$$

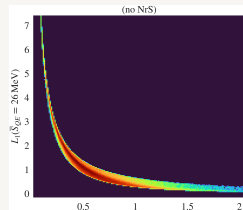
- ❷ In the limit $E_{\nu, \text{LQE}} \gg E_{p, \text{LQE}} - M_p$, the reconstructed nucleon energy reads:

$$E_{p, \text{LQE}} = \omega_{\text{LQE}} + S_{\text{LQE}} = \frac{Q^2}{2M_N} + S_{\text{LQE}}$$

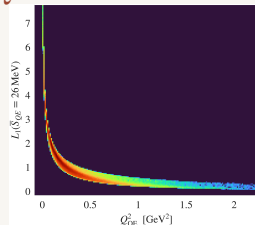
- ❸ Therefore:

$$L_1 \propto \frac{1}{Q^2} \propto \frac{1}{E_p^{\text{LQE}}}$$

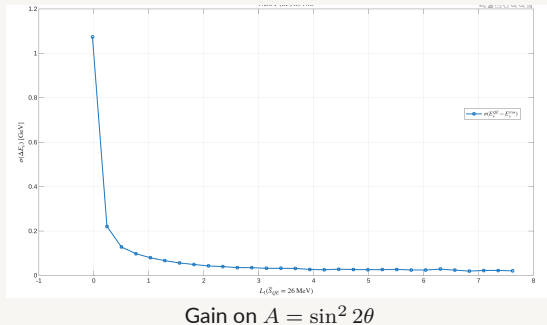
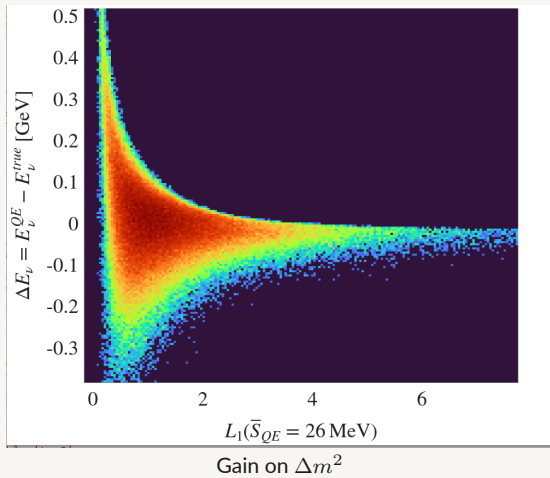
■ L_1 vs proton momentum reco



■ L_1 vs Q^2 reco



L_1 AS A PROXY FOR NEUTRINO ENERGY RECONSTRUCTION QUALITY.



FIRST-ORDER SUPERSCALING CONSTRAINT ON ENERGY RECONSTRUCTION

At first order around the LQE solution,

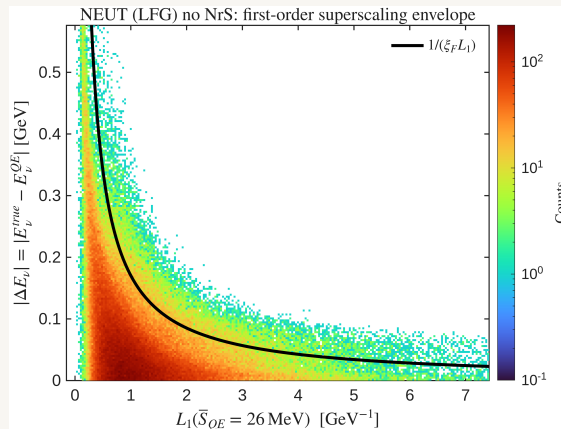
$$\psi' \simeq \chi_F L_1 \Delta E_\nu,$$

The superscaling hypothesis implies that the response is concentrated in

$$|\psi'| \lesssim 1.$$

Therefore,

$$|\Delta E_\nu| \lesssim \frac{1}{\chi_F L_1}$$



SECTION 3

Model comparaison :

