OPENING TALK · QUANTHEP 2026

Lattice Gauge Theory Challenges for Quantum Computers

Where the Euclidean paradigm hits exponential walls — and what unitarity does (and does not) buy

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WHERE THE LATTICE STARTS

Fifty years of the Euclidean paradigm — and five exponential walls

$\langle O \rangle = Z^{-1} \int \mathcal{D}U\, \mathcal{D}\bar{\psi}\, \mathcal{D}\psi\;\; O\, e^{-S}$ importance sampling works iff $e^{-S} \geq 0$ and the chain is ergodic

1 Sign problem

μ ≠ 0, θ ≠ 0: the measure turns complex

2 Real-time

Minkowski weight e^{iS}: no probability at all

3 Signal-to-noise

Variance decays more slowly than the signal

4 Small overlaps

Trial states barely touch the states we want

5 Tunneling times

Ergodicity lost: 1st-order transitions, frozen topology

Five distinct pathologies: 1–2 live in the measure, 3–4 in the estimator, 5 in the Markov chain. All are exponential. None is negotiable.

Classical successes and limits reviewed in e.g. B. Lucini & M. Panero, Phys. Rept. 526 (2013) 93.

What unitarity buys by construction

$|\psi(t)\rangle = e^{-iHt}\,|\psi(0)\rangle$ — no measure, no weights, no configurations to sample

SOLVED BY CONSTRUCTION

1 · Sign problem

Finite μ is just a term in H; θ is just a parameter. Hamiltonian evolution never sees a Boltzmann weight — solved structurally, not by ingenuity.

Classical workarounds (reweighting, complex Langevin, density of states) remain exponentially costly or hard to control; a generic classical solution is NP-hard.

SOLVED BY CONSTRUCTION

2 · Real-time dynamics

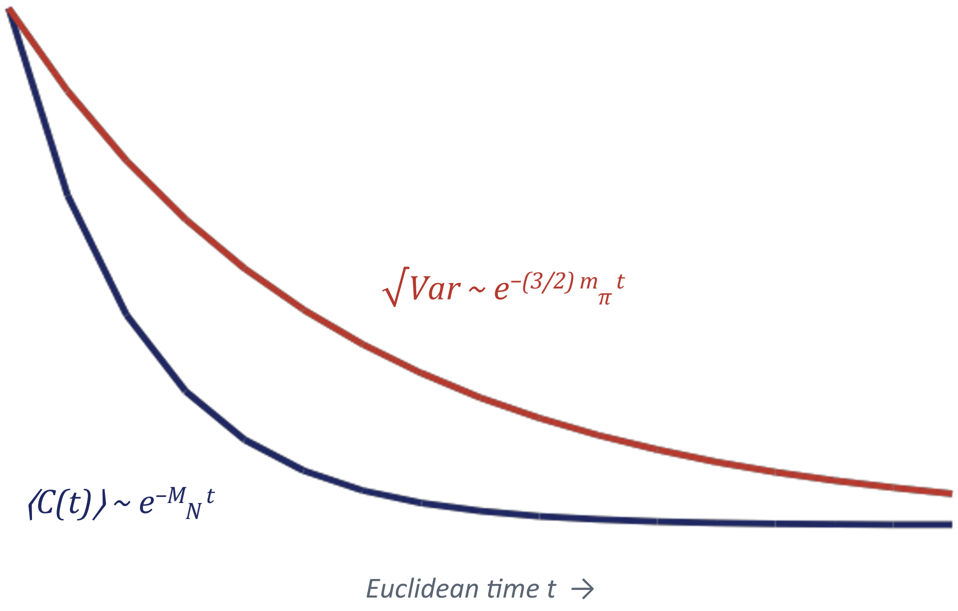
e^{-iHt} is the native hardware operation: Trotterised evolution costs poly(V, t, 1/ε). No analytic continuation, no ill-posed inversion of $G(\tau) = \int d\omega\, e^{-\omega\tau}\rho(\omega)$.

Scattering, transport, thermalisation, fragmentation become direct measurements rather than inverse problems.

Feynman, Int. J. Theor. Phys. 21 (1982) 467 · Jordan, Lee & Preskill, Science 336 (2012) 1130 · NP-hardness: Troyer & Wiese, PRL 94 (2005) 170201 · density of states at finite density: Langfeld & Lucini, PRD 90 (2014) 094502.

