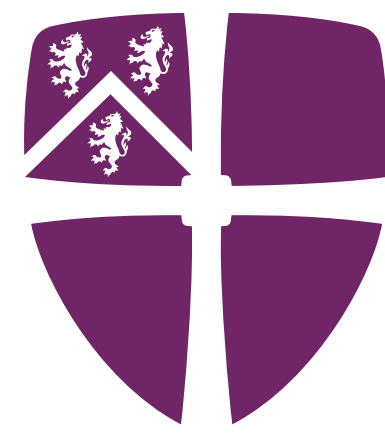


Simulating quantum field theories with continuous-variable quantum computers

Simon Williams

QuantHEP 2026

Queen Mary, University of London



Durham
University

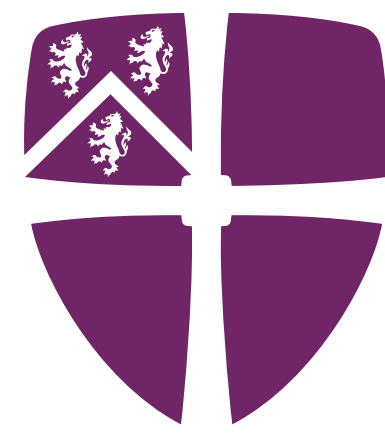


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- **Qubit vs Qumode** : a little help from my photons
- **The qumode lattice** : discretising space
- **Real-time simulation on CVQC** : learning to be non-Gaussian
- Validating the qumode tensor network:
 - **Real-time scattering** processes on the qumode tensor network
 - **False-vacuum decay** on the qumode tensor network

[1] S Abel, M Spannowsky and SW, Real-time scattering processes with continuous-variable quantum computers, [Phys. Rev. A 112, 012614](#)

[2] S Abel, M Spannowsky and SW, Qumode Tensor Networks for False Vacuum Decay in Quantum Field Theory, [arXiv:2506.17388](#)

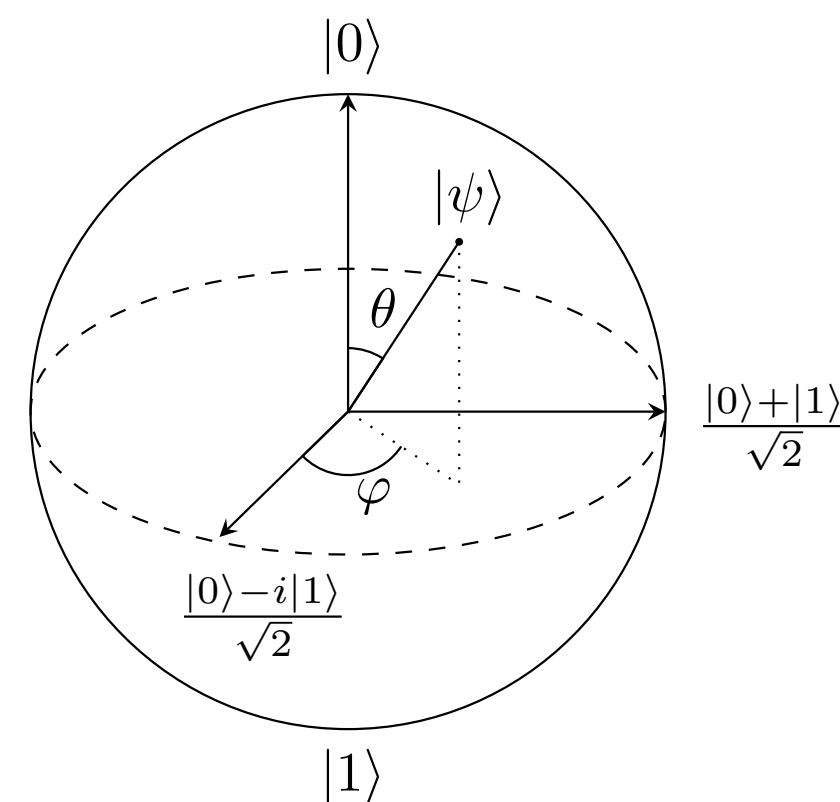
Quantum Computing - Discrete vs Continuous

Discrete-variable quantum computing

Information is encoded in systems with discrete energy levels:

Qubit

Two state quantum system spanned by $|0\rangle$ and $|1\rangle$



Qu(d)it

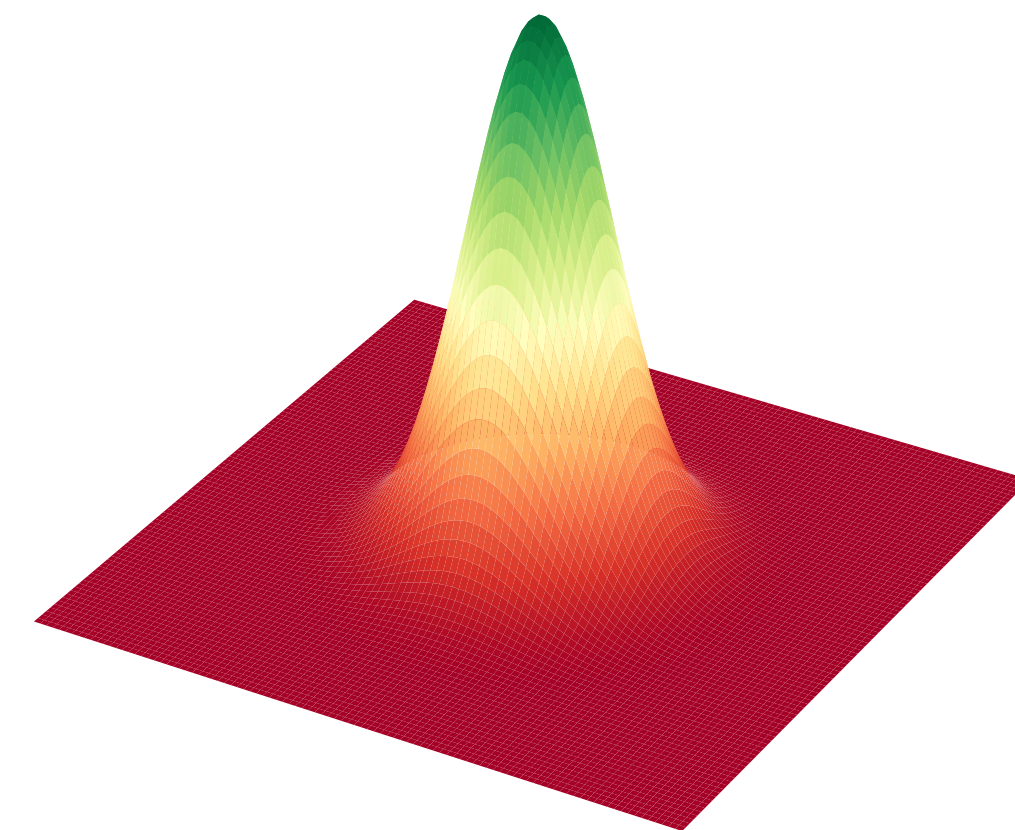
Multilevel systems with d states

Continuous-variable quantum computing

Information is encoded in observables that vary continuously - infinite dimensional Hilbert spaces

Qumode

Information encoded into the quadrature variables of a quantum mechanical oscillator, $\hat{q}$ and $\hat{p}$



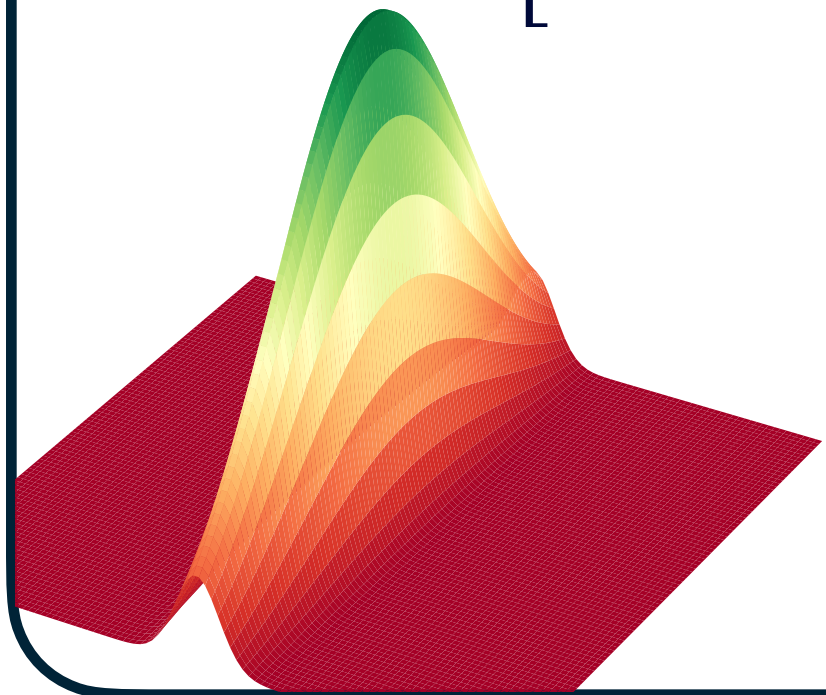
A qumode system is a *tower* of quantum oscillators

With a little help from my photons - photonic devices

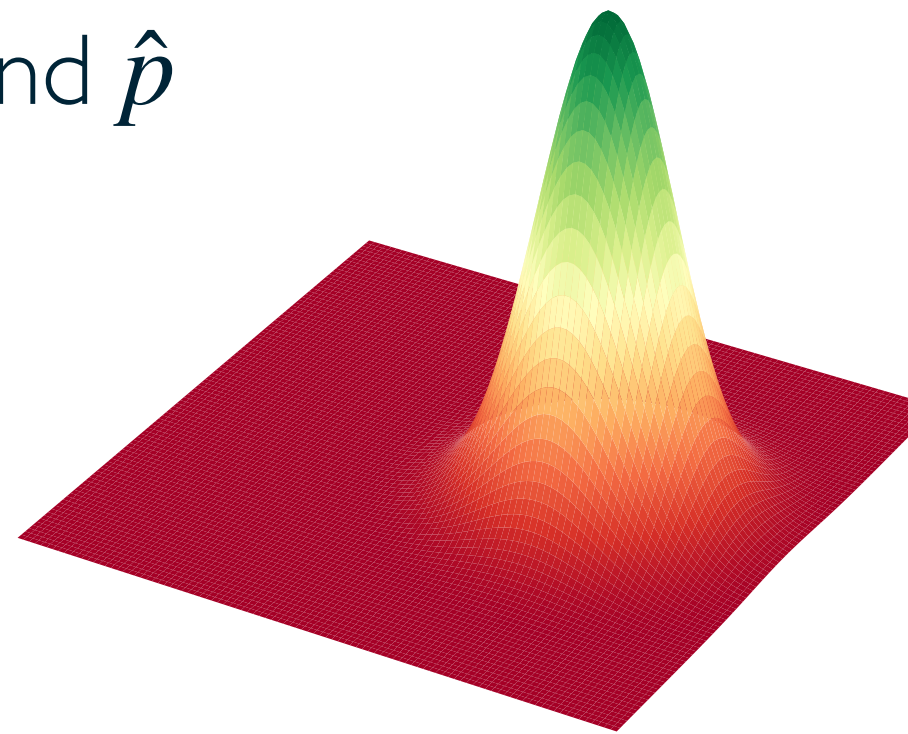
Gaussian operations

Maps Gaussian states to Gaussian states - at most quadratic in quadrature variables $\hat{q}$ and $\hat{p}$

$$D(\alpha) = \exp \left[-i\sqrt{2} (\Re(\alpha)\hat{p} - \Im(\alpha)\hat{x}) \right]$$



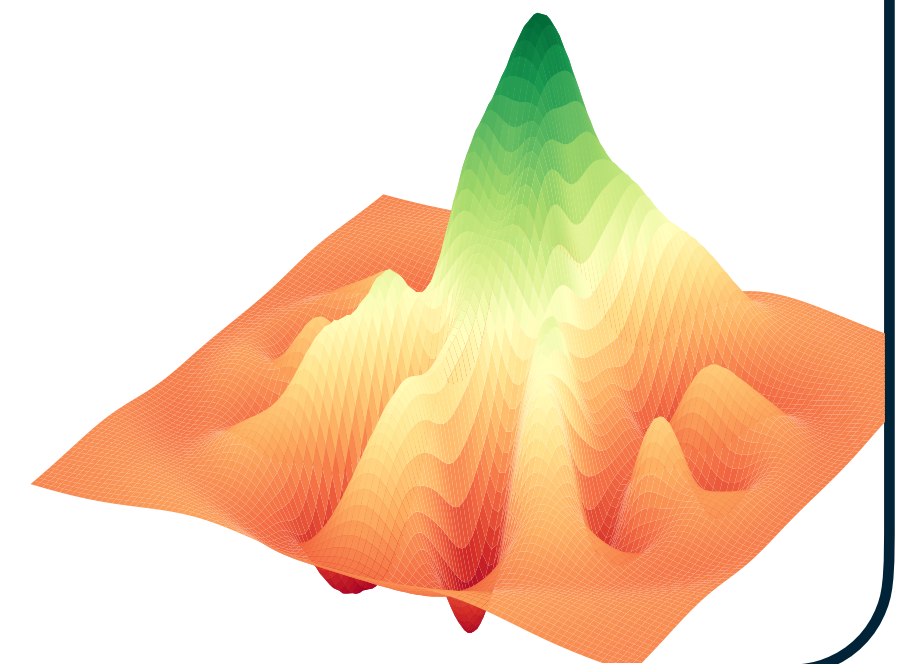
$$S(z) = \exp \left[\frac{1}{2} (z^* a^2 - z a^{\dagger 2}) \right]$$



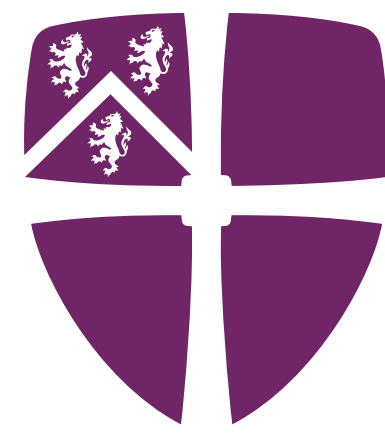
Quantum (digital simulation)

Maps Gaussian states to non-Gaussian states - greater than quadratic in quadrature variables $\hat{q}$ and $\hat{p}$

Optical non-linearities are too weak to introduce required non-Gaussianity



Universal Computation - It is impossible to achieve universal computation in photonics with only Gaussian operations - “no-go” theorem



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Why go quantum for quantum field theory?

Why QFT on a Quantum Device? - the sign problem

Real-time dynamics - many interesting physical processes are dynamical, requiring real-time evolution to extract meaningful physical observables

Euclidean lattice QFT

Euclidean lattice QFT is extremely powerful:

$$Z = \int \mathcal{D}\phi \, e^{-S_E[\phi]/\hbar}$$

Reframes the path integral into a statistical problem, allowing **importance sampling**.

Monte Carlo techniques sample smooth positive distribution

Real-time QFT

Real-time introduces highly oscillatory phase:

$$Z = \int \mathcal{D}\phi \, e^{iS[\phi]/\hbar}$$

Contributions cancel rather than add like probabilities

Problem: real-time dynamics and sign-problem regimes are hard for classical Monte Carlo.

Real-time dynamics are hard

Hamiltonian lattice formulations replace Euclidean path integral with $|\psi(t)\rangle = e^{-iHt} |\psi(t=0)\rangle$

Classical (Tensor networks)

MPS/TDVP works very well for gapped, low-entanglement dynamics

Dynamical, non-perturbative systems drive rapid entanglement growth

Bond dimension χ grows rapidly

Quantum (digital simulation)

Hamiltonian dynamics is native to quantum hardware

Entanglement is represented directly by the quantum state

The cost moves to circuit depth, noise and measurement

The noise budget, not Hilbert-space size, is the limit

In general the unitary operation must be **Trotterised** to make it tractable:

$$\mathcal{U}(t) = \left[\prod_i e^{-iH_i \delta t} \right]^{t/\delta t}$$

Real-time simulation on a quantum device

Quantum simulation is performed in three main steps:

State preparation

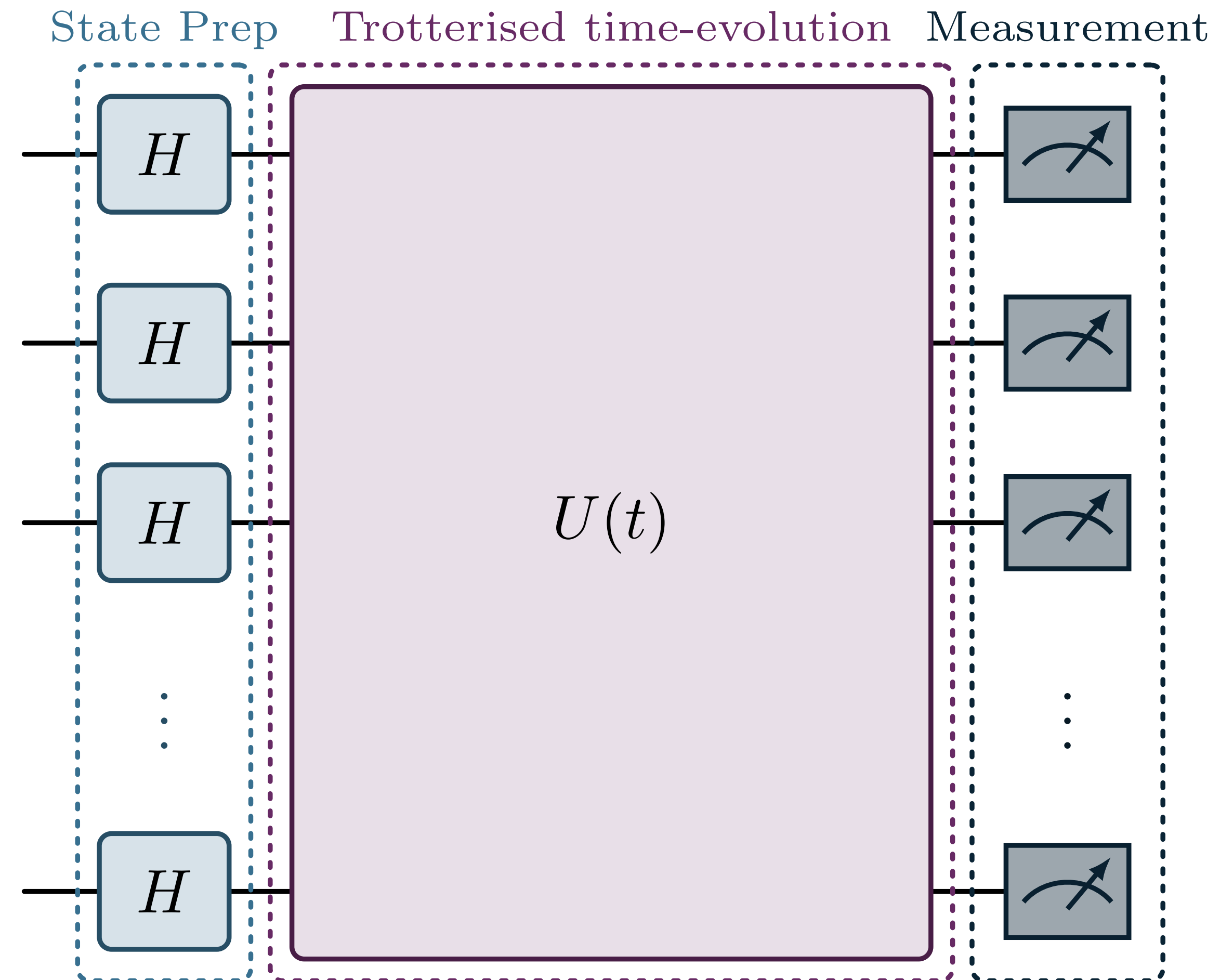
The system must first be prepared in the correct initial state. A common approach is **adiabatic state preparation**.

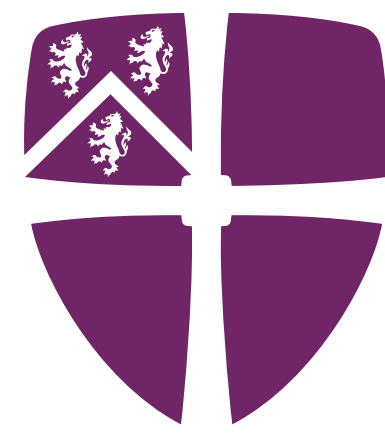
Trotterised time-evolution

The real-time evolution is performed using the n -th order Suzuki-Trotter formalism up to error $\mathcal{O}(\delta t^{n+1})$

Measurement

The system is measured in a suitable basis to retrieve the desired observable

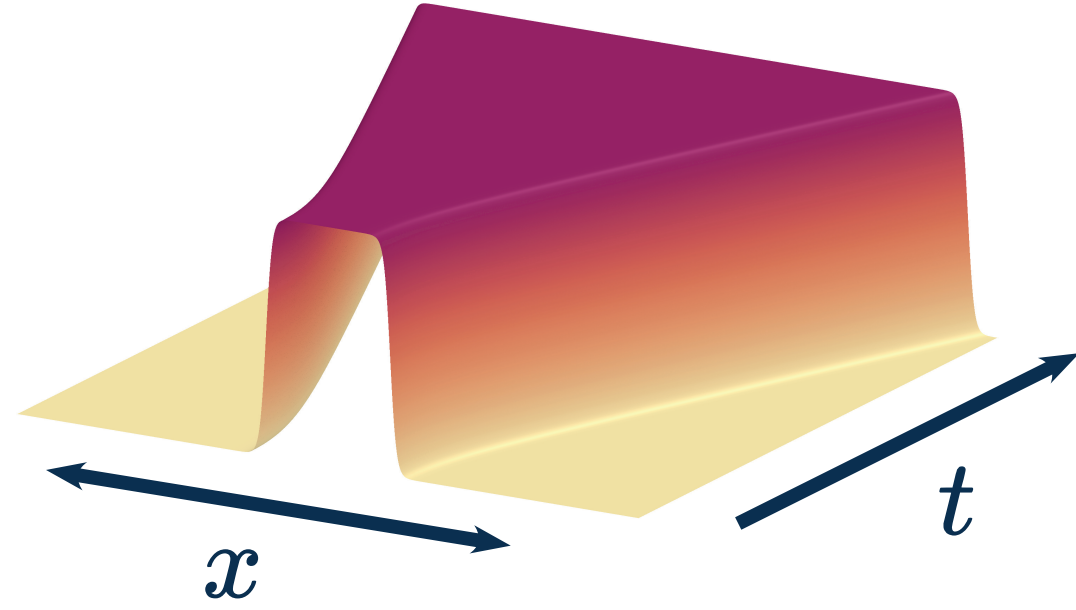




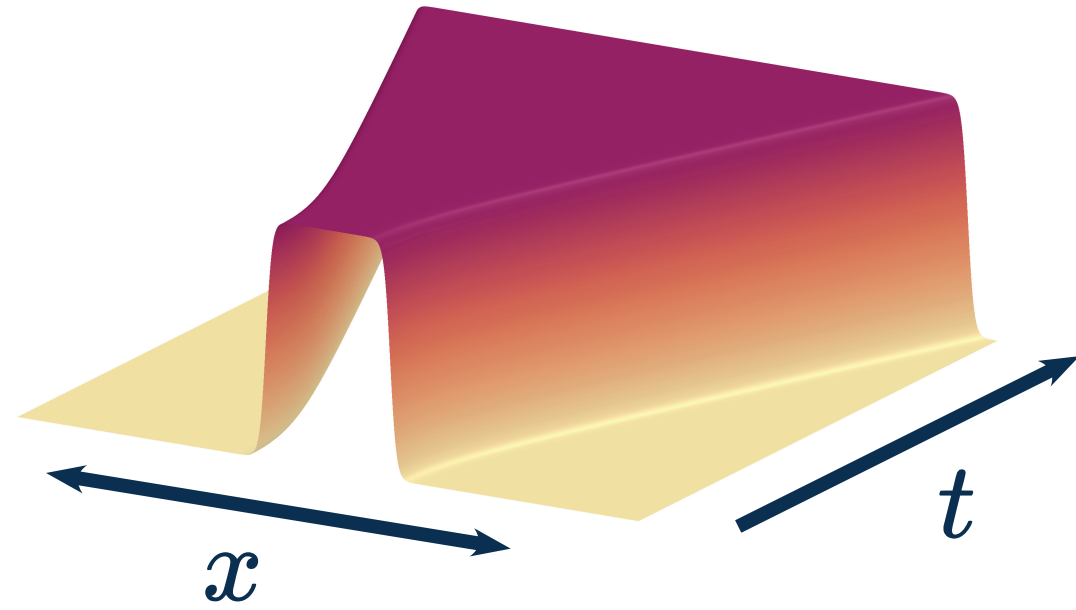
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Scalar field theory on a continuous-variable device

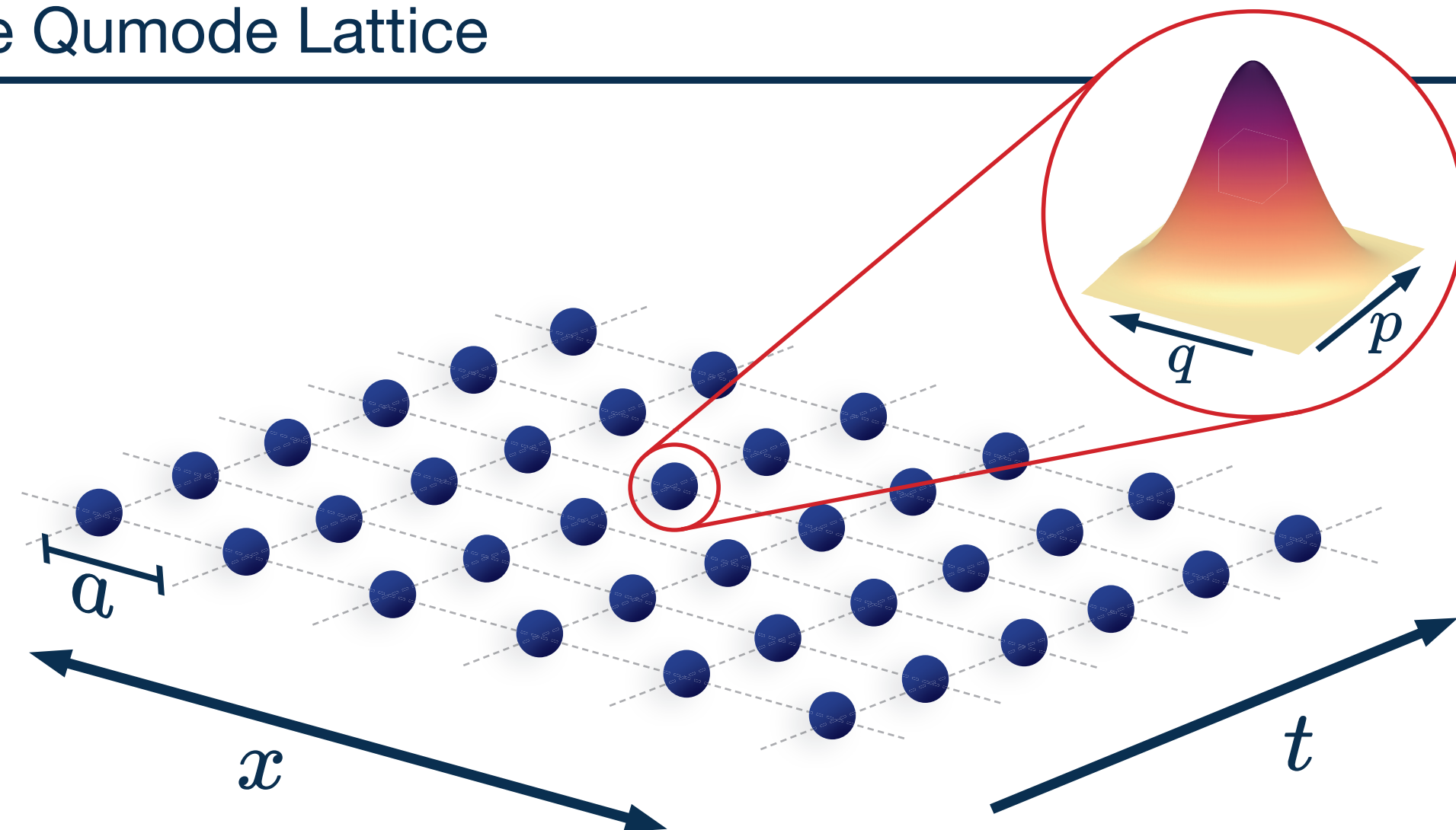


Quantum Field Theory



1

The Qumode Lattice



Free scalar field theory on the qumode lattice

Free-scalar field theory in the continuum:

$$\mathcal{H}(x, t) = \frac{1}{2} (\pi(x, t)^2 + (\nabla \varphi(x, t))^2 + \omega^2 \varphi(x, t)^2) \quad \text{where} \quad \pi(x, t) = \frac{\partial \mathcal{L}}{\partial(\partial_0 \varphi(x, t))} = \partial_0 \varphi(x, t)$$

Latticeise the theory

We construct a spatial lattice $x_n = na$ with periodic boundary conditions $x_N = x_0$

$$H = \frac{a}{2} \sum_{n=0}^{N-1} \left[\pi(x_n, t)^2 + \left(\frac{\varphi(x_{n+1}, t) - \varphi(x_n, t)}{a} \right)^2 + \omega^2 \varphi(x_n, t)^2 \right]$$

By diagonalising the Hamiltonian, one can make the connection

$$\hat{q}_n(t) \leftrightarrow \varphi(x_n, t), \quad \hat{p}_n(t) \leftrightarrow \pi(x_n, t) \quad \text{with} \quad [\hat{q}_n, \hat{p}_m] = i\delta_{nm}$$

Interacting scalar field theory on the qumode lattice

Interacting-scalar field theory in the continuum:

$$\mathcal{H}(x, t) = \frac{1}{2} \left(\pi(x, t)^2 + (\nabla \varphi(x, t))^2 + \omega^2 \varphi(x, t)^2 \right) + \mathcal{V}_I(\varphi(x, t))$$

where $\mathcal{V}_I$ is no longer quadratic in $\varphi(x, t)$, and thus no longer quadratic in q