SUPERSCALING VARIABLES TO COMPARE MODELS FOR A FIXED FLUX

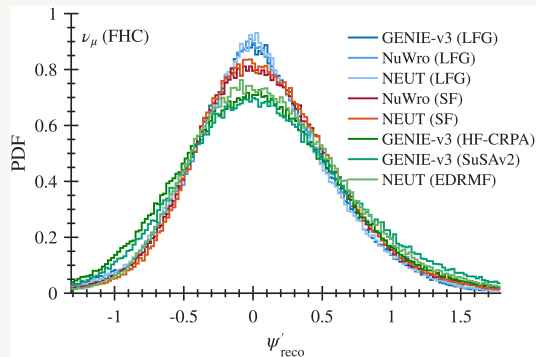
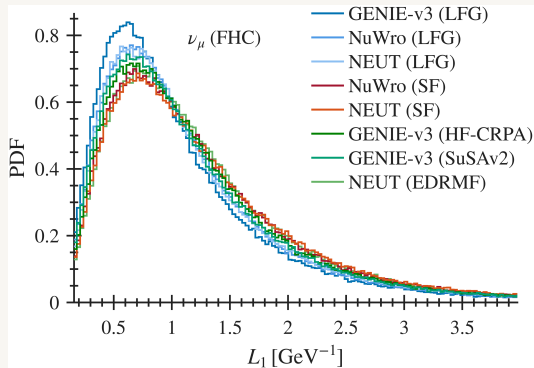


Figure: Comparison of the superscaling-related variables L_1 and reconstructed ψ' for different nuclear models, using the same neutrino flux.

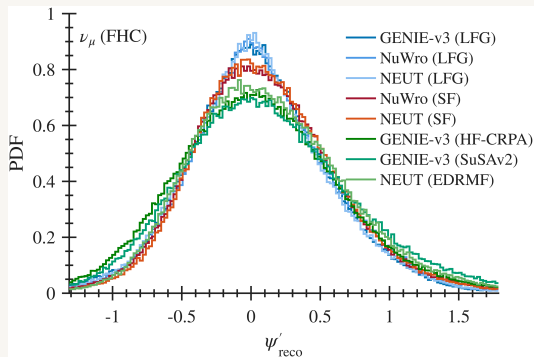
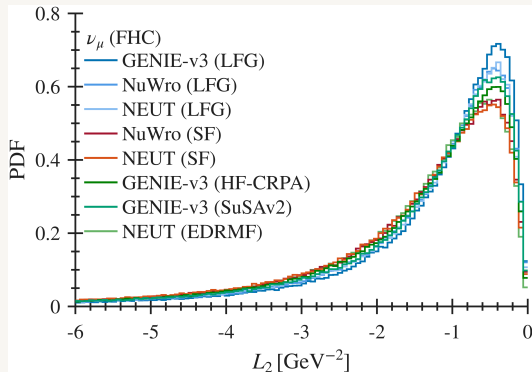


Figure: Comparison of the superscaling-related variables L_2 and reconstructed ψ' for different nuclear models, using the same neutrino flux.

THE DISCRIMINATION IS COMING BOTH FROM THE HADRONIC AND THE MUONIC PART

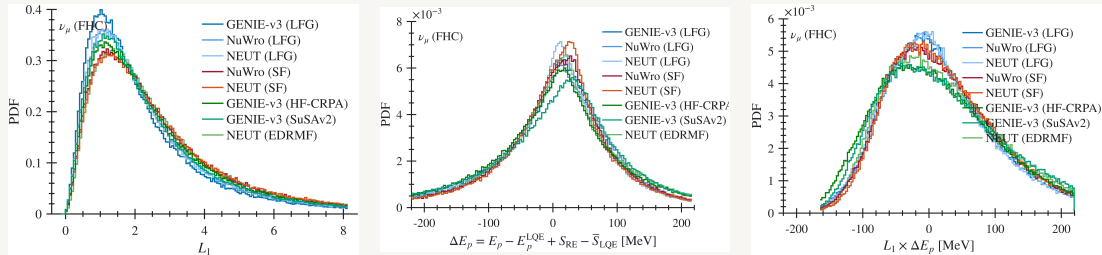


Figure: Comparison of the muonic coefficient L_1 , the hadronic residual ΔE_p , and their product $L_1 \Delta E_p$ for different nuclear models, using the same neutrino flux.

