Towards Quantum Simulation of Neutrino-Nucleus Many-Body Dynamics

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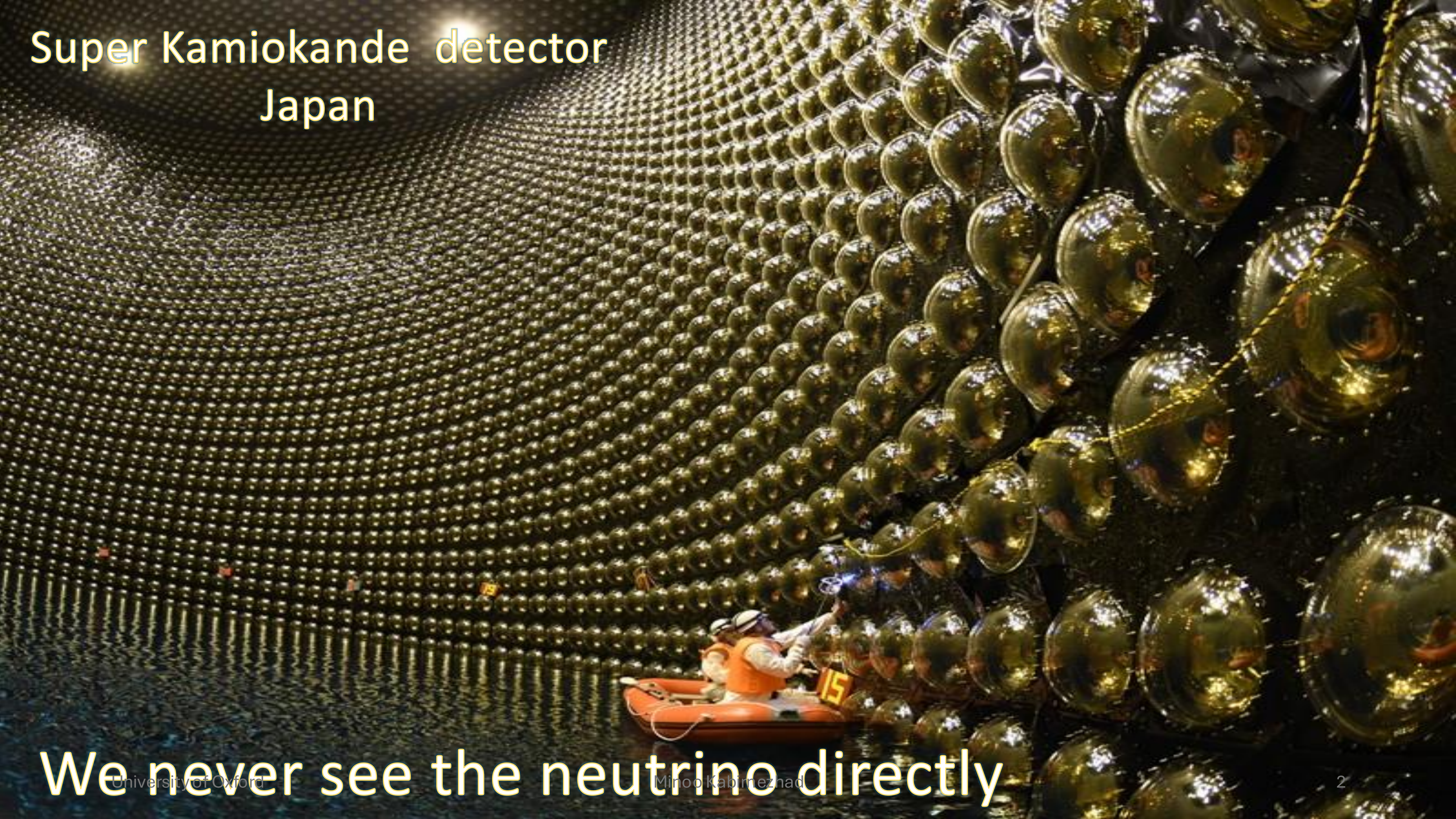
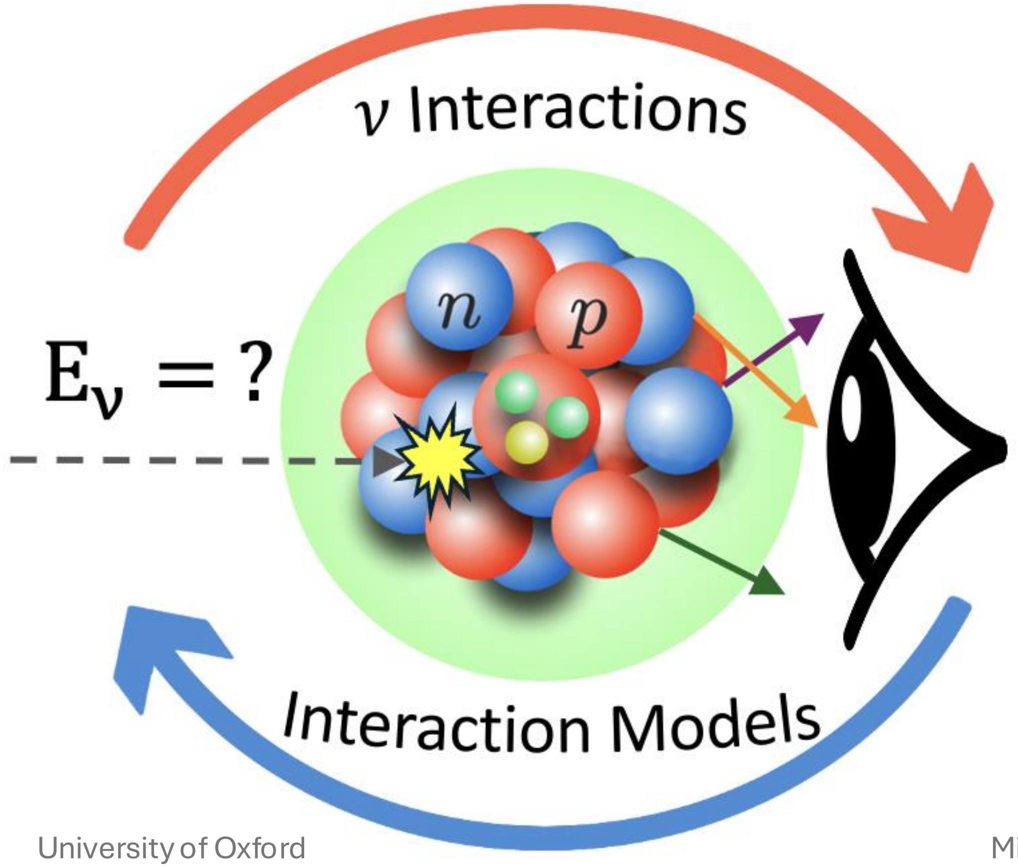
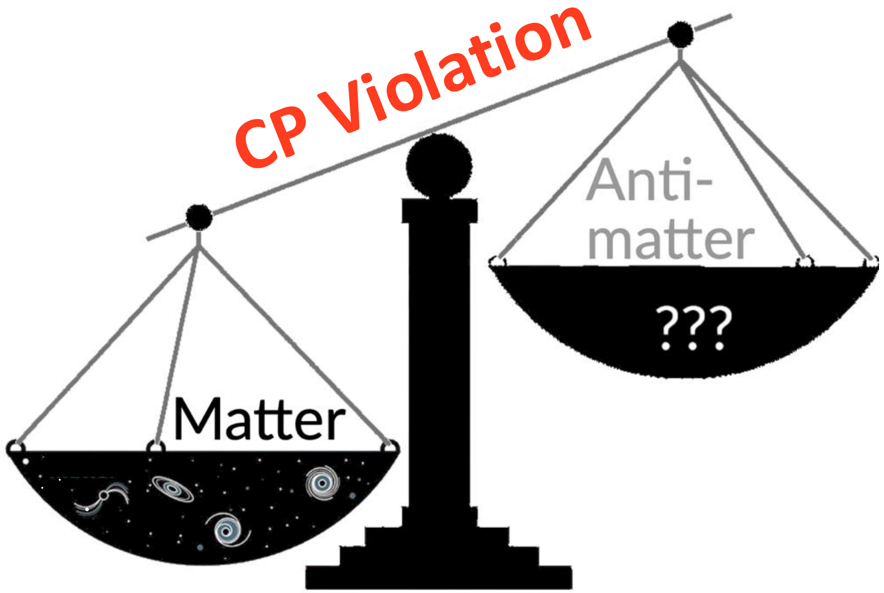

Super Kamiokande detector
Japan
15
We never see the neutrino directly

The challenge:
Seeing the invisible



DUNE

Hyper-K

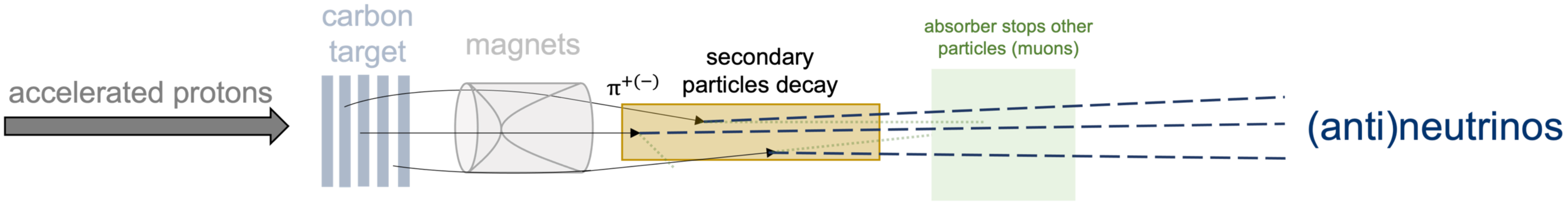


Precision Measurements

Accurate Prediction

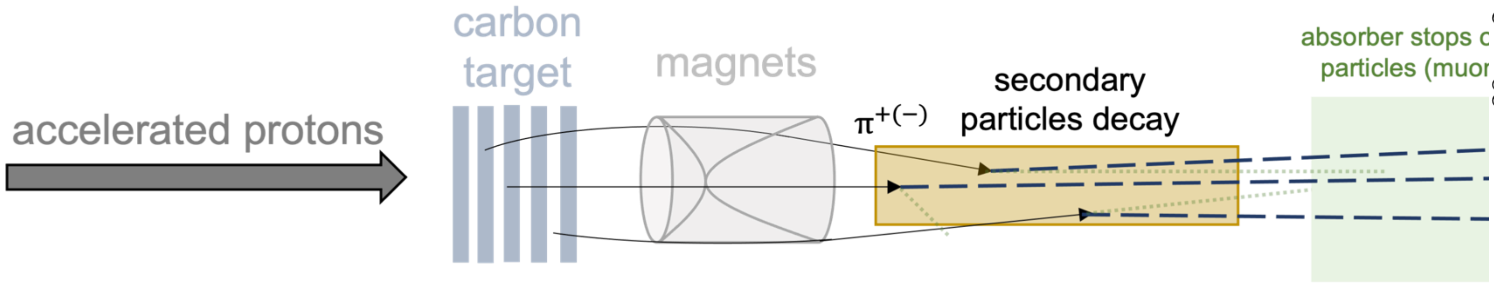
Accelerator-based neutrino experiments

- ν_μ **Beam Source:** Produced from accelerated protons

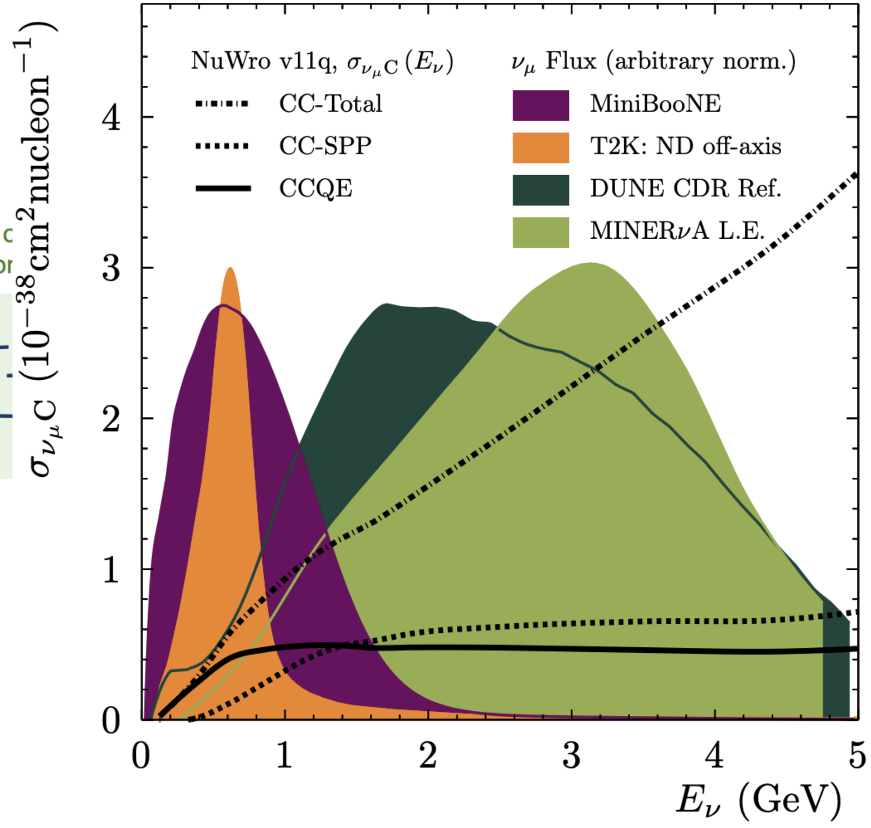


Accelerator-based neutrino experiments

- **ν_μ Beam Source:** Produced from accelerated protons
- **Energy Requirement:** Must be in the few GeV range