Lattice the theory

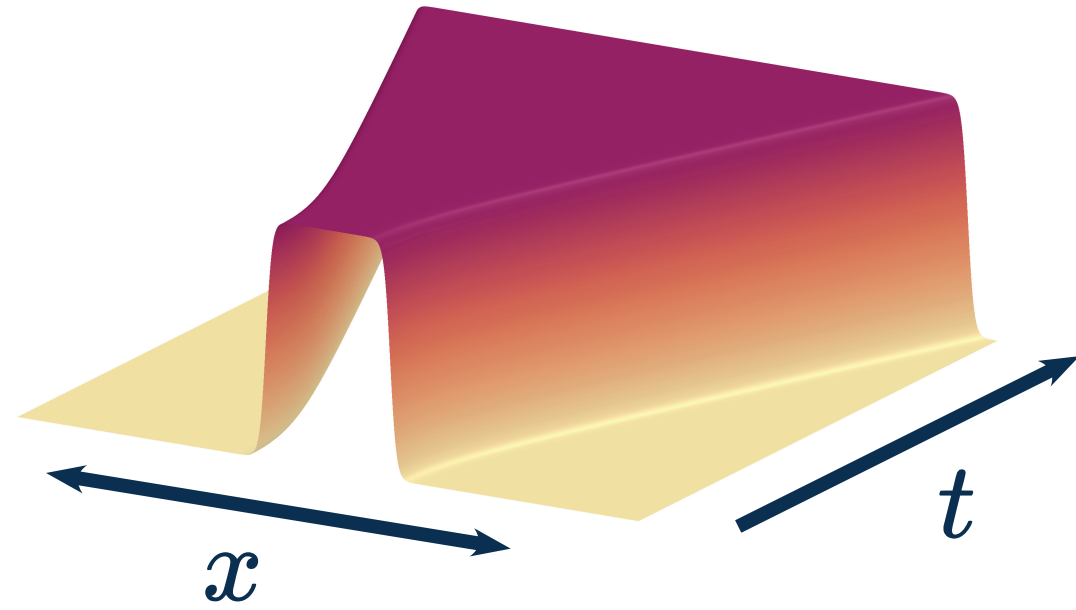
Making the same discretisation as the free theory and expanding terms

$$H a^{-1} = \sum_n \left[\frac{1}{2} (\hat{p}_n(t)^2 + \omega^2 \hat{q}_n(t)^2) + V_I(\hat{q}_n) \right] - \frac{1}{a^2} \sum_n \hat{q}_{n+1} \hat{q}_n \quad \text{where} \quad V_I = \frac{1}{a^2} \hat{q}_n^2 + \mathcal{V}_I(\hat{q}_n)$$

Regardless of the potential, the formulation reduces to a sum of three terms:

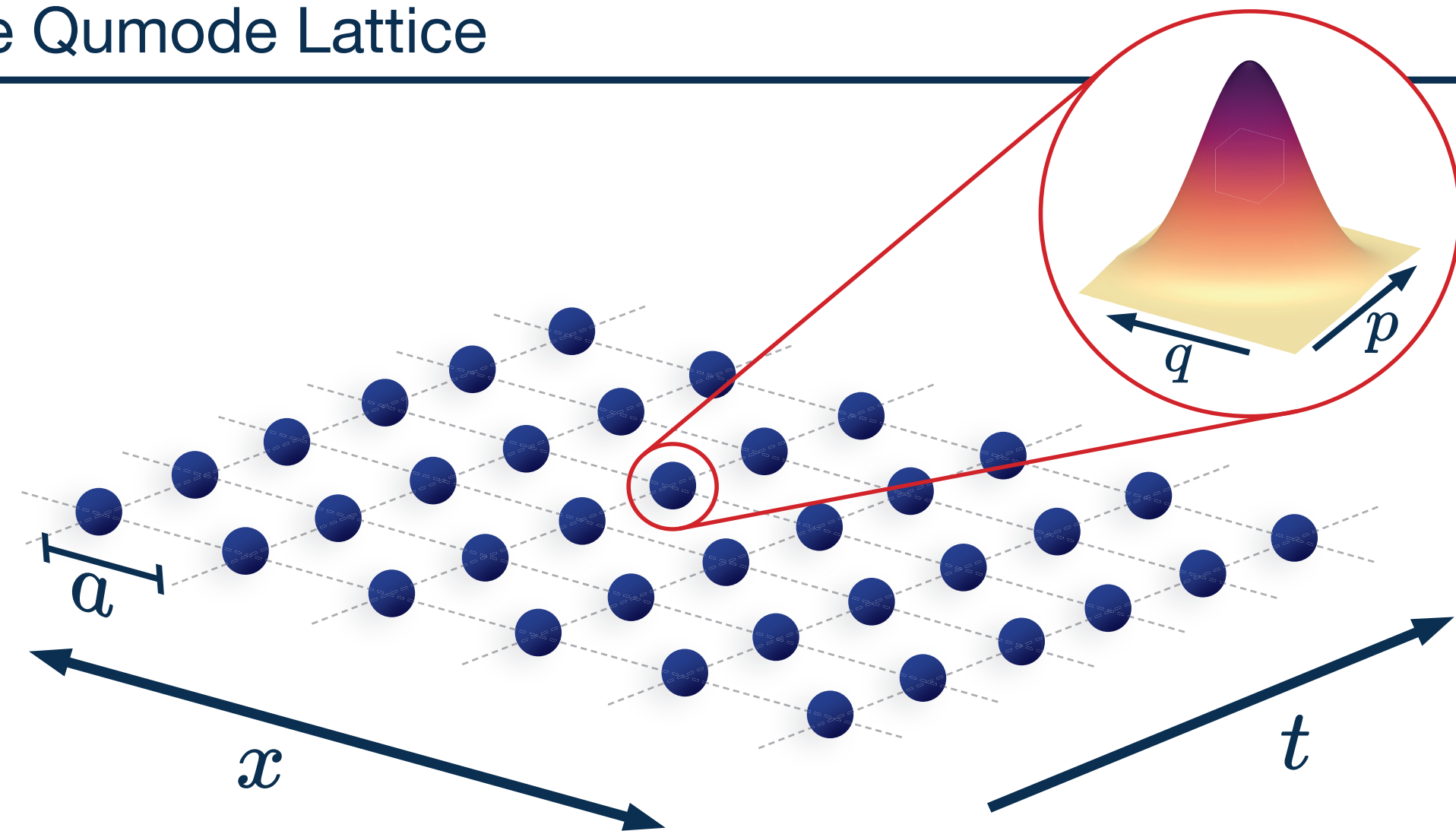
(1) The SHO Hamiltonian, (2) arbitrary potential V_I acting locally on each site, and (3) a simple hopping term

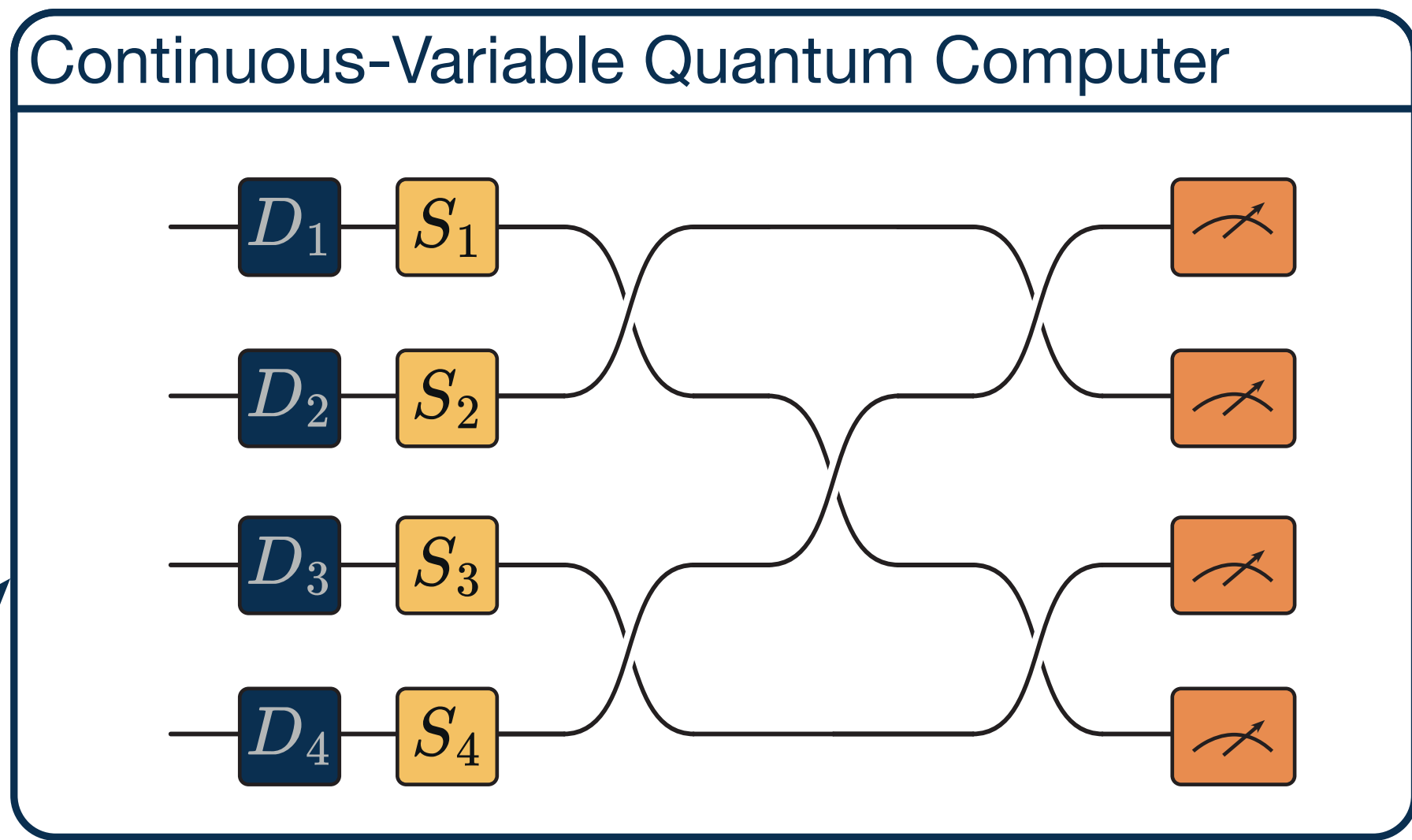
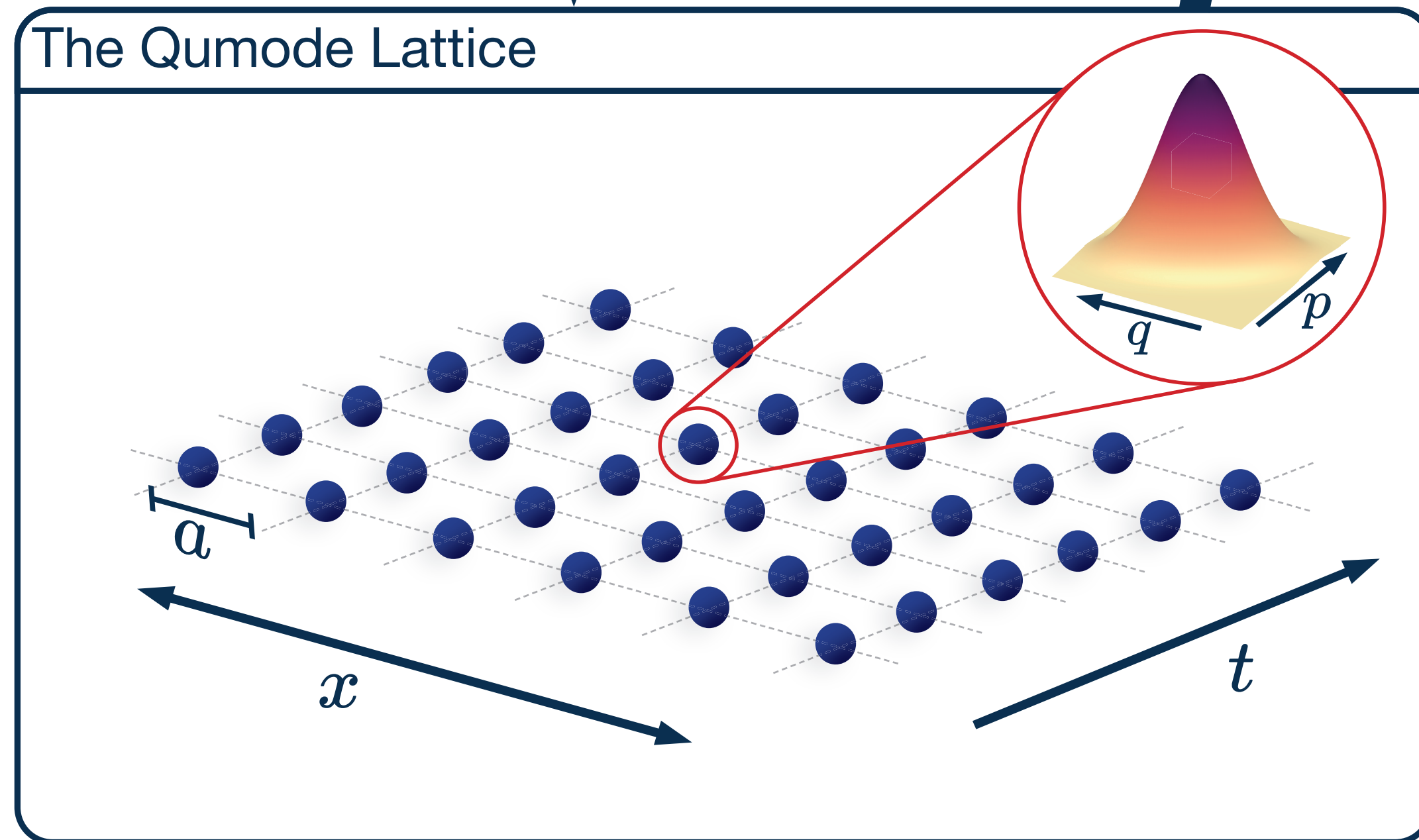
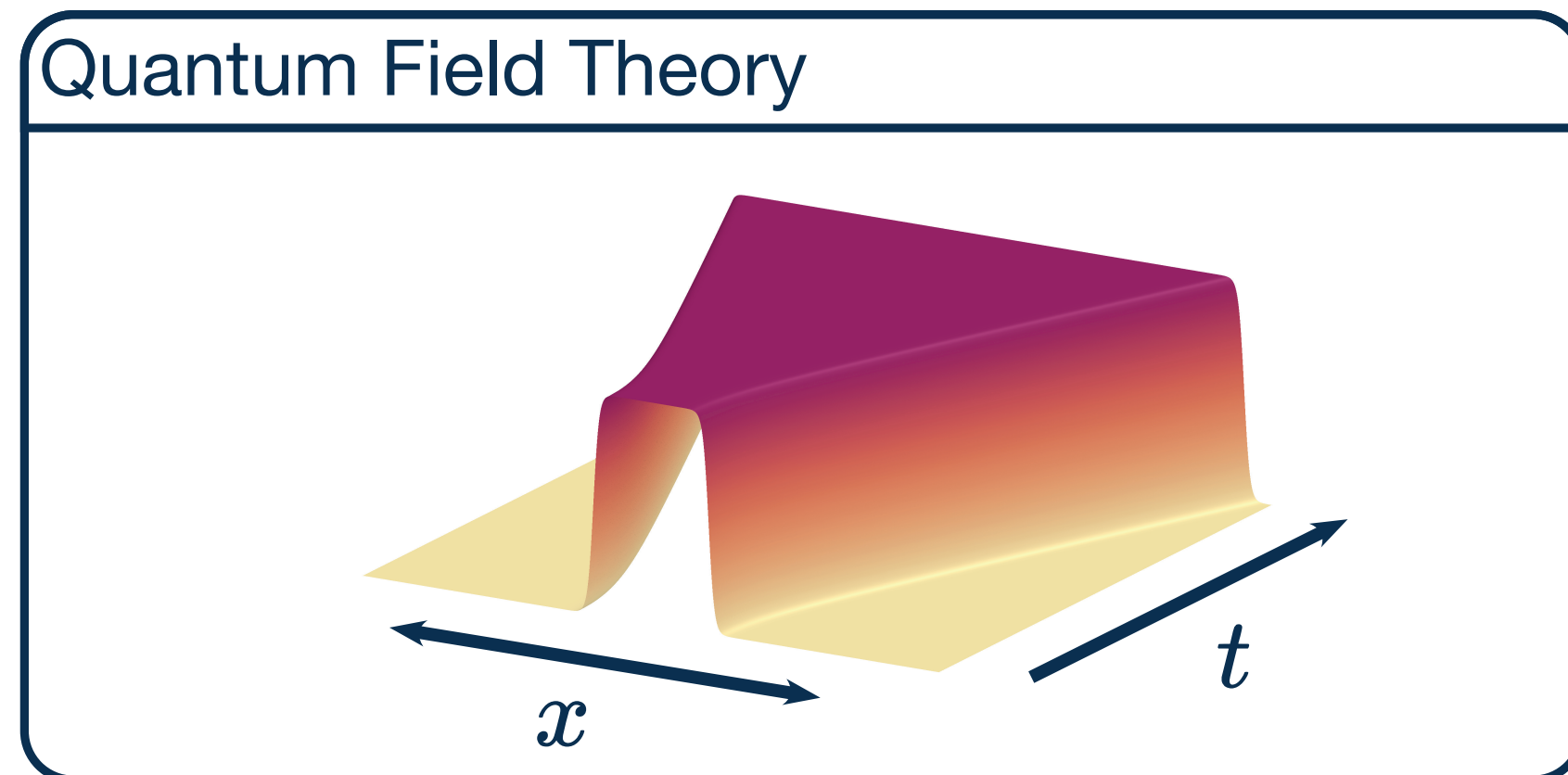
Quantum Field Theory



1

The Qumode Lattice





Real-time Simulation on CVQCs - single qumode evolution

The time-evolution of a single qumode is performed via the diagonal part of H , performed with the **evolver-gadget**

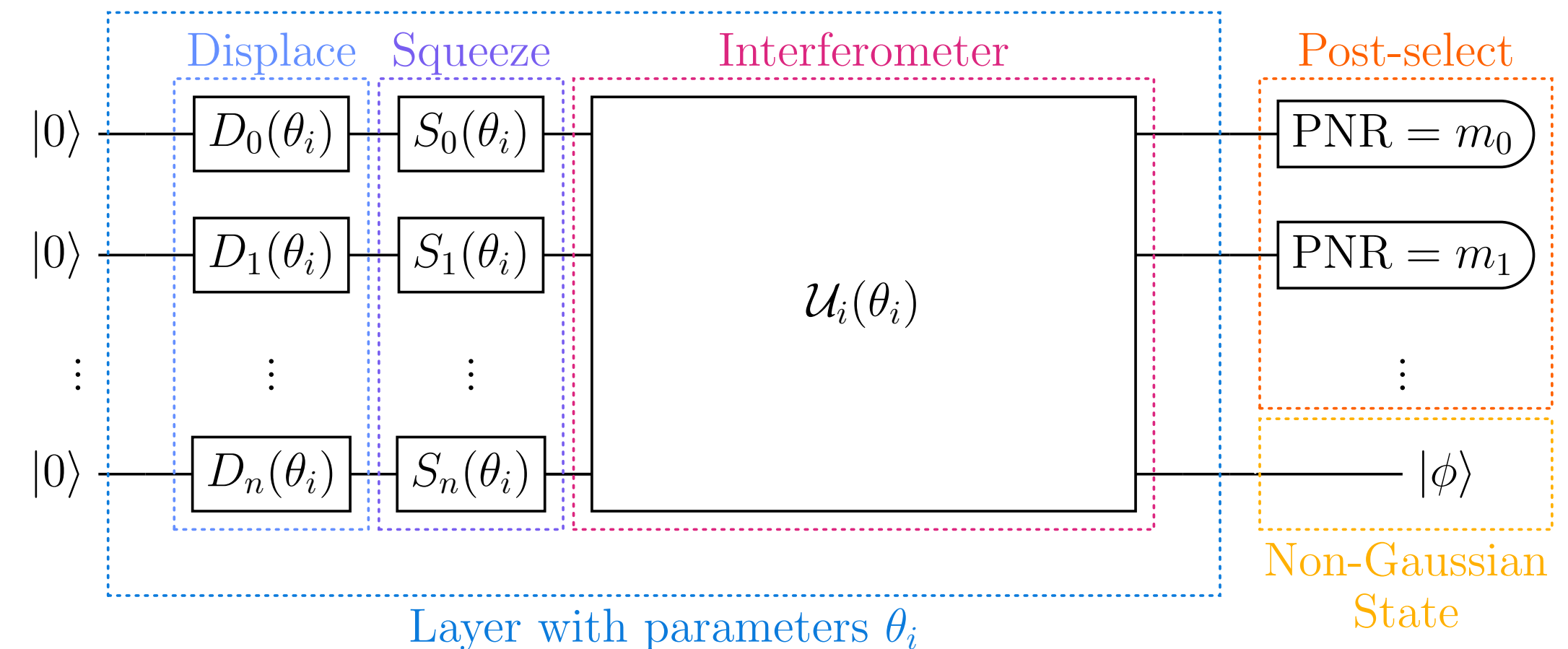
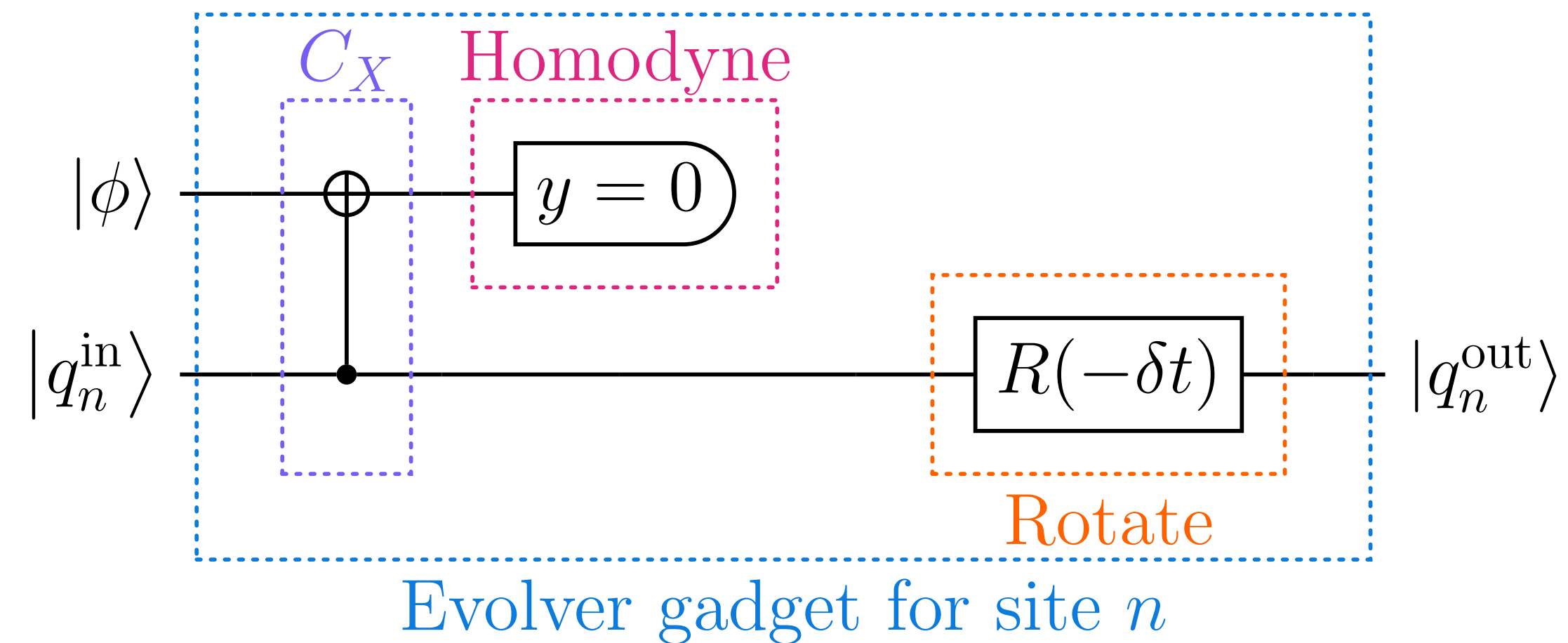
$$\mathcal{U}_{\text{diag}}(\delta t) = \mathcal{U}_R(-\delta t) \mathcal{U}_V(\delta t)$$

The SHO contribution

$$\mathcal{U}_R(-\delta t) = \exp \left(-i \frac{1}{2} (\hat{p}^2 + \omega^2 \hat{q}^2) \delta t \right)$$

Non-Gaussian contribution

The non-Gaussian part of the evolution acts locally on each site and is learned from a machine-learning enhanced state preparation protocol

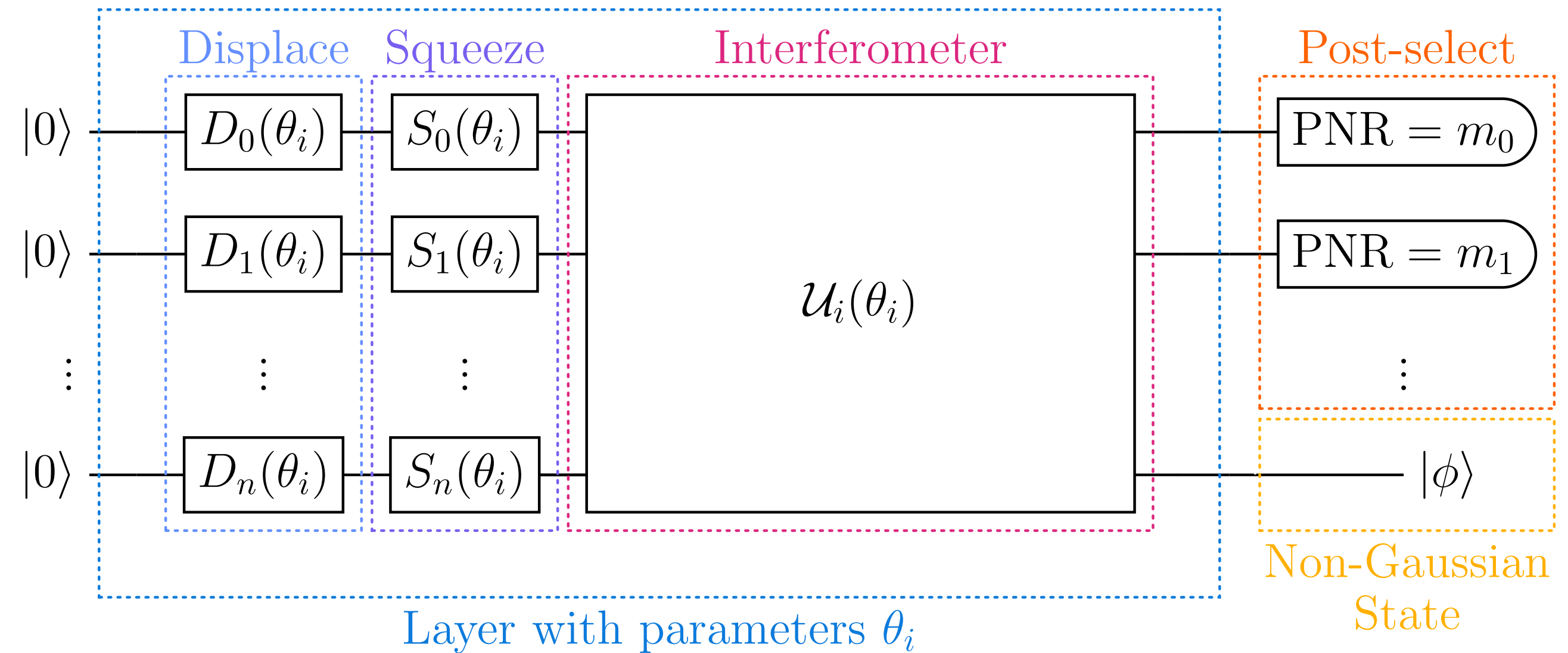


Learning to be non-Gaussian

Non-Gaussian gate operations

constructed through measurement-based quantum computing

Circuit is constructed entirely from Gaussian operations, with measurements inducing a non-Gaussian state on $|\phi\rangle$



Machine-learning enhanced state preparation

A classical machine-learning algorithm can then be used to train the gate parameters to achieve a non-Gaussian target state $|\phi\rangle$ using the truncated expansion in the Fock basis: $|\phi\rangle = \sum_n A_n |n\rangle$

Learning to be non-Gaussian

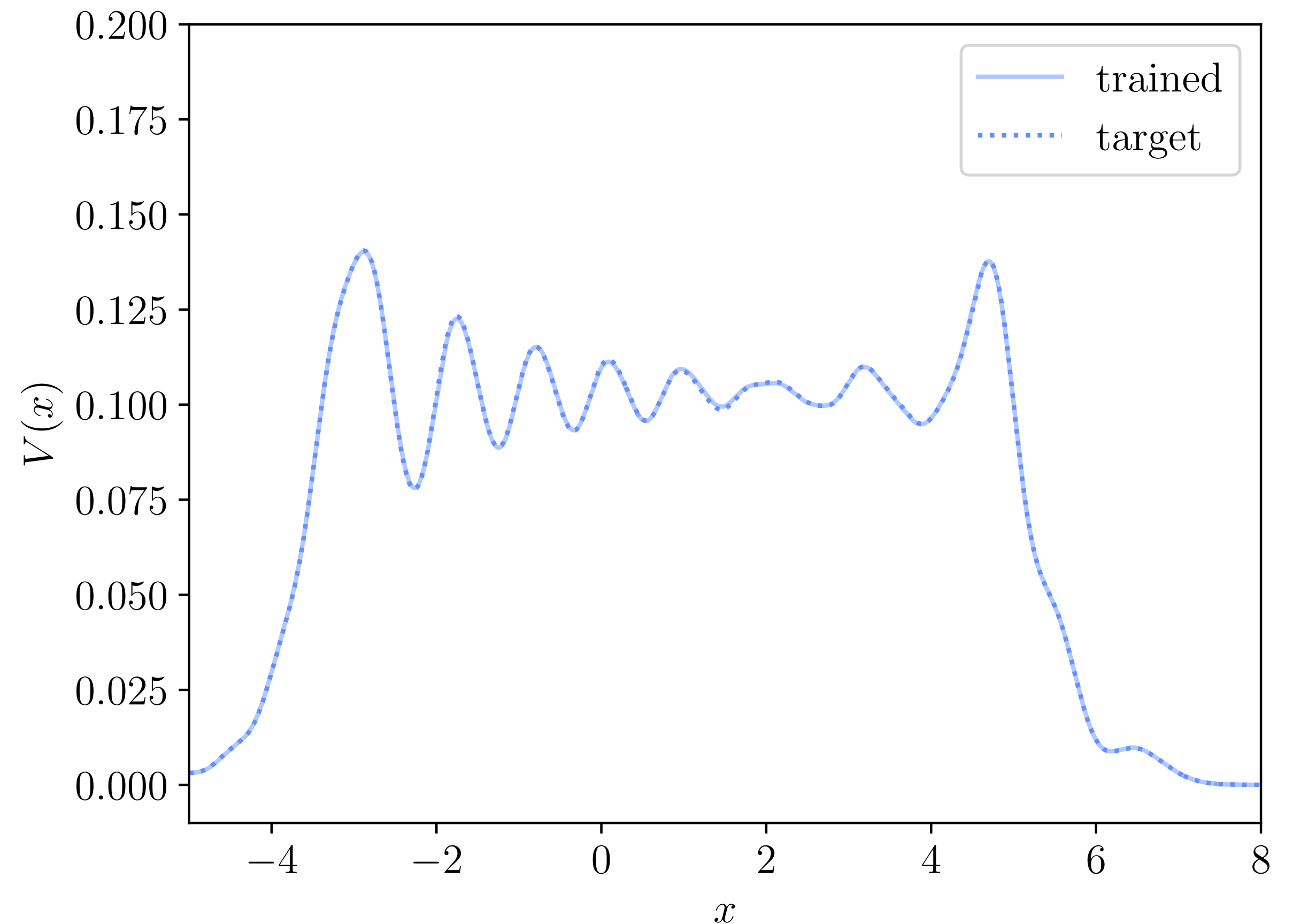
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Real-time Simulation on CVQCs - single qumode evolution