SECTION 4

Universality of ψ' across flux

Universality of ψ' across flux /

SECTION 4.1

Universality accross experiment.

UNIVERSALITY ACROSS TARGET AND FLUX.

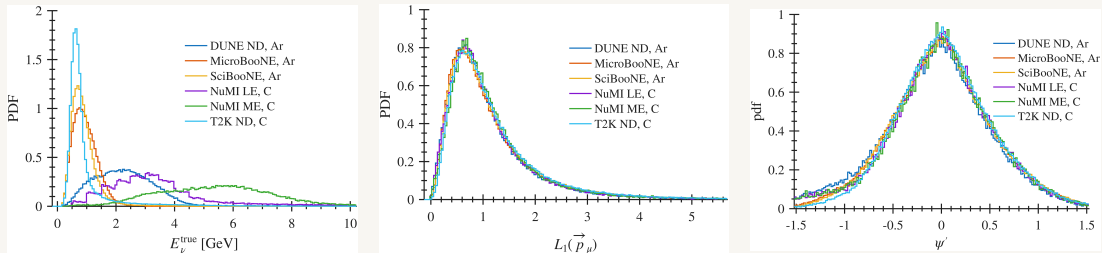


Figure: Comparison of L_1 (top) and tuned ψ' (bottom) distributions for several experimental configurations with different neutrino fluxes and target materials: T2K ND, NuMI ME, and NuMI LE on carbon, and MicroBooNE, SciBooNE, and DUNE ND on argon. To isolate the dependence on the flux and target, all samples are generated with the same theoretical model, the NEUT LFG model.

Universality of ψ' across flux /

SECTION 4.2

Oscillated and Unoscillated flux

OSCILLATED AND UN-OSCILLATED ν_μ FLUX

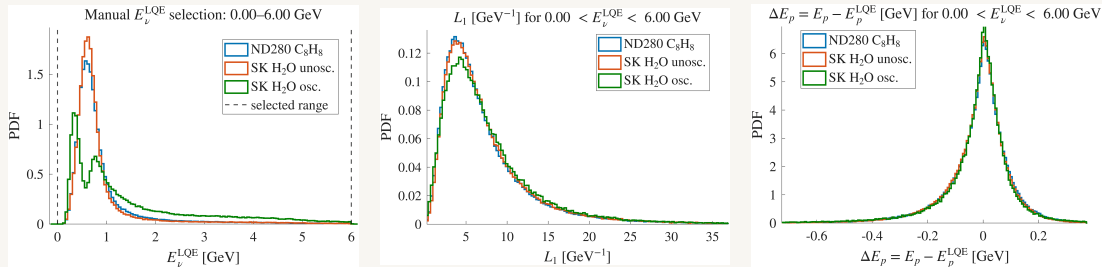


Figure: Comparison of the reconstructed E_ν^{LQE} spectrum, the muon-only coefficient L_1 , and the hadronic residual ΔE_p for the ND, SK unoscillated, and SK oscillated ν_μ samples. No additional kinematic cut is applied here, so L_1 and ΔE_p are shown over the full selected CC1p1h sample.

OSCILLATED AND UN-OSCILLATED ν_μ FLUX, CUT ON THE NEUTRINO ENERGY.