Wide energy neutrino beam



Accelerator-based neutrino experiments

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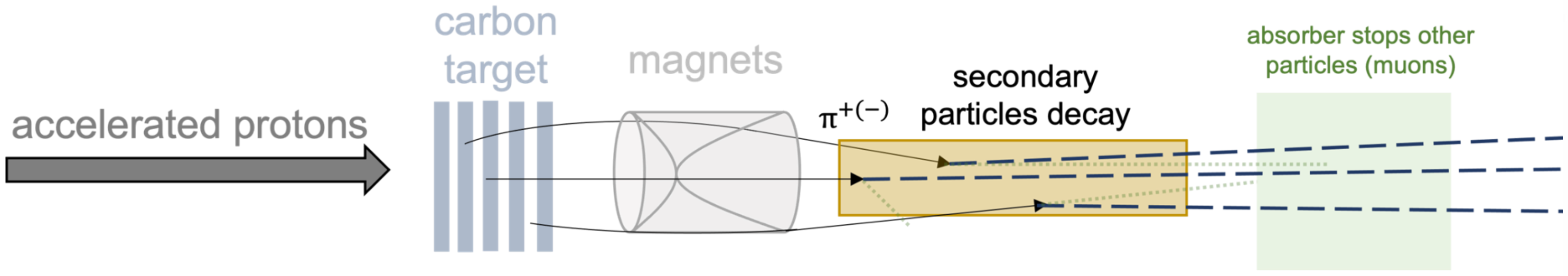
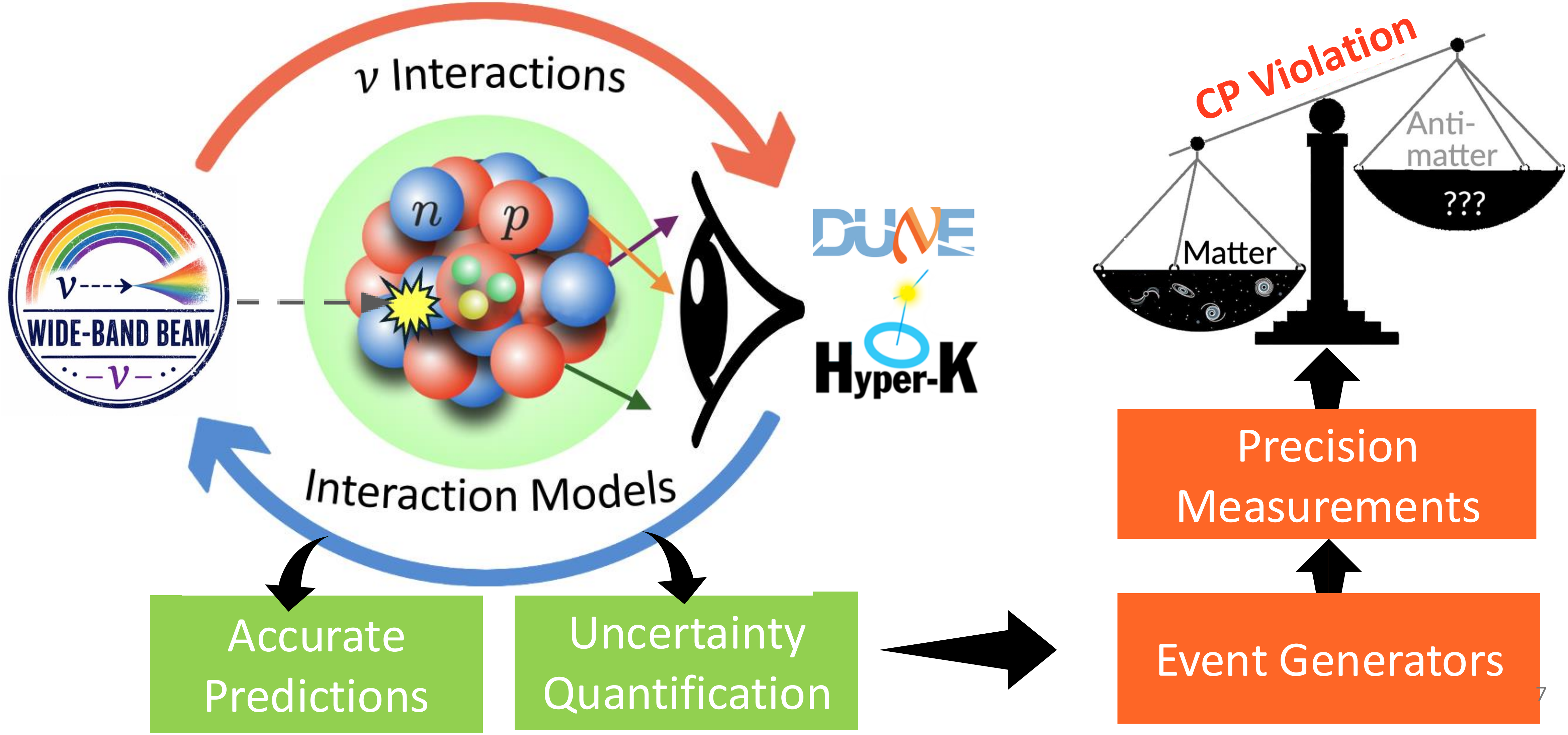
- **Energy Requirement:** Must be in the few GeV range

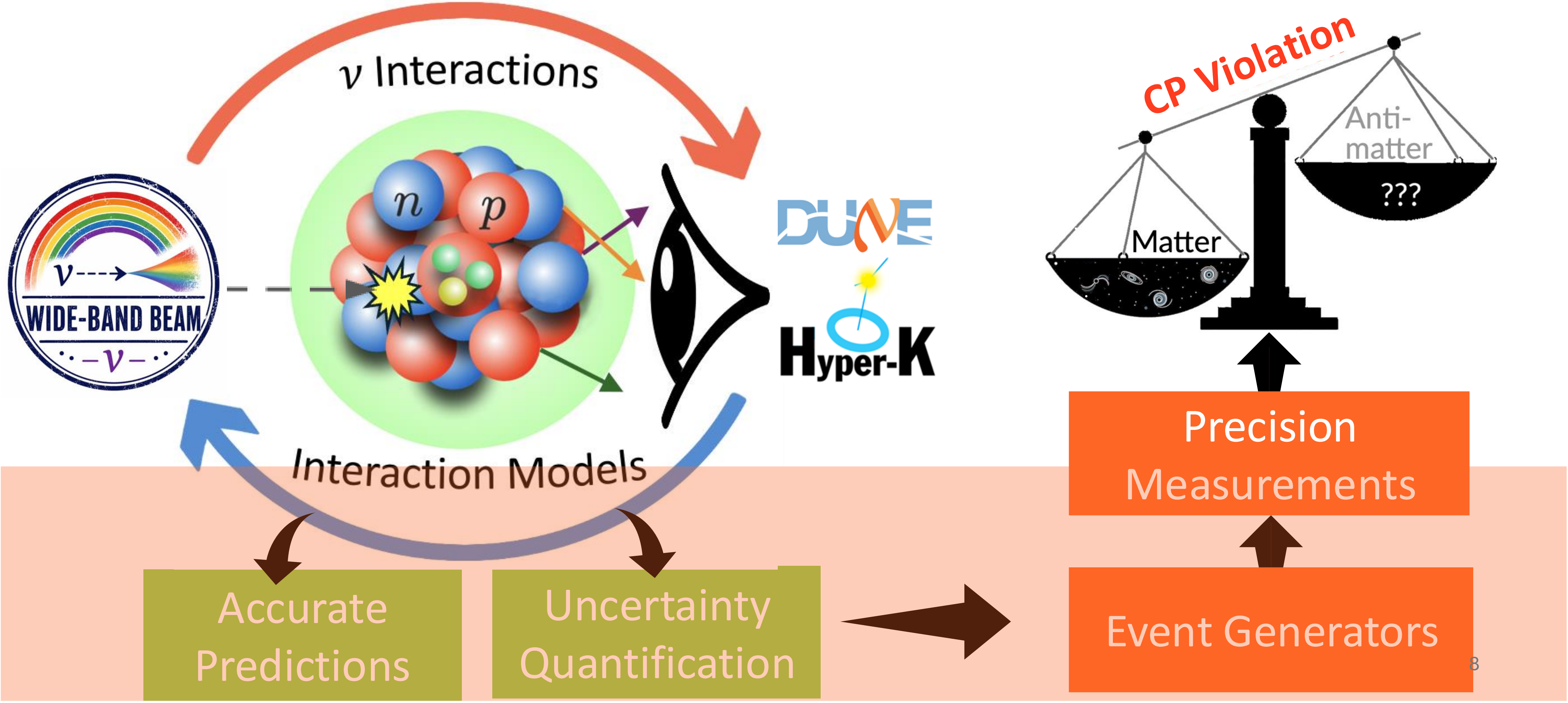


$$E_\nu \in [0.1 - 10]\ \mathrm{GeV}$$

Unveiling the Neutrino

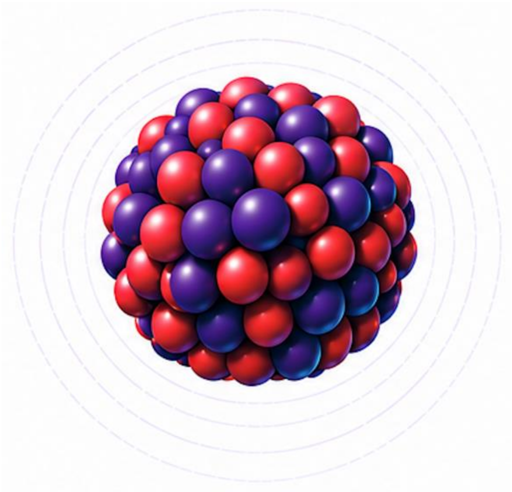


Customised Modelling for Future Discoveries



ν-Nucleus Interaction in Neutrino Experiment

1. Ground state



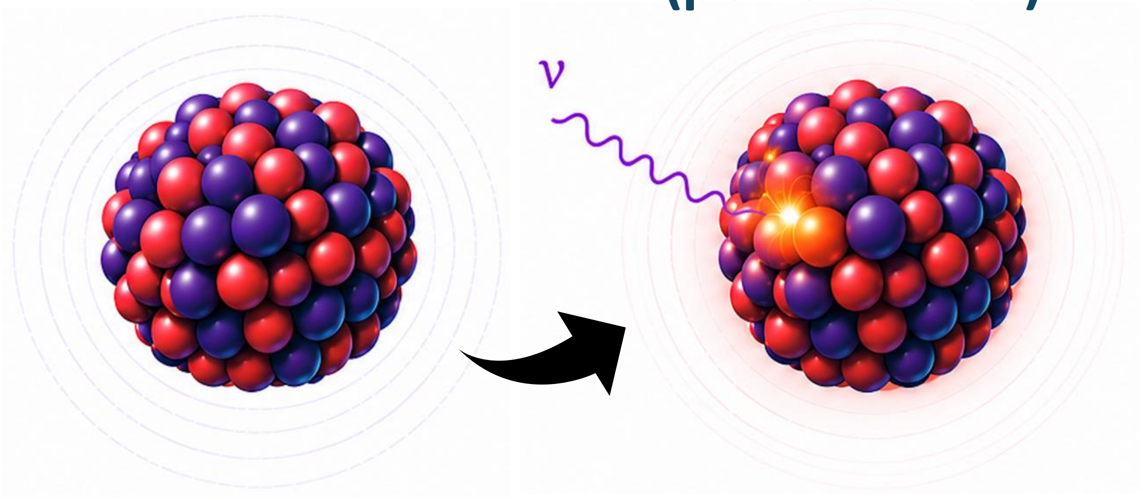
$|\Psi_0\rangle$

- Strongly correlated many-body quantum system
- $\mathrm{H}|\Psi_0\rangle = \mathrm{E}_0|\Psi_0\rangle$

