Implications of the SO(3) parametrization

Elena Abad Díaz

Institut Ruđer Bošković

Standard neutrino parameterization

Neutrino mixing is usually described by 3 angles and 1 Dirac (+2 Majorana) CP-violating phases

PMNS unitary mixing matrix

$$U_{\rm PMNS} = R_x(\theta_{23})R_y(\theta_{13},\delta_{\rm CP})R_z(\theta_{12})U_M(\alpha_1,\alpha_2) = U U_M(\alpha_1,\alpha_2)$$

$$U = \begin{pmatrix} c_{12}\,c_{13} & s_{12}\,c_{13} & s_{13}\,e^{-i\delta_{\rm CP}} \\ -s_{12}\,c_{23} - c_{12}\,s_{13}\,s_{23}\,e^{i\delta_{\rm CP}} & c_{12}\,c_{23} - s_{12}\,s_{13}\,s_{23}\,e^{i\delta_{\rm CP}} & c_{13}\,s_{23} \\ s_{12}\,s_{23} - c_{12}\,s_{13}\,c_{23}\,e^{i\delta_{\rm CP}} & -c_{12}\,s_{23} - s_{12}\,s_{13}\,c_{23}\,e^{i\delta_{\rm CP}} & c_{13}\,c_{23} \end{pmatrix}$$

Order-dependent parametrization

The mixing angles $\theta_{12},\ \theta_{13},\ \theta_{23}$ have been fitted experimentally

To change the order of the rotation angles yields a completely different set of mixing angles

SO(3) parameterization

➢ Introduce an **order independent parameterization** based SO(3) group:

Proposed SO(3) parametrization

$$U_{\mathrm{SO3}} = \begin{pmatrix} \frac{\theta_x^2+\cos\theta\left(\theta_y^2+\theta_z^2\right)}{\theta^2} & \frac{\theta\theta_z\sin\theta-\theta_x\theta_y(\cos\theta-1)}{\theta^2} & -\frac{\theta_x\theta_z(\cos\theta-1)+\theta\theta_y\sin\theta}{\theta^2} \\ -\frac{\theta_x\theta_y(\cos\theta-1)+\theta\theta_z\sin\theta}{\theta^2} & \frac{\cos\theta\left(\theta_x^2+\theta_z^2\right)+\theta_y^2}{\theta^2} & \frac{\theta\theta_x\sin\theta-\theta_y\theta_z(\cos\theta-1)}{\theta^2} \\ \frac{\theta\theta_y\sin\theta-\theta_x\theta_z(\cos\theta-1)}{\theta^2} & -\frac{\theta\theta_x\sin\theta+\theta_y\theta_z(\cos\theta-1)}{\theta^2} & \frac{\cos\theta\left(\theta_x^2+\theta_y^2\right)+\theta_z^2}{\theta^2} \end{pmatrix}$$

Described by 3 rotation angles $(\theta_x, \theta_y, \theta_z)$ with $\theta = \sqrt{\theta_x^2 + \theta_y^2 + \theta_z^2}$

and the **CP-conserving phases** $\delta_{CP} = 0, \pi.$

PMNS $\leftrightarrow$ SO(3)

Given the equivalence of the two parametrizations, we map the matrices in order to express **the PMNS angles as functions of the SO(3) angles:**

PMNS

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\mathrm{CP}}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{\mathrm{CP}}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{\mathrm{CP}}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{\mathrm{CP}}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{\mathrm{CP}}} & c_{23}c_{13} \end{pmatrix}$$

SO(3)

$$\begin{pmatrix} \frac{\theta_x^2+\cos\theta\left(\theta_y^2+\theta_z^2\right)}{\theta^2} & \frac{\theta\theta_z\sin\theta-\theta_x\theta_y(\cos\theta-1)}{\theta^2} & -\frac{\theta_x\theta_z(\cos\theta-1)+\theta\theta_y\sin\theta}{\theta^2} \\ -\frac{\theta_x\theta_y(\cos\theta-1)+\theta\theta_z\sin\theta}{\theta^2} & \frac{\cos\theta\left(\theta_x^2+\theta_z^2\right)+\theta_y^2}{\theta^2} & \frac{\theta\theta_x\sin\theta-\theta_y\theta_z(\cos\theta-1)}{\theta^2} \\ \frac{\theta\theta_y\sin\theta-\theta_x\theta_z(\cos\theta-1)}{\theta^2} & -\frac{\theta\theta_x\sin\theta+\theta_y\theta_z(\cos\theta-1)}{\theta^2} & \frac{\cos\theta\left(\theta_x^2+\theta_y^2\right)+\theta_z^2}{\theta^2} \end{pmatrix}$$

$$c_{12} = \frac{\theta_x^2 + (\theta_y^2 + \theta_z^2)\cos\theta}{\sqrt{\theta^4 - (\theta_x\theta_z(\cos\theta - 1) + \theta\theta_y\sin\theta)^2}}$$

$$s_{23} = \frac{\theta\theta_x\sin\theta - \theta_y\theta_z(\cos\theta - 1)}{\sqrt{\theta^4 - (\theta_x\theta_z(\cos\theta - 1) + \theta\theta_y\sin\theta)^2}}$$

$$\boxed{\delta_{CP} = \pi}$$

$$s_{13} = \pm\frac{\theta_x\theta_z(\cos\theta - 1) + \theta\theta_y\sin\theta}{\theta^2}$$