Signal-to-noise: the Parisi–Lepage wall

- The variance of a correlator is itself a correlator — with its own, lighter, spectrum
- Nucleon: signal falls with M_N, noise with (3/2) m_π
- Glueballs and disconnected diagrams face constant noise — vacuum quantum numbers
- On a quantum computer, Monte Carlo noise becomes shot noise, σ ~ 1/√N_shots — but the variance of the physical estimator can still grow exponentially with t or V



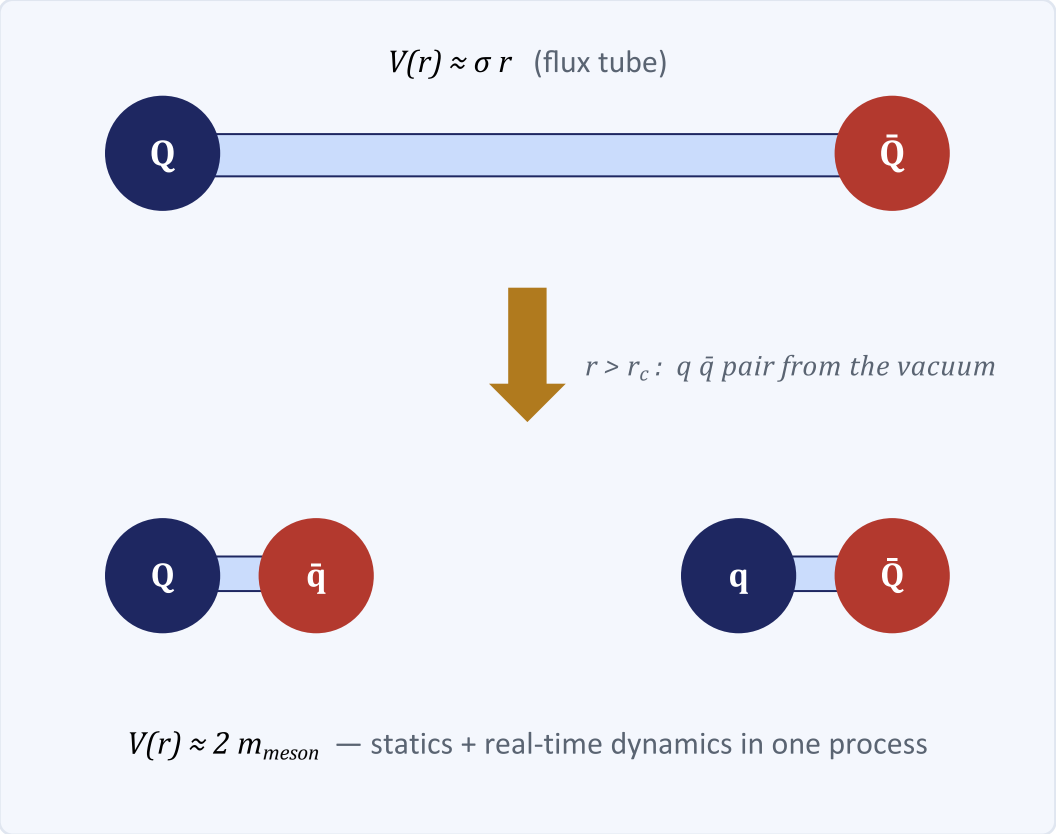
The noise outlives the signal: an exponential race that the signal always loses.

Parisi, Phys. Rept. 103 (1984) 203 · Lepage, TASI (1989) · quantum reading: overlap & shot-noise cost of measurement replaces the estimator variance.

Small overlaps: string breaking as the showcase

- Spectroscopy rests on trial operators: contamination dies only like e^{−ΔE t}
- String breaking is the classic case: a Wilson-line operator has tiny overlap onto the broken two-meson state — it took ~20 years of classical effort to resolve
- On quantum hardware the problem returns as state-preparation cost: repetitions ∝ 1/|⟨ψ_trial|Ω⟩|²; amplitude amplification helps only quadratically
- First quantum-simulator observations of string breaking appeared in 2024–25 — in simplified models, far from continuum QCD: the benchmark is set, not met