ν-Nucleus Interaction in Neutrino Experiment

1. Ground state

2. Weak interaction (perturbation)



$|\Psi_0\rangle$

$\left|\Psi_f(0)\right\rangle = \hat{O}|\Psi_0\rangle$

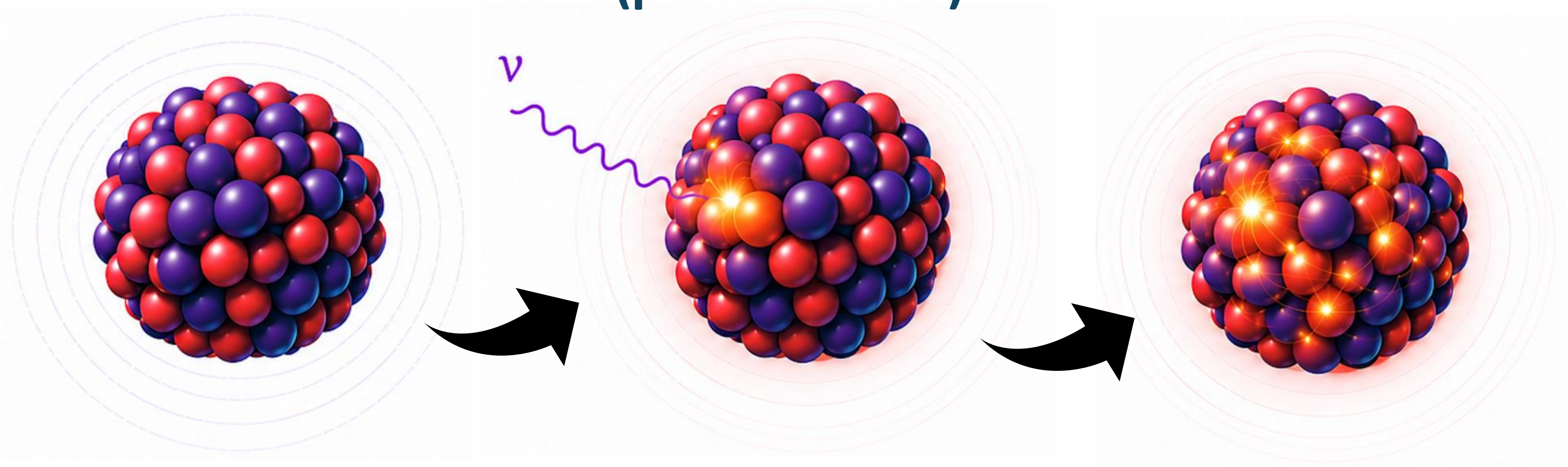
- A neutrino interacts via the weak force and excite nucleus locally

ν-Nucleus Interaction in Neutrino Experiment

1. Ground state

2. Weak interaction (perturbation)

3. Propagation



- The excitation propagates through the many-body system

$|\Psi_0\rangle$

$|\Psi_f(0)\rangle = \hat{O}|\Psi_0\rangle$

$|\Psi_f(t)\rangle = e^{-iHt}\,\hat{O}|\Psi_0\rangle$

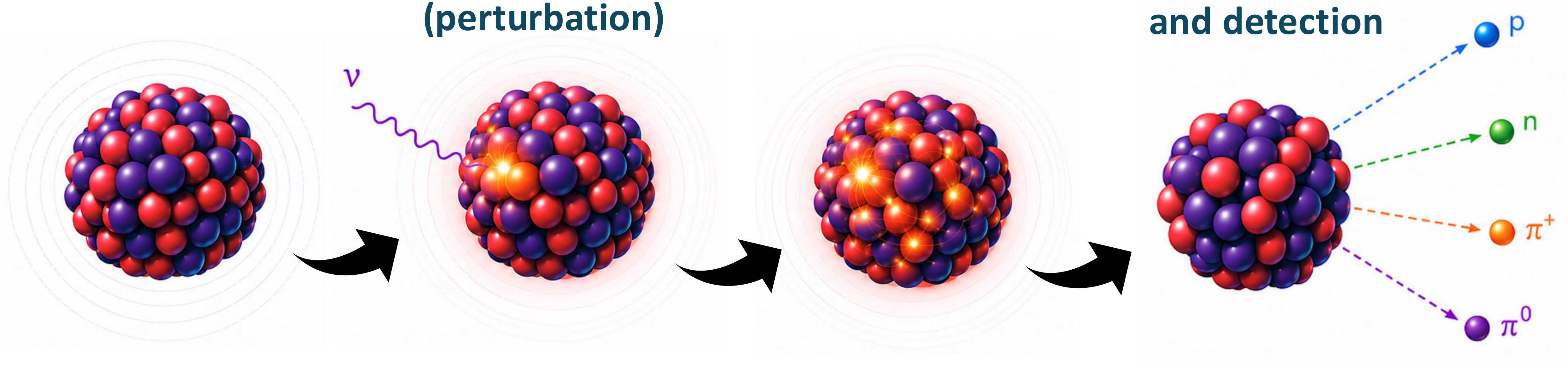
ν-Nucleus Interaction in Neutrino Experiment

1. Ground state

2. Weak interaction (perturbation)

3. Propagation

4. Particle emission and detection



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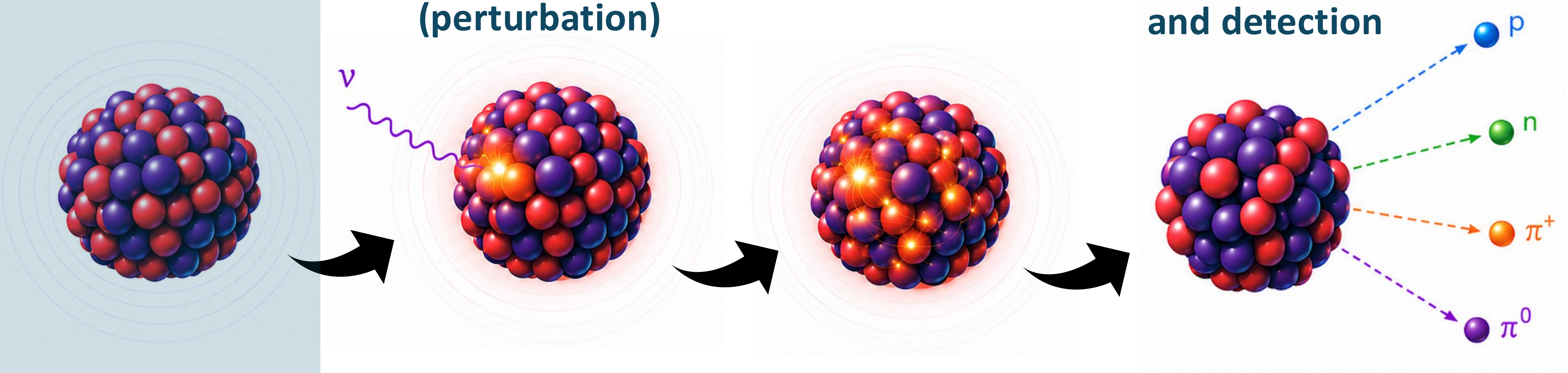
1. Preparing the Ground state

1. Ground state | **2. Weak interaction (perturbation)** | **3. Propagation** | **4. Particle emission and detection**



$|\Psi_0\rangle$

$\left|\Psi_f(0)\right\rangle = \hat{O}|\Psi_0\rangle$

$|\Psi_f(t)\rangle = e^{-iHt}\,\hat{O}|\Psi_0\rangle$

Not the biggest challenge!

1. Preparing the Ground state

- A nucleus in its ground state is stationary: all measurable quantities remain constant because the state is an energy eigenstate.

$$H \mid \Psi_0\rangle = E_0 \mid \Psi_0\rangle$$

$$H = T + V_{NN} + V_{3N} + \ \ldots$$

T: kinetic energy of the nucleons

V_{NN}: two-nucleon interactions

V_{3N}: three-nucleon interactions

1. Preparing the Ground state

- The interacting nuclear ground state is a quantum superposition of many-body configurations.
- The number of possible configurations grows exponentially with the size of the nucleus.
- Classical calculations explicitly represent the many-body wavefunction

$$| \Psi_0\rangle = \sum_i c_i \, | \Phi_i\rangle, \text{ solving for all the } c_i$$

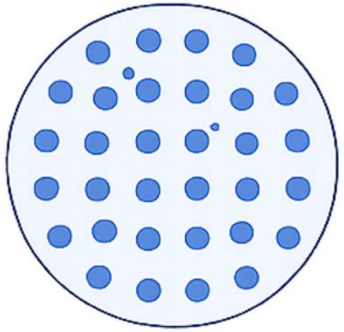
- For ^{40}Ar, assuming 100 single-particle states, the Hilbert space contains

$$\binom{100}{40} \approx 10^{28} \text{ configurations.}$$

Simple case: Non-interacting nucleon

$$H = T$$

- There is only one configuration:



$$|\Psi_0\rangle = |\text{Fermi sea}\rangle$$

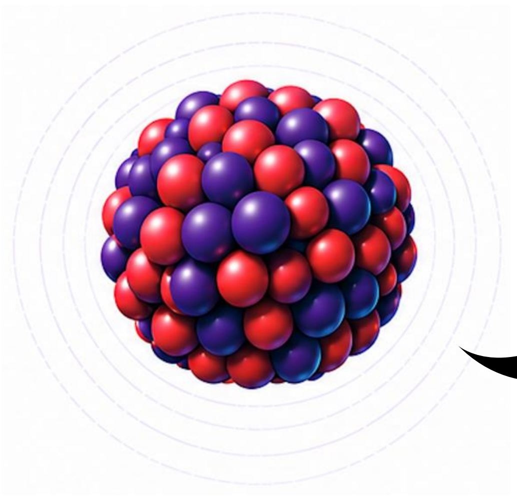
Occupied ($|k| < k_F$)

Unoccupied ($|k| > k_F$)

- To achieve the computational speed required for event generation, most neutrino interaction models rely on simplified descriptions of the nuclear dynamics.
- Additional mechanisms are then incorporated to account for missing many-body effects and improve agreement with experimental data.

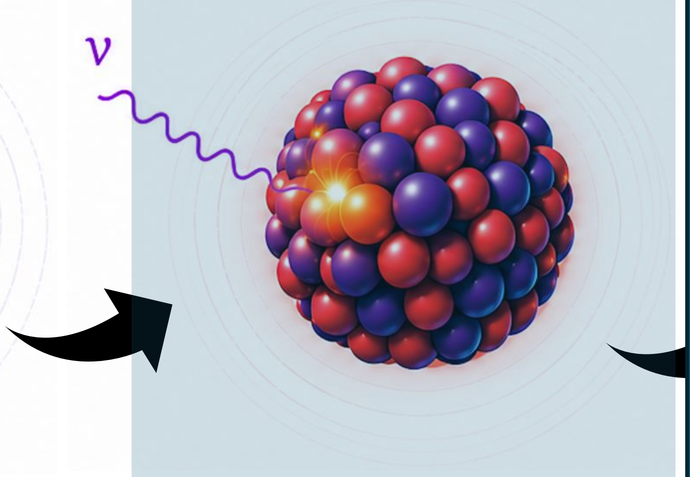
2. Excited state

1. Ground state



$|\Psi_0\rangle$

2. Weak interaction (perturbation)



$$|\Psi_f(0)\rangle = \hat{O}|\Psi_0\rangle$$

- The weak transition operator $\hat{O}(q,\omega)$ must be modelled over a broad kinematic region.
- Applying $\hat{O}$ drives the nucleus out of equilibrium, creating a superposition of many excited states.
- The resulting state, $|\Psi_f(0)\rangle$, is not an eigenstate of the nuclear Hamiltonian and therefore evolves in time.