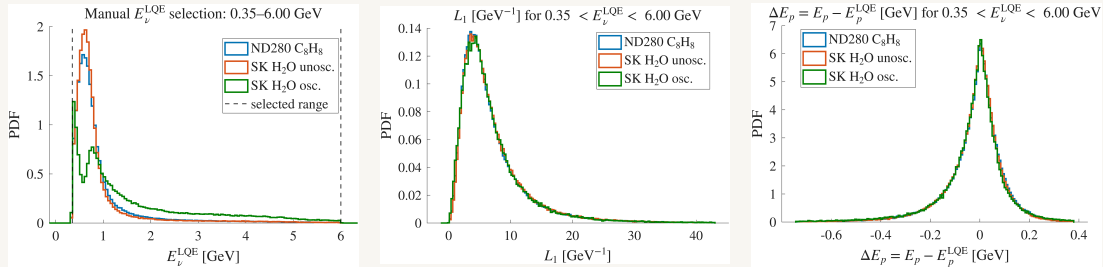


Figure: Comparison of the reconstructed E_ν^{LQE} spectrum, the muon-only coefficient L_1 , and the hadronic residual ΔE_p for the ND, SK unoscillated, and SK oscillated ν_μ samples. A low-energy cut on E_ν^{LQE} is applied to remove the region where the superscaling expansion is most sensitive to the low- $p_{T\mu}$ divergence.

THE MODEL TENSION IS STILL PRESENT AFTER THE LOW ENERGY CUT.

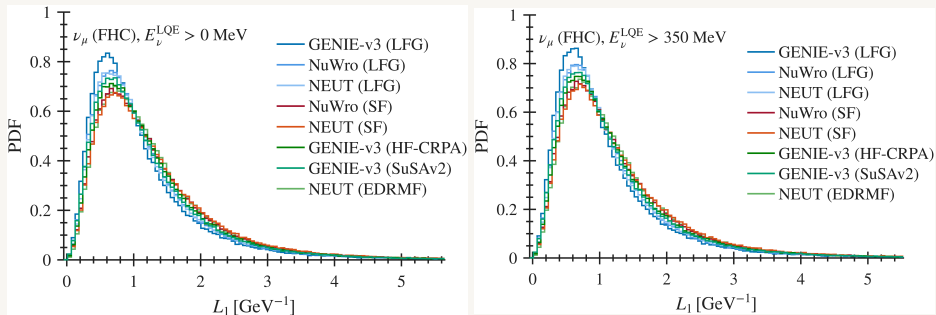


Figure: Comparison of the L_1 distributions between different nuclear models before and after applying a low-energy cut on E_ν^{LQE} .

TAKE-AWAY: L_1 UNIVERSALITY AND LOW- $p_{T\mu}$ LIMITATION

L_1 as a superscaling observable

L_1 behaves as a muon-only superscaling variable:

- ◉ mostly stable under flux changes;
- ◉ still sensitive to nuclear-model differences;
- ◉ therefore useful to compare nuclear models with muon kinematics only.

Two different effects

One should distinguish:

- ◉ genuine nuclear-model tension;
- ◉ low-energy flux effects amplified by the $p_{T\mu} \rightarrow 0$ divergence.

Low- $p_{T\mu}$

$$L_n \propto \frac{1}{p_{T\mu}^{n-\frac{1}{2}}},$$

so small flux, MC, or reconstruction differences can generate large changes in L_1 and ψ' .

Practical strategy

For oscillation-flux studies, apply a low-momentum cut:

$$E_\nu^{\text{LQE}} \gtrsim 350 \text{ MeV}.$$

For ψ' , the S_{RE} tuning partly absorbs the flux-induced peak shift.

SECTION 5

MAPS : NINJA and ESS ν B

FLUX CONFIGURATIONS: NINJA AND ESS ν SB

I NINJA

NINJA is a T2K near-detector experiment using nuclear-emulsion detectors to measure neutrino–nucleus interactions with very high tracking resolution.

We consider different detector positions with different off-axis angles:

- ◉ SS floor: on-axis flux;
- ◉ B2 floor: intermediate off-axis flux;
- ◉ Ground floor: large off-axis flux;

I ESS ν SB

ESS ν SB is a future long-baseline neutrino project based on the European Spallation Source beam.

I Low-energy limitation

ESS ν SB has a low-energy neutrino flux. This makes it interesting for oscillation physics, but not optimal for an L_1/ψ' measurement, since the low- $p_{T\mu}$ divergence can amplify flux and reconstruction effects.

I Purpose here

Use NINJA and ESS ν SB fluxes to test whether the same muon-only observable remains stable across very different beam spectra.