$$N_{shots} \sim 1 \,/\, |\langle\psi_{trial}|\Omega\rangle|^2$$

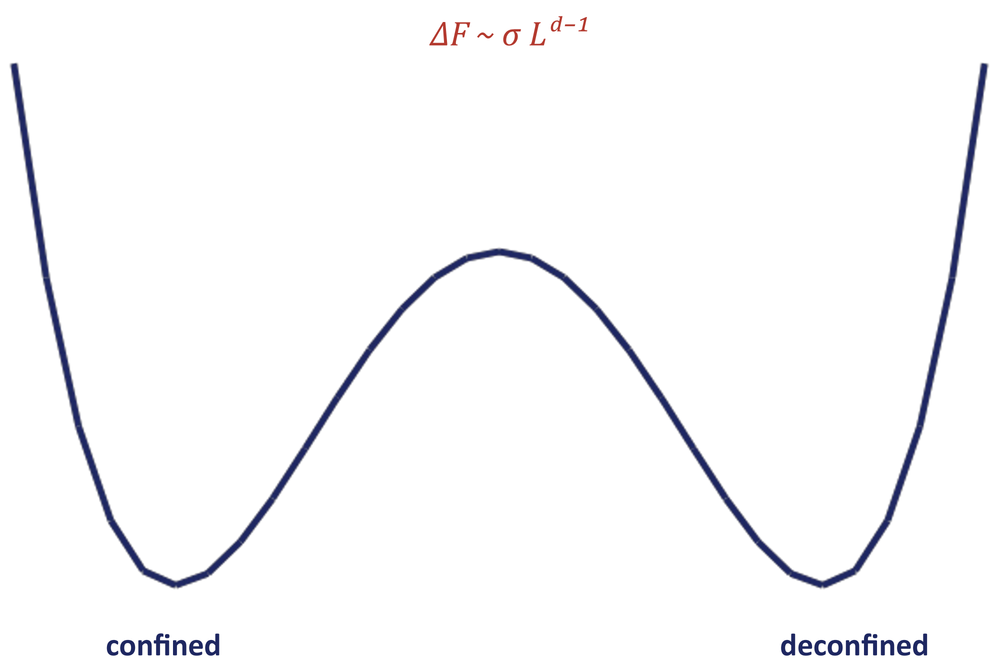


Classical: Bali et al. (SESAM), PRD 71 (2005) 114513 · Quantum simulators: González-Cuadra et al., Nature (2025) [arXiv:2410.16558]; trapped-ion (2+1)D QED, Commun. Phys. (2025).

Exponential tunneling times & broken ergodicity

- First-order transitions: phase flips suppressed by the interface free energy — supercritical slowing down (SU(N) deconfinement is first order for $N \geq 3$)
- Topological freezing: τ_Q diverges as $a \to 0$; the Markov chain gets stuck in one sector
- Classical lesson: algorithmic ingenuity can flatten barriers — density-of-states (LLR) methods sample across coexisting phases where importance sampling fails
- On quantum hardware the barrier returns through state preparation: adiabatic evolution needs $T \gtrsim \Delta^{-2}$, and at a first-order transition the gap closes exponentially

$$\tau_{flip} \sim e^{2\sigma L^{d-1}}, \quad \Delta_{min} \sim e^{-c\, L^{d-1}}$$



The classical tunneling barrier returns as an exponentially small level splitting.

SU(N) first-order deconfinement: Lucini, Teper & Wenger, JHEP 02 (2005) 033 · LLR: Langfeld, Lucini & Rago, PRL 109 (2012) 111601; Lucini et al., PRD 108 (2023) 074517 · freezing: Del Debbio, Panagopoulos & Vicari, JHEP 08 (2002) 044.

Scoreboard: classical vs quantum

Challenge	Classical Monte Carlo	Quantum computer
1 · Sign problem	exp(V) reweighting; generically NP-hard	**SOLVED — no weights in unitary evolution**
2 · Real-time dynamics	ill-posed inversion of Euclidean correlators	**SOLVED — e^(−iHt) is native**
3 · Signal-to-noise	$S/N \sim \exp(-(M_N - 3/2\, m_\pi)t)$	**REPHRASED — shot cost of measurement**
4 · Small overlaps	excited-state contamination, operator bases	**REPHRASED — state preparation $\propto 1/\lvert\text{overlap}\rvert^2$**
5 · Tunneling times	$\exp(\sigma L^{(d-1)})$ flips; frozen topology	**REPHRASED — adiabatic gap closes exponentially**

2 solved structurally, 3 transformed: the exponentials reappear elsewhere — from the measure and the Markov chain into state preparation and measurement.

“The sign problem dies — the other exponentials just get reframed”

A showcase: string breaking

Statics (overlaps) plus dynamics (real time) in one process. 2024–25 simulator results set the benchmark — now towards non-Abelian theories and controlled errors.

Honest benchmarks

Compare against the best classical method: in 1+1D and much of 2+1D, tensor networks also evade the sign problem. Advantage is a race, not a right.

A verification culture

If the payoff is regimes no classical method reaches, cross-check in classically accessible corners — with error bars that mean what lattice error bars mean.

Welcome to QuantHEP 2026 — the lattice community is watching, with sympathy and with error bars.
Have a great conference!