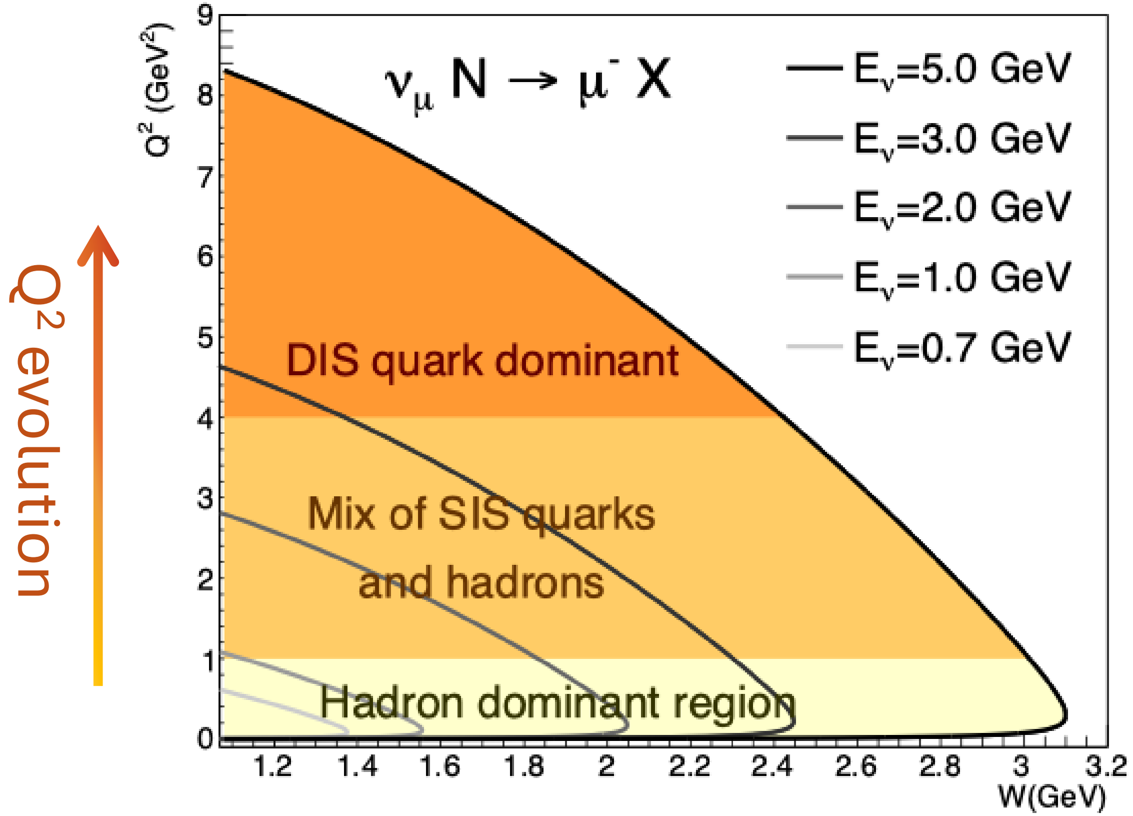
Broad neutrino energy spectrum

↓

wide kinematic coverage

↓

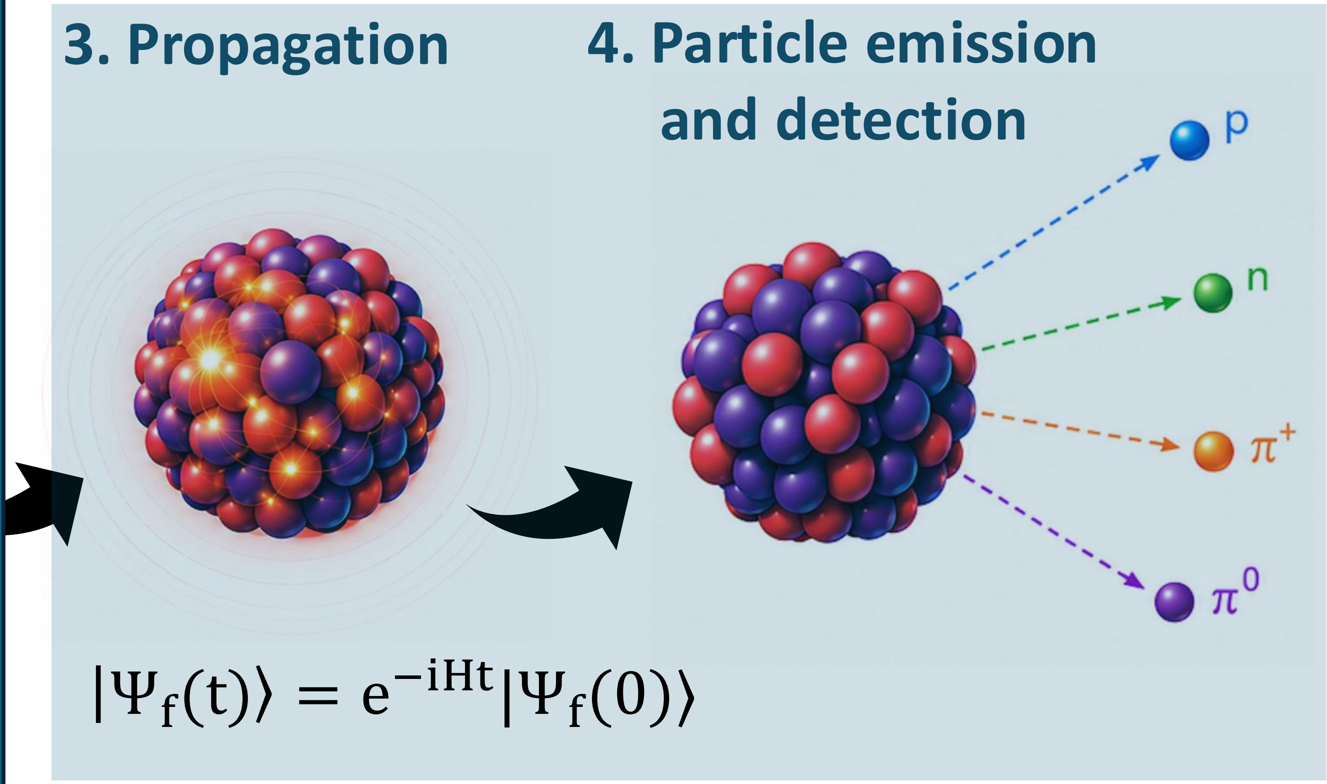
multiple interaction mechanisms contribute



3. Time evolution and particle emission

- What is the time-evolved many-body state, $|\Psi_f(t)\rangle$?

- How does its quantum evolution give rise to the final-state particles observed in the detector?



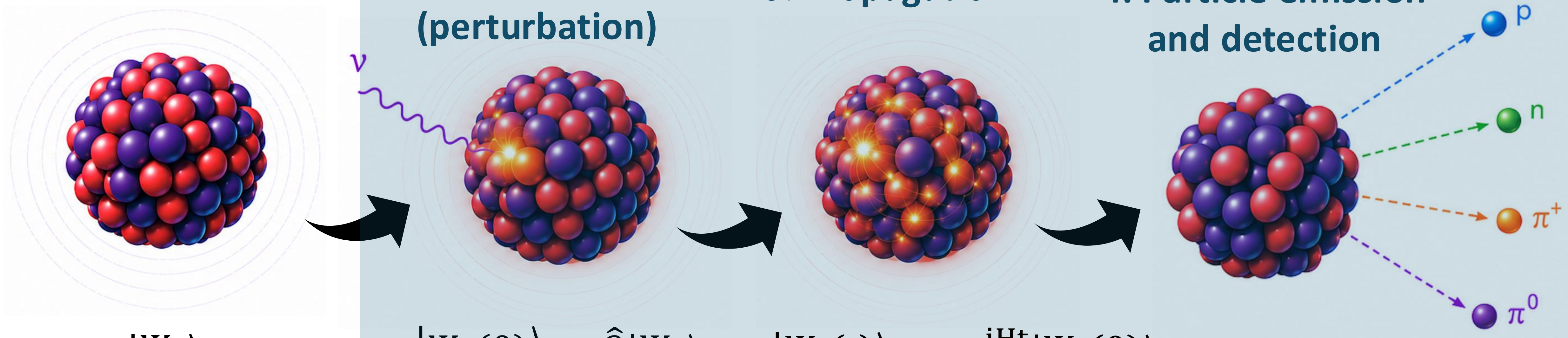
ν-Nucleus Interaction in Neutrino Experiment

1. Ground state

2. Weak interaction (perturbation)

3. Propagation

4. Particle emission and detection



$|\Psi_0\rangle$

$|\Psi_f(0)\rangle = \hat{O}|\Psi_0\rangle$

$|\Psi_f(t)\rangle = e^{-iHt}|\Psi_f(0)\rangle$

The central challenge is predicting how a local quantum excitation propagates through a strongly correlated finite nucleus and ultimately emerges as the particles observed in the detector.

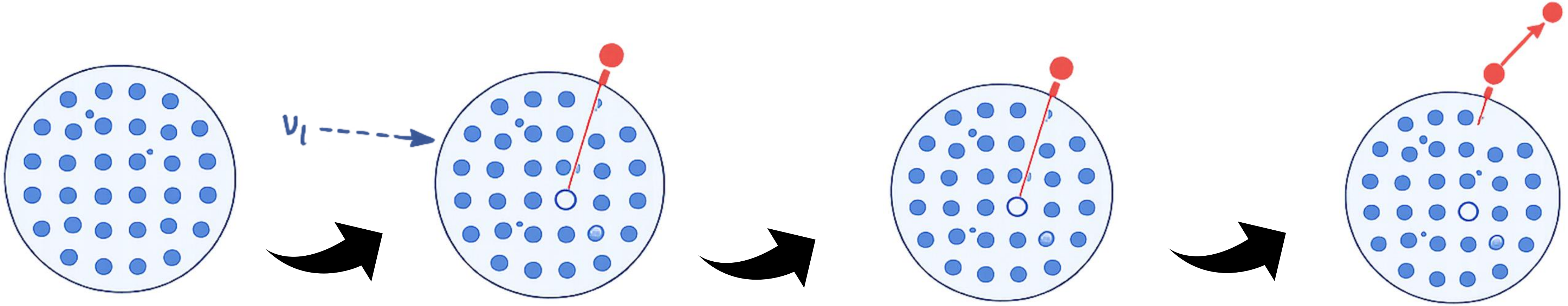
The most simplest case: H=T

1. Ground state
Fermi sea

2. Weak interaction
One nucleon is excited

3. Time evolution
Under H=T

4. Particle emission and detection



Fermi sea

A 1p1h configuration
One particle above K_F
and one hole below K_F

No interaction ->
no scattering,
no Mixing

One nucleon leaves

- Occupied ($|k| < k_F$)
- Unoccupied ($|k| > k_F$)

Quantum Advantage for Nuclear Physics

- Quantum technologies offer a fundamentally new way to simulate strongly correlated many-body systems.
- Universal fault-tolerant quantum computers could, in principle, perform exact simulations of nuclear dynamics.
- However, realistic nuclei such as $^{40}\mathrm{Ar}$ require a very large number of logical qubits and deep quantum circuits.

A. Roggero *et al.*, Phys. Rev. D, 101(7), 2020.

J. D. Watson *et al.*, arXiv:2312.05344 (2023).

Quantum Advantage for Nuclear Physics

- Quantum technologies offer a fundamentally new way to simulate strongly correlated many-body systems.
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- However, realistic nuclei such as $^{40}\mathrm{Ar}$ require a very large number of logical qubits and deep quantum circuits.
- Large-scale fault-tolerant quantum computers are unlikely to be available in the near future.
- This motivates exploring analogue quantum simulation as a near-term alternative.

Why Analogue Quantum Simulation?