$$\boxed{\delta_{CP} = 0}$$

Allowed values of $\theta_x, \theta_y, \theta_z$

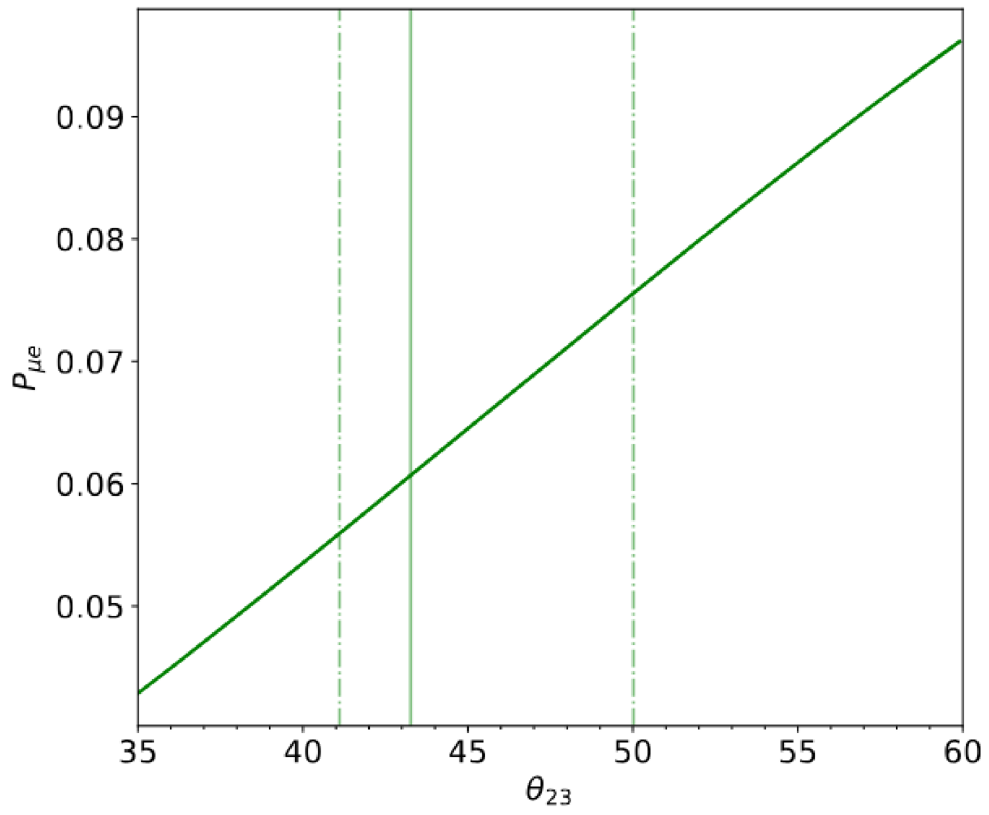
To determine the allowed values of SO(3) parametrization we **fit them to NuFIT data** results using the relations between the two parametrization,

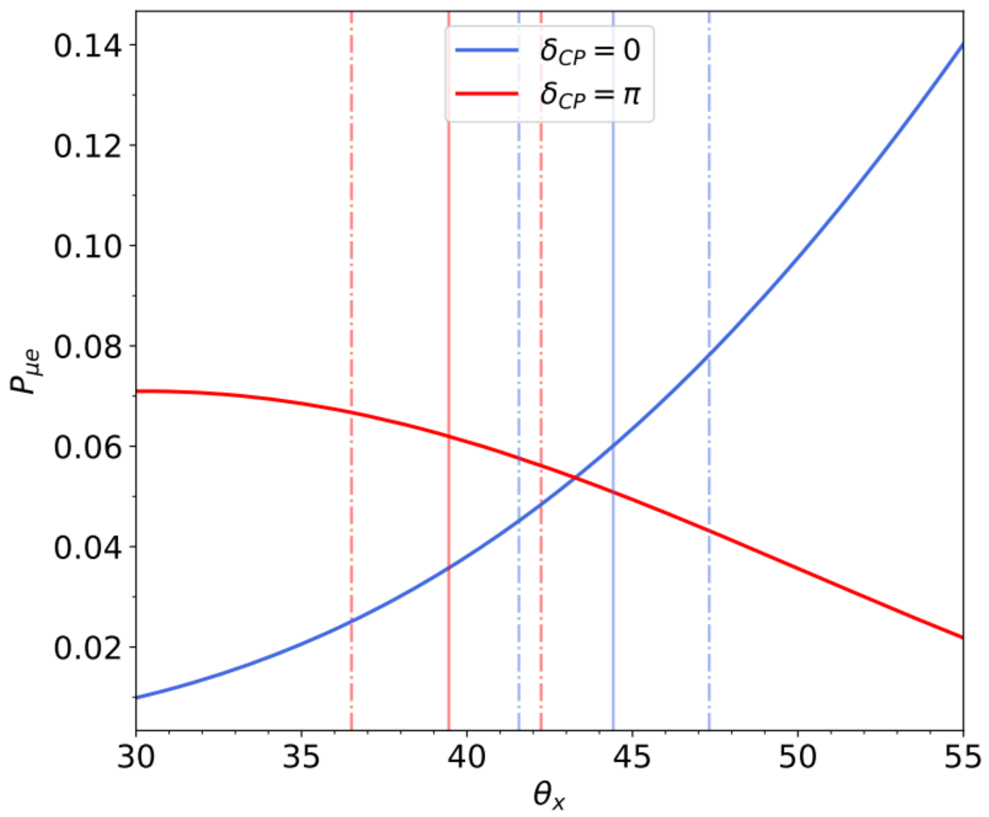
$$\chi^2 = \left(\frac{\theta_{23}^{\text{nufit}} - \theta_{23}(\theta_x, \theta_y, \theta_z)}{\sigma(\theta_{23}^{\text{nufit}})}\right)^2 + \left(\frac{\theta_{12}^{\text{nufit}} - \theta_{12}(\theta_x, \theta_y, \theta_z)}{\sigma(\theta_{12}^{\text{nufit}})}\right)^2 + \left(\frac{\theta_{13}^{\text{nufit}} - \theta_{13}(\theta_x, \theta_y, \theta_z)}{\sigma(\theta_{13}^{\text{nufit}})}\right)^2$$