Heralding the non-Gaussian state

The non-Gaussian part is transferred to the target qumode via the controlled-X gate

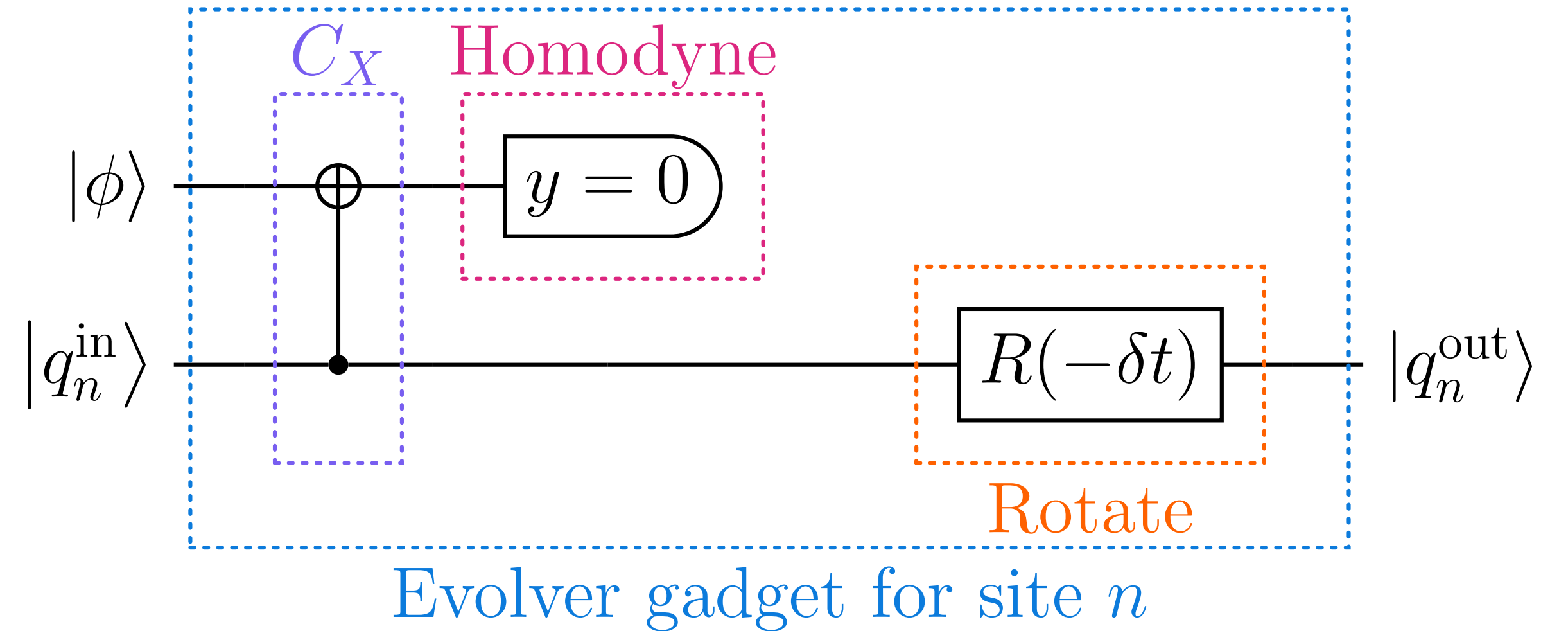
$$\hat{C}_X(-s; \hat{x}, \hat{y}) = e^{is\hat{x}\hat{p}_y/\hbar}$$

Inducing a shift $y \rightarrow y + sx$

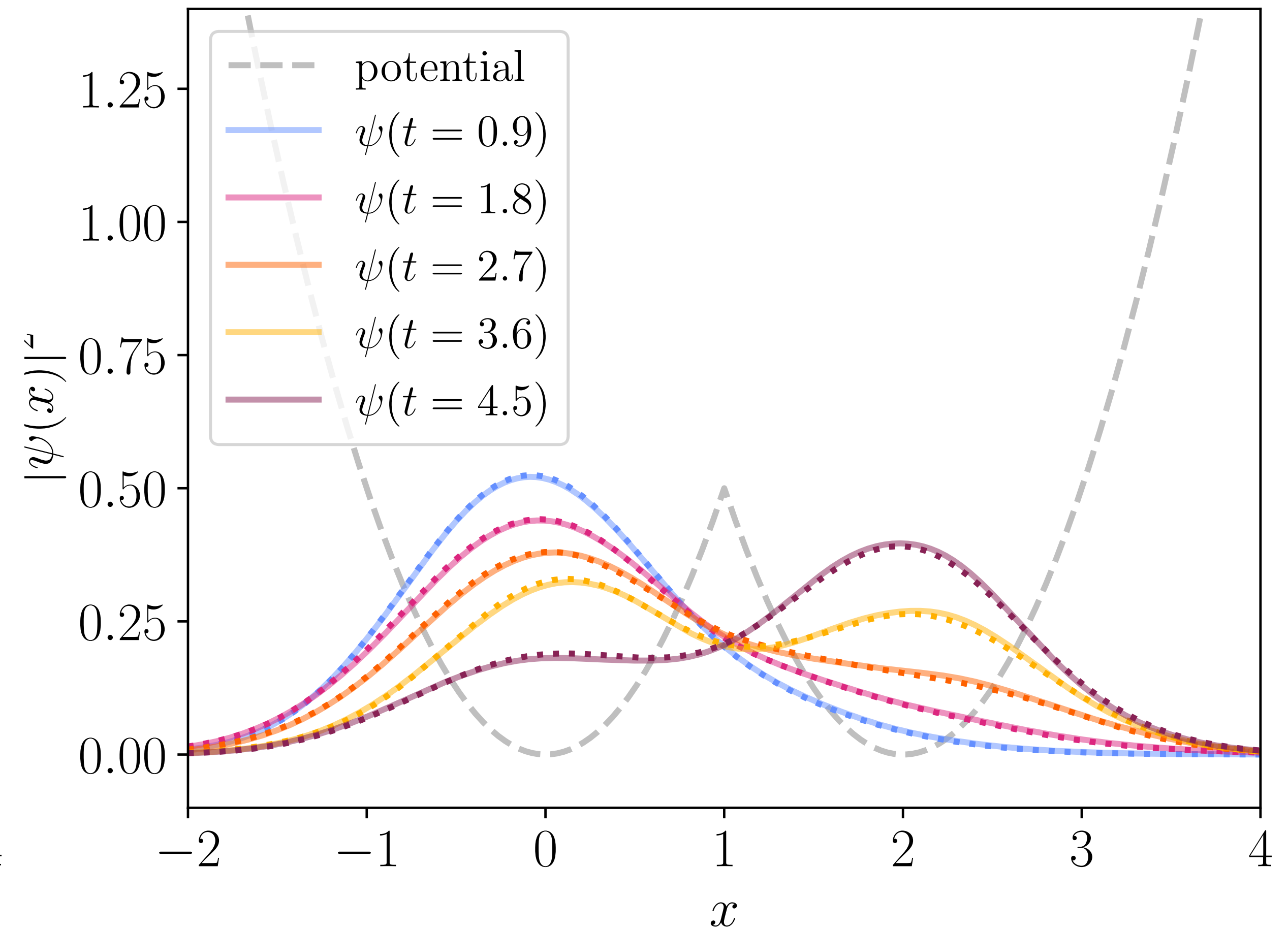
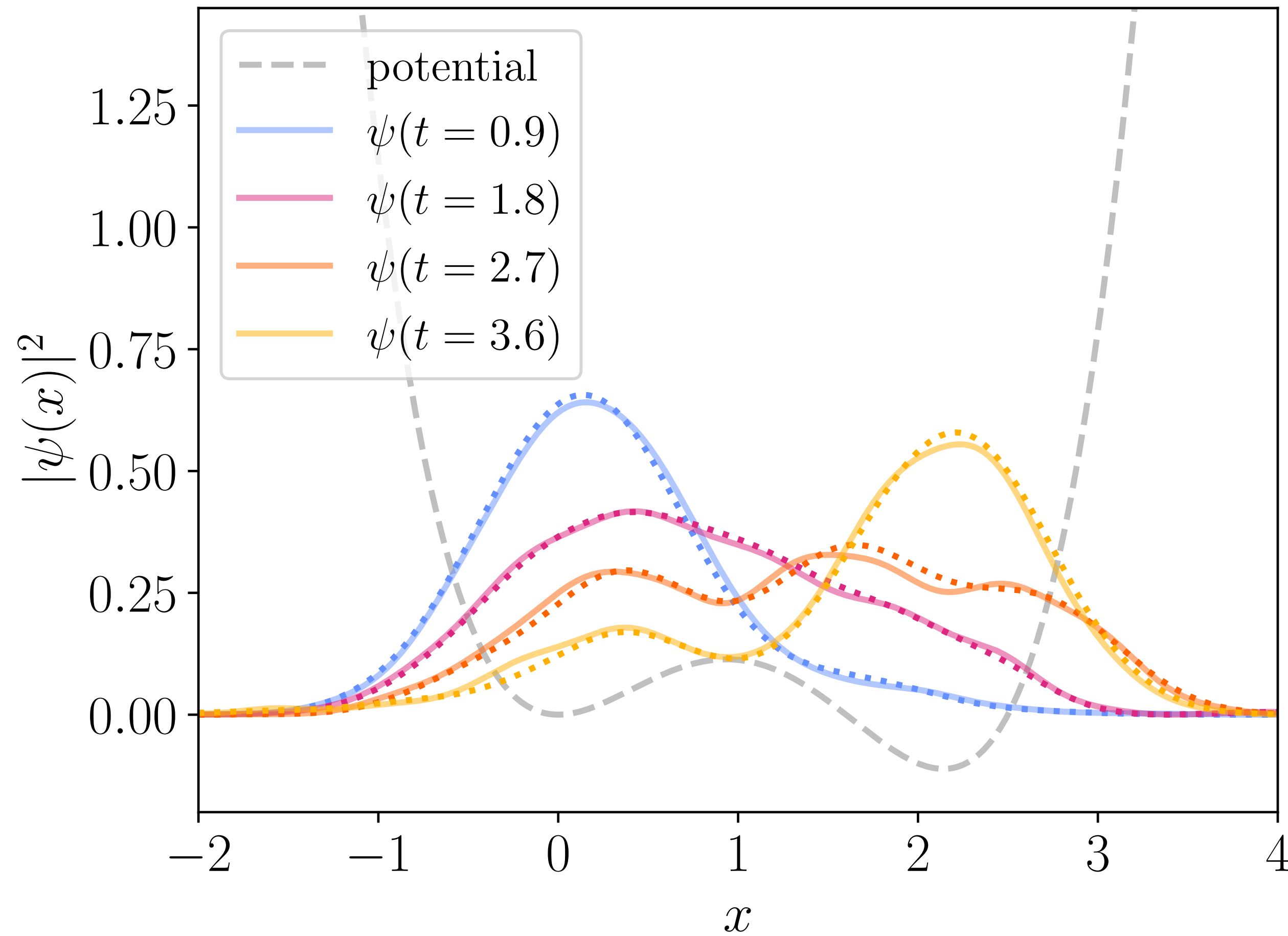
Post-selecting a Homodyne measurement on $y = 0$ heralds the non-Gaussian state:

$$\langle x | \psi(\delta t) \rangle = \exp \left[-\frac{i}{2} (\hat{p}^2 + \hat{x}^2) \delta t \right] \exp \left[-iV_1(\hat{x})\delta t \right] \langle qx | \phi_0 \rangle \langle x | \psi_{\text{in}} \rangle$$

The Trotterised time-evolution can now be performed using the *evolver-gadget*



Time-evolution using photons



Real-time Simulation on CVQCs

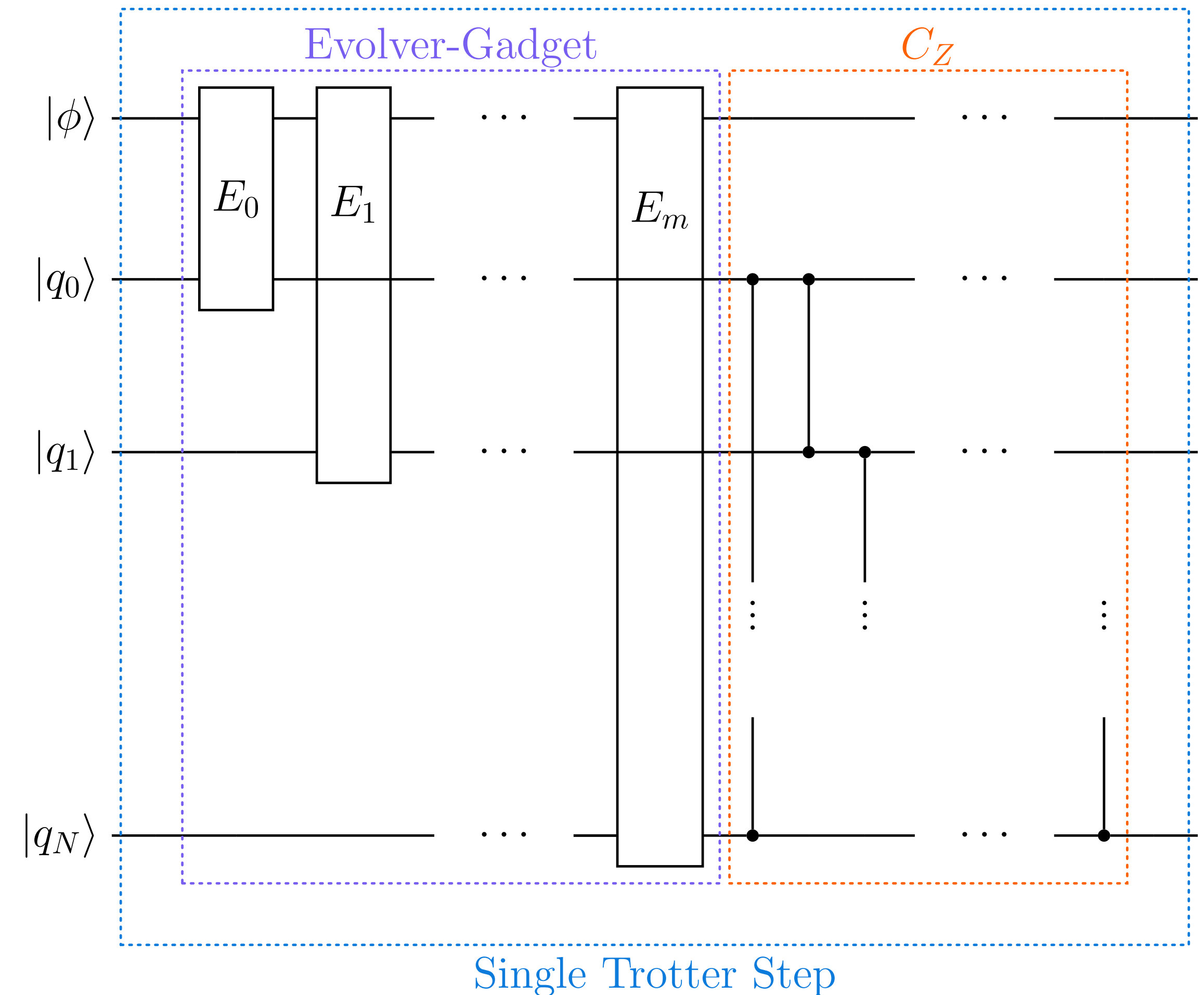
Extending to scalar field theory

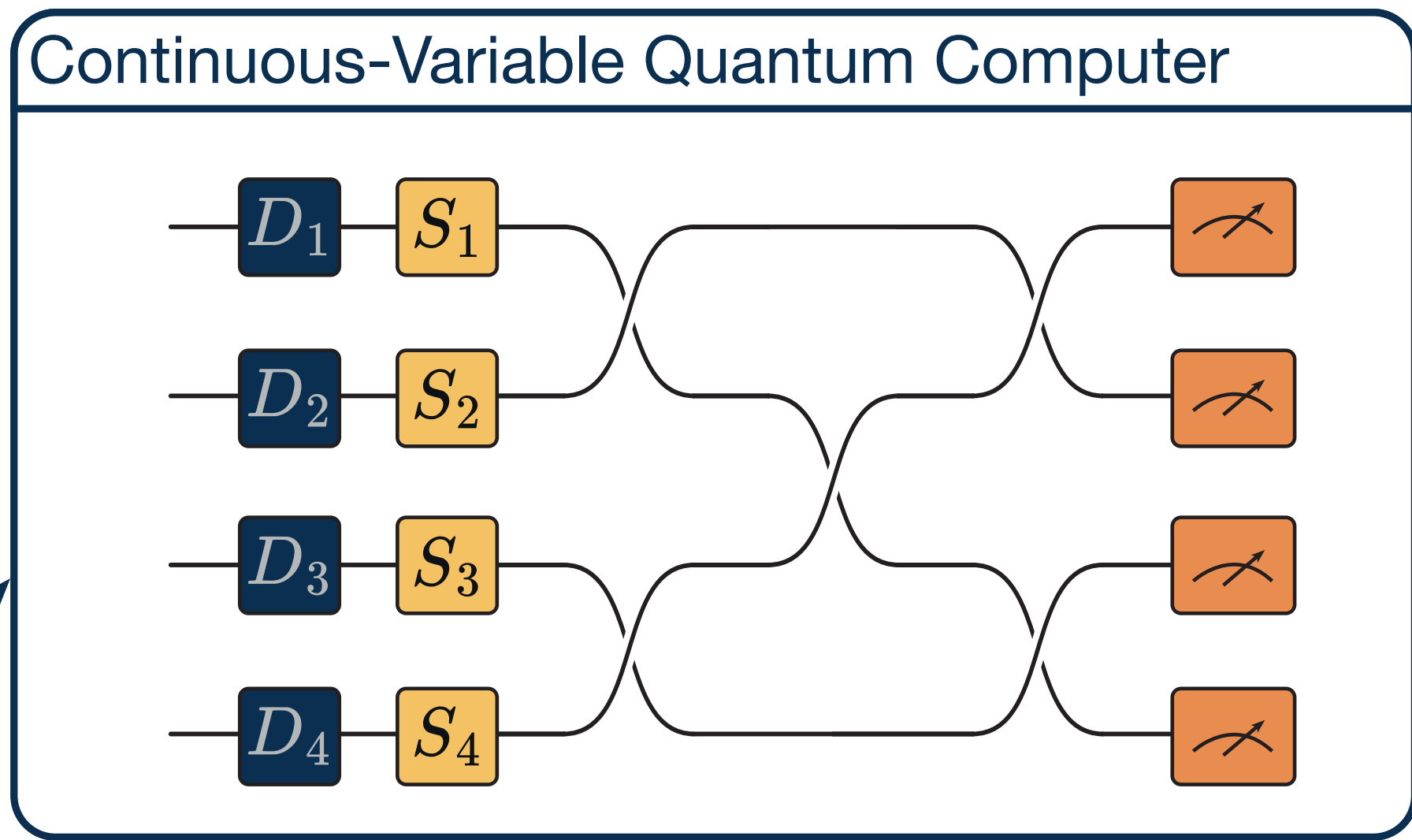
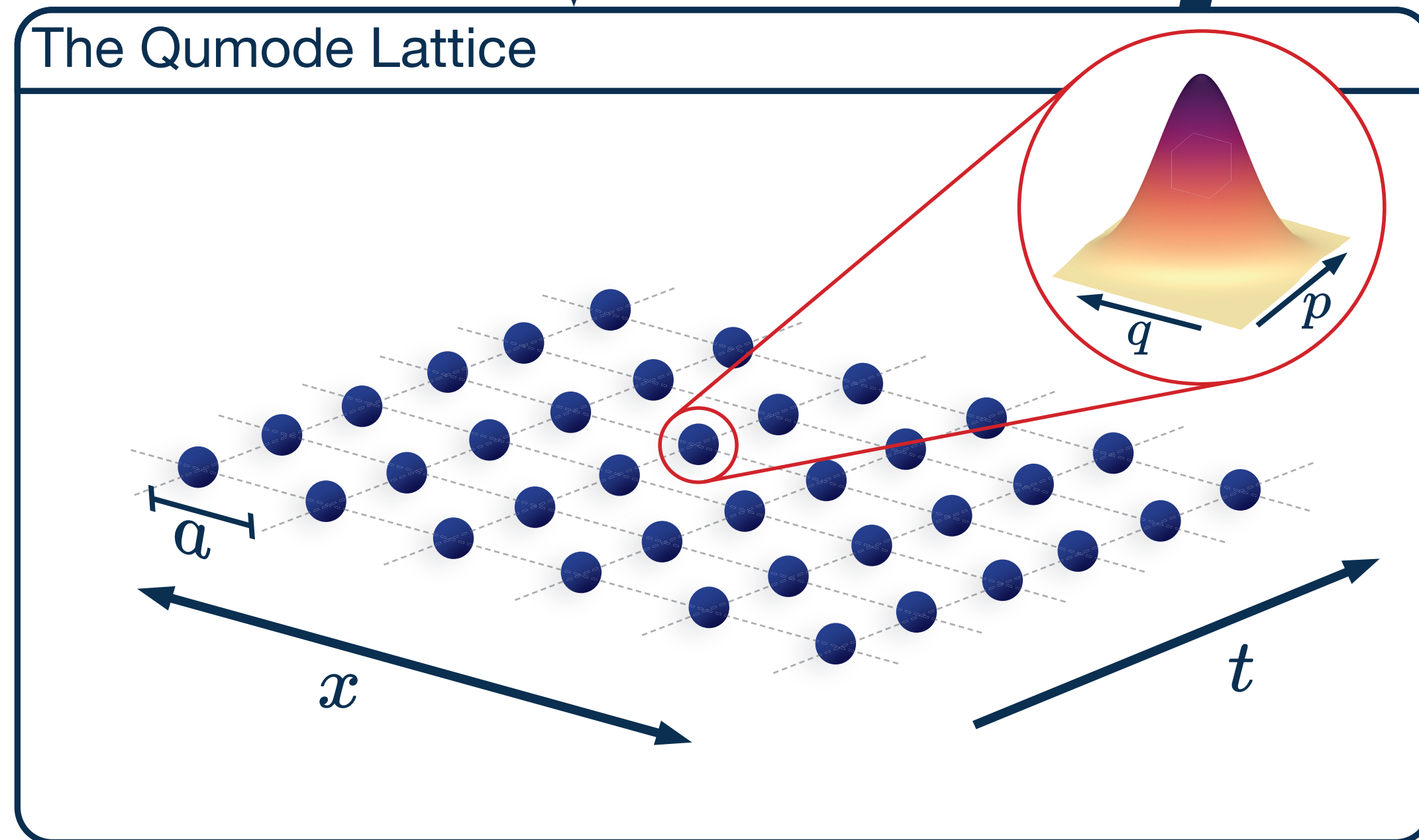
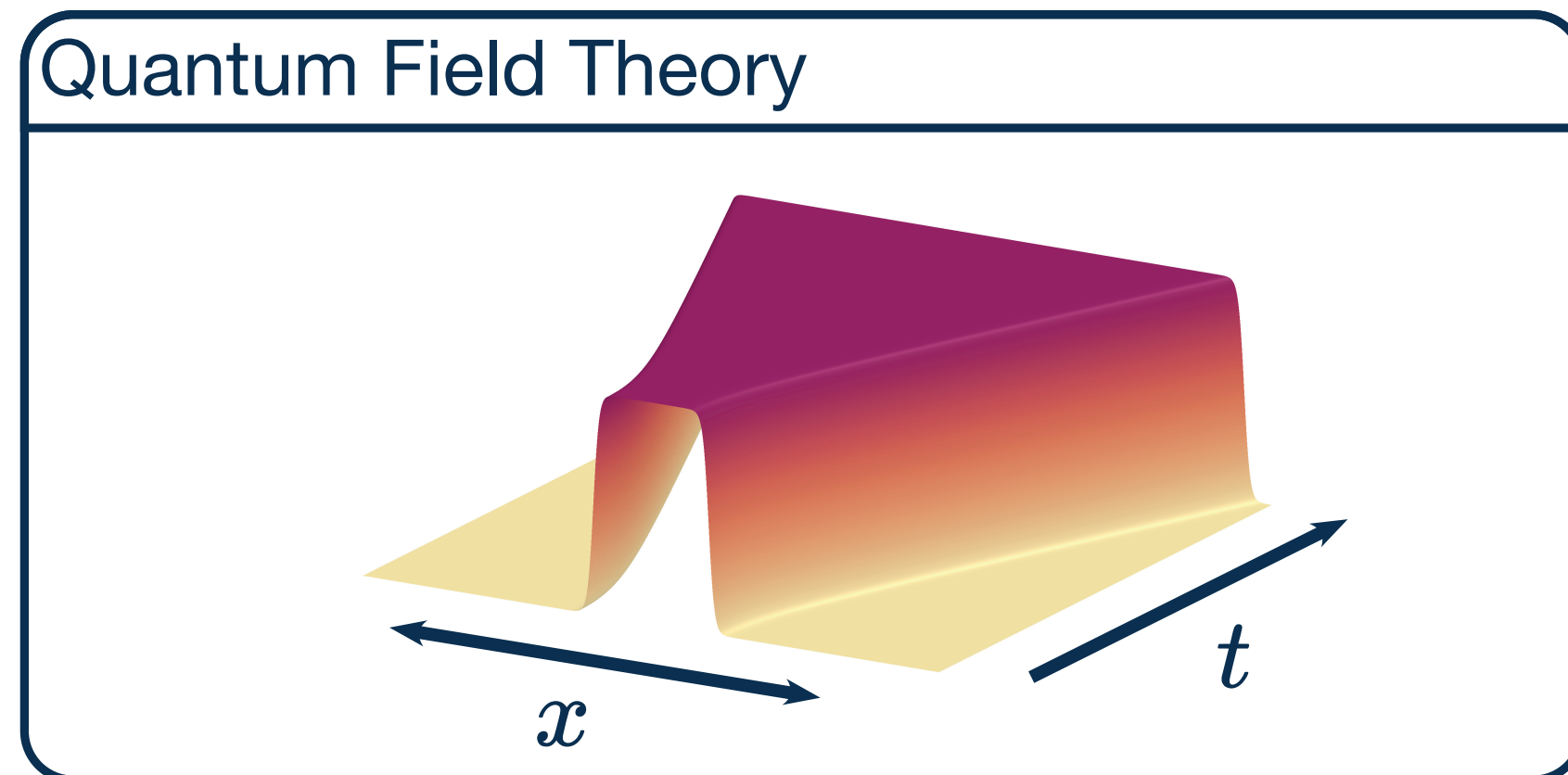
The extension to a full QFT simulation is simply achieved by performing nearest neighbour hopping terms:

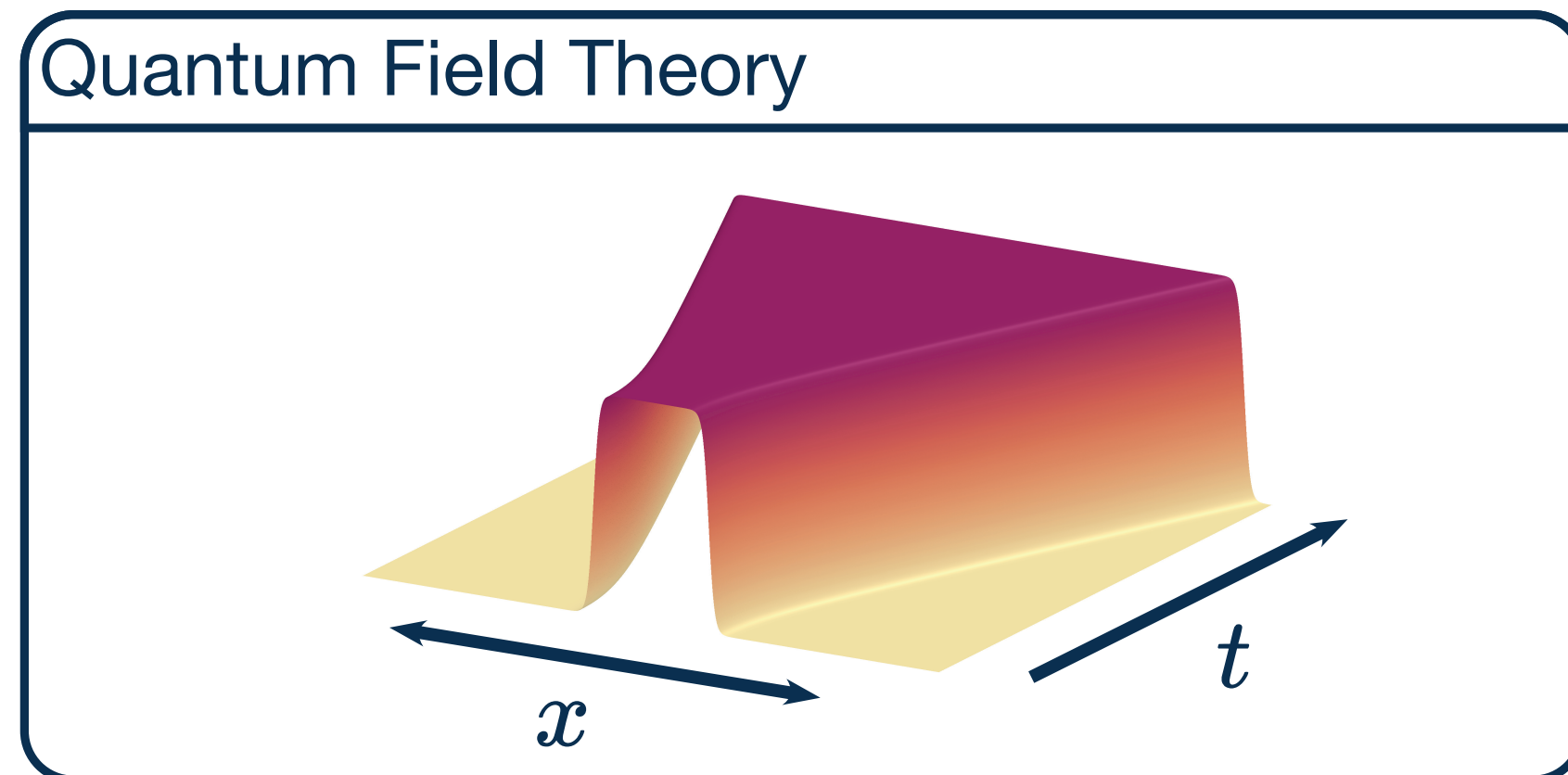
$$\mathcal{U}_{\text{hop}} = \prod_{n=1}^N e^{ia^{-2} \hat{q}_{n+1} \hat{q}_n \delta t}$$