- No exponentially large many-body wavefunction needs to be stored.

$$|\Psi_0\rangle = \sum_i c_i \,|\, i\rangle$$

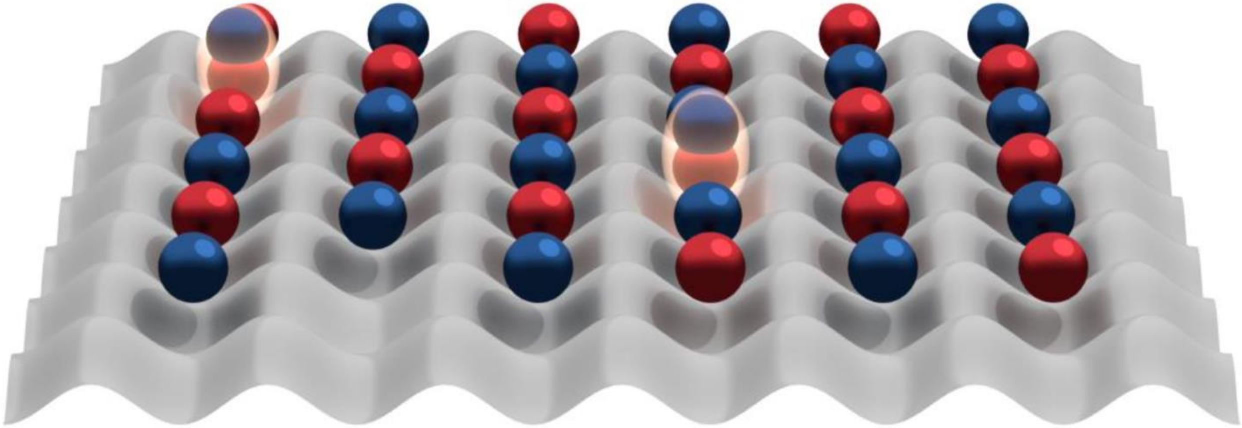
Classical representation

- Instead, the atoms themselves realize the quantum state.
- The many-body dynamics are realized physically, rather than computed numerically.

Why Ultracold Atoms?

- Largest controllable quantum many-body systems available today ($10^3 - 10^6$ atoms).
- Highly tunable interactions, trapping geometries, and internal degrees of freedom.
- Natural implementation of Hamiltonian time evolution. Direct access to real-time many-body dynamics.
- An ideal platform for exploring nuclear many-body dynamics beyond current classical capabilities.

Can Nature Simulate Itself?



Analogue quantum simulation with ultracold atoms

Map a complex nuclear system onto a highly controllable quantum platform that obeys the same many-body physics.

Universality Beyond Energy Scales

A. J. Daley *et al.*,
Nature **607**, 667–676 (2022).

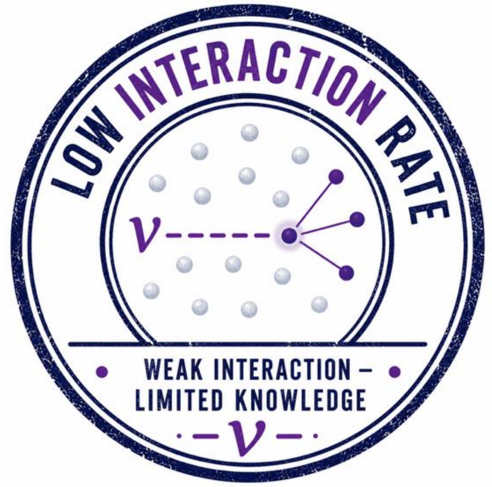
- Nuclei and ultracold atoms differ by many orders of magnitude in energy, length, and time scales.
- Both systems reside close to the **unitary regime.**

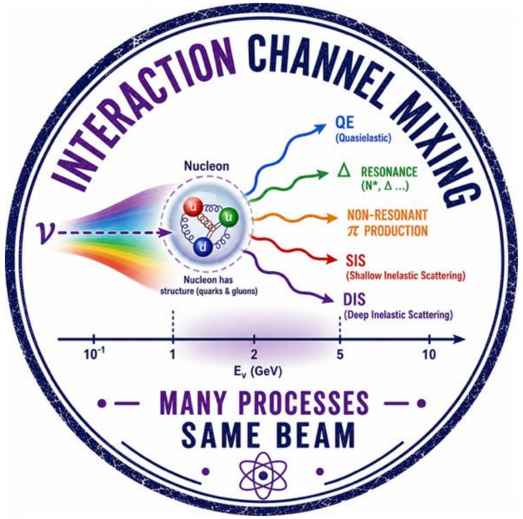
$$K_F|\mathrm{a}| \gg 1$$

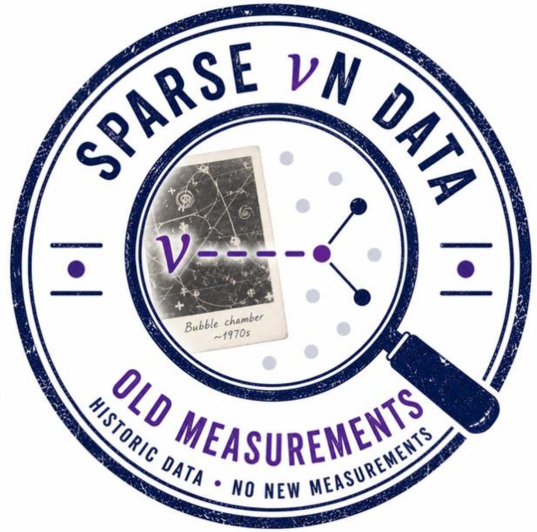
- Near unitarity, many-body dynamics become largely universal and insensitive to the microscopic interaction.
- Matching the relevant dimensionless parameters enables ultracold atoms to reproduce essential aspects of nuclear many-body dynamics.

Conclusion & Outlook

- Understanding neutrino-nucleus interactions is essential for precision neutrino oscillation measurements and future discoveries.
- The weak interaction creates a local excitation in a strongly correlated quantum many-body nucleus.







Conclusion & Outlook

- Understanding neutrino-nucleus interactions is essential for precision neutrino oscillation measurements and future discoveries.
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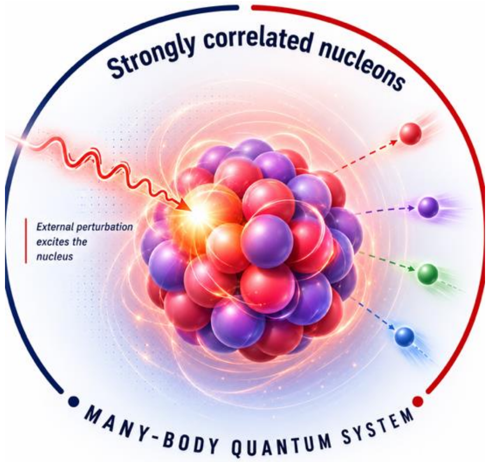
- Predicting the observed final-state particles requires understanding the real-time evolution of this many-body quantum system.

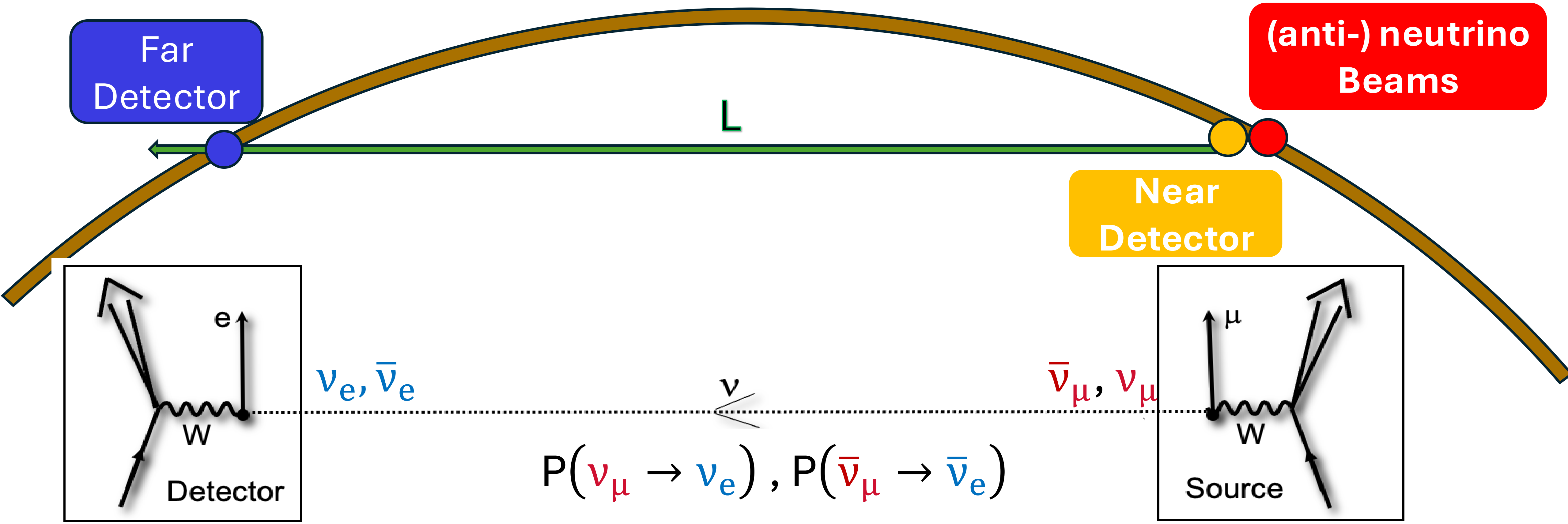


Conclusion & Outlook

- Understanding neutrino-nucleus interactions is essential for precision neutrino oscillation measurements and future discoveries.
- The weak interaction creates a local excitation in a strongly correlated quantum many-body nucleus.
- Predicting the observed final-state particles requires understanding the real-time evolution of this many-body quantum system.
- Ultracold atoms provide a highly controllable platform to explore universal many-body dynamics beyond the reach of current classical methods.