We obtain **two sets of solutions**, for $\delta_{CP} = 0$ and $\delta_{CP} = \pi$. The corresponding best-fit values and 3σ ranges are

	$\delta_{CP} = \pi$		$\delta_{CP} = 0$	
	bfp	3σ range	bfp	3σ range
$\theta_x/^\circ$	39.45	36.51 – 42.24	44.42	41.57 – 47.32
$\theta_y/^\circ$	20.64	19.65 – 21.59	4.76	3.68 – 5.87
$\theta_z/^\circ$	28.82	27.56 – 30.08	35.32	34.14 – 36.49

Appearence probability for DUNE



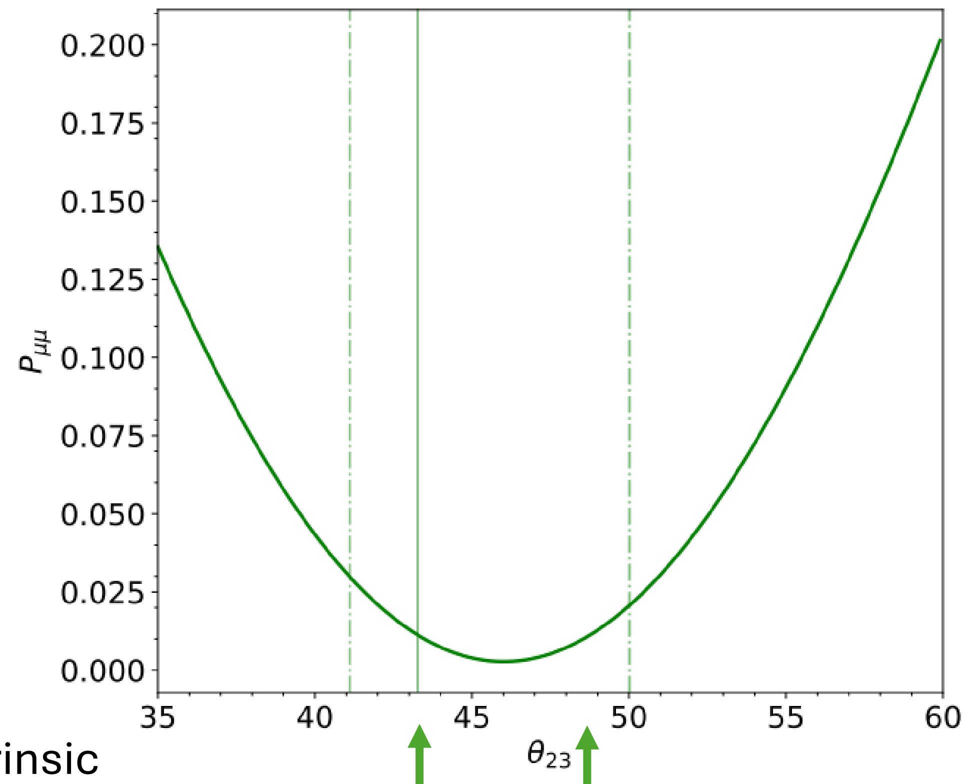


- Increases with θ_{23}

- $\delta_{\mathrm{CP}} = \pi$ $\rightarrow$ decreases with θ_{x}
- $\delta_{\mathrm{CP}} = 0$ $\rightarrow$ increases with θ_{x}

Disappearence probability for DUNE

$$P_{\mu\mu} = \ 1 - \sin^2 2\theta_{23} \sin^2 \Delta$$

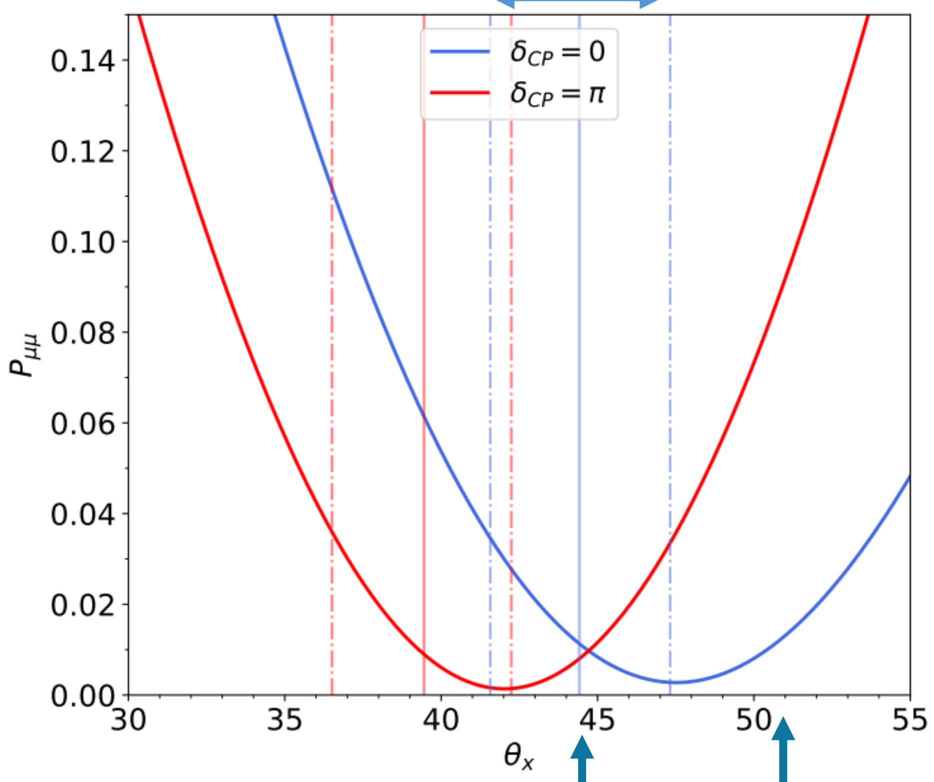


intrinsic octant degeneracy

$$\underbrace{P_{\mu\mu}(\theta_{23}) = P_{\mu\mu}(\pi/2 - \theta_{23})}_{\text{Both inside } 3\sigma}$$