NINJA AND ESS ν B FLUX.

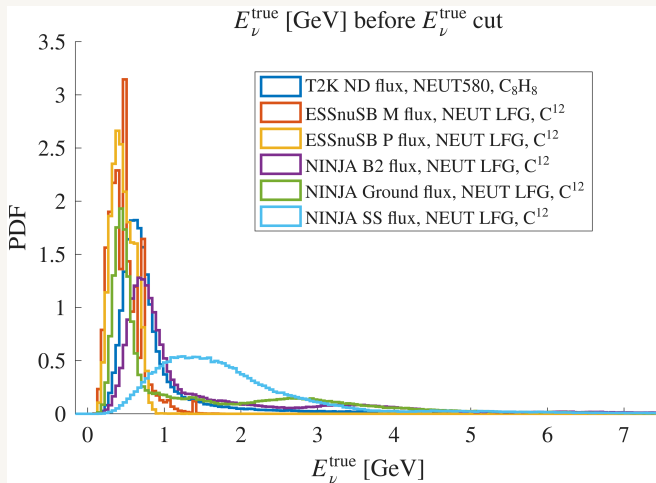


Figure: True neutrino energy distributions for the T2K ND, NINJA, and ESS ν B flux configurations.

SUPERSCALING VARIABLES L_1 AND ψ' BEFORE CUT AT 450 MEV.

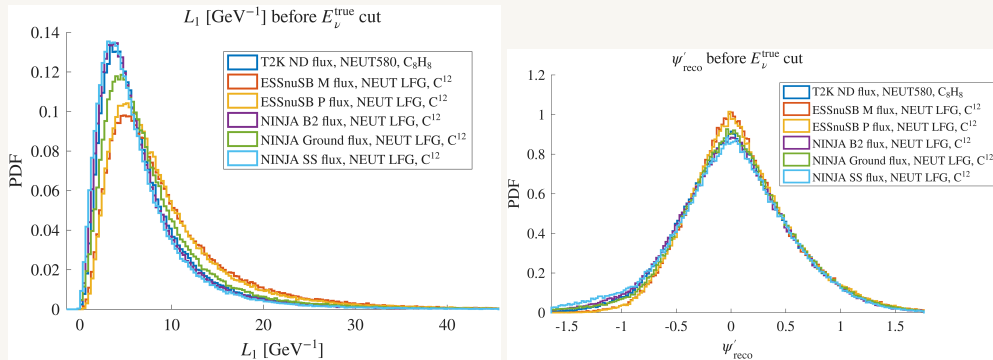


Figure: Comparison of the L_1 and reconstructed ψ' distributions for the T2K ND, NINJA, and ESS ν B flux configurations before applying the neutrino-energy cut.

SUPERSCALING VARIABLES L_1 AND ψ' BEFORE CUT AT 450 MEV.

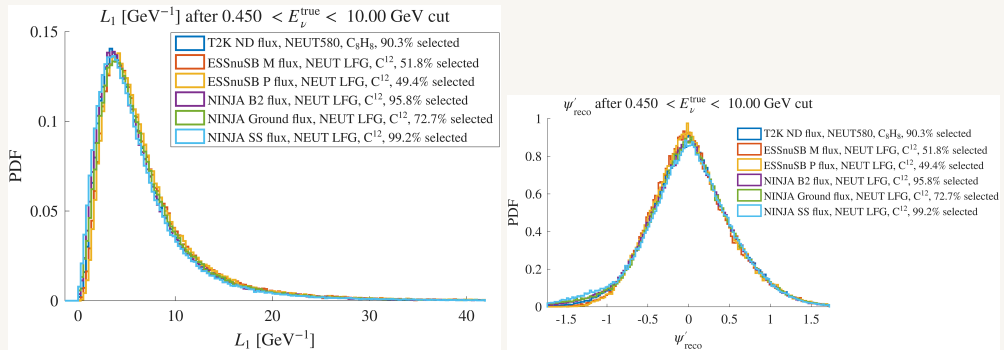
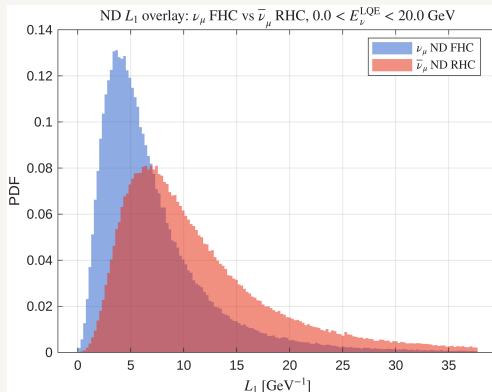