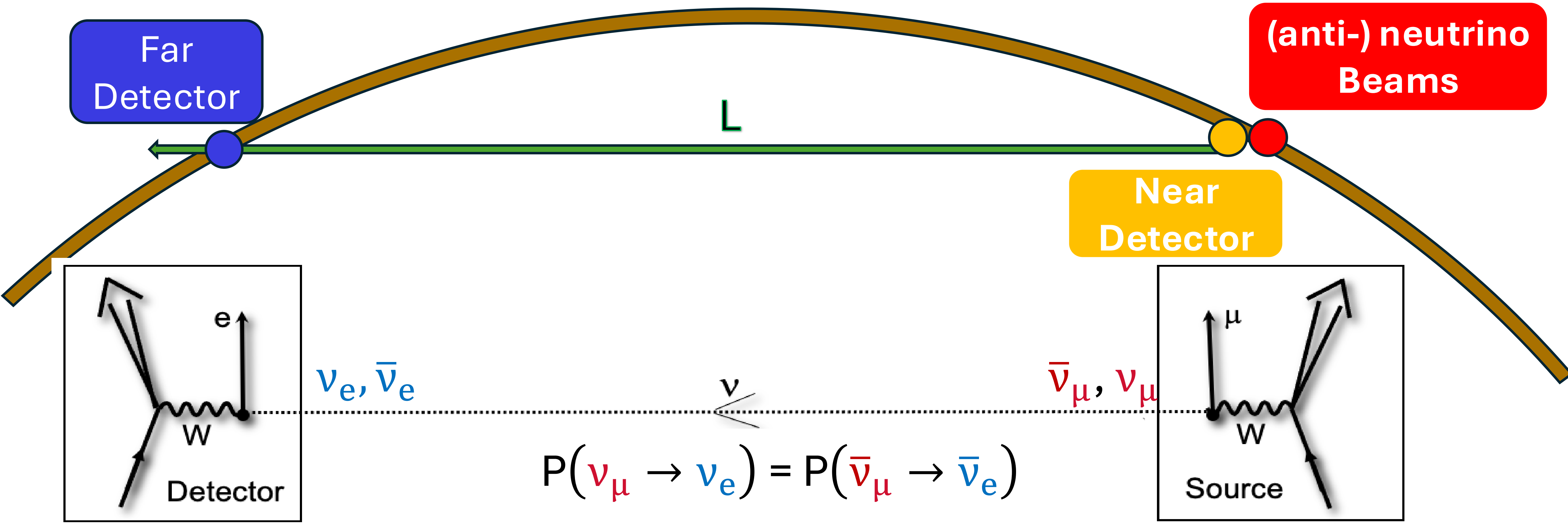
Backup

Neutrino Oscillation experiments



$$P(\nu_\mu \to \nu_e)\,,\, P(\bar{\nu}_\mu \to \bar{\nu}_e)$$

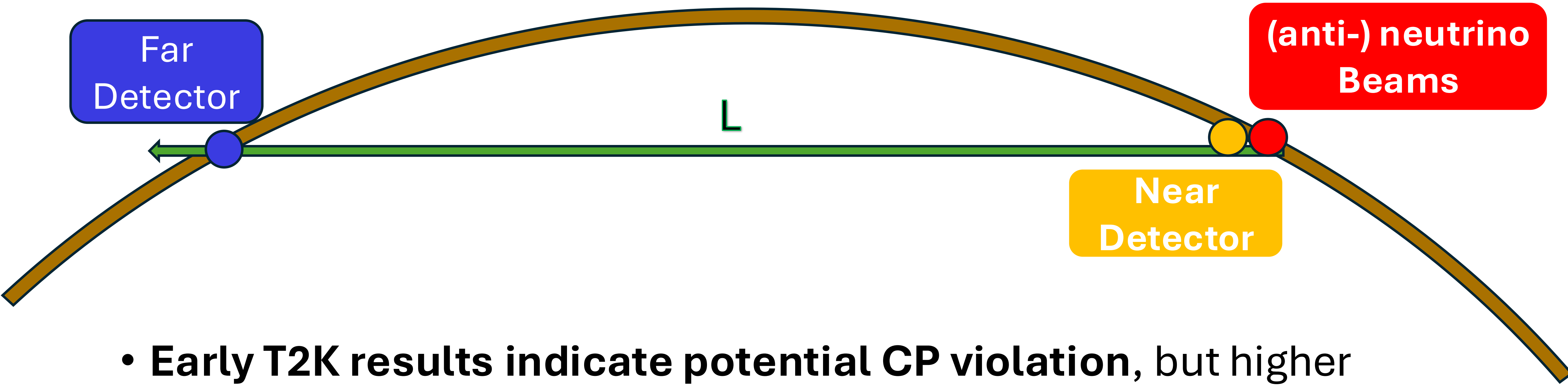
Neutrino Oscillation experiments



$$P(\nu_\mu \to \nu_e) = P(\bar{\nu}_\mu \to \bar{\nu}_e)$$

CP conservation $(\delta_{CP} = 0)$

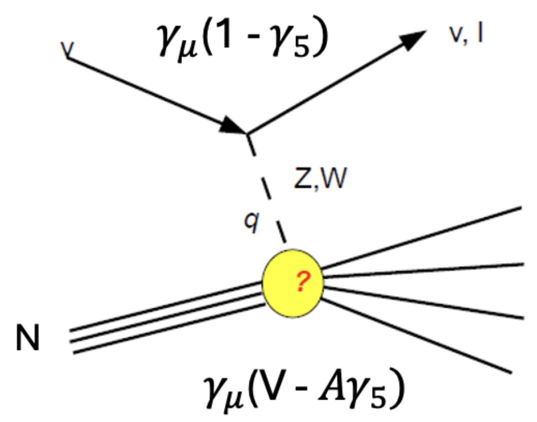
Neutrino Oscillation experiments



- **Early T2K results indicate potential CP violation**, but higher precision measurements are required to confirm this.
- **Cross-section uncertainties remain a major challenge**, impacting the precision of oscillation parameter measurements.

ν-nucleon interaction in the GeV regime

- The nucleon's internal structure modifies the weak interaction.
- The weak current couples through vector and axial-vector currents.
- Form factors determine how the scattering differs from that of a point-like particle.
- Lattice QCD provides first-principles calculations of these form factors, but is currently limited by computational cost and accessible kinematics.
- Experimental data remain essential to determine, constrain, and validate the form factors across the relevant kinematic region.



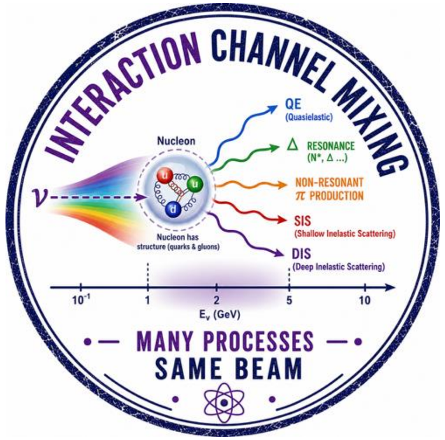
Bubble Chambers

- **Targets:** Hydrogen & deuterium
- **Role:** Pioneered neutrino interaction studies (1960s–1980s)
- **Key Contribution:** Early cross-section measurements
- Now retired, often found in national lab car parks

Big European Bubble Chamber (BEBC) CERN

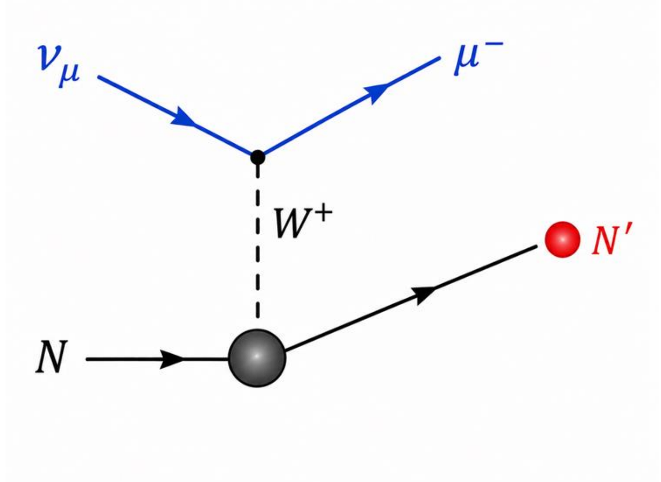
The 15-foot Bubble Chamber FNAL

ν-nucleon interactions in the GeV regime



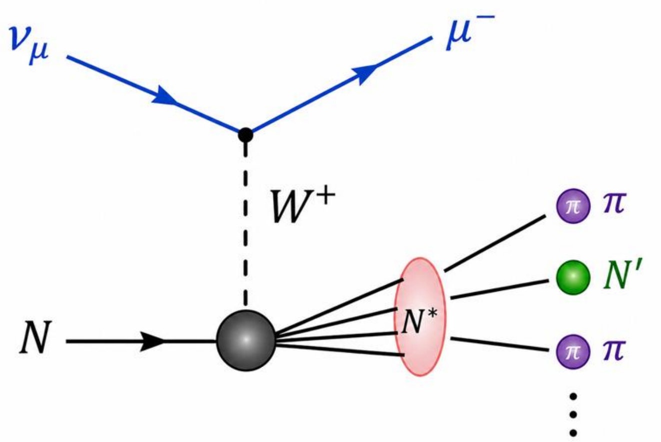
- Wide-Band Beam Drives Many processes

(Quasi-) Elastic scattering

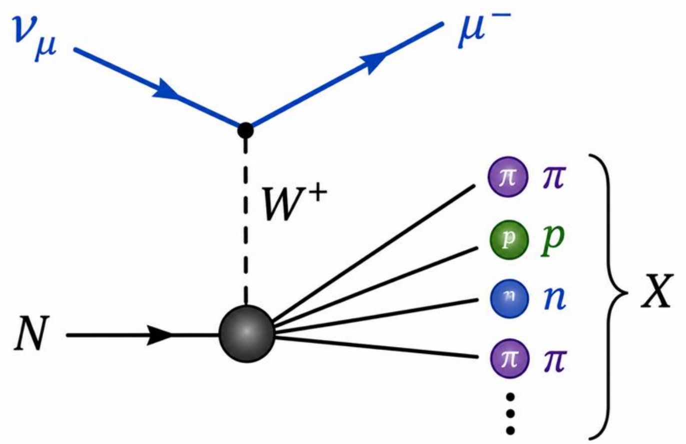


Nucleon remains intact

Inelastic scattering



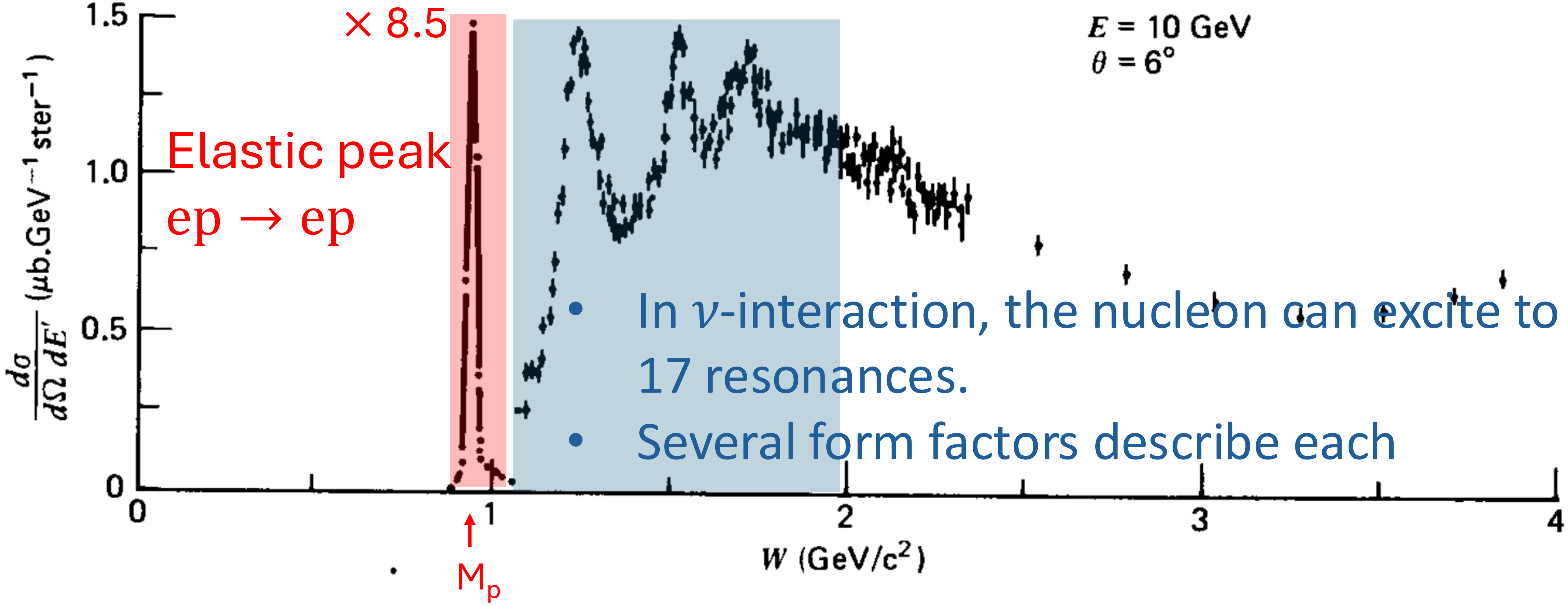
Nucleon excitation



Shallow or deep Inelastic scattering

ep → eX cross-section

Proton excitations ep → eN(mesons)



- In ν-interaction, the nucleon can excite to 17 resonances.
- Several form factors describe each