Both inside 3σ

3σ



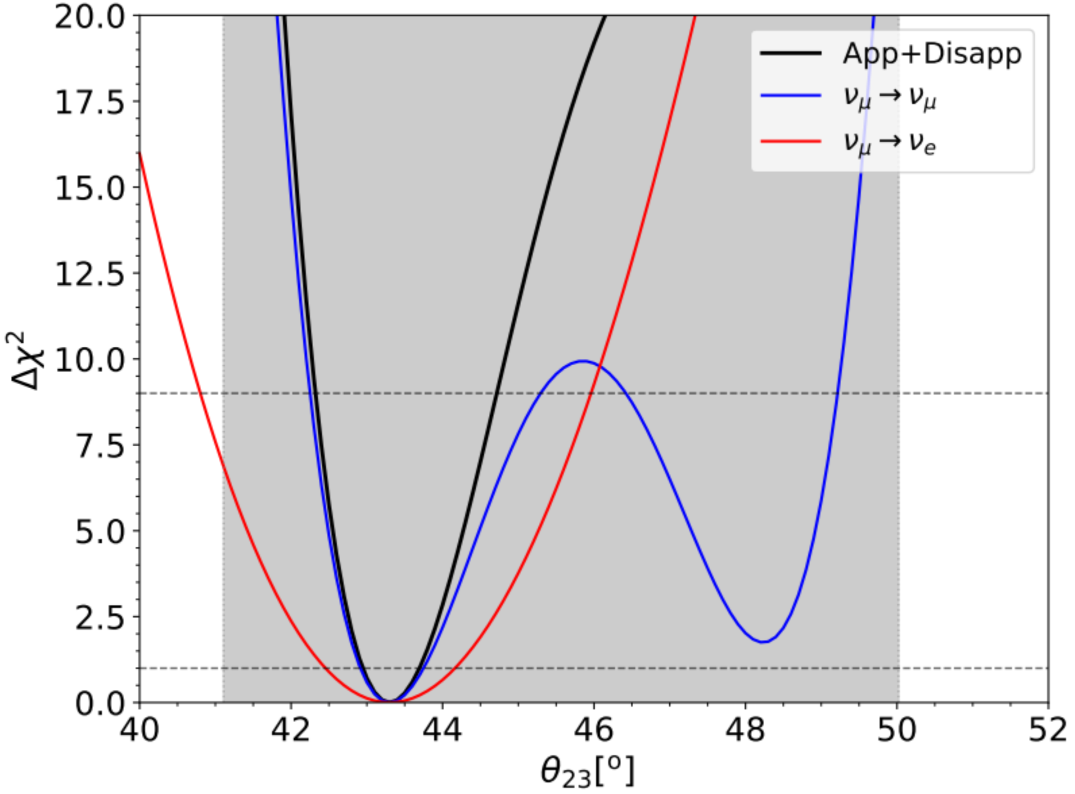
Best-fit

Clone solution → outside 3σ

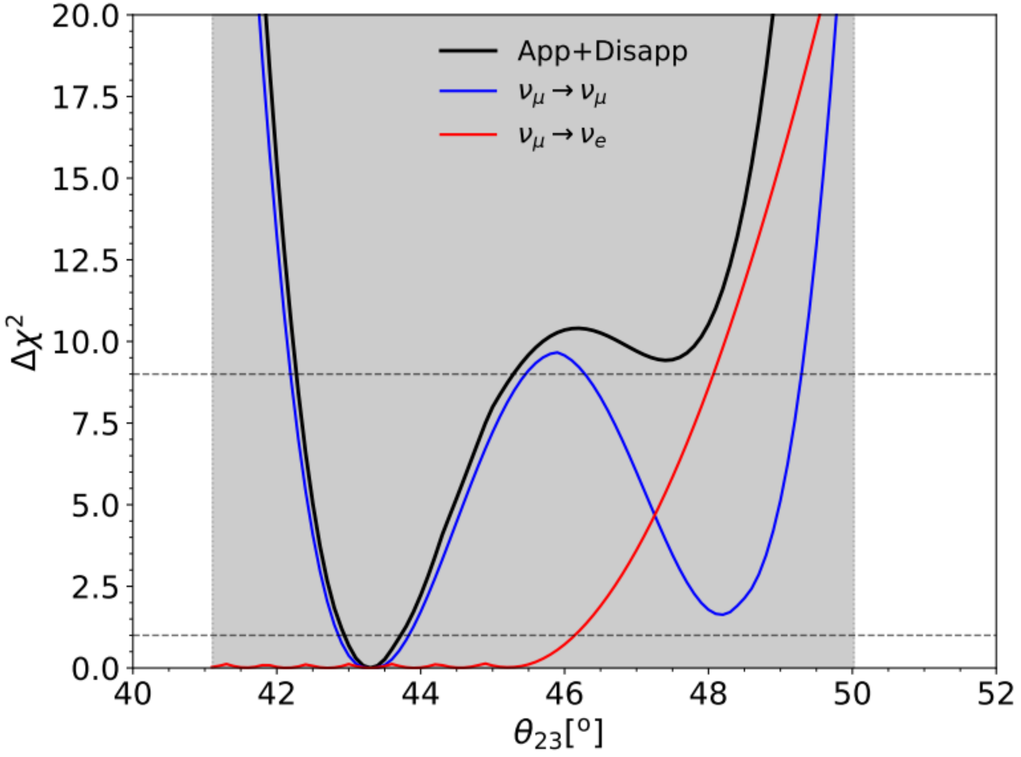
Octant sensitivity for θ_{23}

$$\chi^2_{\rm stat} = \frac{\left(N^{\rm true}(\theta_{23} = 43.27°) - N^{\rm false}(\theta_{23} = 40° \to 52°)\right)^2}{N^{\rm true}}$$