Evolution without truncation

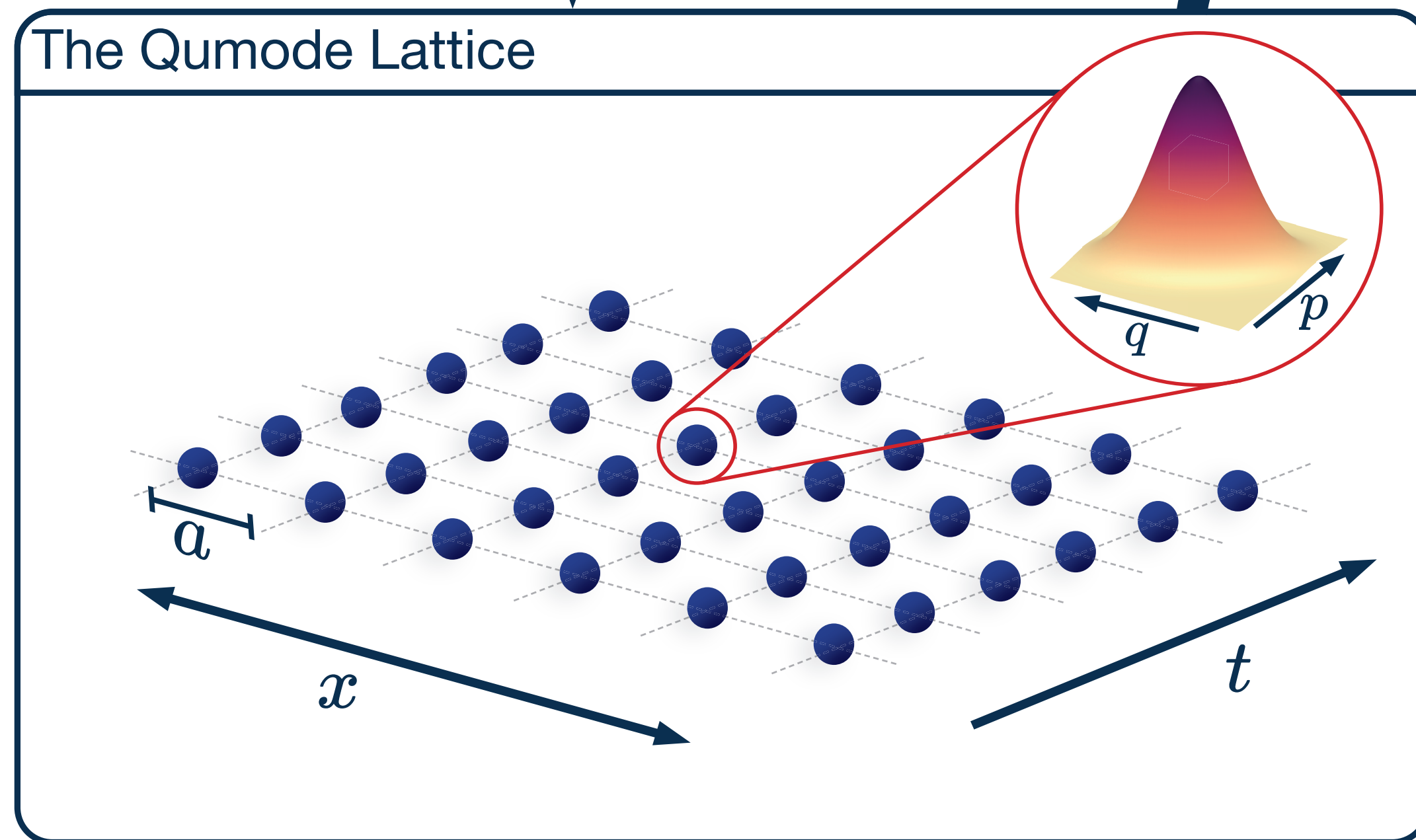
Combined, the quantum qumode lattice can simulate the real-time dynamics without the need of entanglement truncation or field digitisation on a CVQC



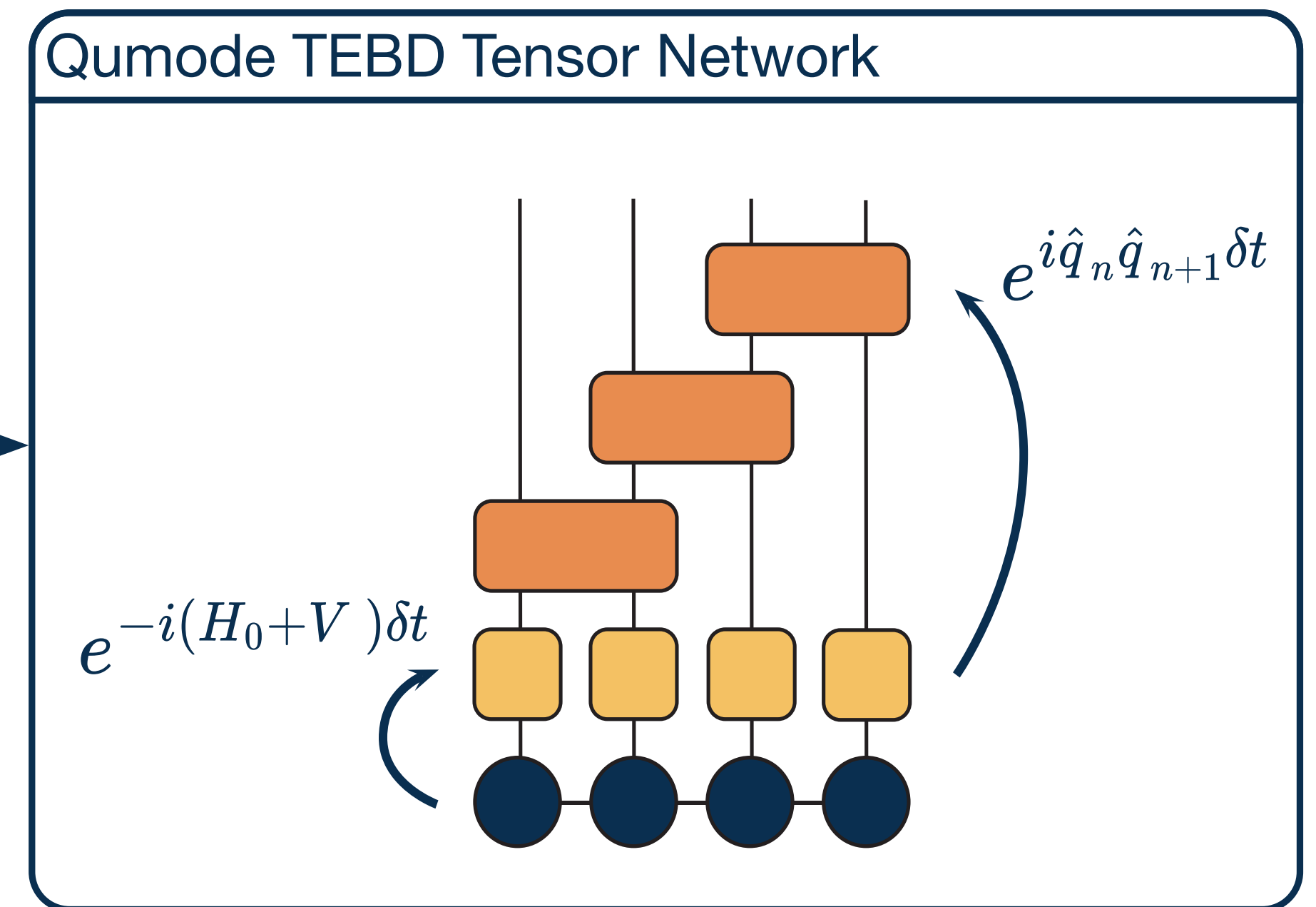
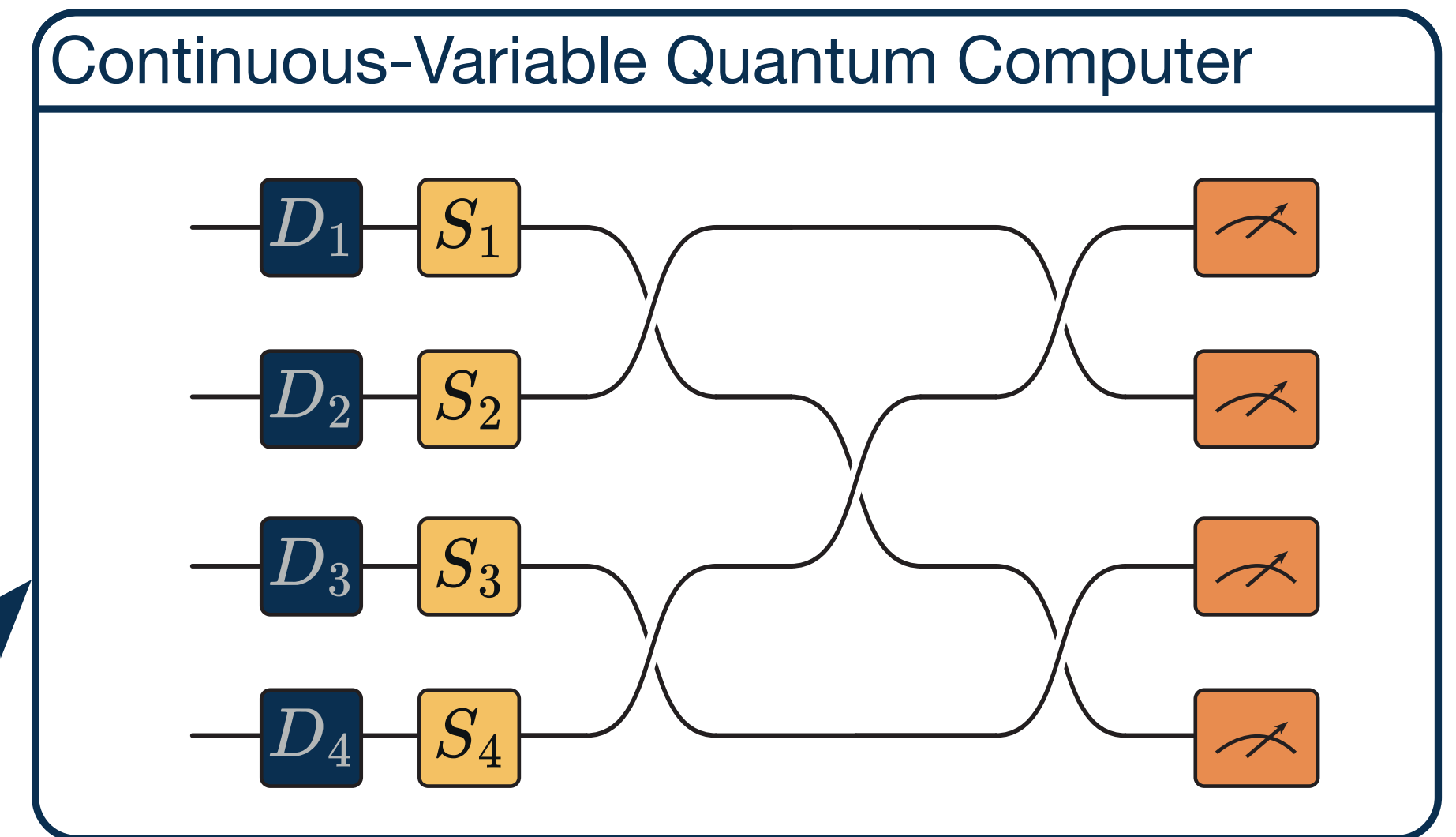


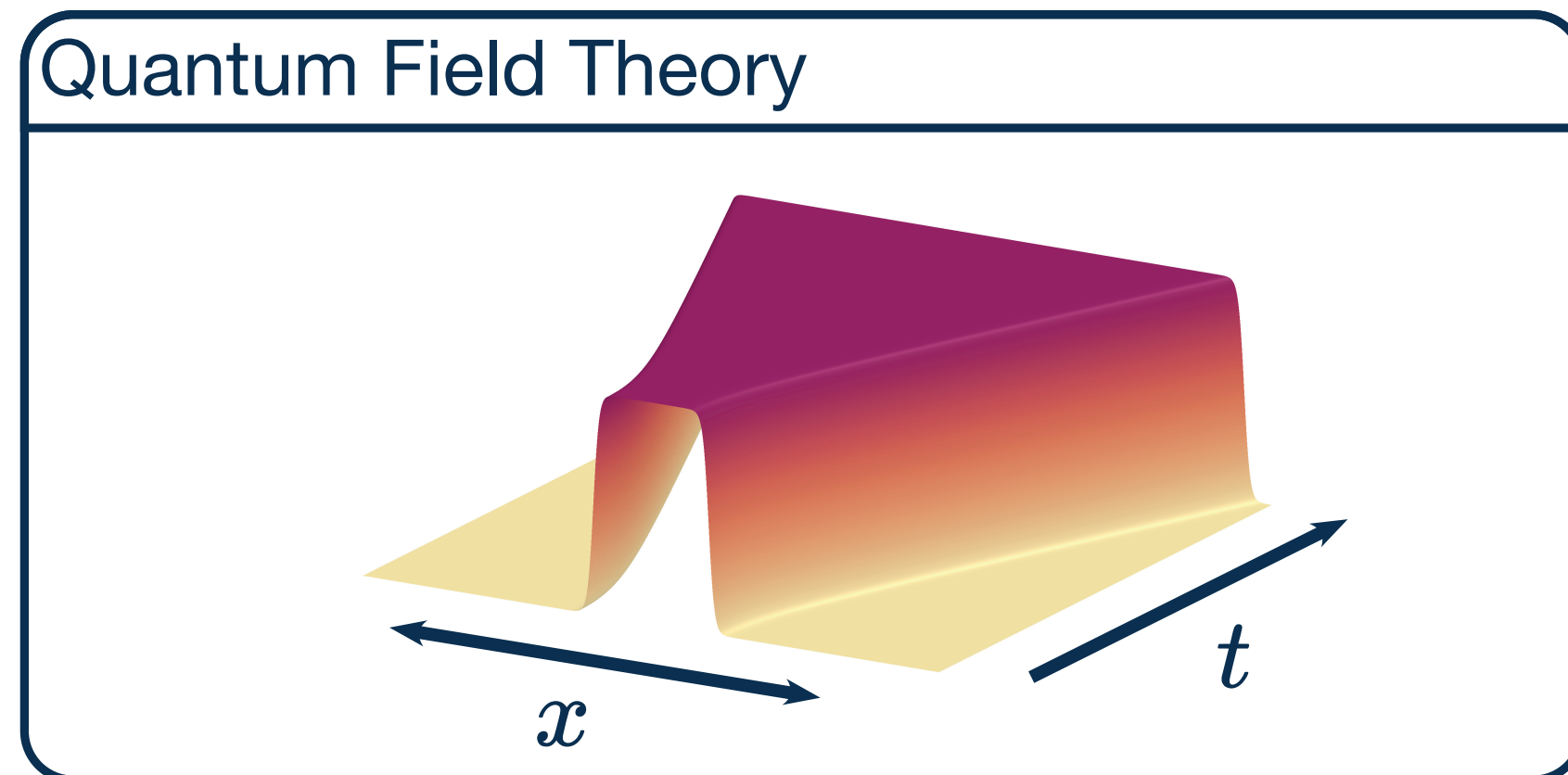


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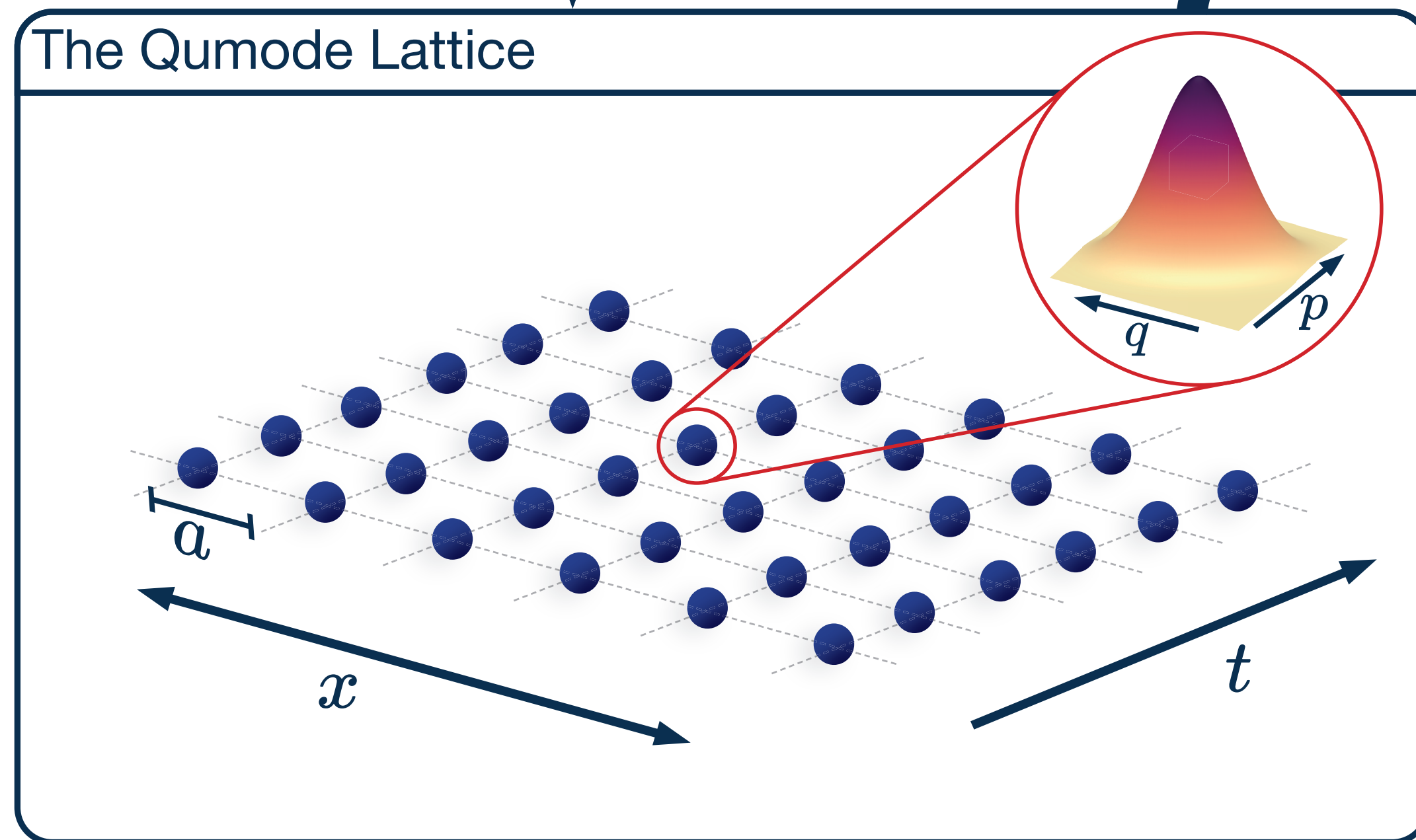


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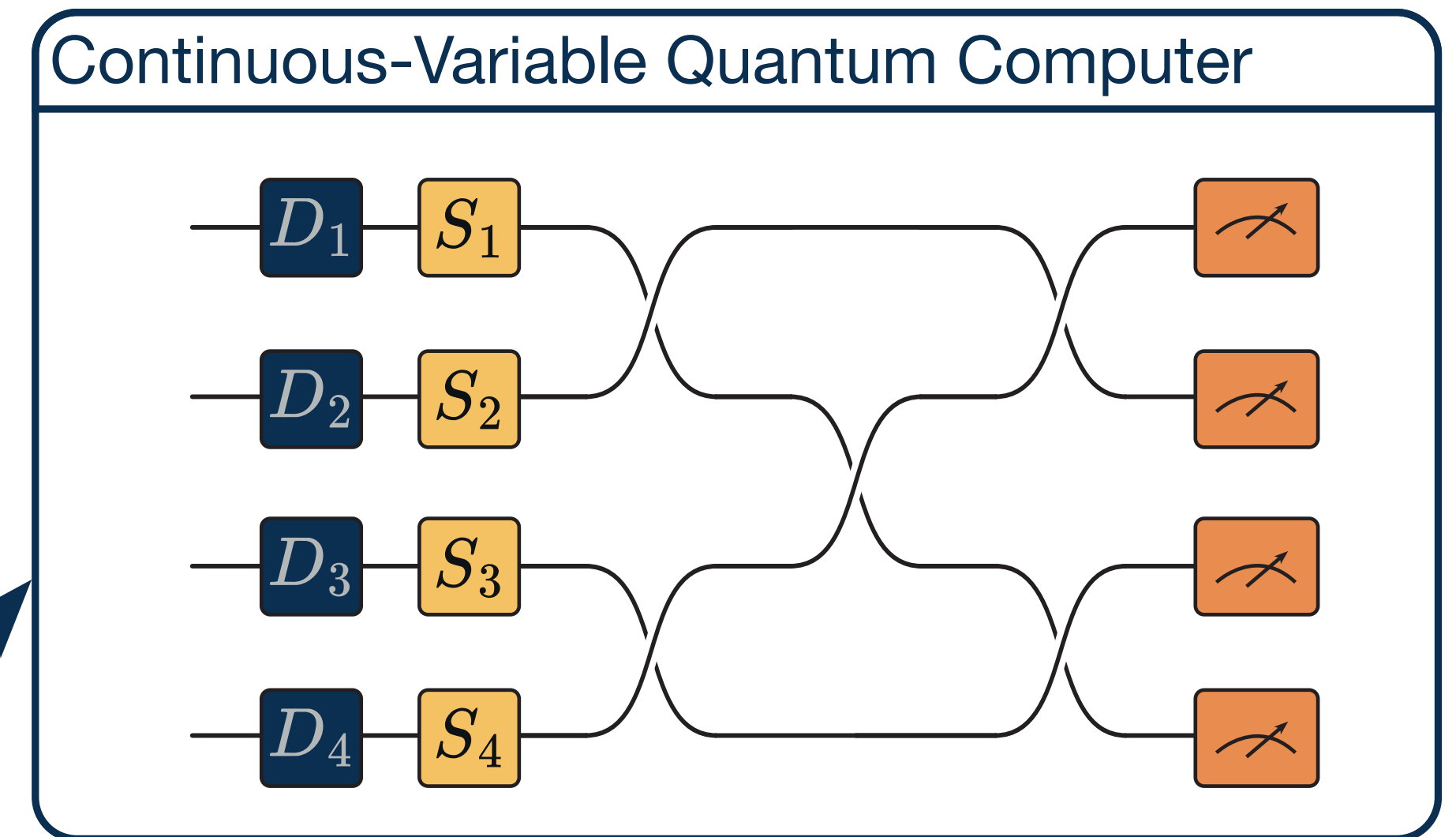




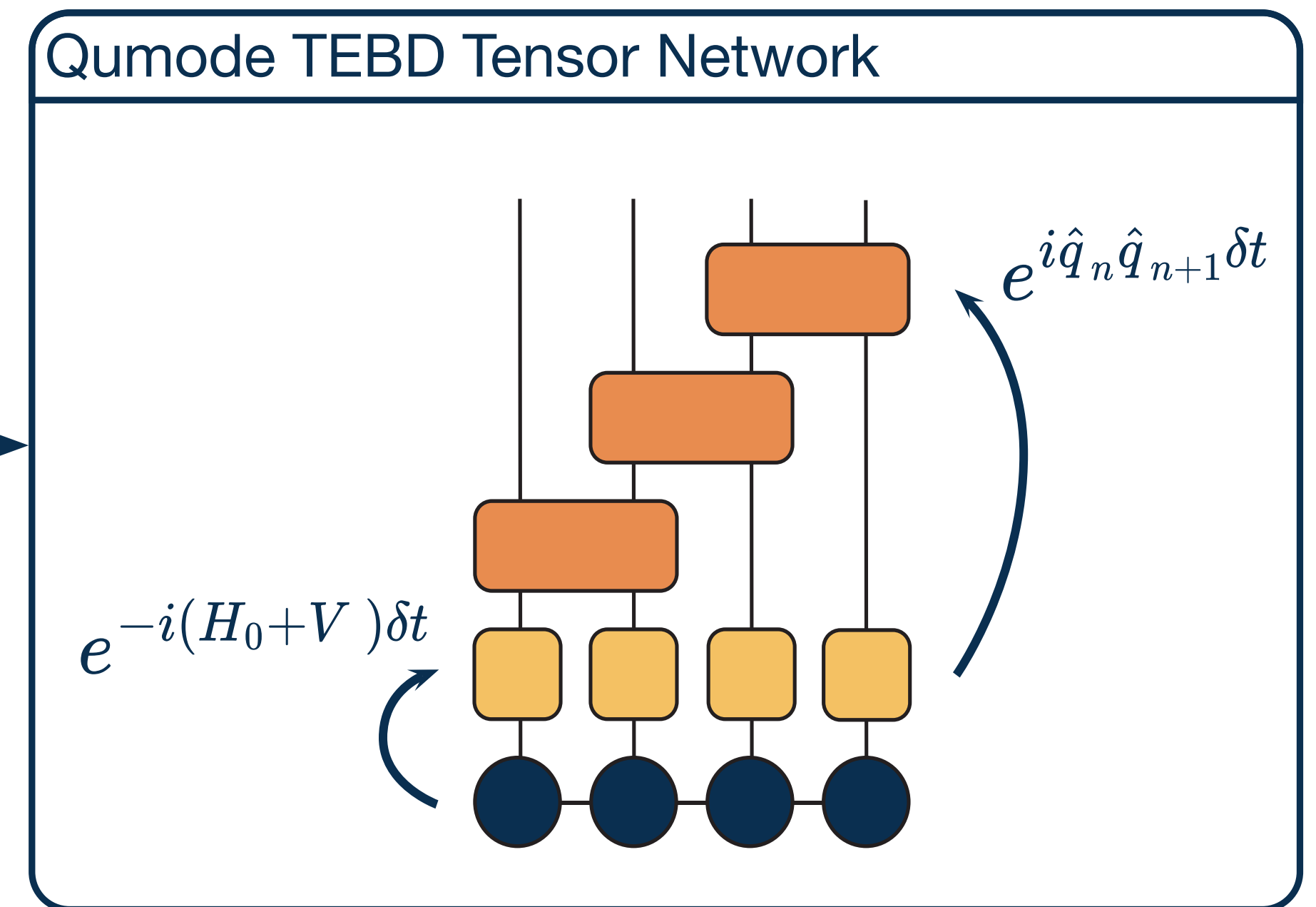
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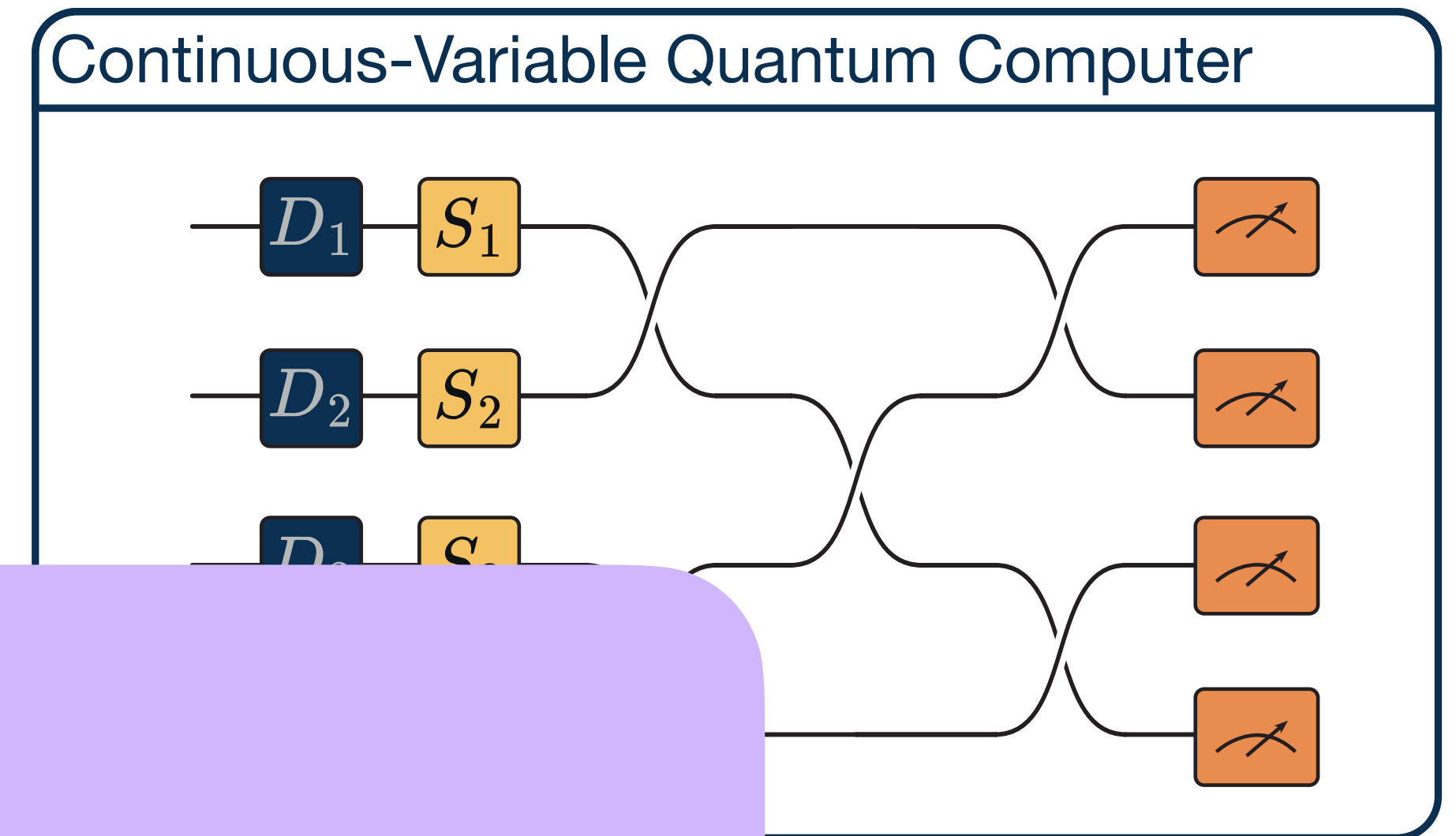
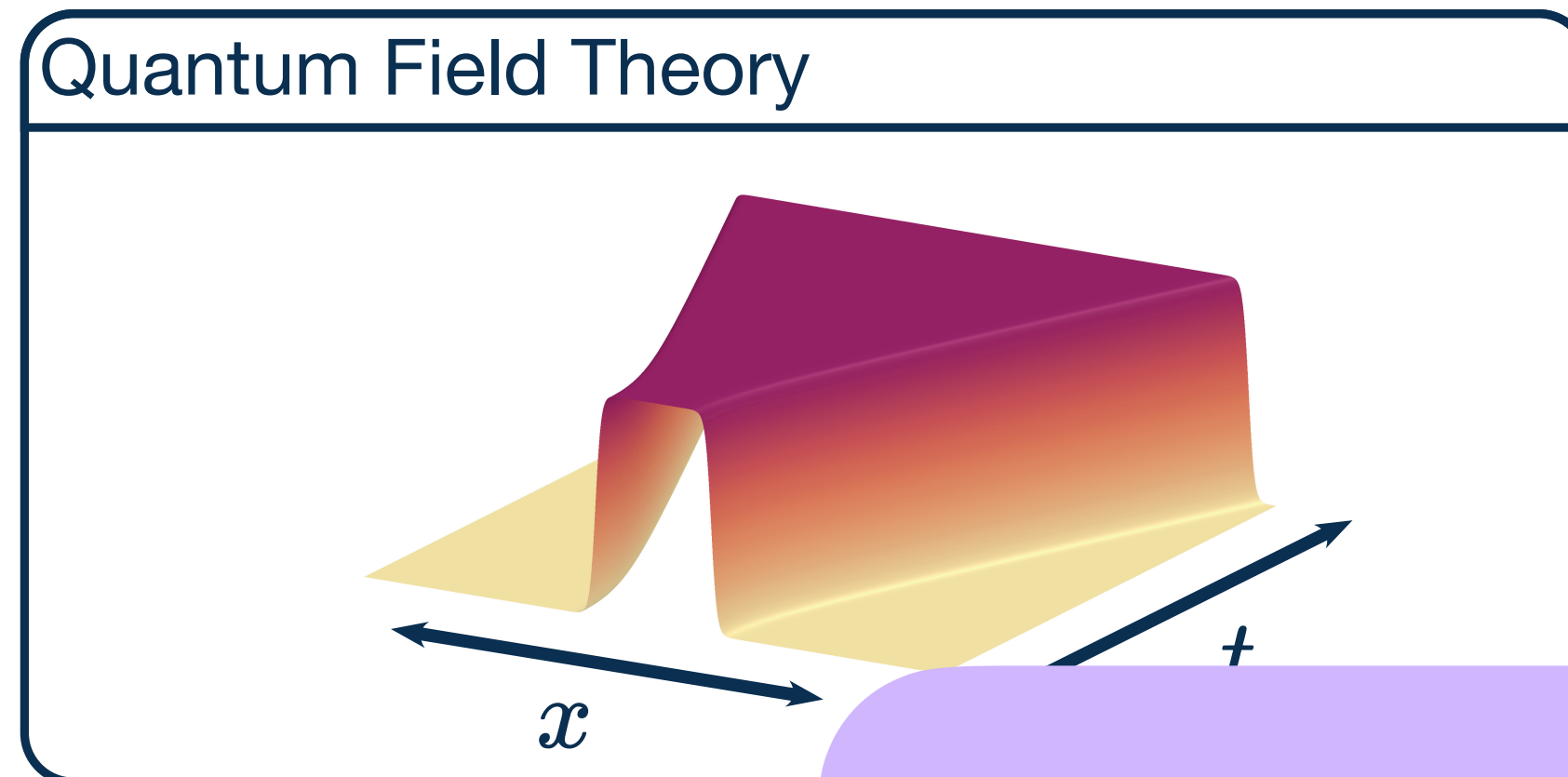