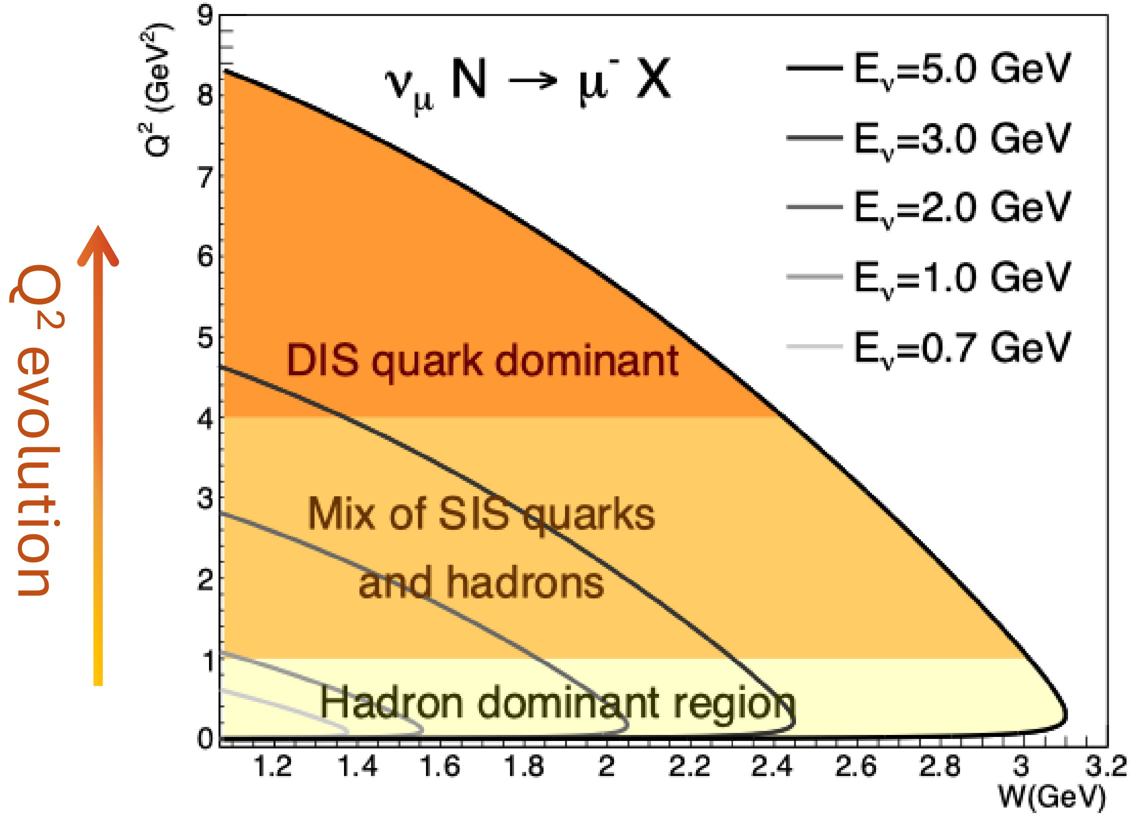
Broad neutrino energy spectrum

↓

wide kinematic coverage

↓

multiple interaction mechanisms contribute

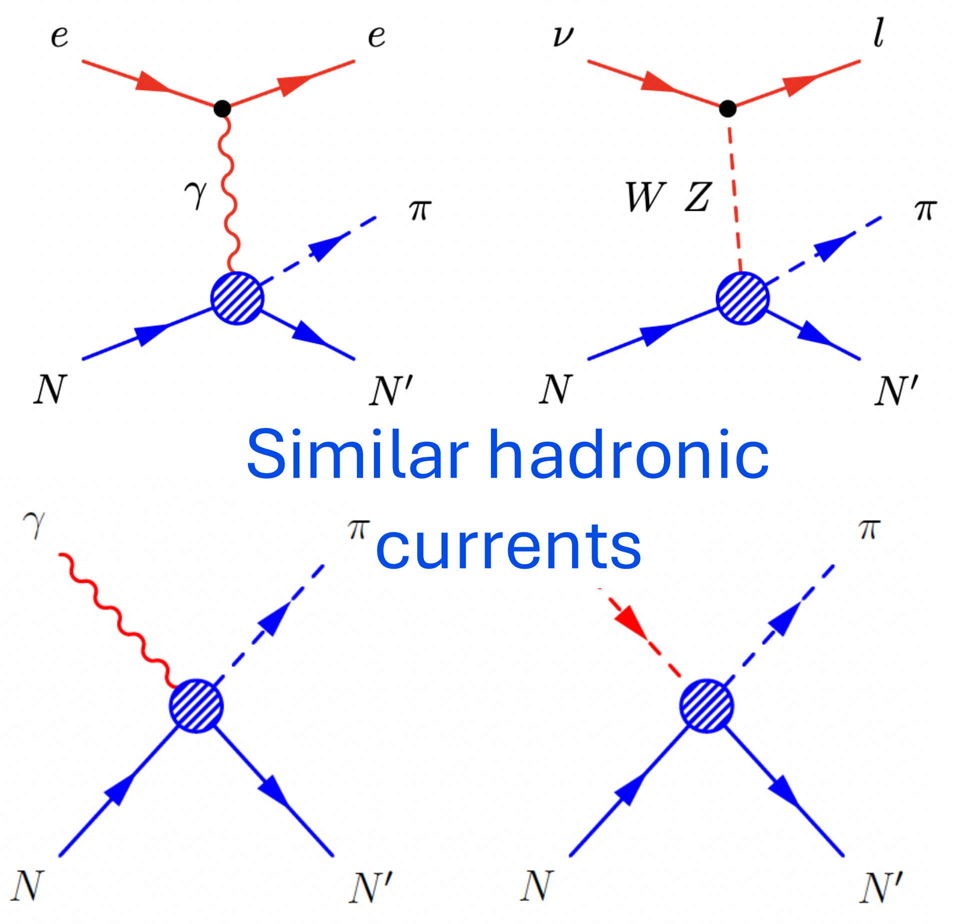


Challenges and Uncertainties in the complex GeV Region

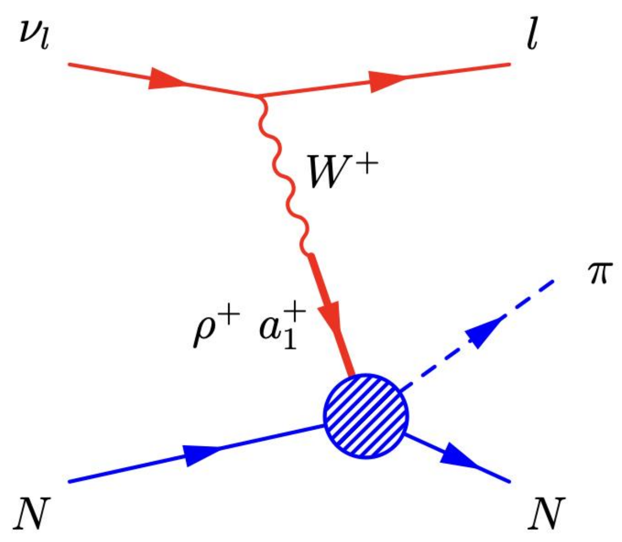
- **Multiple Processes**: The GeV energy range involves a complex interplay of interaction mechanisms on the nucleon, requiring precise knowledge of form factors for each process.
- **Quark-Hadron Transition**: Begins as early as Q^2, requiring careful treatment.
- **Parametrisation**: Essential for incorporating unknown physics and handling interfering processes.
- **Uncertainty Estimation**: Accurate assessment of theoretical uncertainties is a crucial input for neutrino measurements.
- **Limited data**

Solution for single pion production: MK model

- The MK model comprehensively describes single-pion production in interactions involving **photons, electrons, and neutrinos** with nucleons.
- Phenomenological models in this region must account for numerous processes and parameters.
- A unified model is essential for interpreting all interactions and **maximising data utilisation**.



Form Factors in the model



- The Meson dominance model is rooted in the effective Lagrangian of quantum field theory.

1. J. J. Sakurai, Annals Phys.11, 1 (1960)
2. M. Gell-Mann and F. Zachariasen, Phys. Rev. 124, 953 (1961)

- This framework explains the interaction between neutrinos and nucleons through meson exchange.
- The number of parameters in several form factors is reduced by imposing all symmetries and unitarity.
- At large Q2, resonance form factors must align with the perturbative QCD constraints.

Data used in the Joint analysis

# data point	Photon, electron, pion, Neutrino Channels	Q^2 Range $(GeV/C)^2$	W Range GeV	Form Factors	
≈ 9800	$\gamma\, p \to n + \pi^+ ,\ \gamma p \to p + \pi^0$	0	1.08 – 2.0	Proton	Vector
≈ 31000	$ep \to en + \pi^+,\ ep \to ep + \pi^0$	0.16 – 6.0	1.08 – 2.0		
≈ 2500	$\gamma n \to p + \pi^-$	0	1.08 – 2.0	Neutron	
≈ 700	NEW $en \to ep + \pi^-$	0.4 – 1.0	1.08 – 1.8		
≈ 400	$\pi^+ p \to p + \pi^+,\ \pi^- p \to p + \pi^-$	0	1.08 – 2.0	Axial-Vector	
<100	$\nu N \to l^- N + \pi\ , \bar{\nu} N \to l^+ N + \pi$	$Q^2>0$ Integrated	1.08 – 2.0 Integrated		

Data used in the Joint analysis (Over 6 years!)

# data point				Form ctors	
≈ 9800				on	Vector
≈ 31000					
≈ 2500				on	
≈ 700					
≈ 400				-Vector	
<100		d			

- These data sets are systematically incorporated into the analysis to parameterise the model.
- Through analysis, parameters are evaluated, and the inherent systematic uncertainties in the model are addressed.
- Reduced $\chi 2 \approx 1$

M. Kabirnezhad
Phys. Rev. D **97** (2018)
Phys. Rev. D **102** (2020)
Phys.Rev.C **107** (2023)
e-Print: 2409.02890(2026)

MK model

The MK model comprehensively describes single-pion production in interactions involving **photons, electrons, and neutrinos** with nucleons.

- Meson Dominance (MD) form factor: Maintains **unitarity** and integrates **QCD principles** for both resonant and non-resonant interactions.
- **CVC and PCAC** fulfilment: Ensures model consistency at low Q^2.
- Q^2 evolution: Utilises QCD calculations and **quark-hadron duality**.
- W evolution: Applies **Regge trajectory** and the Hybrid model.

R. González-Jiménez, *et al*
Phys. Rev. D **95** (2017)

Meson Dominance (MD) model

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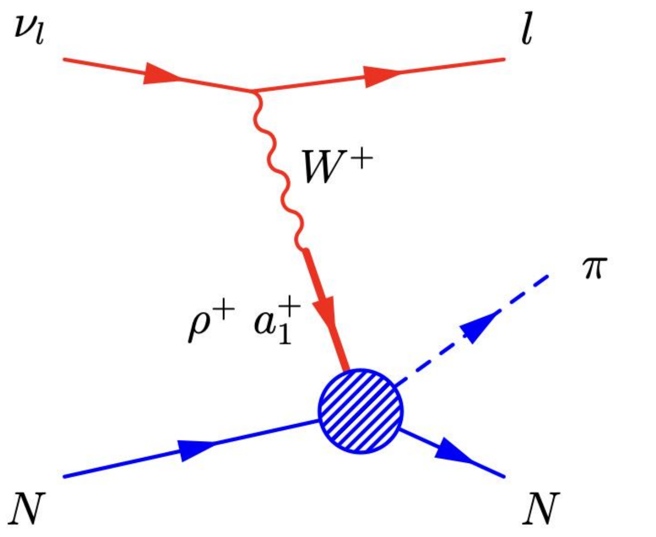
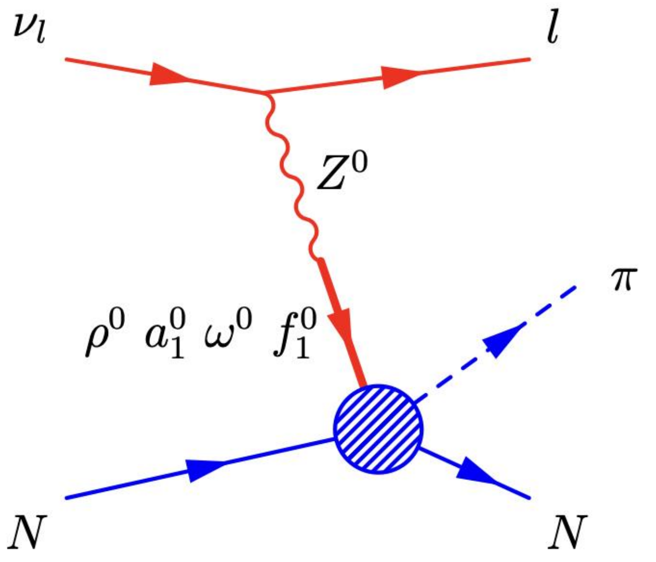
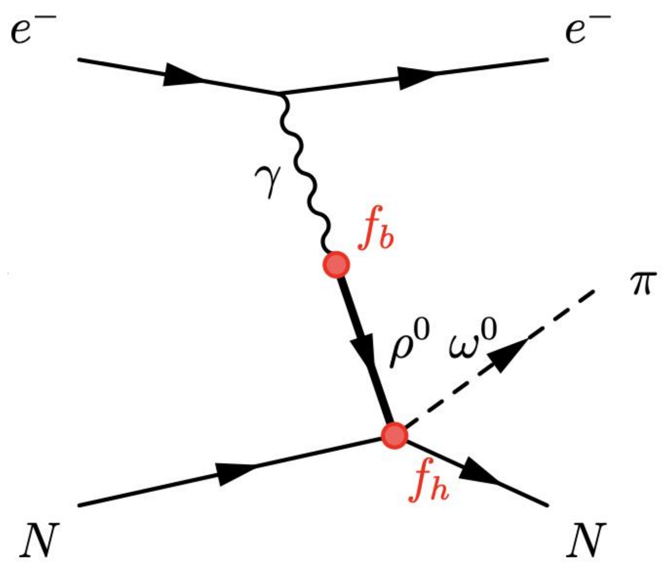
- It establishes connections between vector and axial currents and corresponding meson fields with analogous quantum properties.
- This framework explains the interaction between neutrinos and nucleons through meson exchange.