Figure: Comparison of the L_1 and reconstructed ψ' distributions for the T2K ND, NINJA, and ESS ν B flux configurations after applying the neutrino-energy cut.

L_1 TO DISCRIMINATE BETWEEN ν AND $\bar{\nu}$.



SECTION 6

Using the L_1 variable to constrain oscillation.

1D ENERGY MIGRATION

For each true-energy bin j , the oscillated event rate is

$$N_j^{\text{true,osc}} = \Phi_\alpha(E_j) P_{\alpha \rightarrow \beta}(E_j) \sigma_\beta(E_j) \Delta E_j \times \mathcal{N}.$$

For a two-flavour disappearance approximation,

$$P_{\mu\mu}(E_j) = 1 - A \sin^2 \left(1.267 \frac{\Delta m^2 L}{E_j} \right).$$

The 1D prediction in reconstructed energy is obtained with the migration matrix

$$M_{ij}^{1D} = P(E_\nu^{\text{reco}} \in i \mid E_\nu^{\text{true}} \in j),$$

so that

$$N_i^{1D} = \sum_j M_{ij}^{1D} N_j^{\text{true,osc}}.$$

$$\sum_i M_{ij}^{1D} = 1 \quad \text{for each populated true-energy bin } j.$$

2D MIGRATION WITH E_ν^{LQE} AND L_1

The sample is split into L_1 bins, labelled by q . For each bin, we define

$$M_{ij}^{(q)} = P(E_\nu^{\text{reco}} \in i \mid E_\nu^{\text{true}} \in j, L_1 \in q),$$

and the corresponding L_1 fraction

$$\epsilon_q(j) = P(L_1 \in q \mid E_\nu^{\text{true}} \in j), \quad \sum_q \epsilon_q(j) = 1.$$

The predicted reconstructed spectrum is

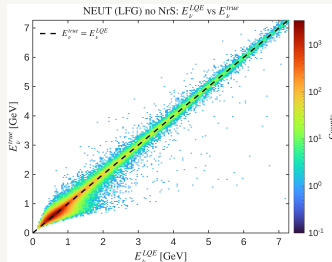
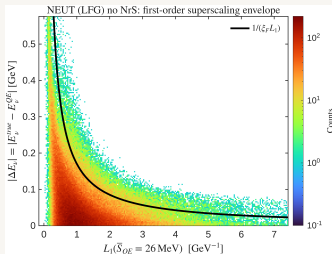
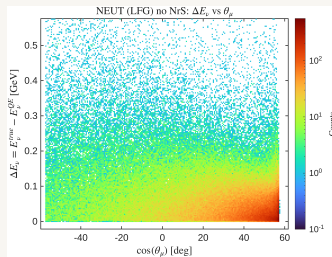
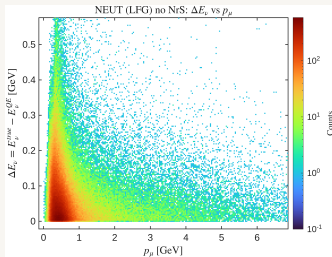
$$N_{iq}^{2\text{D}} = \sum_j M_{ij}^{(q)} \epsilon_q(j) N_j^{\text{true,osc}}.$$