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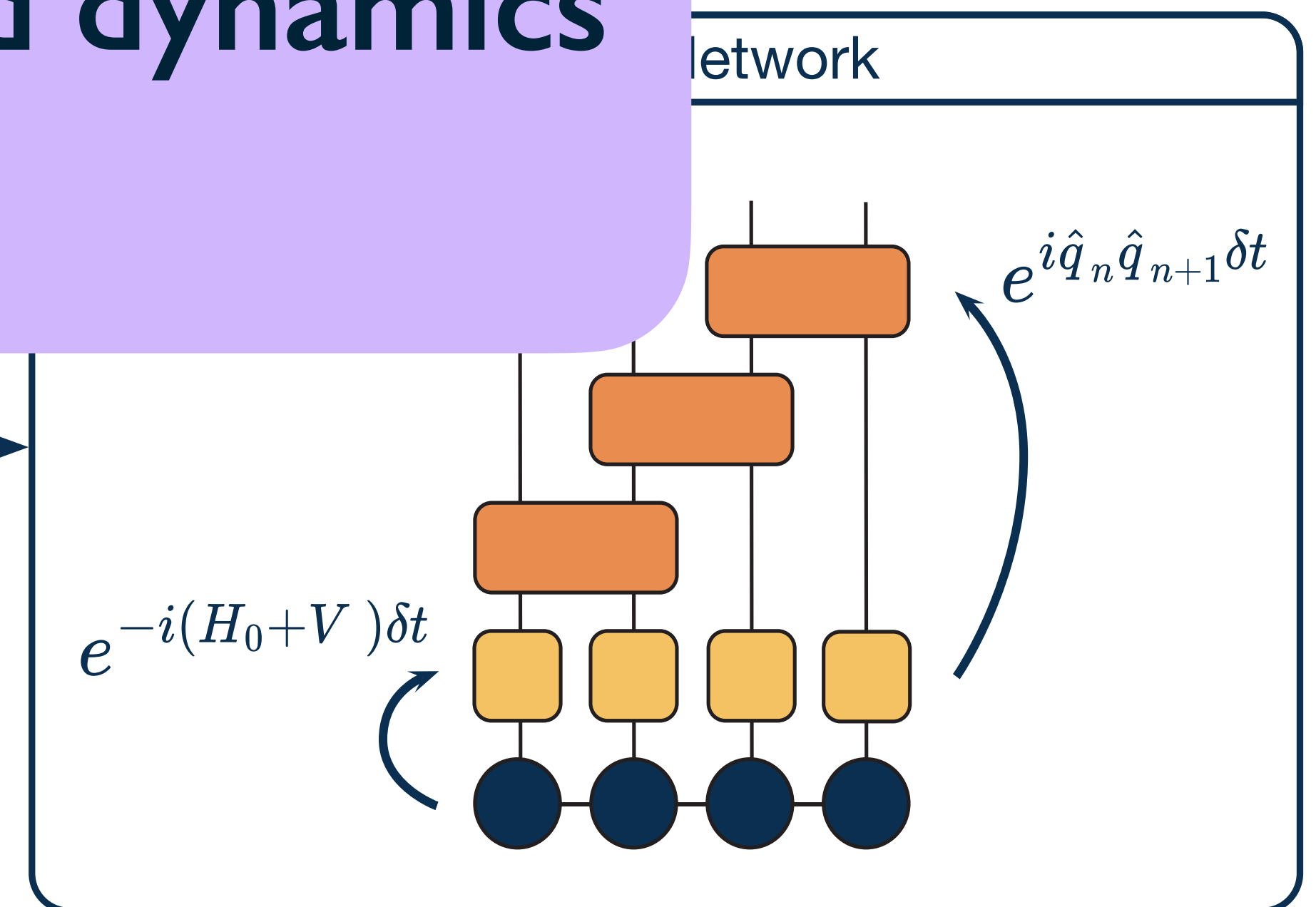
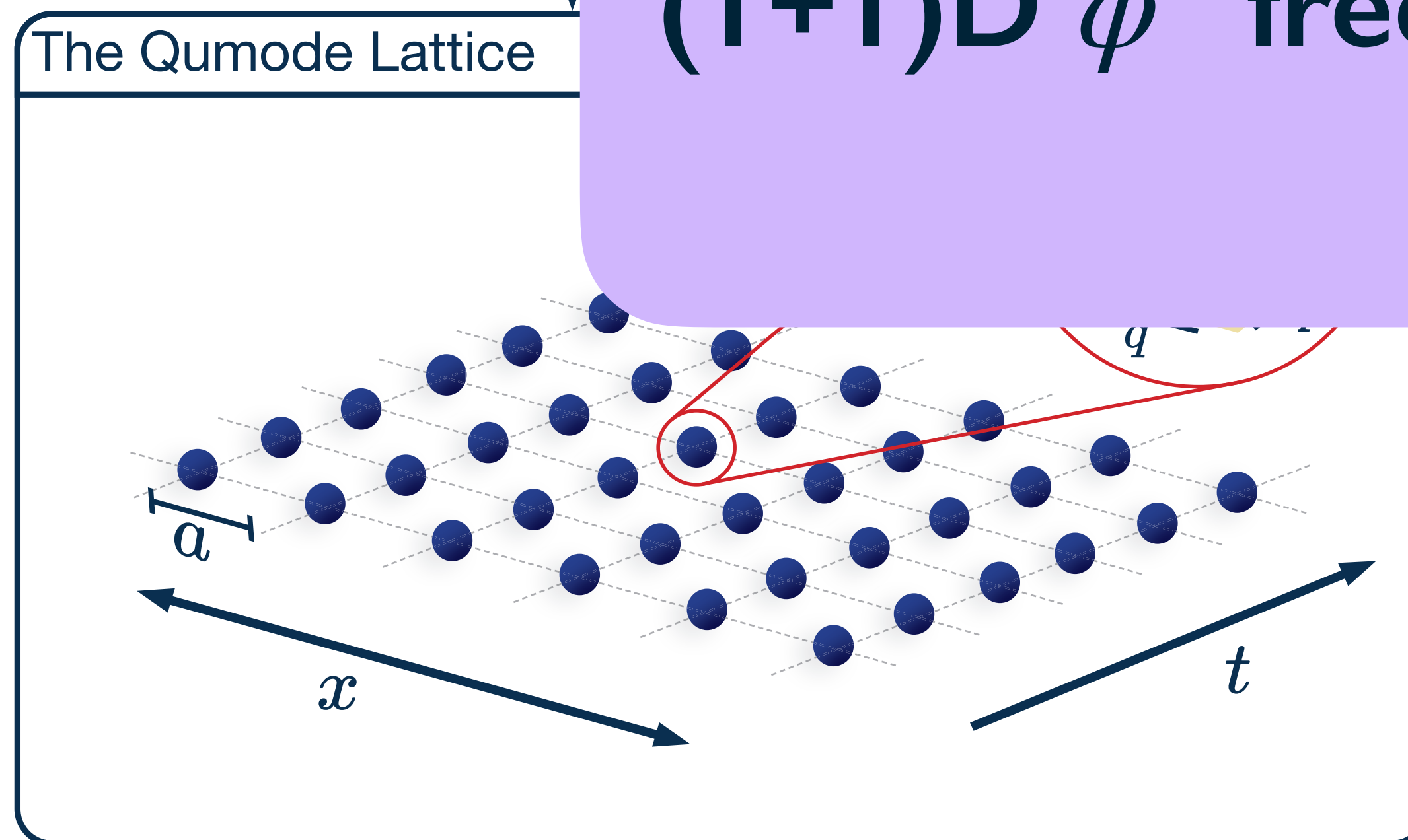


2





$(1+1)\text{D } \phi^4$ free field dynamics



The propagator in scalar QFT

The free-field propagator:

$$\tilde{D}_R = \frac{i}{p^2 - m^2 + i\epsilon p_0}$$

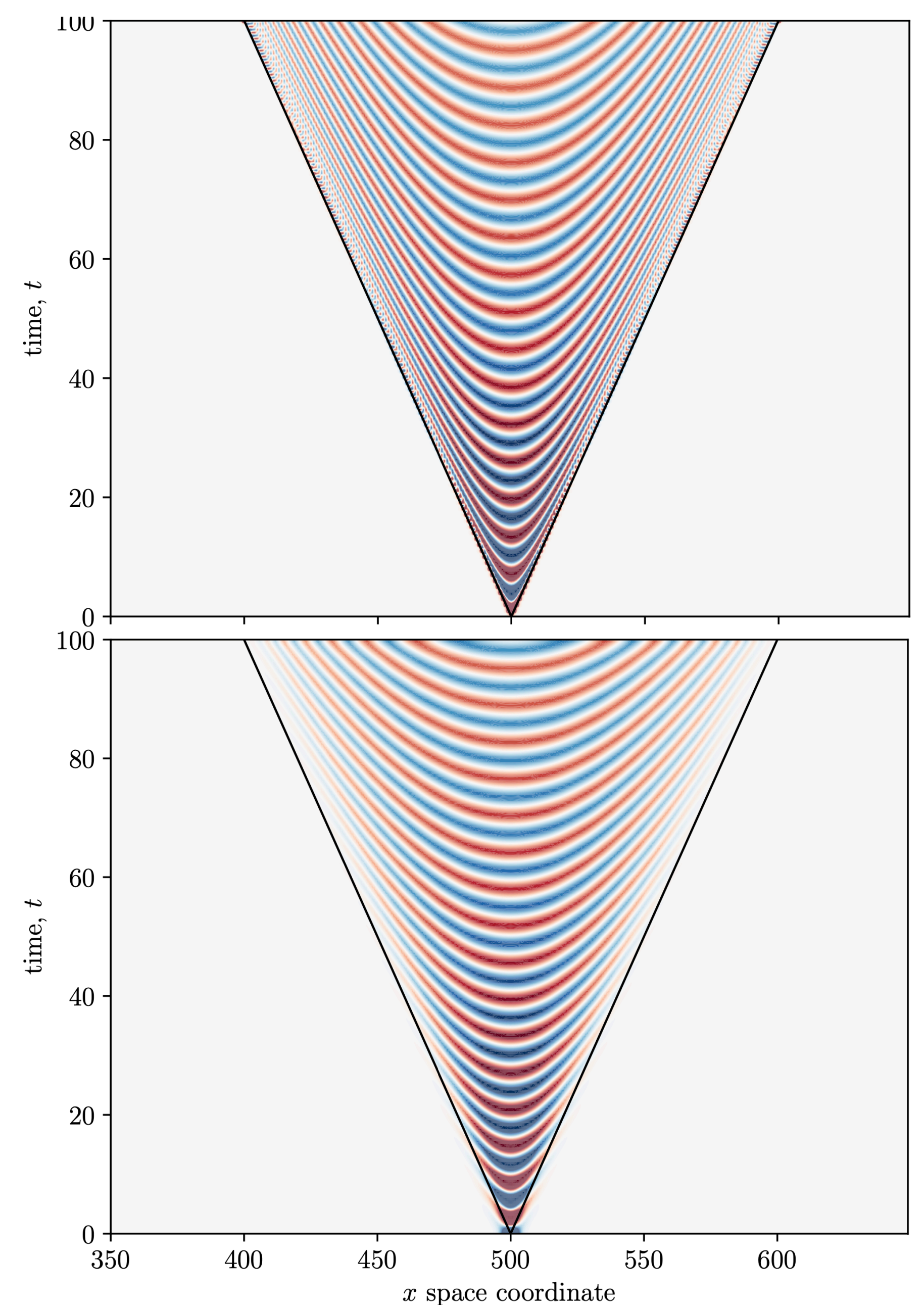
In (1+1)-dimensions the propagator takes the closed-form solution

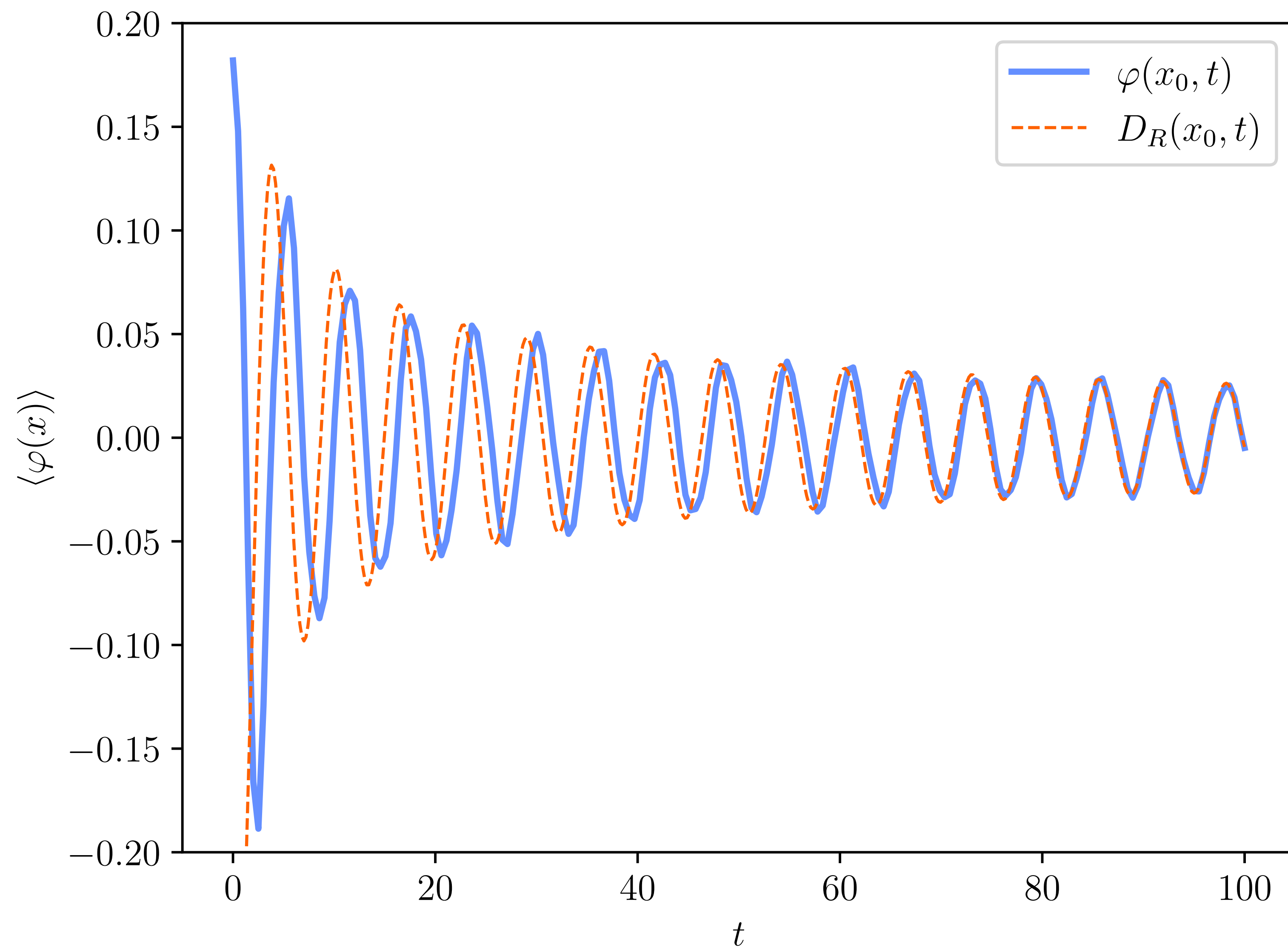
$$D_R(t - t', x - x') = \frac{1}{2} \Theta(t - t') \Theta(\tau^2) J_0(m\tau)$$

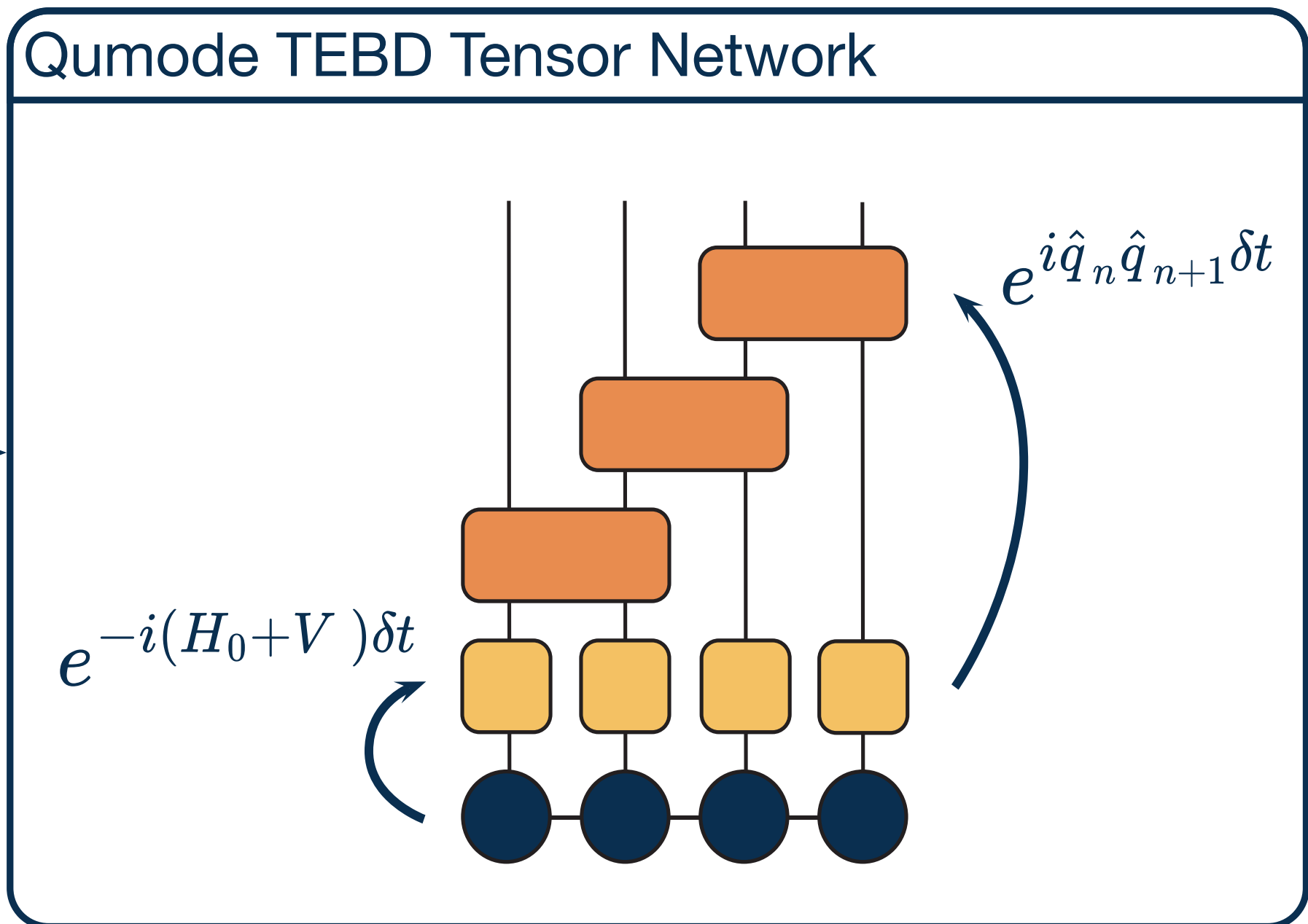
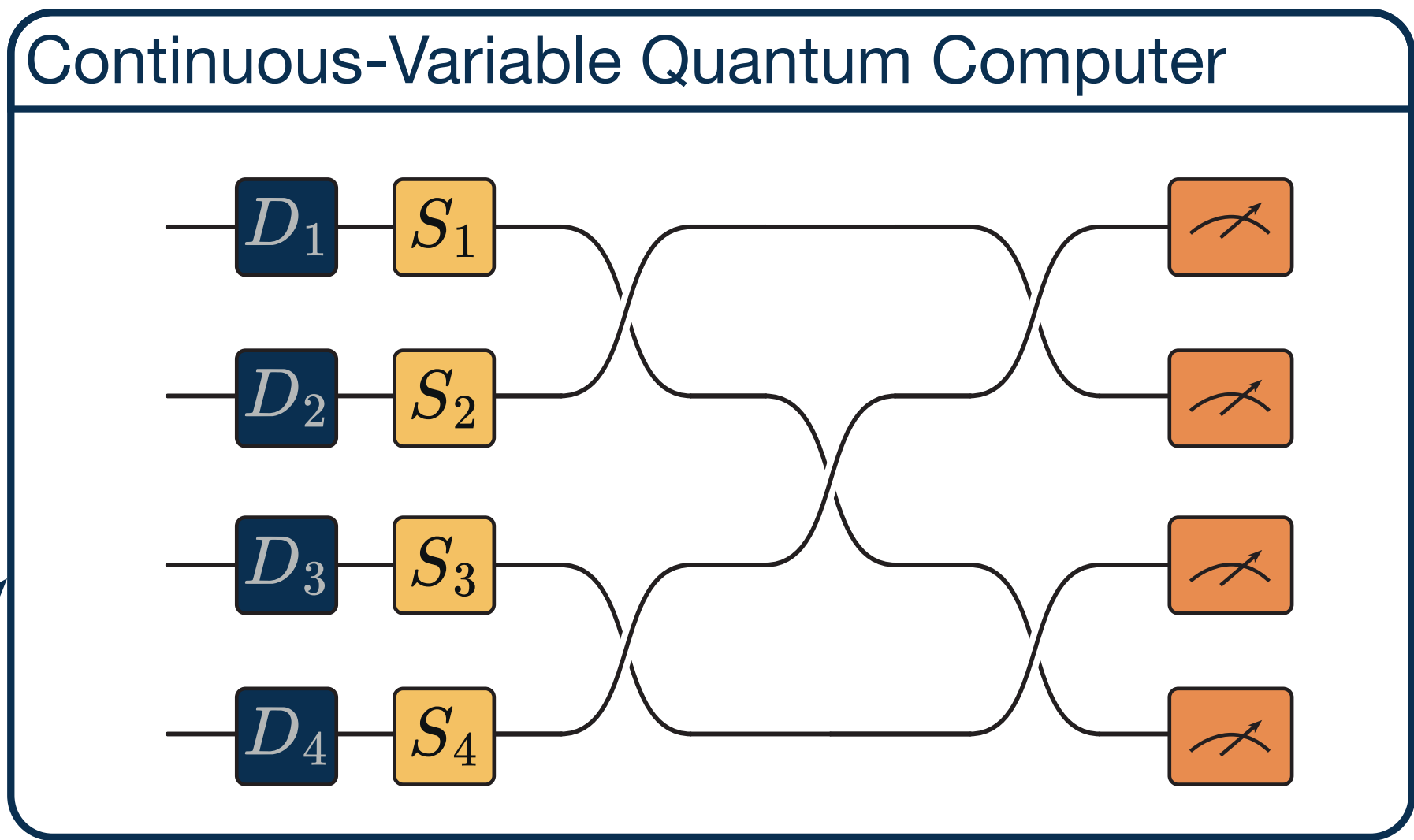
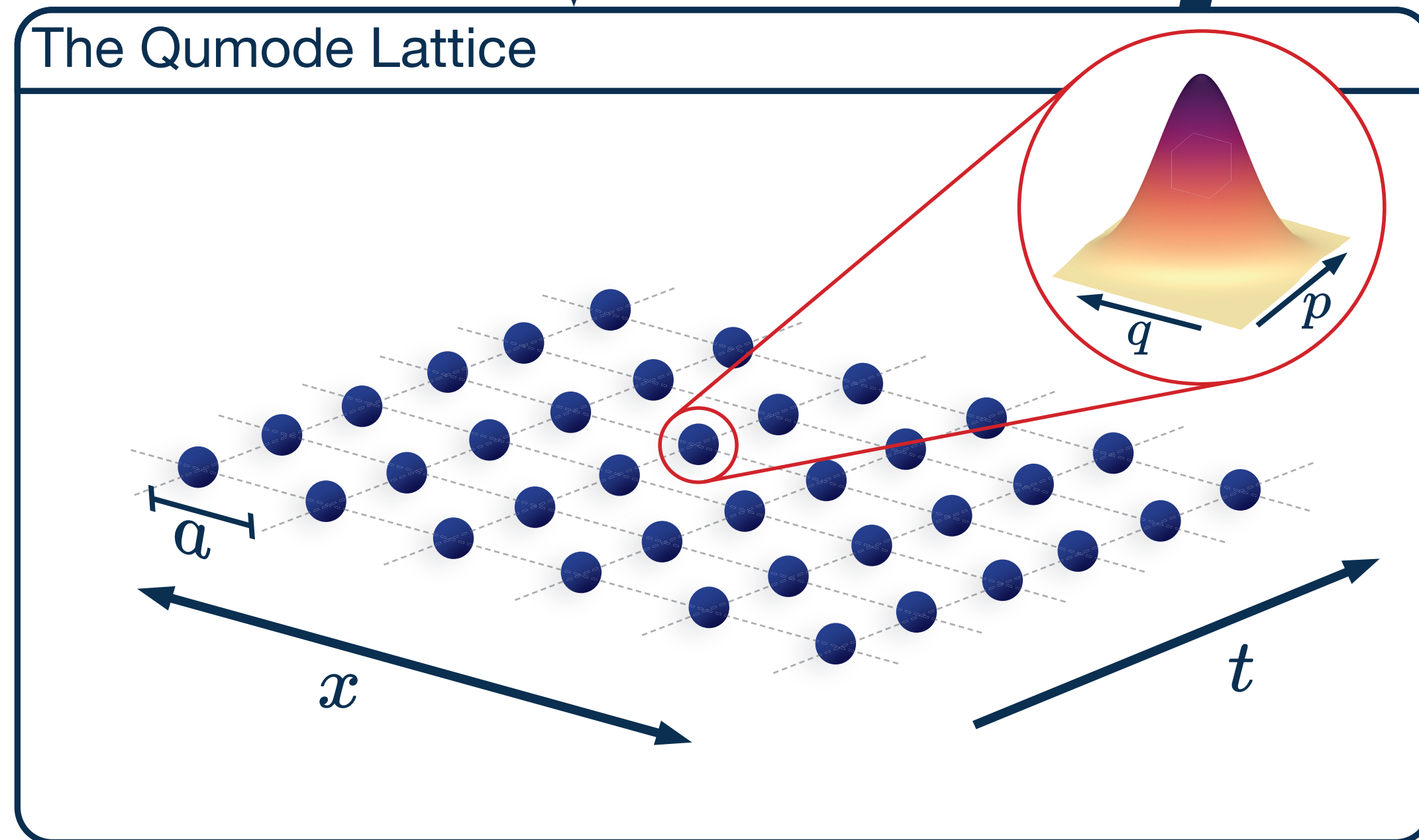
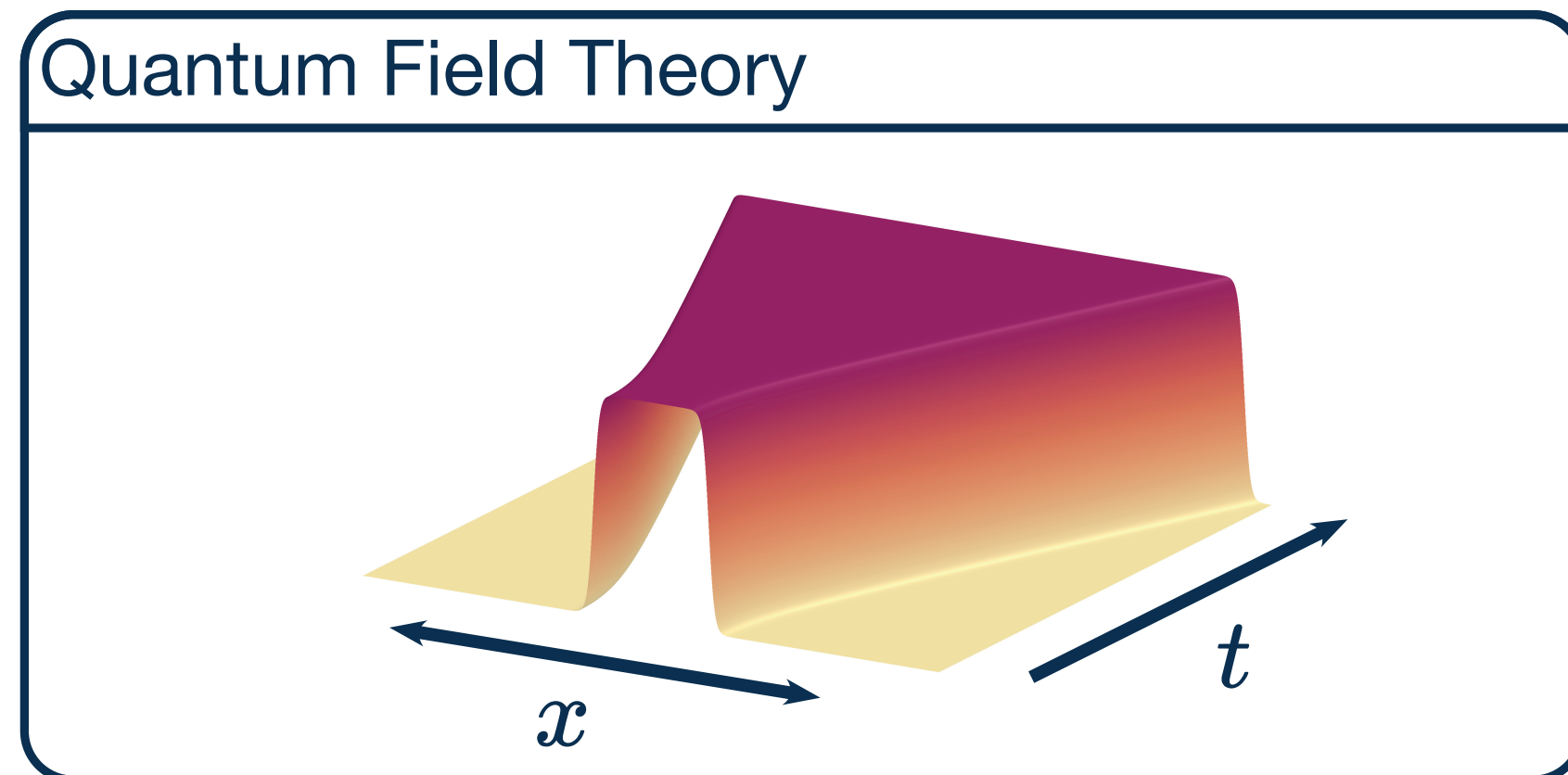
where $\tau^2 = (t - t')^2 - (x - x')^2$