Meson Dominance (MD) model



- MD form factors can be expressed in terms of the meson masses and the coupling strengths, summing over all possible mesons:

$$\mathrm{F_N}(Q^2) = \sum_{j=1}^{n} \frac{m_j^2}{m_j^2 + Q^2} \left(\frac{f_h}{f_b}\right)$$

- Although they do not inherently comply to the unitarity condition (analytic model) or accurately predict behaviour at high Q^2, they can be **imposed**!

C. Adamuscin *et al.* Eur. Phys. J. C 28, 115 (2003)

k	ρ -group	$m_{(\rho)k}$[GeV]	ω -group	$m_{(\omega)k}$[GeV]
1	$\rho(770)$	0.77526	$\omega(782)$	0.78265
2	$\rho(1450)$	1.465	$\omega(1420)$	1.410
3	$\rho(1700)$	1.720	$\omega(1650)$	1.670
4	$\rho(1900)$	1.885	$\omega(1960)$	1.960
5	$\rho(2150)$	2.150	$\omega(2205)$	2.205
k	a_1 -group	$m_{(a_1)k}$[GeV]	f_1 -group	$m_{(f_1)k}$[GeV]
1	$a_1(1260)$	1.230	$f_1(1285)$	1.2819
2	$a_1(1420)$	1.411	$f_1(1420)$	1.4263
3	$a_1(1640)$	1.655	$f_1(1510)$	1.518
4	$a_1(2095)$	2.096	$f_1(1970)$	1.1971

Asymptotic behaviour of form factor

- At large Q2, resonance form factors must align with the perturbative QCD constraints.

$L_\alpha(Q^2)$ incorporates QCD contributions in the asymptotic region

$$\boxed{L_\alpha(Q^2)} \to \begin{cases} 1, & Q^2 \to 0 \\ C_\alpha \ln^{n_\alpha} \dfrac{Q^2}{\Lambda_{QCD}}, & Q^2 \to \infty \end{cases}$$

G. Vereshkov and N. Volchanskiy (PRD 2007)

$$L_\alpha(Q^2) = 1 + \boxed{g_\alpha} \ln\left(1 + \frac{Q^2}{\Lambda_{QCD}^2}\right)^{n_\alpha}$$

g_α defined as a free parameter in the fit

$\Lambda_{QCD} \in [0.19 - 0.24]$ GeV

$n_3 > n_1 > n_2$

MD form factors used in the model

- For spin 3/2 resonance:

$$F_{\alpha}^{\{A,V\}}(Q^2) = \frac{f_{\alpha}^{\{A,V\}}}{L_{\alpha}(Q^2)} \sum_{k=1}^{K} \frac{a_{\alpha k}^{\{A,V\}} m_k^2}{m_k^2 + Q^2}, \qquad (\alpha = 1-3)$$

$$L_{\alpha}(Q^2) = 1 + g_{\alpha} \ln\left(1 + \frac{Q^2}{\Lambda_{QCD}^2}\right)^{n_{\alpha}}$$

$$\boxed{g_{\alpha} = g_{1_{\alpha}} + \frac{g_{2_{\alpha}}}{Q^2} + \frac{g_{3_{\alpha}}}{Q^4} + \cdots}$$

Inspired by the twist expansion

- $a_{\alpha k}$ are constrained by unitarity conditions that also satisfy asymptotic QCD requirements.

New Features of the MK model

- **SIS region (effective treatment):** Previously included only leading (scaling) contributions; now incorporates nonleading (higher-twist) effects.
- **Resonance modelling:** All resonances are described consistently using MD form factors.
  - **Spin-3/2 resonances** are described by Rarita-Schwinger formalism.
  - **Higher-spin resonances:** States such as $F_{15}(1680)$ and $F_{37}(1950)$ are treated within the same framework as spin-3/2.
- **Non-resonant interaction:** Modelled consistently with the Hybrid approach.

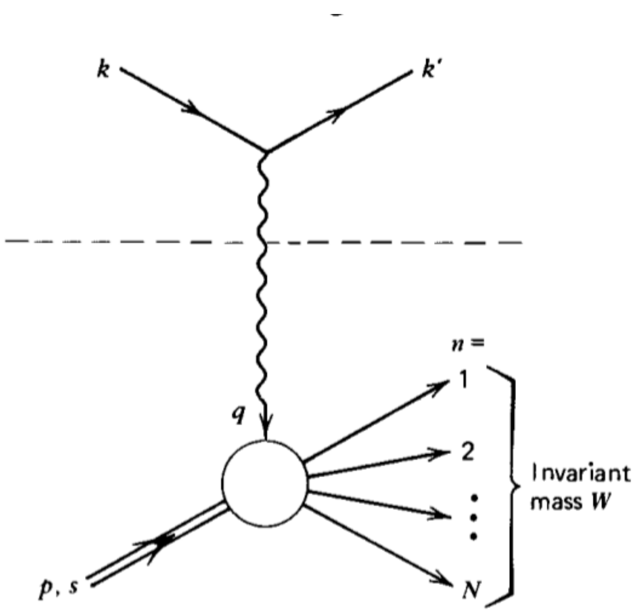
Isospin relation for vector form factor

- If we know proton and neutron form factors, we know vector CC and NC form factors.

reaction	replace $\mathcal{F}_i$ in Eq. (4.7) with
$\ell^- p \to \ell^- p$	F_i^p
$\ell^- n \to \ell^- n$	F_i^n
$\nu n \to \ell^- p$	$F_i^V = F_i^p - F_i^n$
$\nu p \to \nu p$	$\tilde{F}_i^p = (\frac{1}{2} - 2\sin^2\theta_W)F_i^p - \frac{1}{2}F_i^n - \frac{1}{2}F_i^s$
$\nu n \to \nu n$	$\tilde{F}_i^n = (\frac{1}{2} - 2\sin^2\theta_W)F_i^n - \frac{1}{2}F_i^p - \frac{1}{2}F_i^s$

Inelastic scattering

- By Increasing the $Q^2 = -q^2$ we can take a detailed look at the structure of nucleon.
- This can be done by requiring a large energy loss of the bombarding lepton.
- At very large transfer of energy, proton with break up and looses its identity.
- At modest Q^2 proton get excited to baryon resonances such as Δ resonance. The excited state promptly decay to a nucleon and mesons.



Inelastic scattering

- The main challenge now is that we have more than one hadron in the final states and it is not straightforward to calculate the hadron current. Therefore, J^{μ} has a more complex structure than Elastic scattering

Elastic

$$\mathrm{d}\sigma \propto \mathrm{L}^{\mathrm{e}}_{\mu\nu}\, \mathrm{H}^{\mu\nu}$$

Lepton tenson $L^{e}_{\mu\nu} = \sum\sum j^{\dagger}_{\mu}\, j_{\nu}$

Hadron tensor $\mathrm{H}_{\mu\nu} = \sum\sum J^{\dagger}_{\mu}\, J_{\nu}$

Hadron current $J^{\mu} \propto \bar{u}\Gamma^{\mu}u$

Inelastic

$$\mathrm{d}\sigma \propto \mathrm{L}^{\mathrm{e}}_{\mu\nu}\, \mathrm{W}^{\mu\nu}$$

Hadron tensor $W_{\mu\nu} = \sum\sum J_{\mu}{}^{\dagger}\, J_{\nu}$

There are more than single hadron in the final state and the current is complicated.

Inelastic scattering

- The hadronic tensor W serves to parametrise our total ignorance of the form of inelastic hadron current.
- The most general form of the tensor W must be constructed from $g^{\mu\nu}$ and independent momenta p and q.

$$\begin{aligned}\mathcal{W}^{\mu\nu} &= \frac{1}{2m_N}\sum\langle p|J^\mu(0)|\Delta\rangle\langle\Delta|J^\nu(0)|p\rangle\delta(W^2-M_R^2)\\ &= -\mathcal{W}_1 g^{\mu\nu} + \frac{\mathcal{W}_2}{m_N^2}p^\mu p^\nu - i\varepsilon^{\mu\nu\sigma\lambda}p_\sigma q_\lambda\frac{\mathcal{W}_3}{2m_N^2}\\ &+\frac{\mathcal{W}_4}{m_N^2}q^\mu q^\nu + \frac{\mathcal{W}_5}{m_N^2}(p^\mu q^\nu + q^\mu p^\nu) + i\frac{\mathcal{W}_6}{m_N^2}(p^\mu q^\nu - q^\mu p^\nu)\end{aligned}$$

γ^μ is not included as we are parametrizing the cross section which is already summed and averaged over spins.

Inelastic scattering

- W_i s are function of the Lorentz scalar variables that can be constructed from four-momenta at the hadronic vertex.
- Unlike elastic scattering, there are two independent variables.

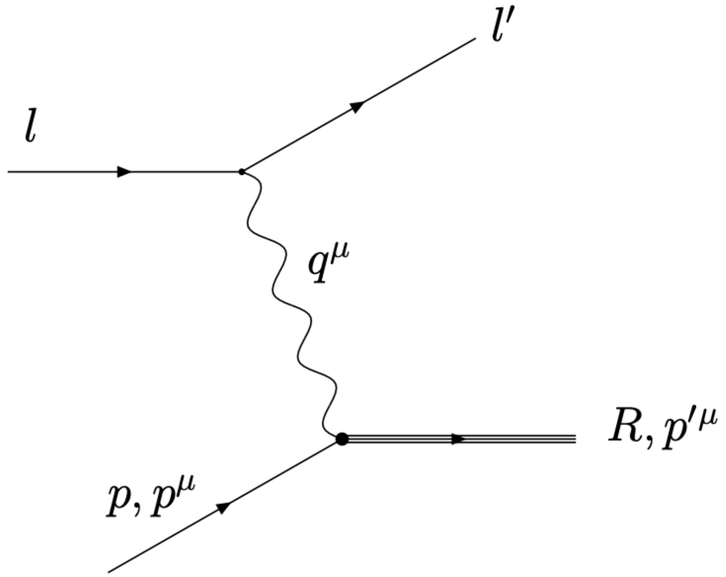
$$\mathcal{W}^{\mu\nu} = \frac{1}{2m_N}\sum\langle p|J^\mu(0)|\Delta\rangle\langle\Delta|J^\nu(0)|p\rangle\delta(W^2 - M_R^2)$$
$$= -\mathcal{W}_1 g^{\mu\nu} + \frac{\mathcal{W}_2}{m_N^2}p^\mu p^\nu - i\varepsilon^{\mu\nu\sigma\lambda}p_\sigma q_\lambda\frac{\mathcal{W}_3}{2m_N^2}$$
$$+\frac{\mathcal{W}_4}{m_N^2}q^\mu q^\nu + \frac{\mathcal{W}_5}{m_N^2}(p^\mu q^\nu + q^\mu p^\nu) + i\frac{\mathcal{W}_6}{m_N^2}(p^\mu q^\nu - q^\mu p^\nu)$$

Resonance production

- $\nu_l N \to lR$
- Resonance mass: M_R
- Resonance width: $\Gamma 0$
- $R = l_{2I,2J}(M_R)$,

$$S{:}\, l = 0,$$
$$P{:}\, l = 1,$$
$$\text{etc}$$

- Parity: $P = -(-1)^l$



Resonance	M_R	Γ_0	χ_E
$P_{33}(1232)$	1232	117	1
$P_{11}(1440)$	1430	350	0.65
$D_{13}(1520)$	1515	115	0.60
$S_{11}(1535)$	1535	150	0.45
$P_{33}(1600)$	1600	320	0.18
$S_{31}(1620)$	1630	140	0.25
$S_{11}(1650)$	1655	140	0.70
$D_{15}(1675)$	1675	150	0.40
$F_{15}(1680)$	1685	130	0.67
$D_{13}(1700)$	1700	150	0.12
$D_{33}(1700)$	1700	300	0.15
$P_{11}(1710)$	1710	100	0.12
$P_{13}(1720)$	1720	250	0.11
$F_{35}(1905)$	1880	330	0.12
$P_{31}(1910)$	1890	280	0.22
$P_{33}(1920)$	1920	260	0.12
$F_{37}(1950)$	1930	285	0.40