θ_{12}, θ_{13} fixed



θ_{12}, θ_{13} marginalized



$P_{\mu\mu} = 1 - \sin^2 2\theta_{23} \sin^2 \Delta$ $\rightarrow$ two minima inside the 3σ

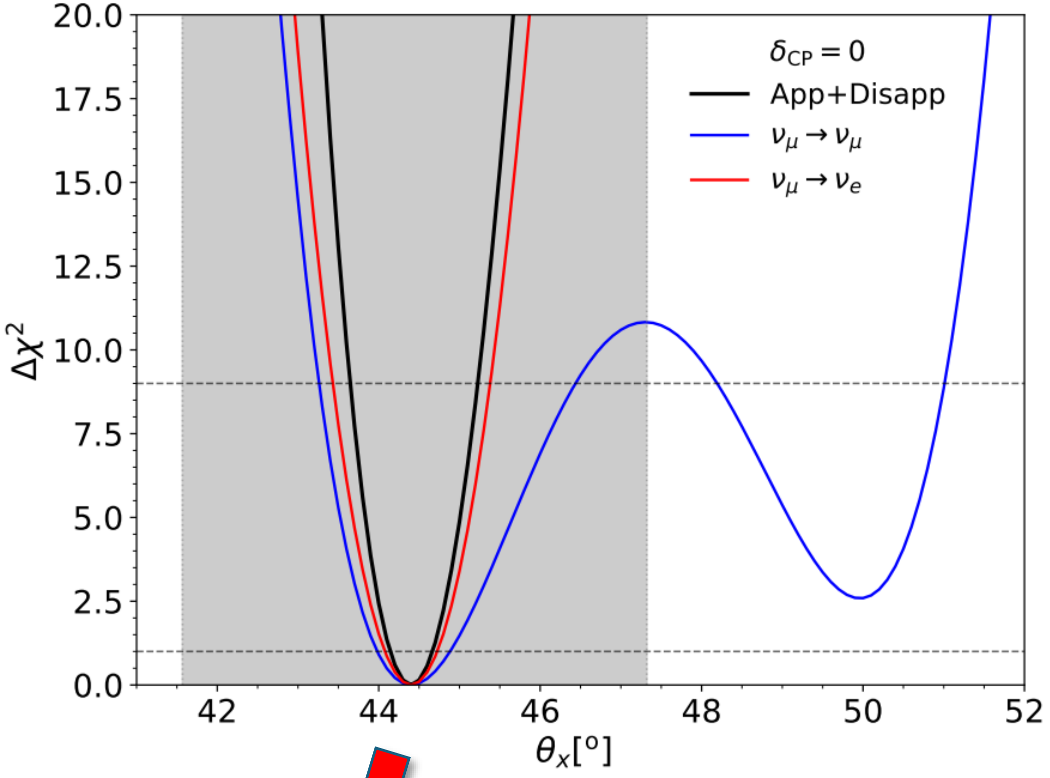
$P_{\mu e} \propto \sin^2 \theta_{13} \sin^2 \theta_{23}$ $\rightarrow$ no octant degeneracy