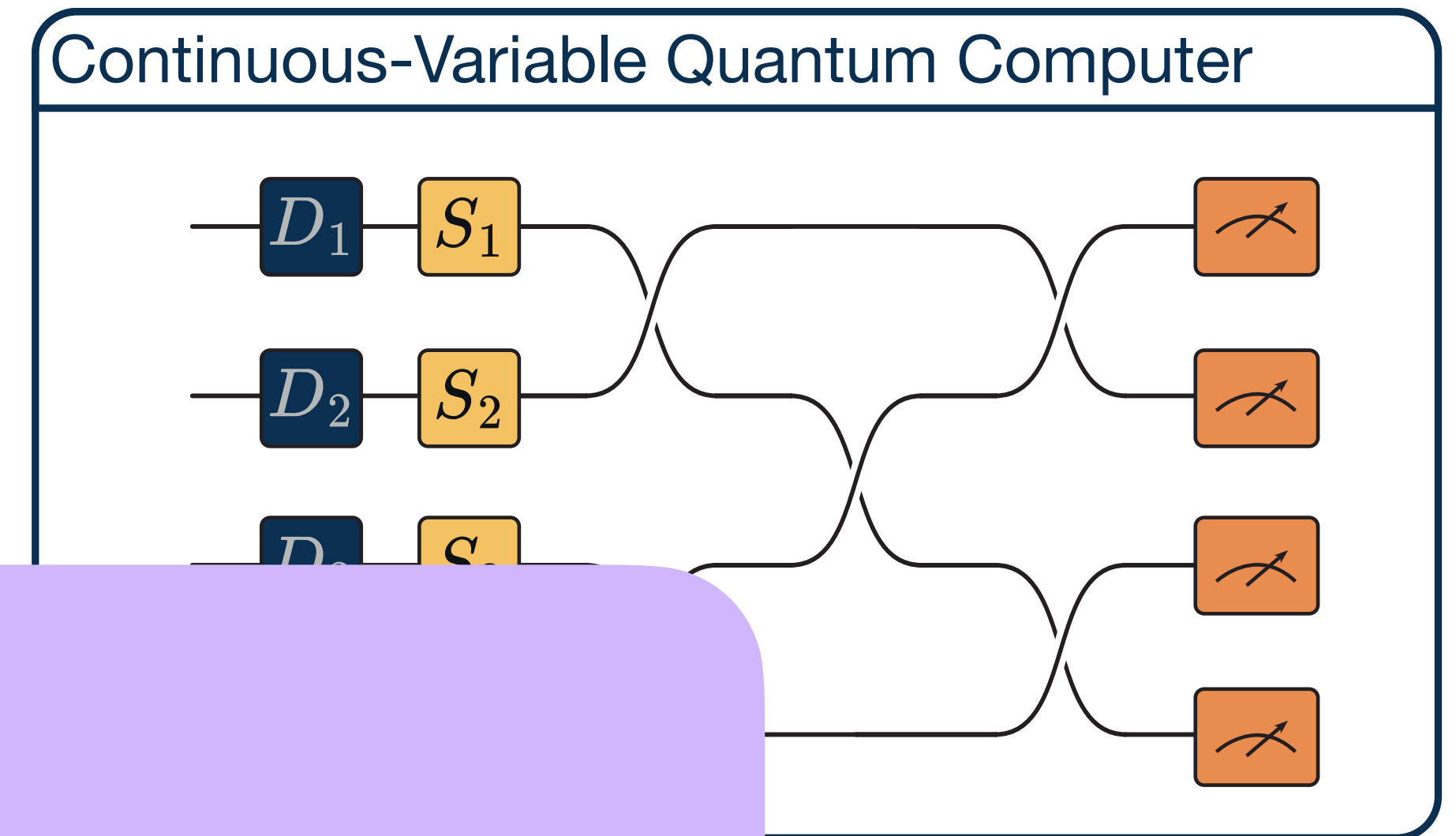
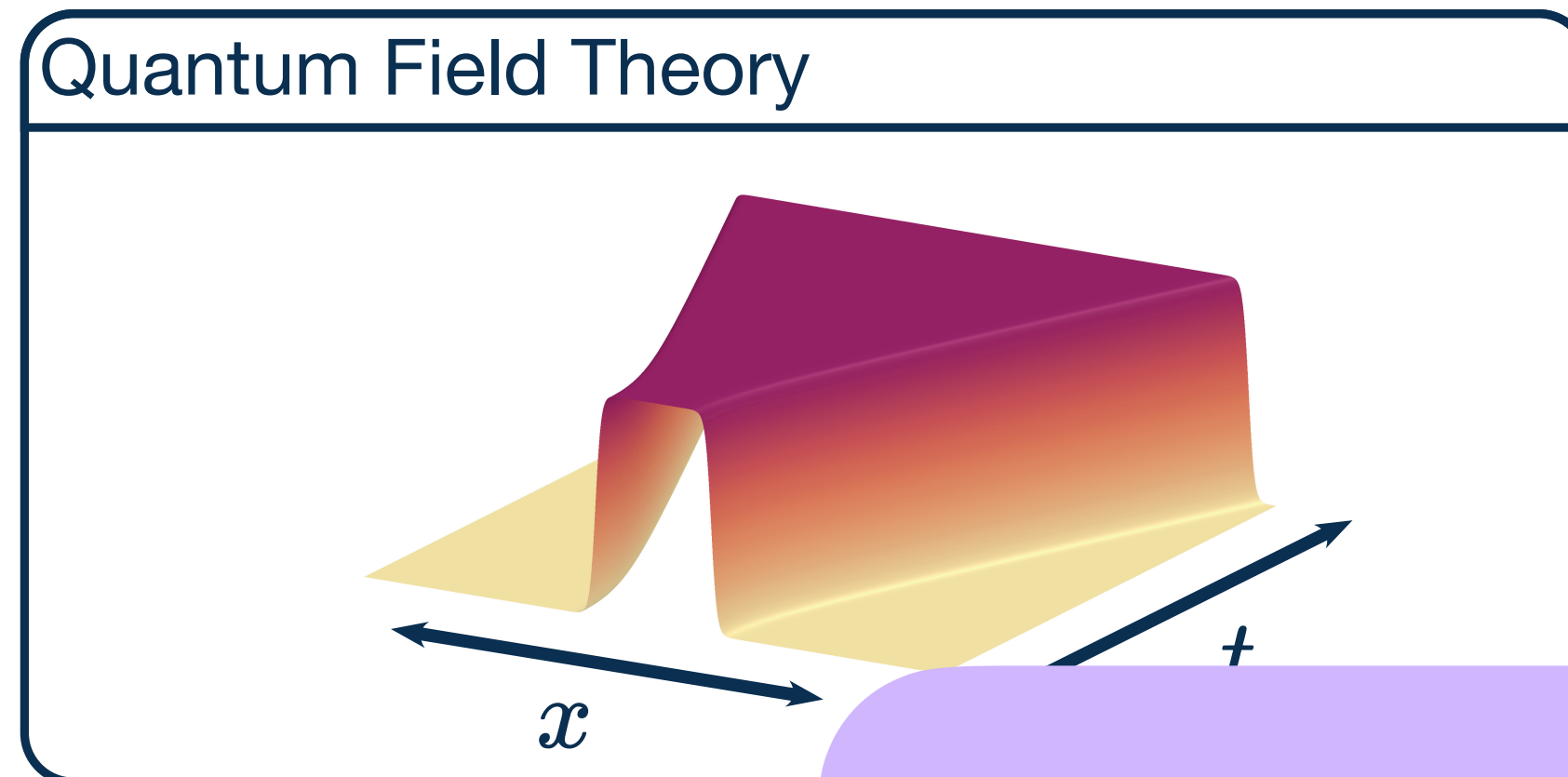
Examining the response of the qumode lattice to a perturbation of the field with a delta function. The solution at time $t > 0$

$$\begin{aligned} \phi(x, t) &= \int dx' D_R(x - x', t) \phi(x', 0) \\ &= D_R(x, t) , \end{aligned}$$

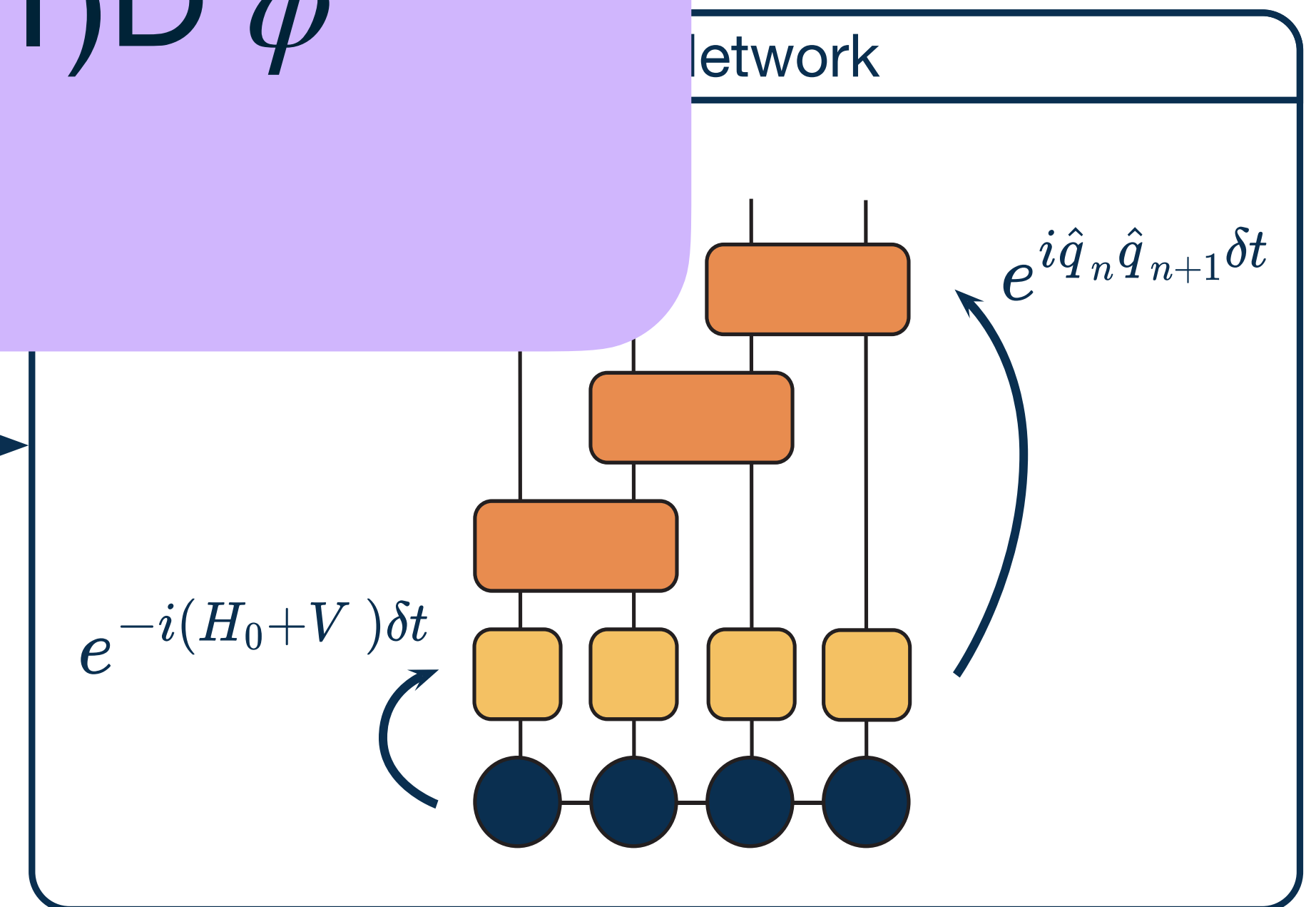
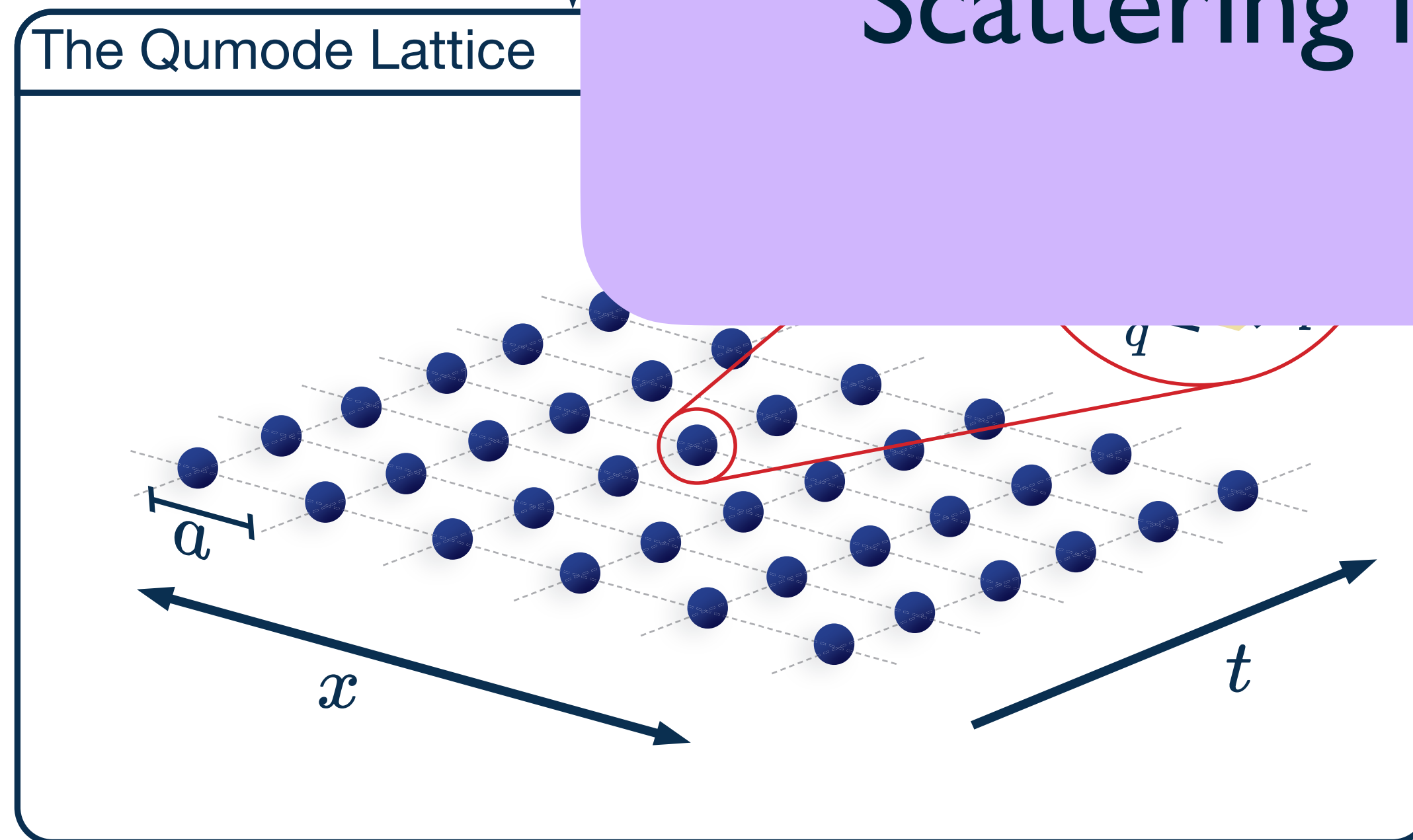








Scattering in $(1+1)\text{D } \phi^4$



Scattering in φ^4 - initial state preparation

(1+1)-dimensional φ^4 scalar field theory:

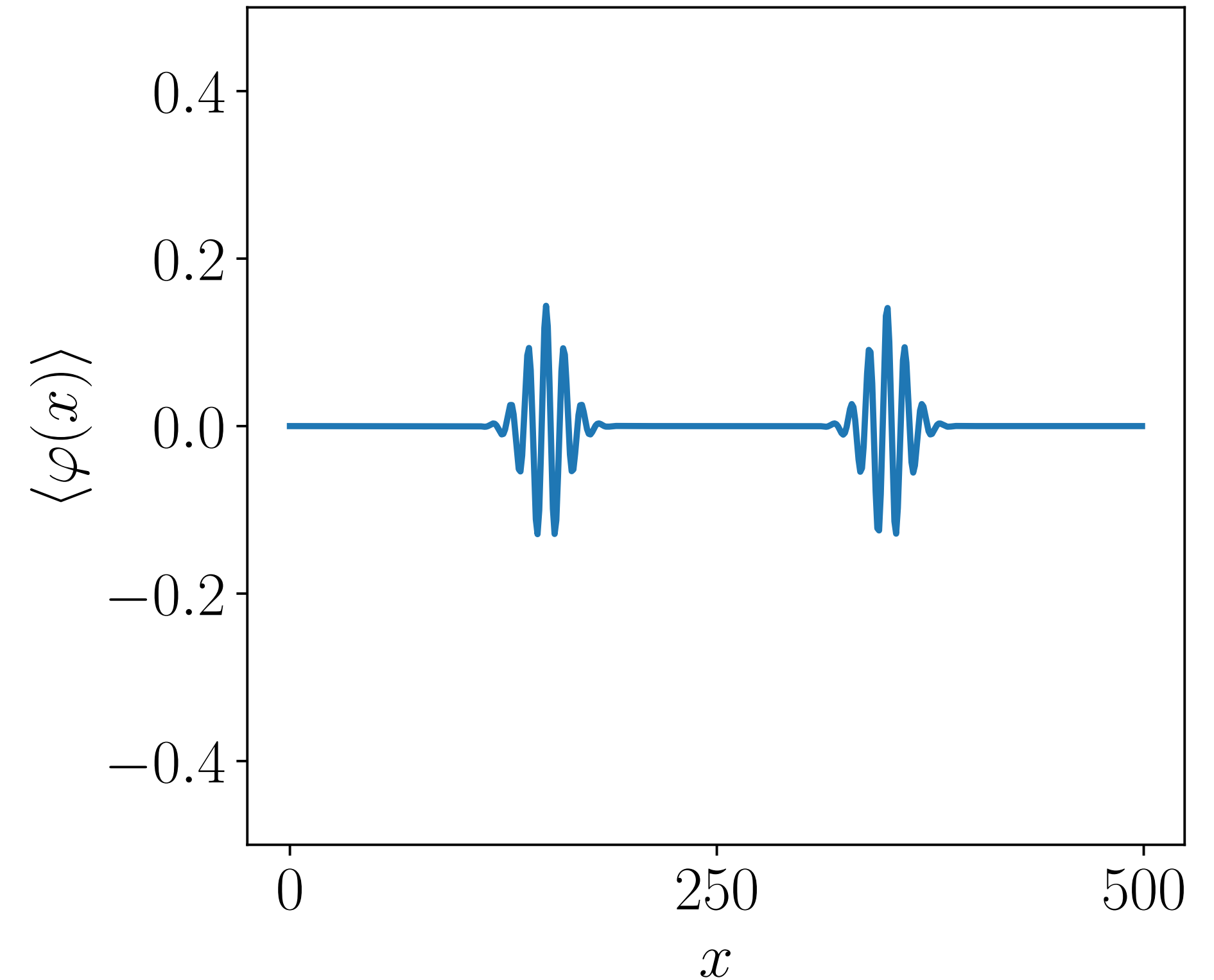
$$\mathcal{V}_I(\varphi) = \lambda\varphi^4$$

The effective local potential therefore becomes:

$$V_I = \frac{1}{a^2}\hat{q}_n^2 + \frac{\lambda}{4!}\hat{q}_n^4$$

Restricting to the non-relativistic approximation the normalised interacting wavepackets have the form:

$$|\psi_n\rangle = \left(A^0 + \sum_{\alpha=0}^{N-1} \frac{1}{\sqrt{\mathcal{N}_\alpha}} \exp\left(-\frac{(k_\alpha - \bar{k})^2}{2\sigma^2}\right) \exp\left(ik_\alpha(x_n - \bar{x})\right) \hat{a}_n^\dagger \right) |0\rangle$$

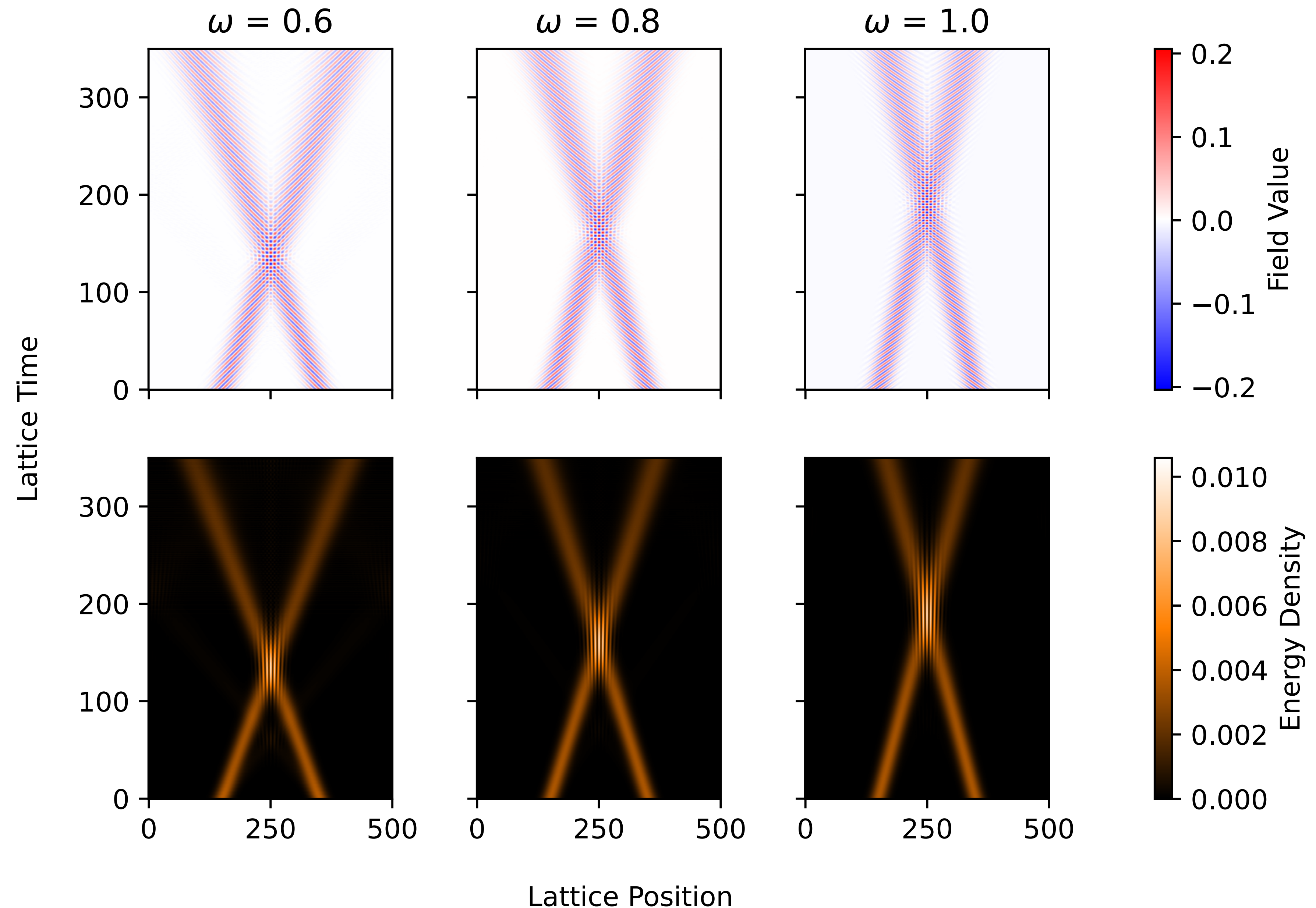


Scattering in φ^4 theory:

Wavepackets initialised with $\sigma = 0.09$ and $k_a = 0.3$, with Trotter step $dt = 0.01$

The lattice is constructed from **500 qumodes** each of which is discretised with $M = 200$