Equivalently,

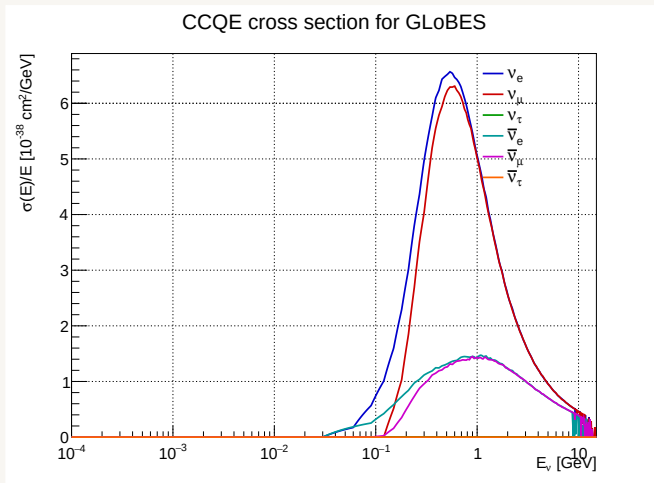
$$K_{ij}^{(q)} = \epsilon_q(j) M_{ij}^{(q)}, \quad N_{iq}^{2\text{D}} = \sum_j K_{ij}^{(q)} N_j^{\text{true,osc}}.$$

$$\sum_{q,i} K_{ij}^{(q)} = 1 \quad \text{for each populated true-energy bin.}$$

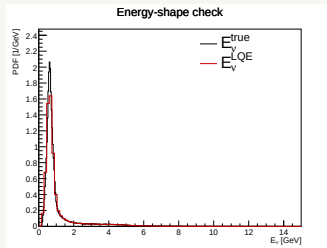
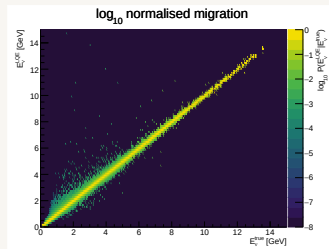
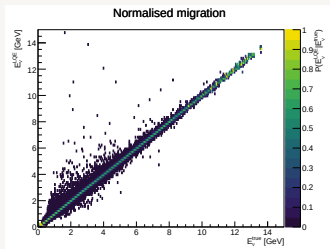
CHANGING FROM (p_μ, θ_μ) TO $(E_\nu^{\text{reco}}, L_1)$