DUNE puts stronger bounds on θ_{23}

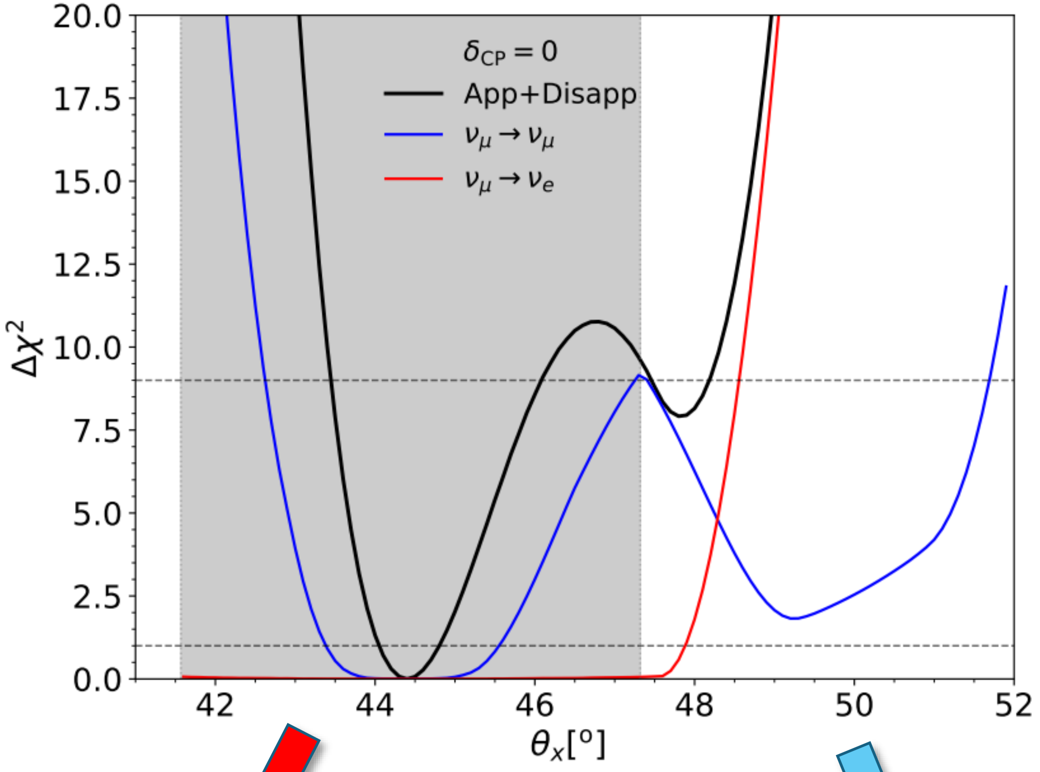
Octant sensitivity for θ_x

$\delta_{CP} = 0$

θ_y, θ_z fixed



θ_y, θ_z marginalized



With **better measurements of θ_y and θ_z** DUNE can improve current 3σ

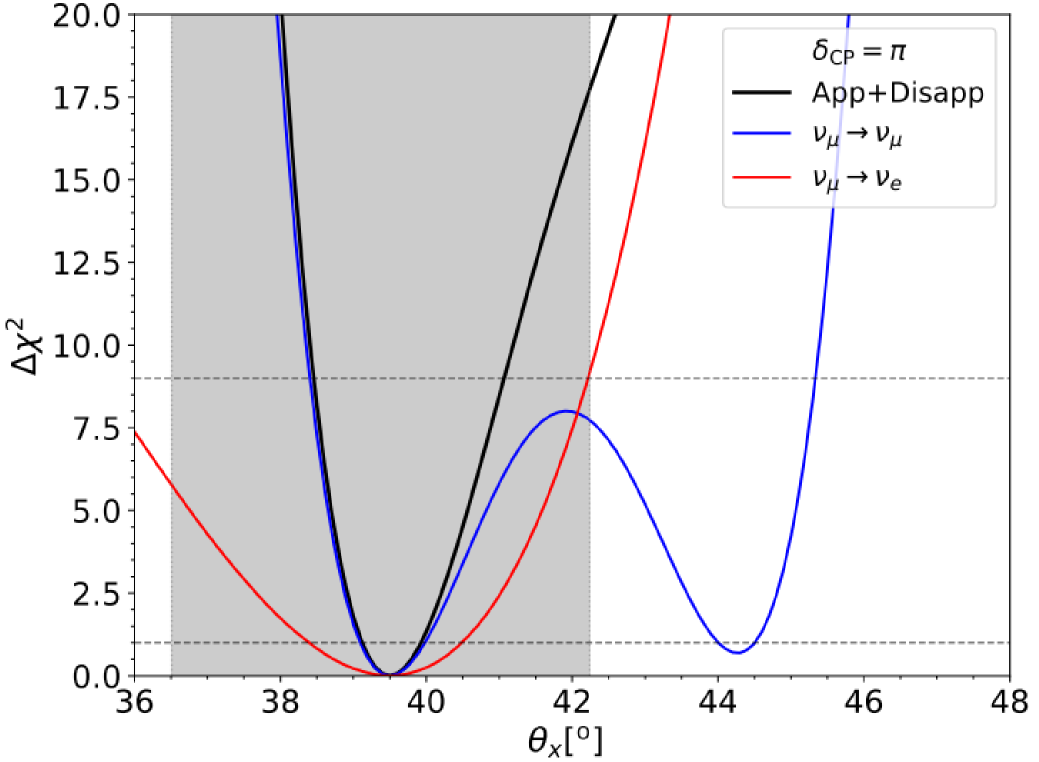
Degeneracy with θ_y and θ_z makes appearance curve really flat