The coupling is fixed at $\lambda = 0.2$ and the mass of the particles is varied

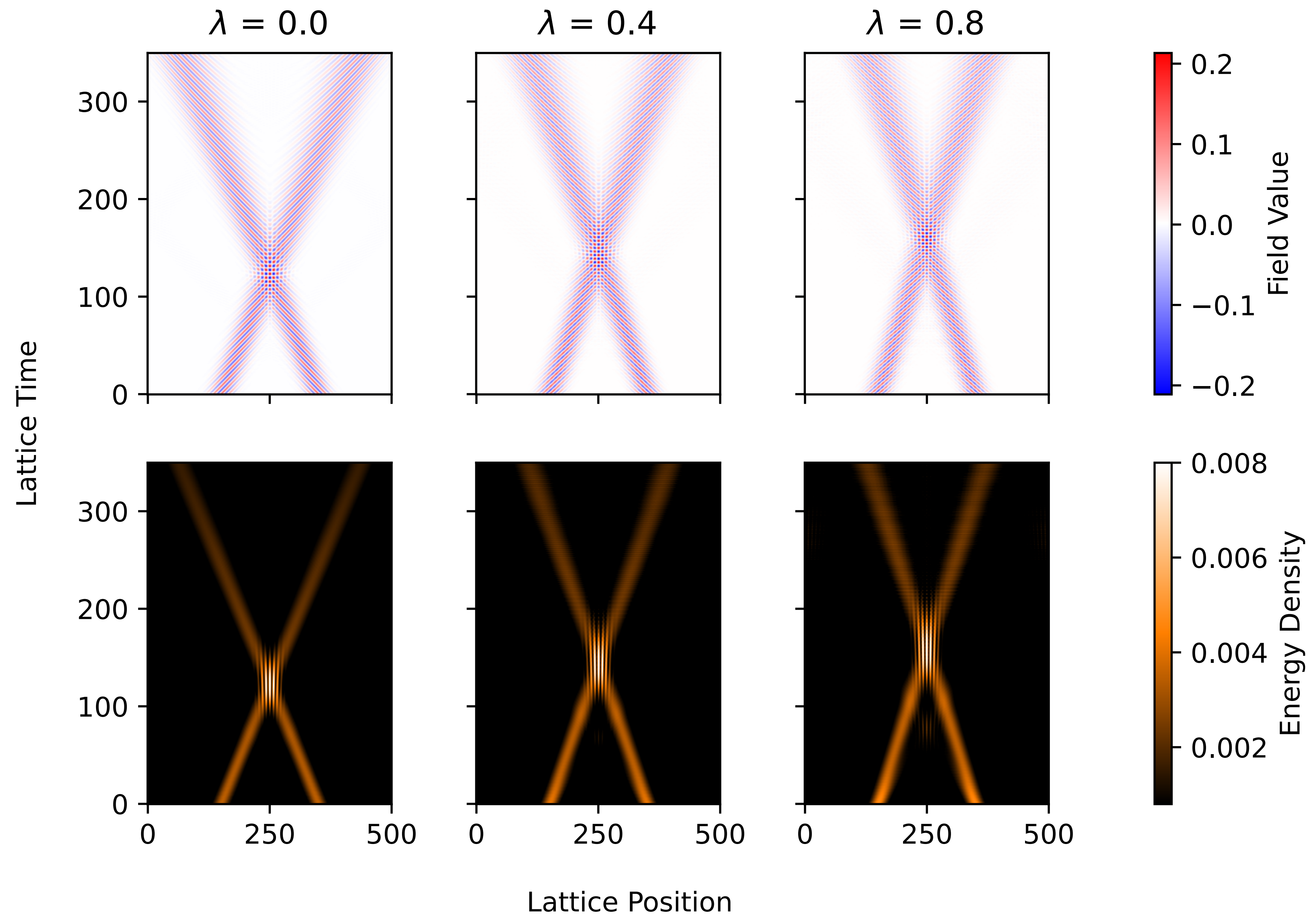


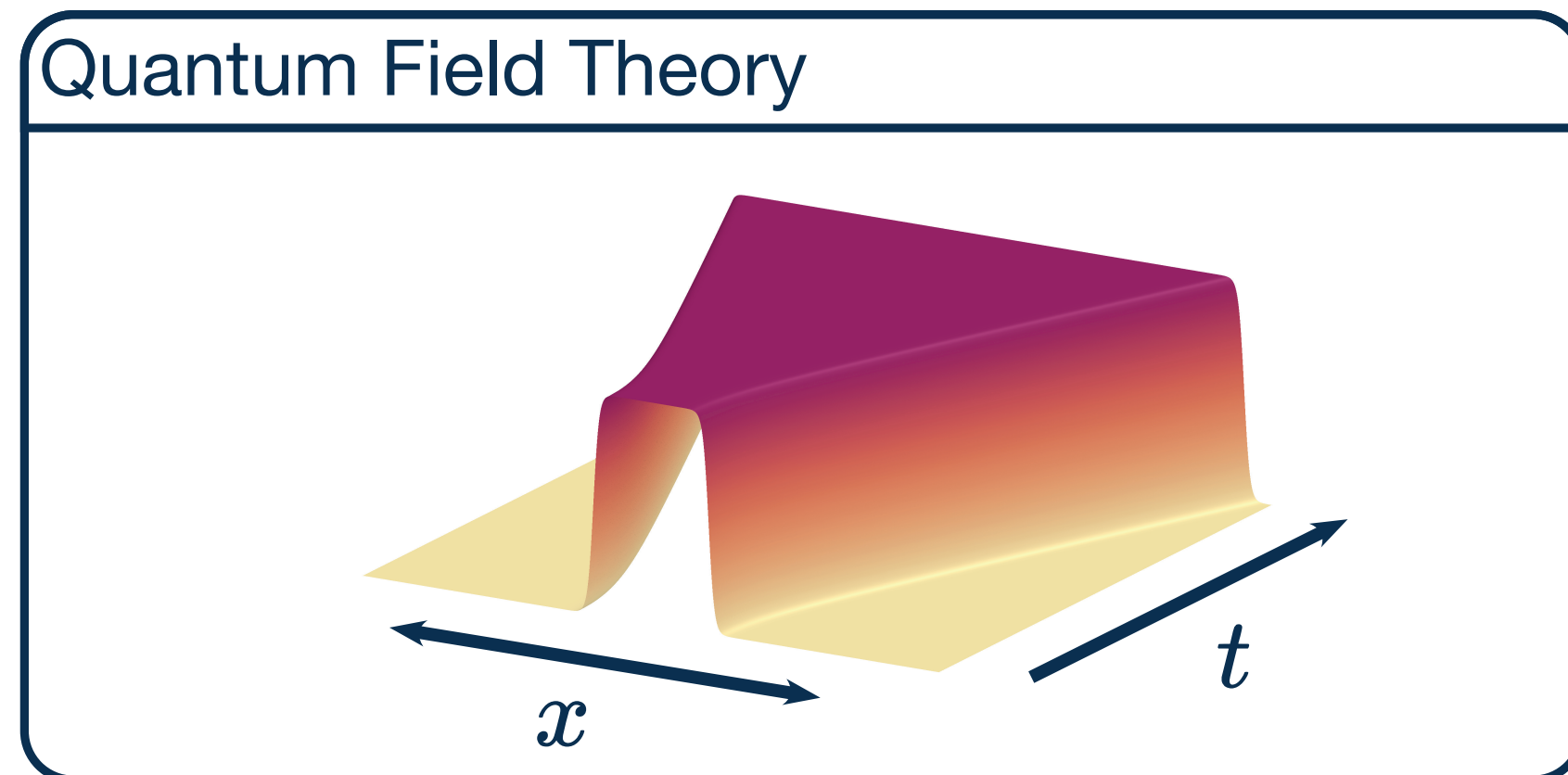
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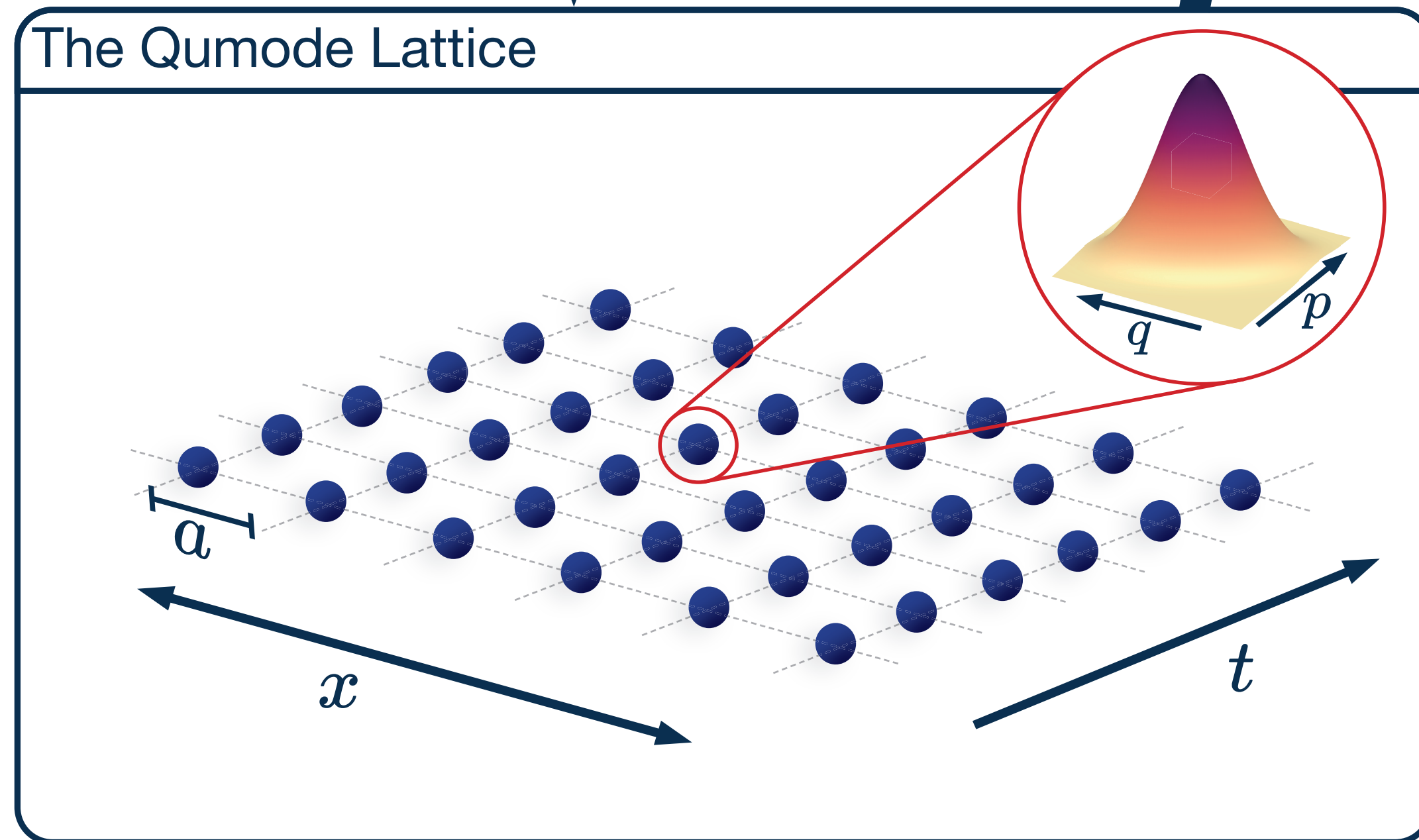
The lattice is constructed from **500 qumodes** each of which is discretised with $M = 200$

The particle mass is fixed at $\omega = 0.6$ and the coupling is varied

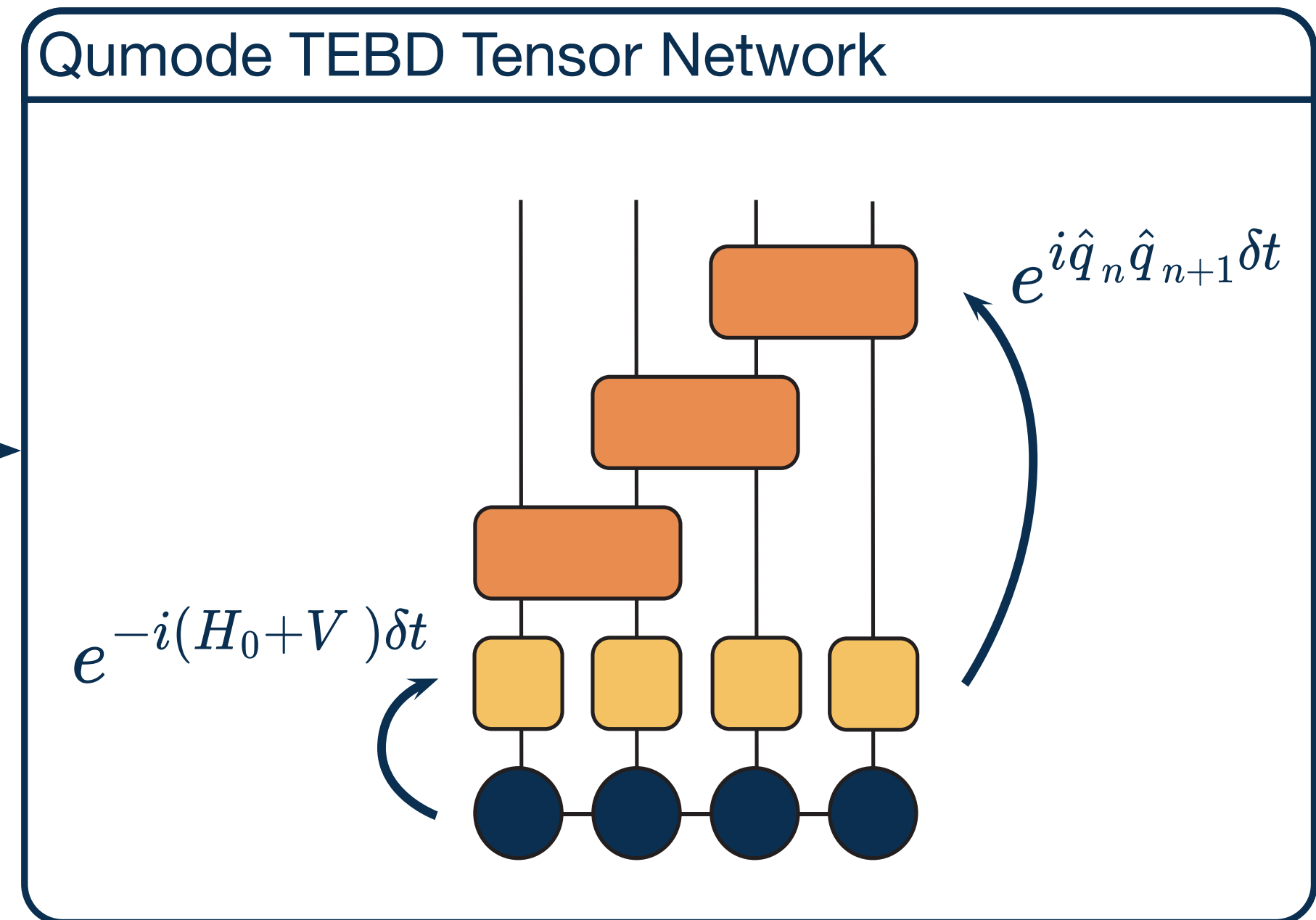
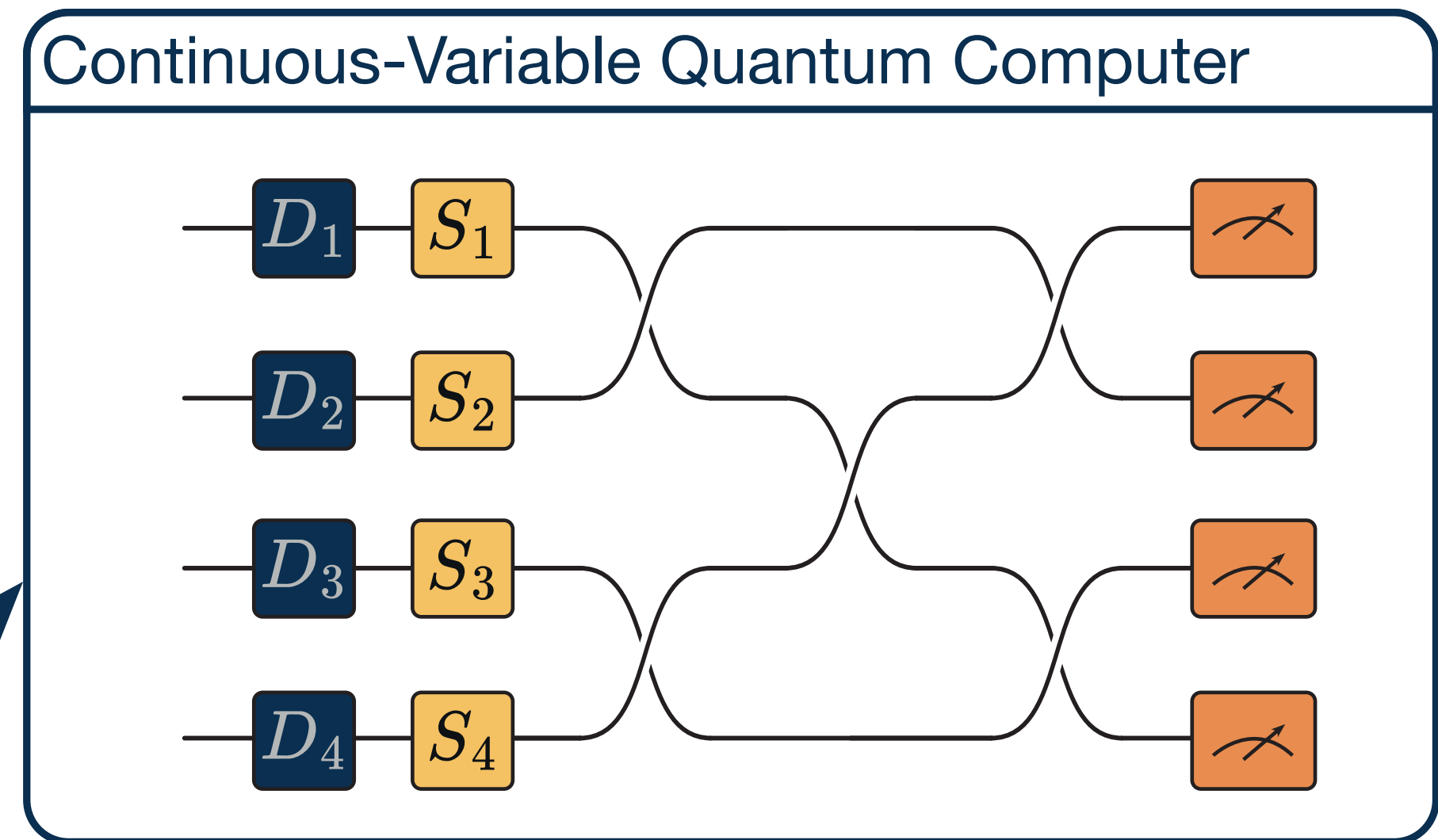


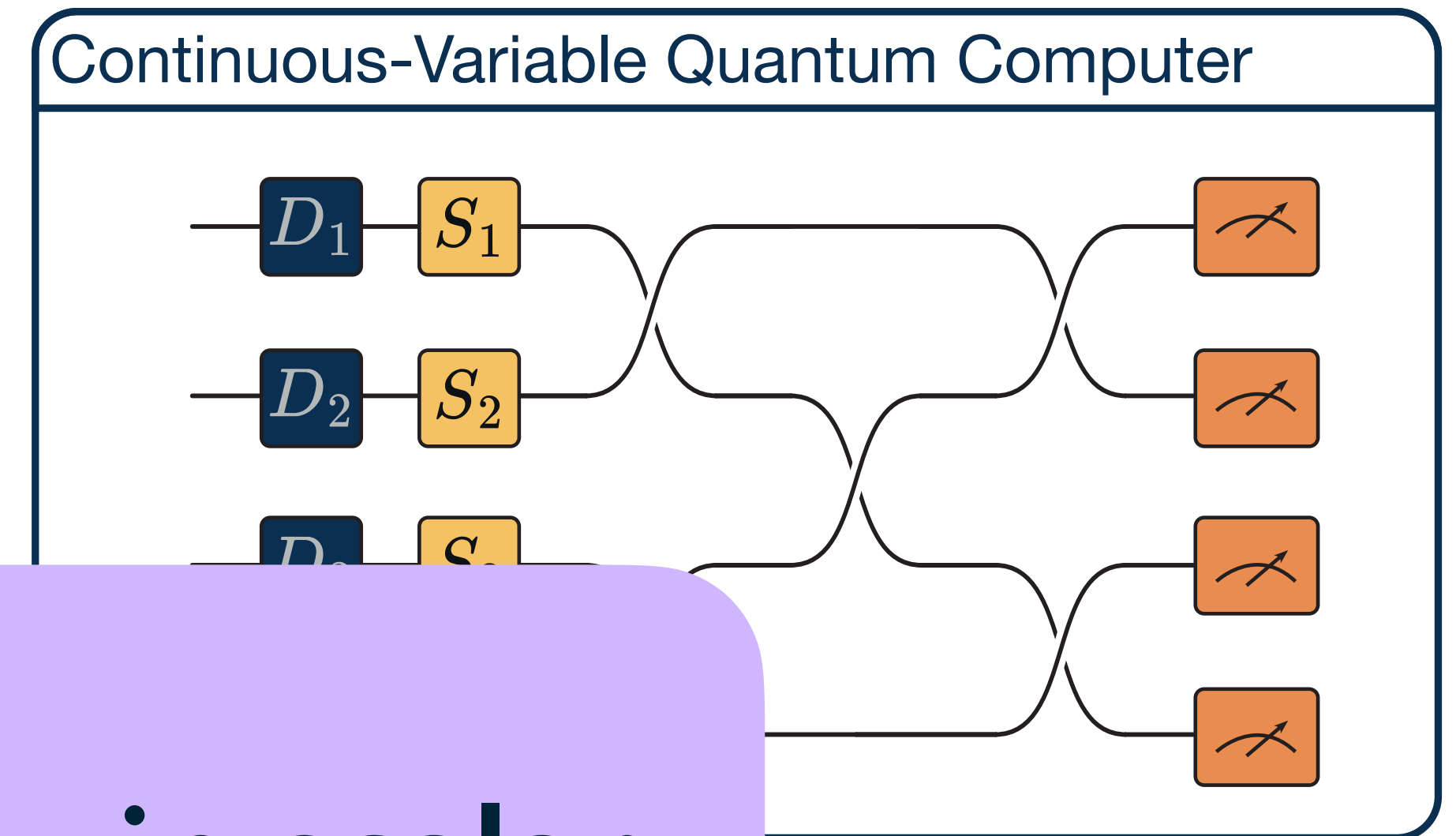
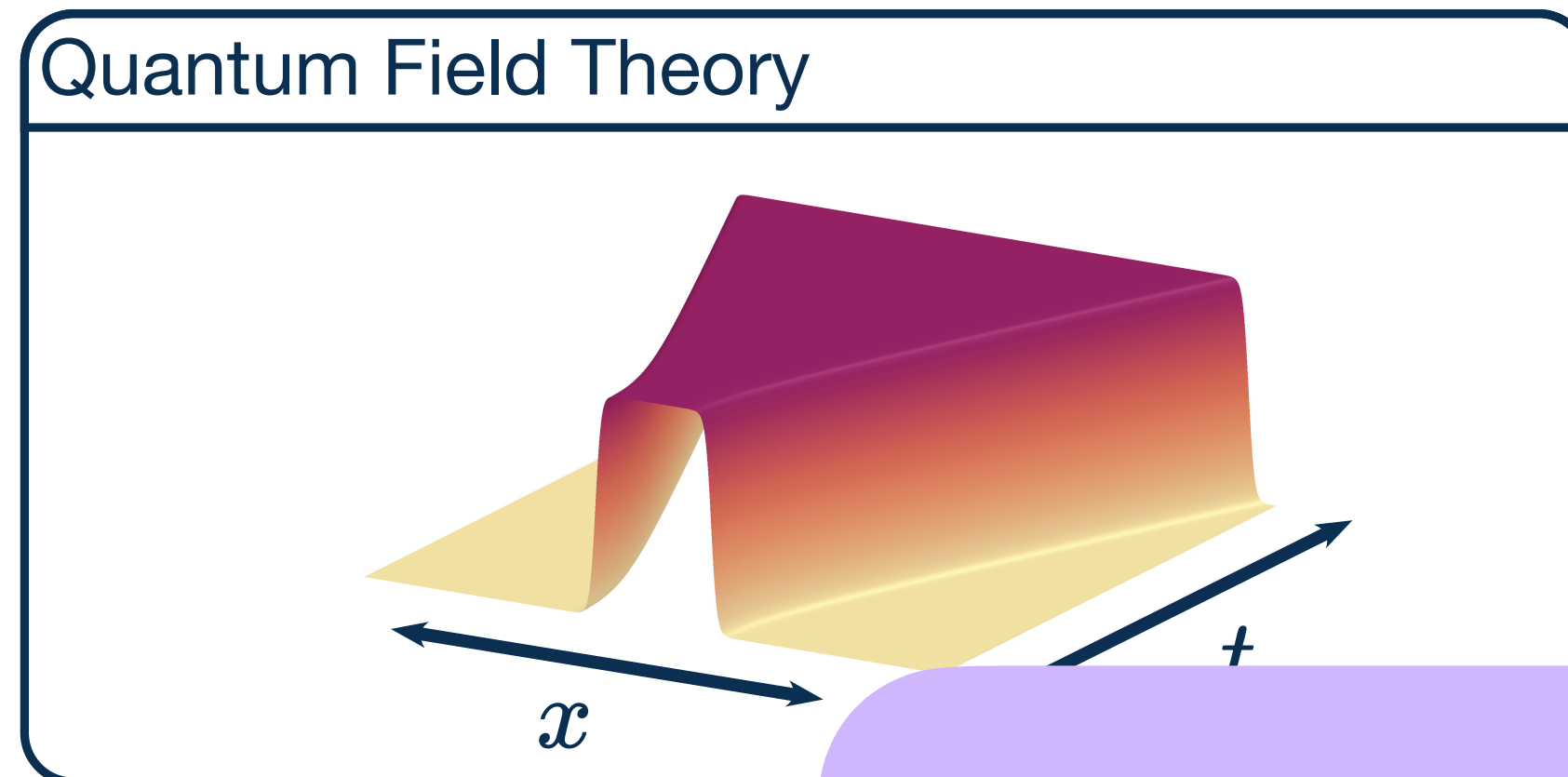


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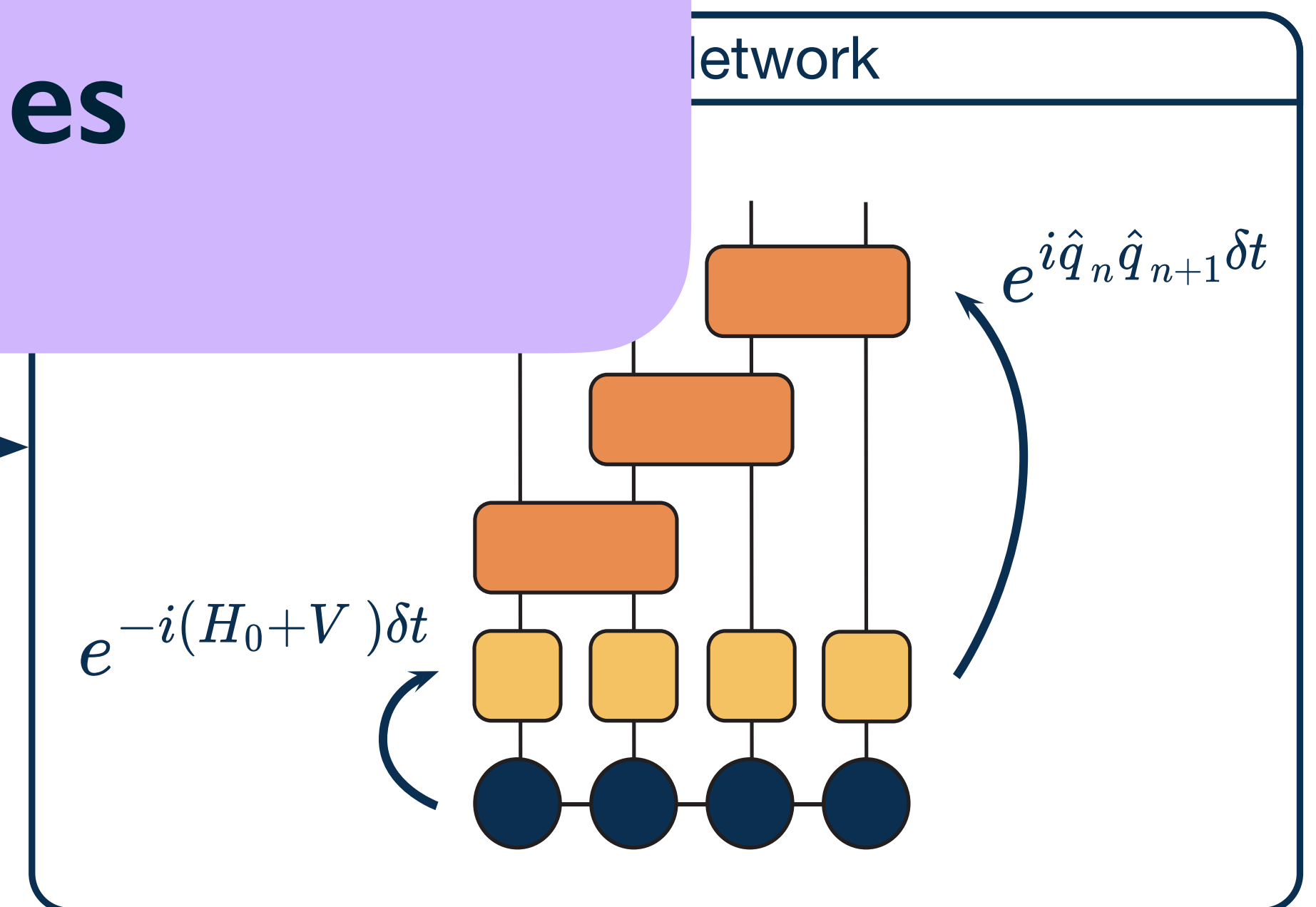
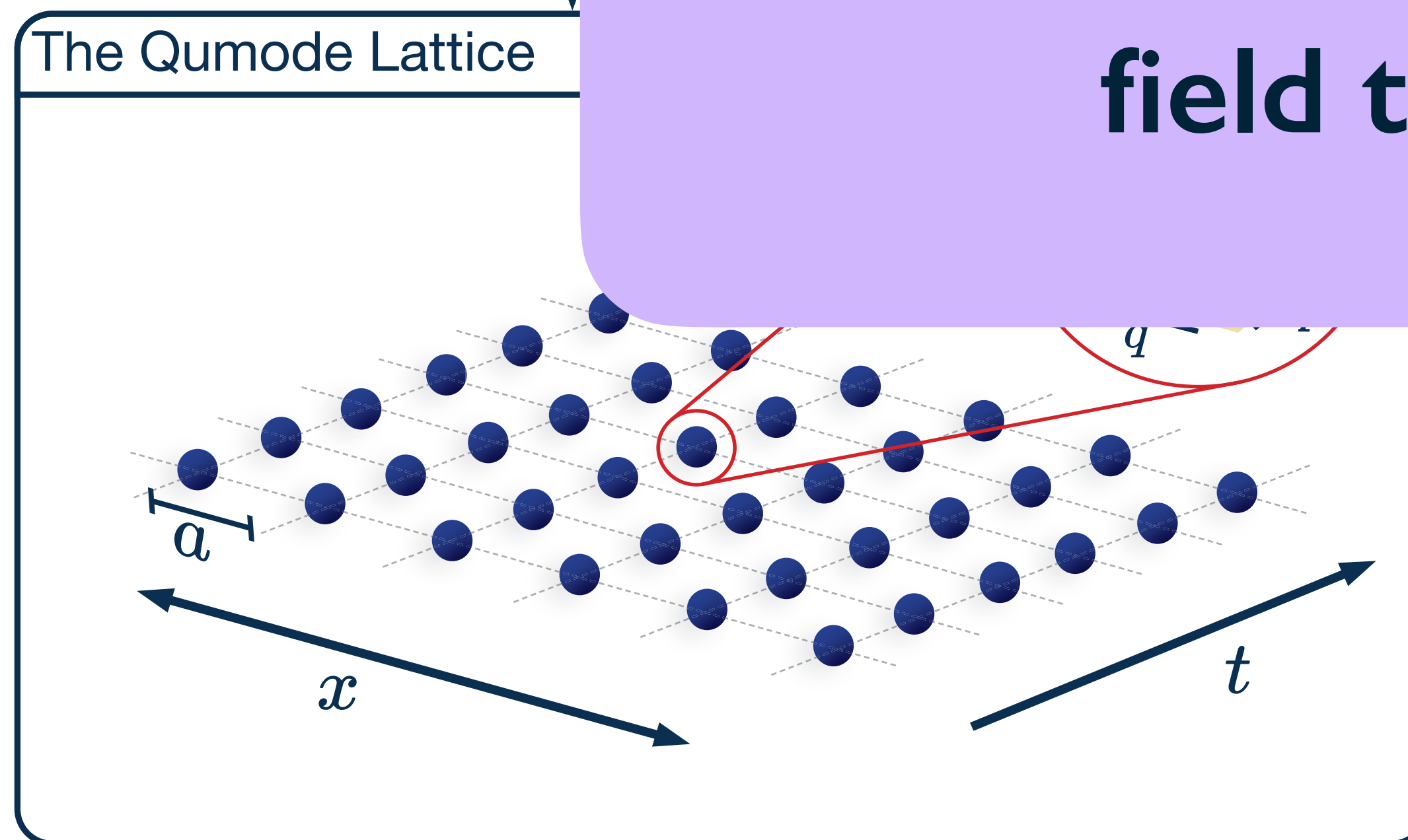


2





False-vacuum decay in scalar field theories



Why false vacuum decay?