GENIE AR23 T2HK C12 CCQE CROSS SECTION

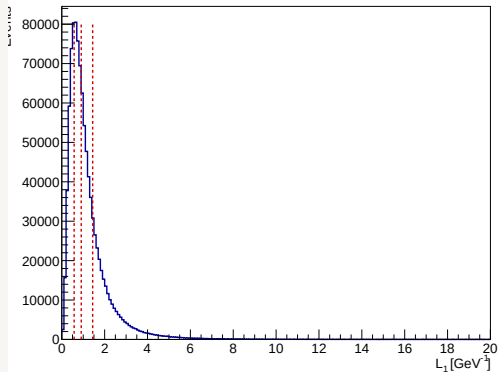


$\bar{\nu}_\mu$ ENULQE MIGRATION MATRIX

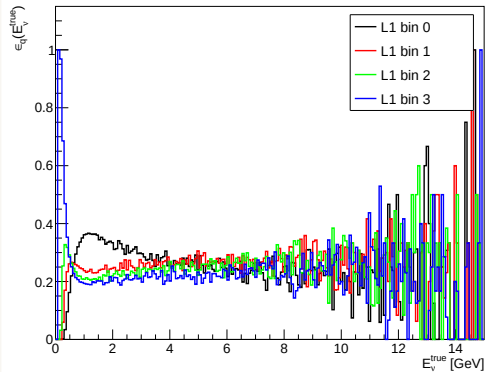


ν_μ EL1 MIGRATION: L_1 BINNING AND EFFICIENCIES

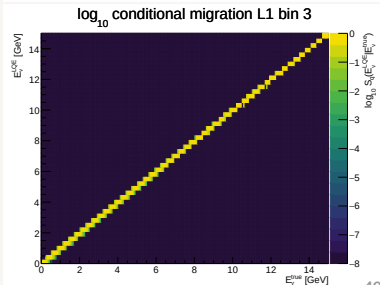
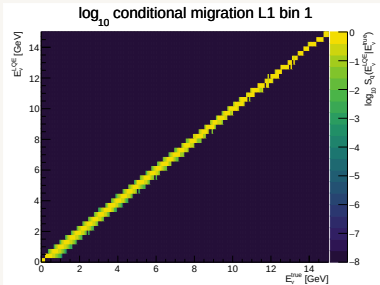
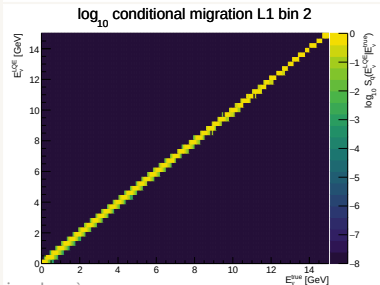
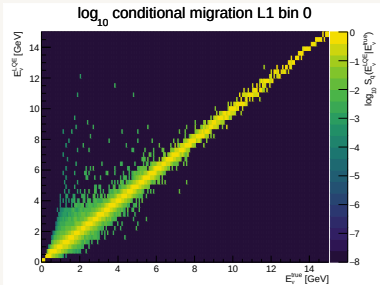
L_1 distribution



L_1 sample efficiencies



ν_μ EL1 MIGRATION BY L_1 BIN



CONCLUSION AND OUTLOOK: SUPERSCALING FRAMEWORK

1. Superscaling measurement

Test superscaling directly in ν and $\bar{\nu}$ scattering by superposing ψ'/L_1 distributions from different fluxes and targets: ND280, NINJA B2/SS, and MINERvA.

2. Model comparison

Use ψ' and L_1 to compare and discriminate Monte Carlo nuclear models: ψ' probes the reconstructed nuclear response, while L_1 isolates the muon-kinematic contribution.

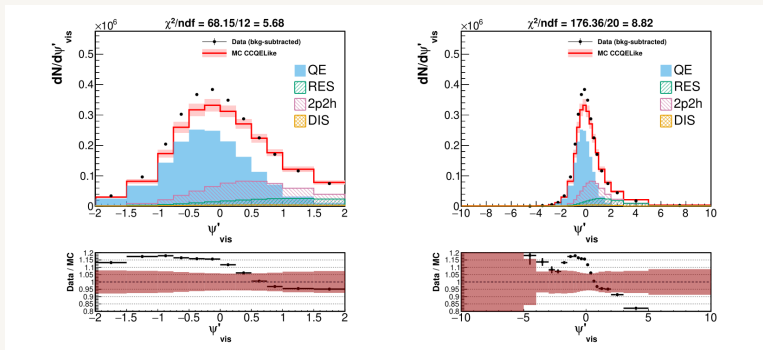
3. Low-energy frontier

Study the low-energy region as an extreme test of the method, especially for ESS ν SB and the projected NINJA Ground-Floor flux, where low- $p_{T\mu}$ effects may strongly affect L_1/ψ' .

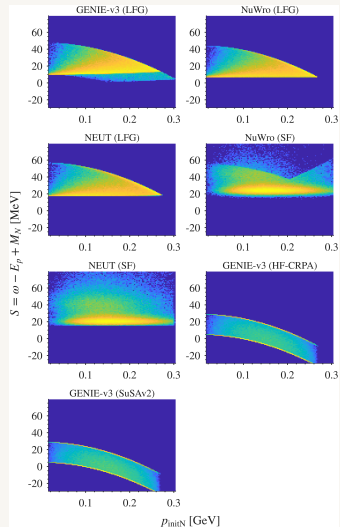
4. Oscillation application

Investigate whether a near-far comparison of L_1 at ND280 and SK/HK can provide a constraint on oscillation predictions.

MEASUREMENT OF THE ψ' WITH MINER ν A DATA.



REMOVAL ENERGY ACROSS MODEL.



REMOVAL ENERGY UNDER TARGET AT REST ASSUMPTION ACROSS MODEL.