Second minimum excluded by our fit to current neutrino oscillation data

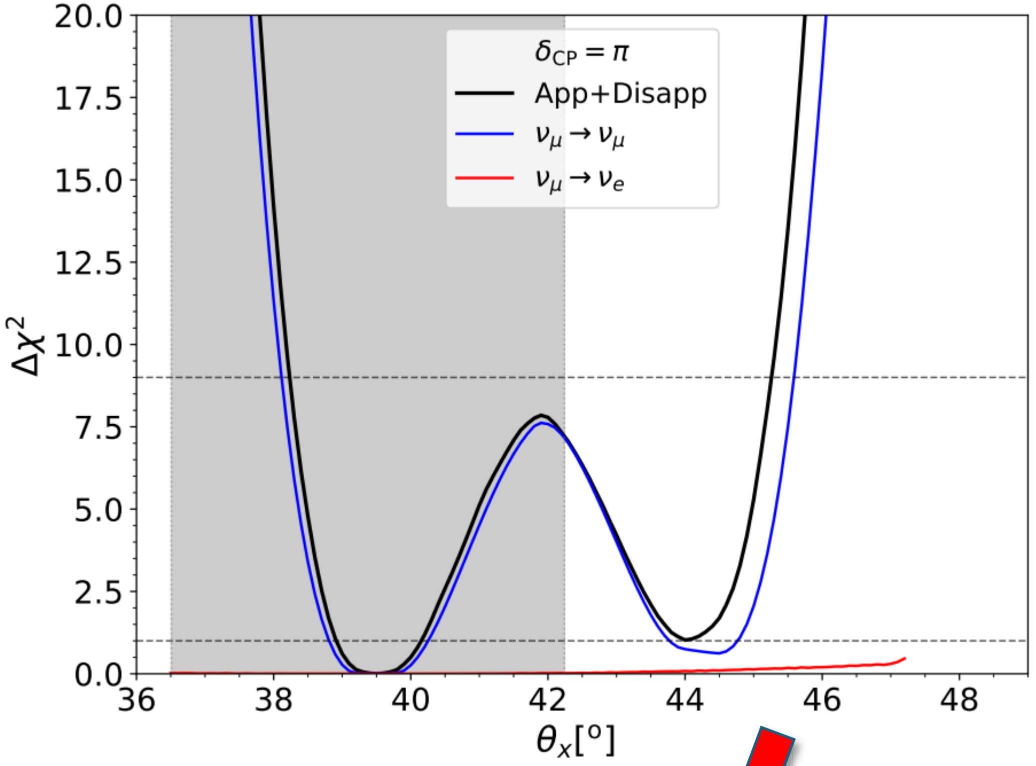
Octant sensitivity for θ_x

$\delta_{CP} = \pi$

θ_y, θ_z fixed



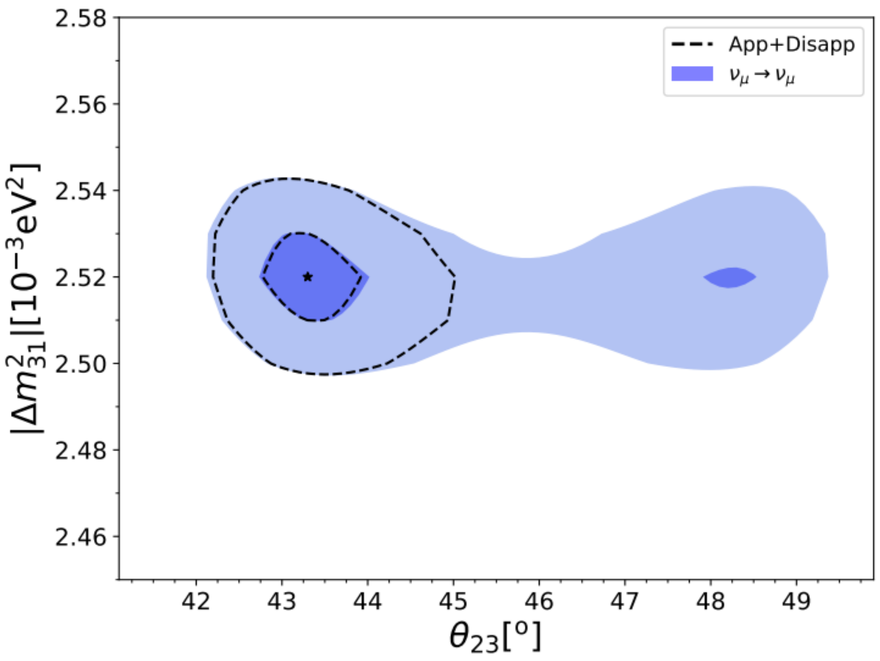
θ_y, θ_z marginalized

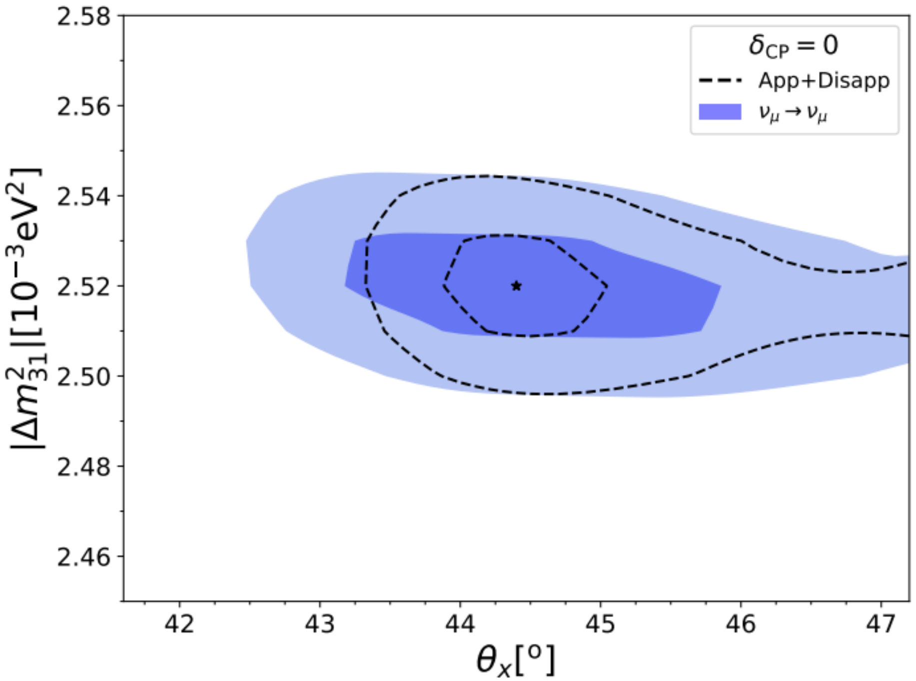


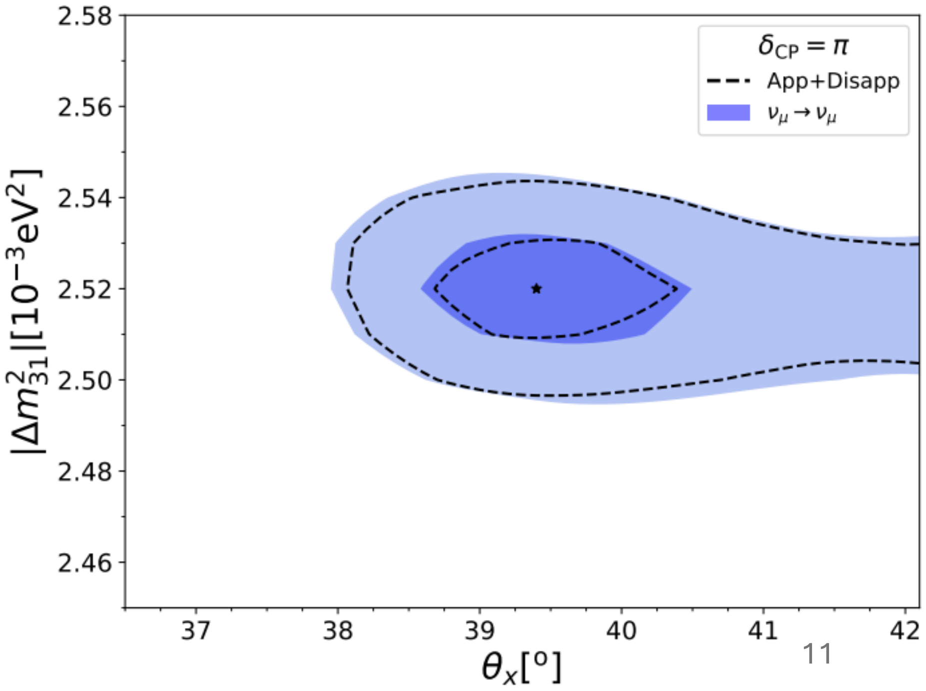
→ DUNE cannot prove θ_x octant unless θ_y and θ_z are fixed

More severe degeneracy with θ_y and θ_z makes the appearance curve completely flat

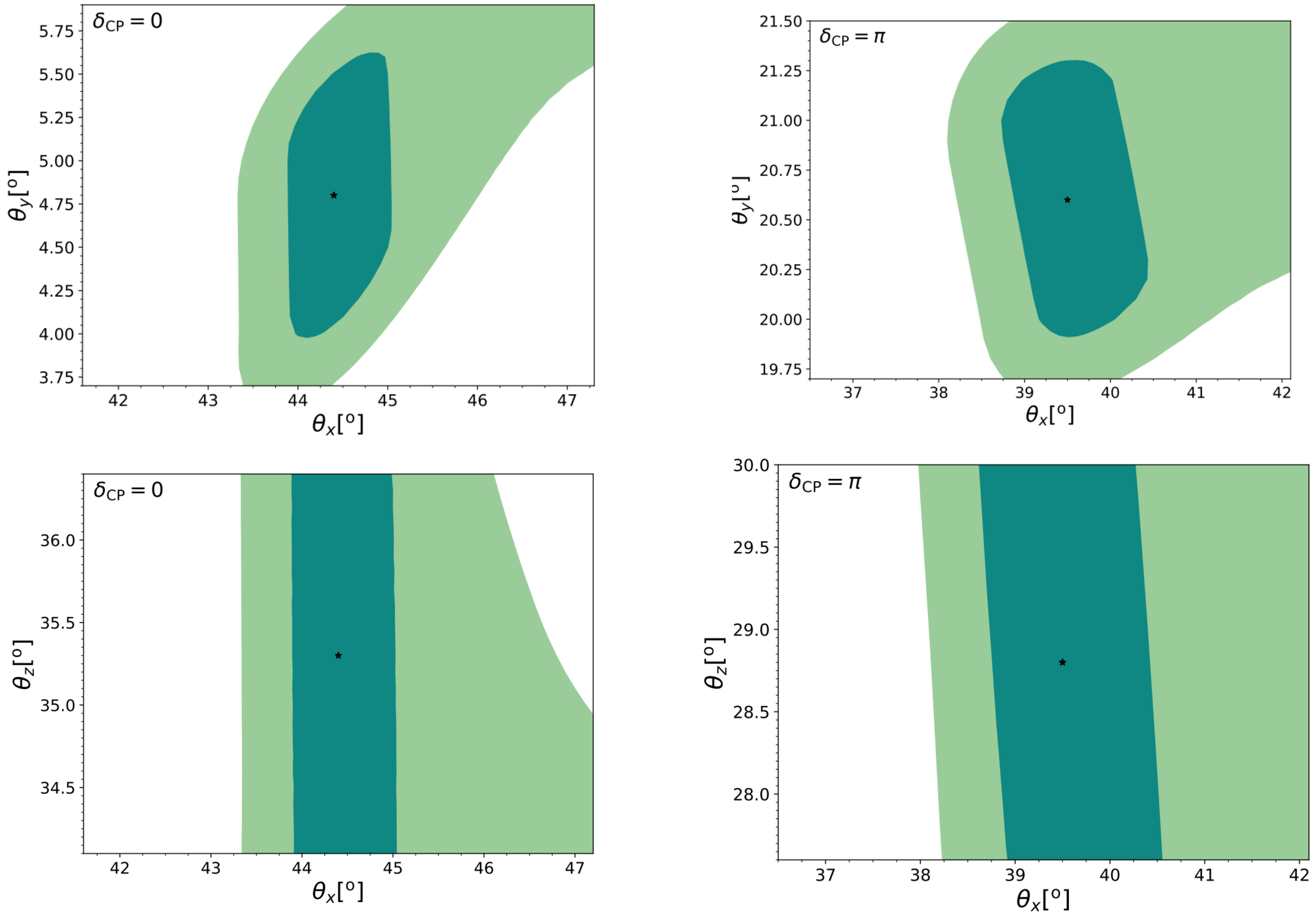
Precision: Δm_{31}^2 vs $\{\theta_{23}, \theta_x\}$







Precision: $\{\theta_y, \theta_z\}$ vs θ_x



Conclusions

- SO(3), an order independent parametrization described by 3 different mixing angle and CP-conserving phases
- Given the equivalence of these two parametrization matrices, we express the angles of the PMNS parametrization (θ23, θ13, θ12) in terms of the angles of the SO(3) parametrization (θx, θy , θz)
- There is not intrinsic octant degeneracy for θ_{x} with current oscillation data because the clone solutions lie outside the 3σ C.L.
- Octant sensitivity with DUNE $\rightarrow \theta_{23}$: can prove the octant with disapp+app stronger bounds

  $\rightarrow \theta_{\mathrm{x}}$: doesn't give better results that current data without a more prescise measurement of θ_{y} and θ_{z} due to degeneracies.

- Disappearence channel can measure Δm_{31}^2, but adding the appearence channel improves θ_{23} and θ_{x} 3σ C.L.
- DUNE has no sensitivity to θ_{y} and θ_{z} $\rightarrow$ Sensitivity studies in the solar neutrino sector with DUNE