A central process in cosmology and statistical physics:

Metastable vacuum decay

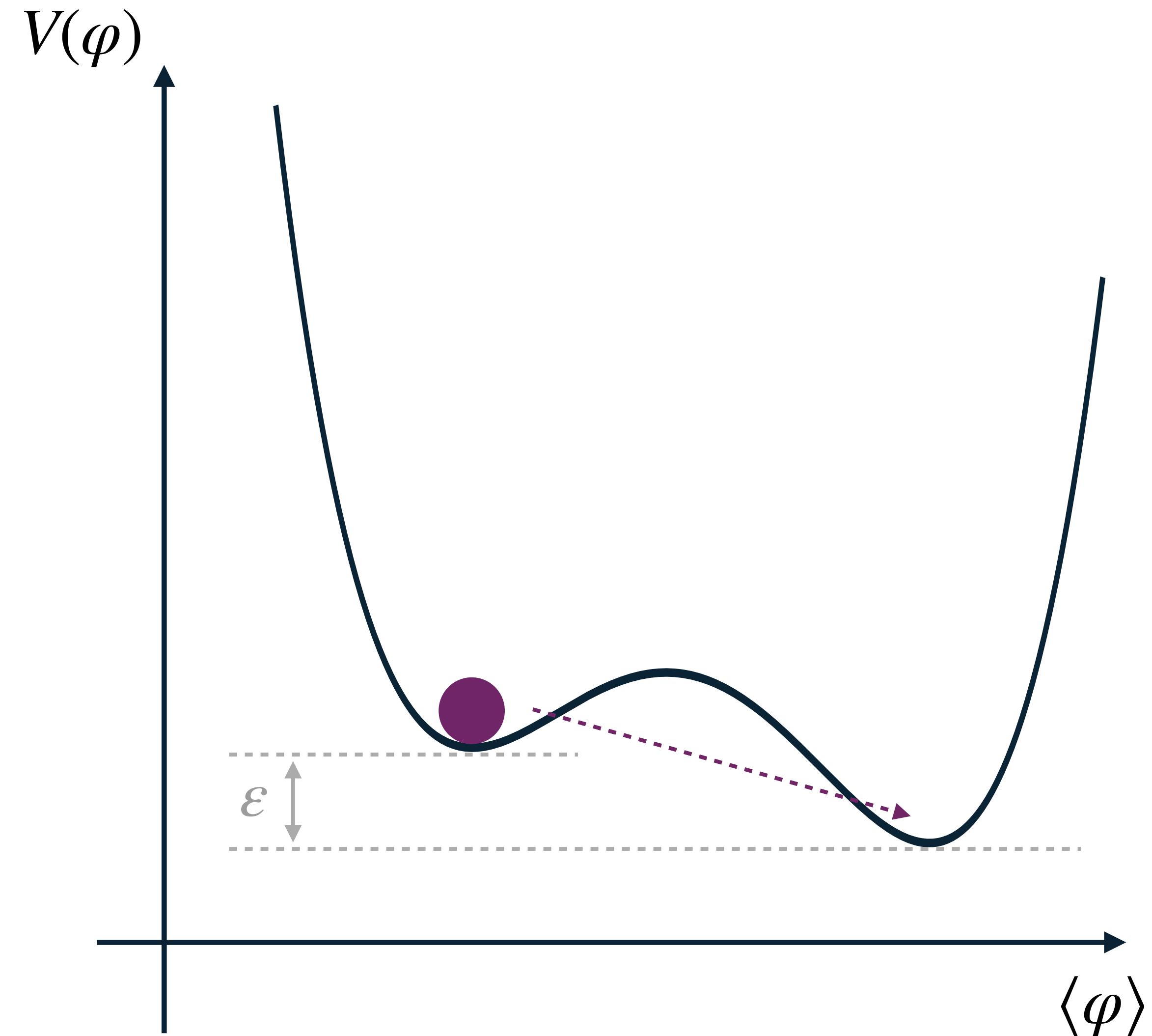
A system can sit in a false minimum before bubble nucleation drives a transition to the true vacuum

First-order phase transitions

Electroweak baryogenesis, gravitational-wave backgrounds, condensed-matter analogues

Real-time, far-from-equilibrium

We need the time evolution, not just static or thermal observables. Euclidean methods only get us so far



Preparing the metastable vacuum

Preparing the metastable vacuum is crucial to extracting true tunnelling observables:

Adiabatic state preparation

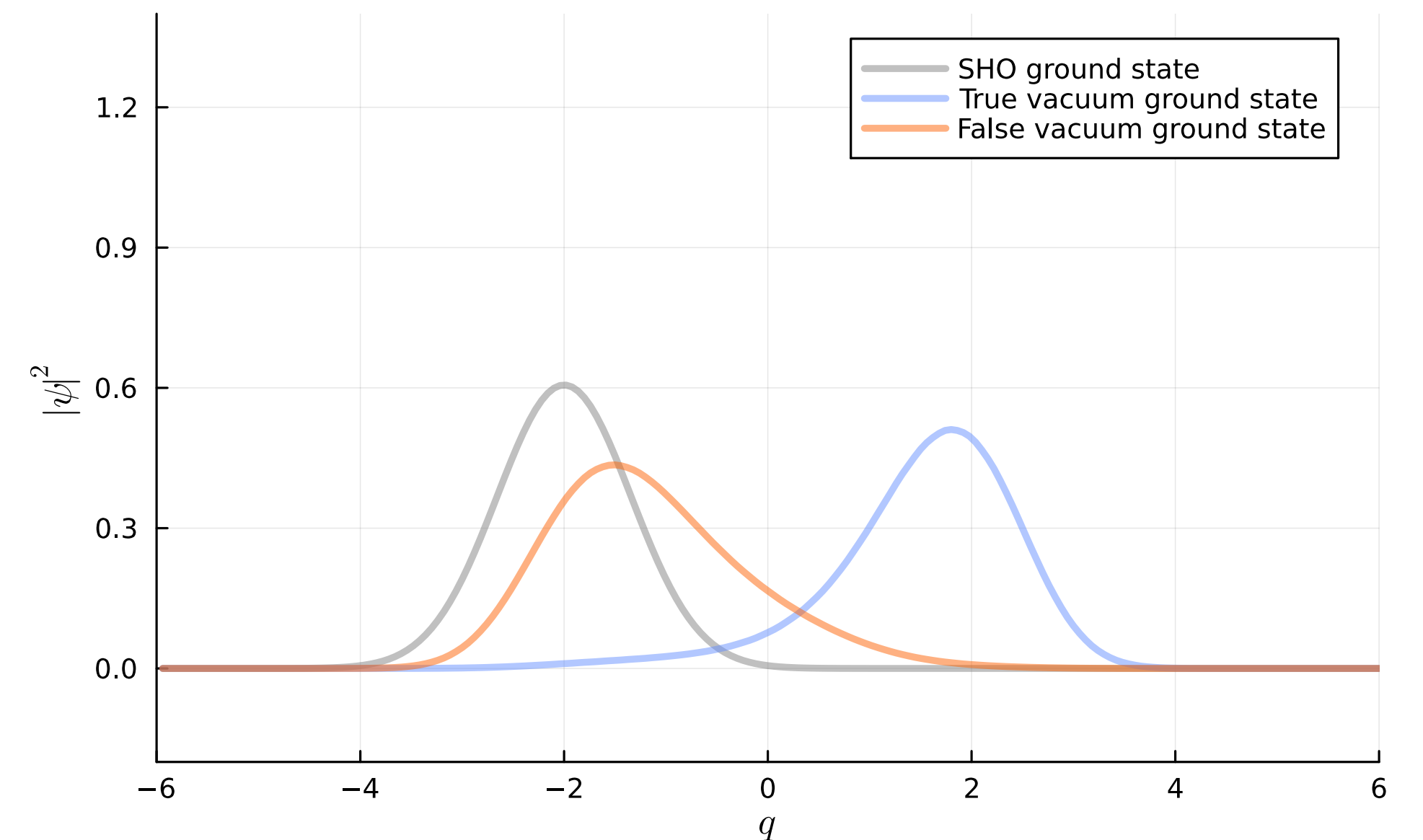
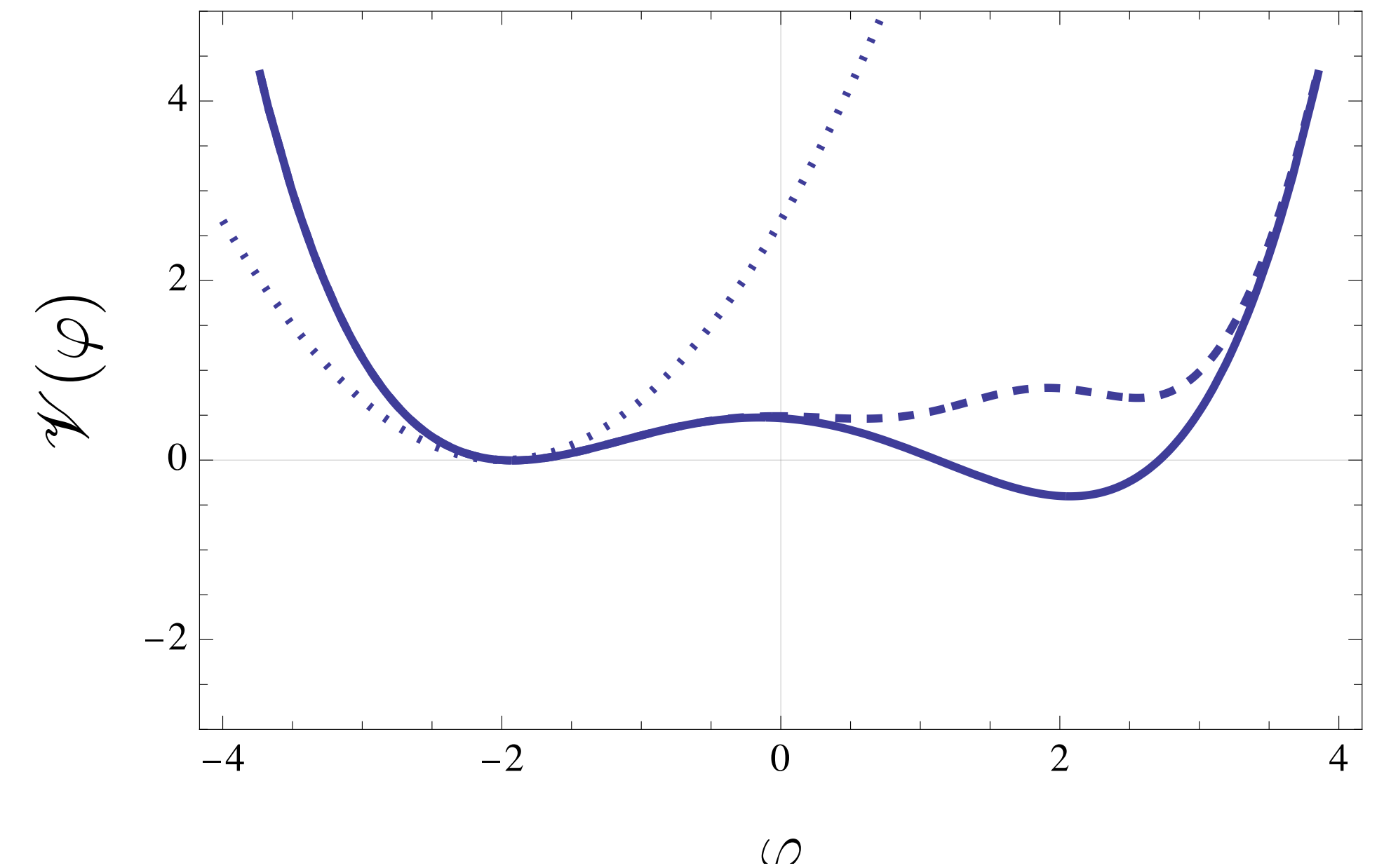
Use adiabatic principle to prepare initial state by slowly turning on interacting potential, whilst lifting true vacuum

Initialise MPS in false-vacuum

Start each qumode in the false-vacuum state

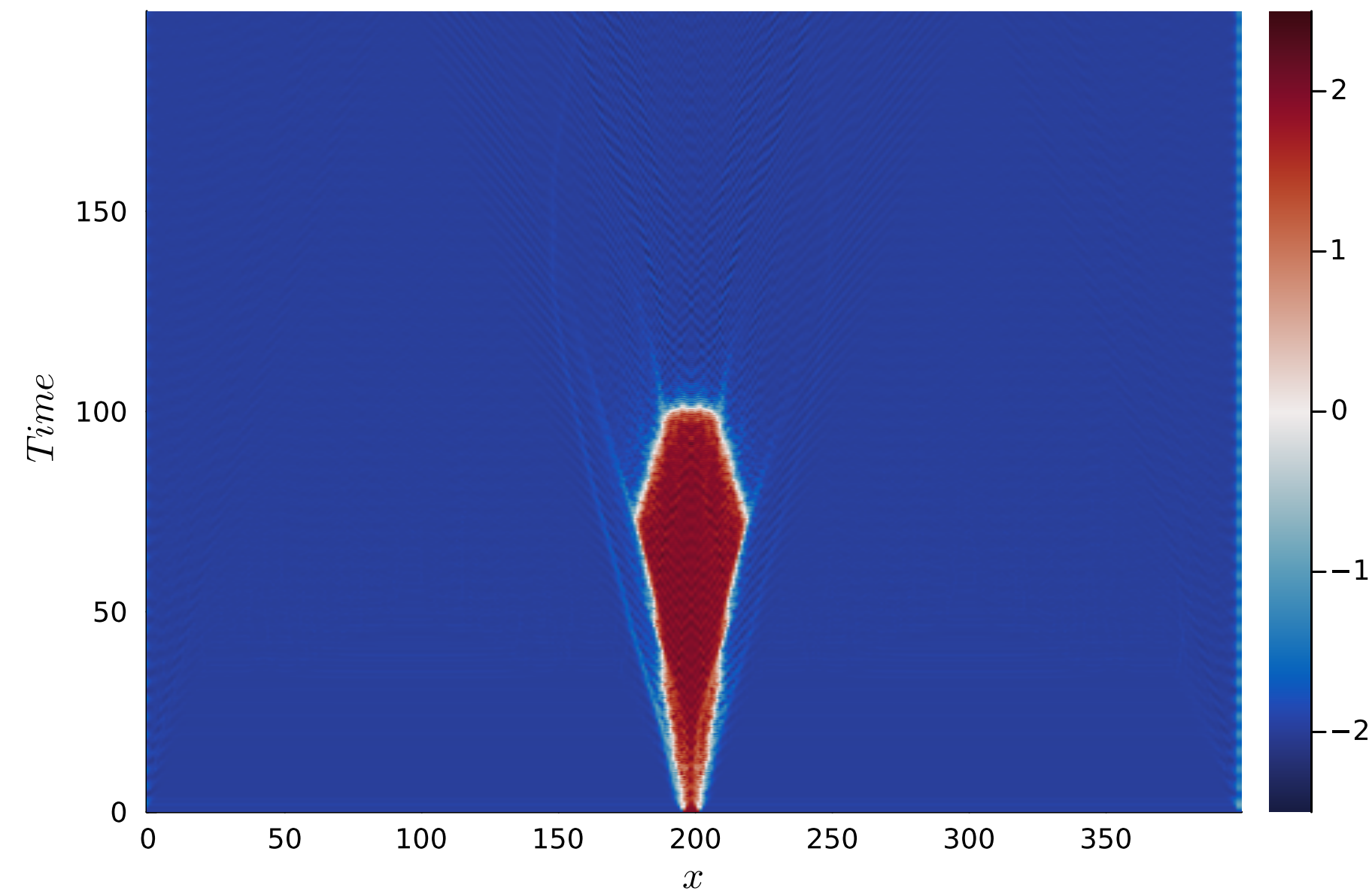
Seed bubble

To initiate the bubble nucleation, one must seed a bubble of true vacuum in a controlled way



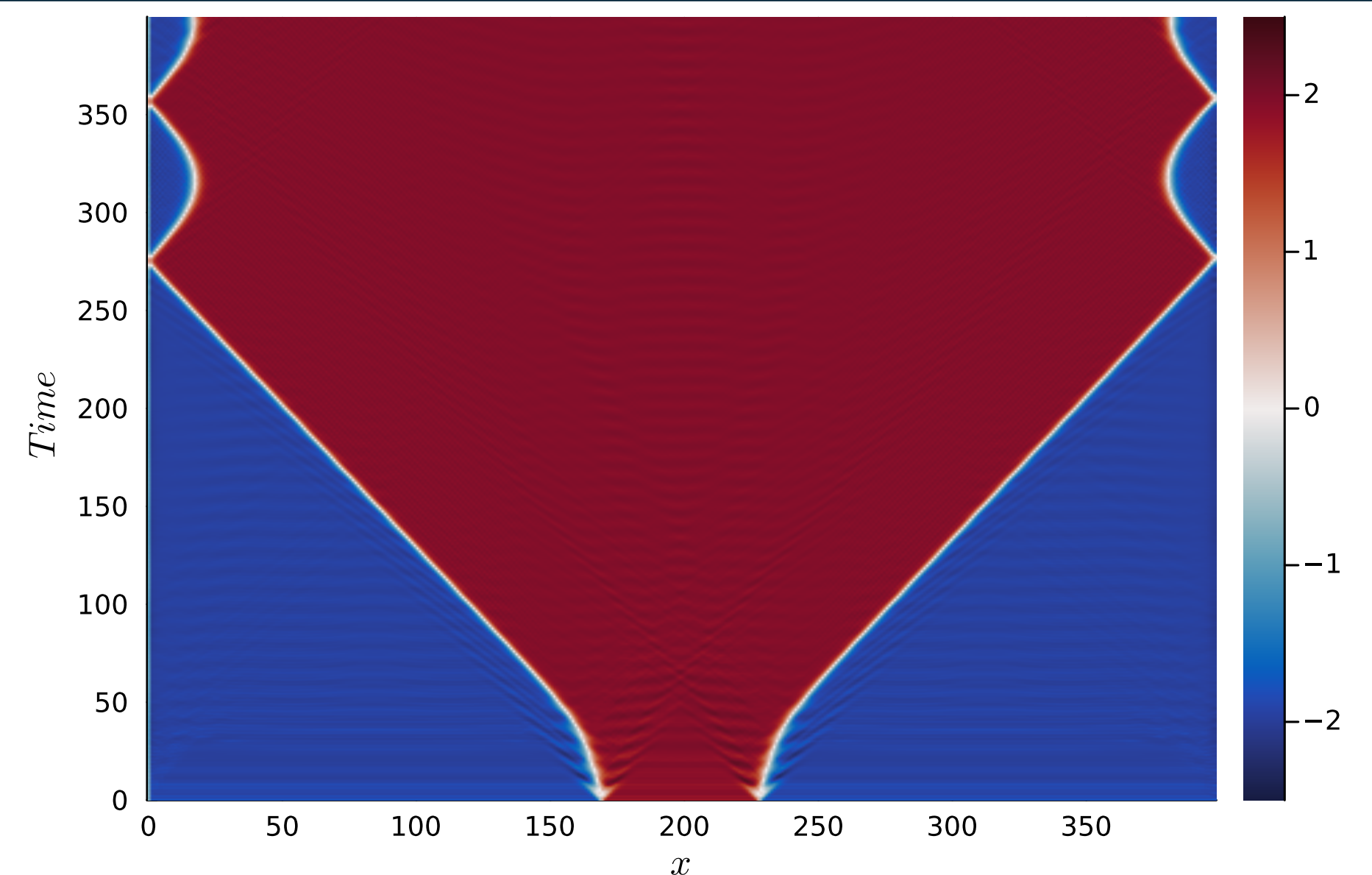
Super-critical and sub-critical bubbles

Sub-critical bubble



Sub-critical bubble has a radius $R < R_c$ - too small for the volume-energy gain to overcome the surface-tension cost

Qumode Tensor Network Vacuum



Super-critical bubble has a radius $R > R_c$

$$m_{\text{wall}} \approx \frac{2\varepsilon d_{\text{bounce}}}{v_{\text{term}}^2} \approx 7.7$$

Order of magnitude of analytical calculations: $m \approx 2.74$

Estimating the vacuum for phase transitions - classical fluctuations

Currently, we model the ground state as a “perfect” product state

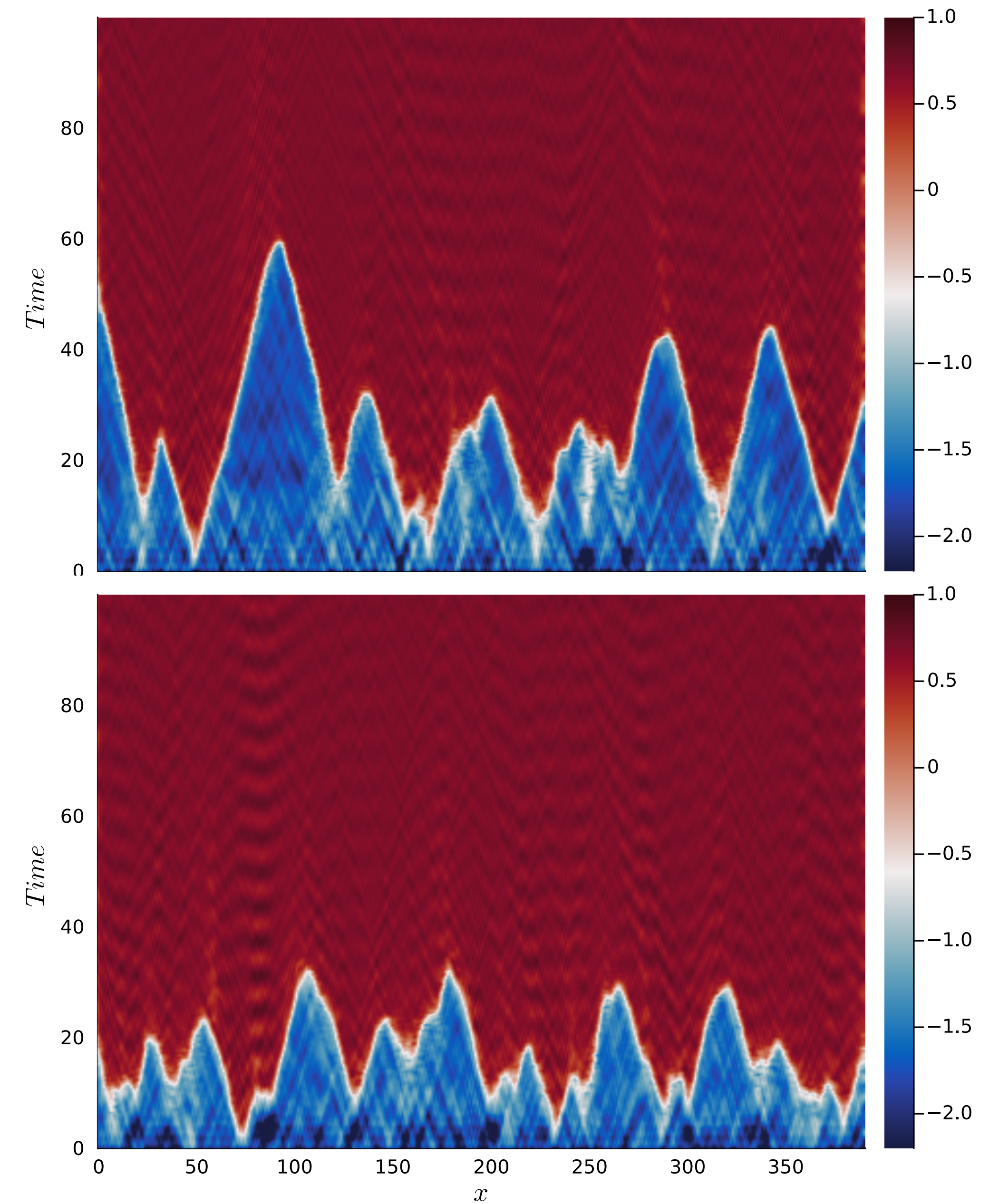
Classical seeding

We “dress” the qumode ground state with long-range fluctuations by classically sampling covariance matrices:

$$\Delta q_n = \sum_{\alpha=0}^{N-1} \frac{1}{\sqrt{2N}} \sqrt{\left(\frac{1}{\omega} - \frac{1}{\omega_\alpha}\right)} \cos(2\pi n\alpha/N) \eta_{n,\alpha} ,$$

$$\Delta p_n = \sum_{\alpha=0}^{N-1} \sqrt{\frac{\omega_\alpha - \omega}{2N}} \cos(2\pi n\alpha/N) \eta'_{n,\alpha} ,$$

Vary the time-scale of the phase transition by altering ε



Seeding bubble nucleation with the negative mode

We want to stimulate a tunnelling event without inserting a large classical bubble by hand

Bounce fluctuation spectrum

Linearise around the Euclidean bounce configuration. The fluctuation operator has one unstable direction:

$$\left[-\partial_x^2 + V''(\phi_{\text{bounce}}) \right] \eta_-(x) = -\kappa^2 \eta_-(x).$$

This is the **negative mode**

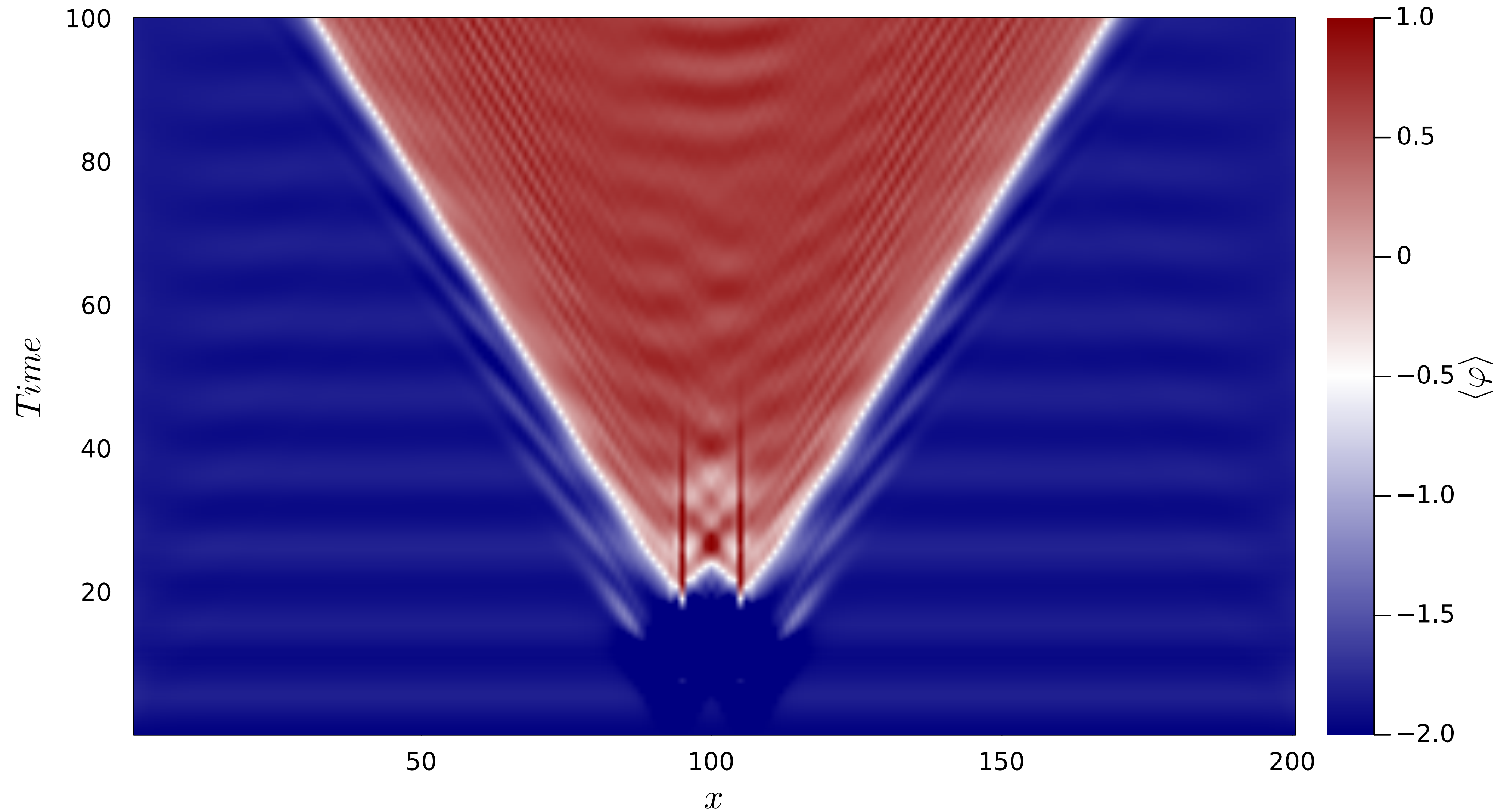
Implementation

The negative mode can be excited by applying a small momentum kick along this unstable direction

Takeaway

The bubble is seeded quantum-mechanically: the walls appear first, then pull the interior towards the true vacuum.

Seeding bubble nucleation with the negative mode





We introduced a **qumode tensor-network** framework for real-time dynamics in interacting scalar QFTs.

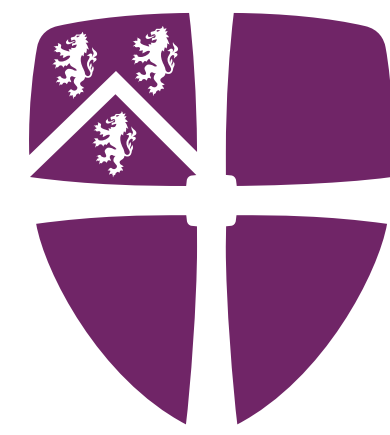
By truncating each qumode, **TEBD** can classically emulate continuous-variable photonic quantum devices.

We demonstrated the framework in two non-equilibrium processes:

1. **Scattering in (1+1)-dimensional φ^4 theory**: validated against analytical two-point correlation functions.

2. **False-vacuum decay**: capturing the bubble nucleation and growth.

Qumodes provide a natural language for bosonic quantum fields, and tensor networks let us test that programme today.



Durham
University



Backup Slides

Near-term classical benchmarking with Tensor Networks

Controlled truncation

To facilitate a quantum emulation, we must discretise the quadrature variable:

$$q_j = j \xi - L/2, \quad j \in [0, M-1], \quad \xi = L/M.$$

Each qumode has a corresponding wavefunction such that

$$\psi_n(q_n) \equiv \langle q_n | \psi_n \rangle$$

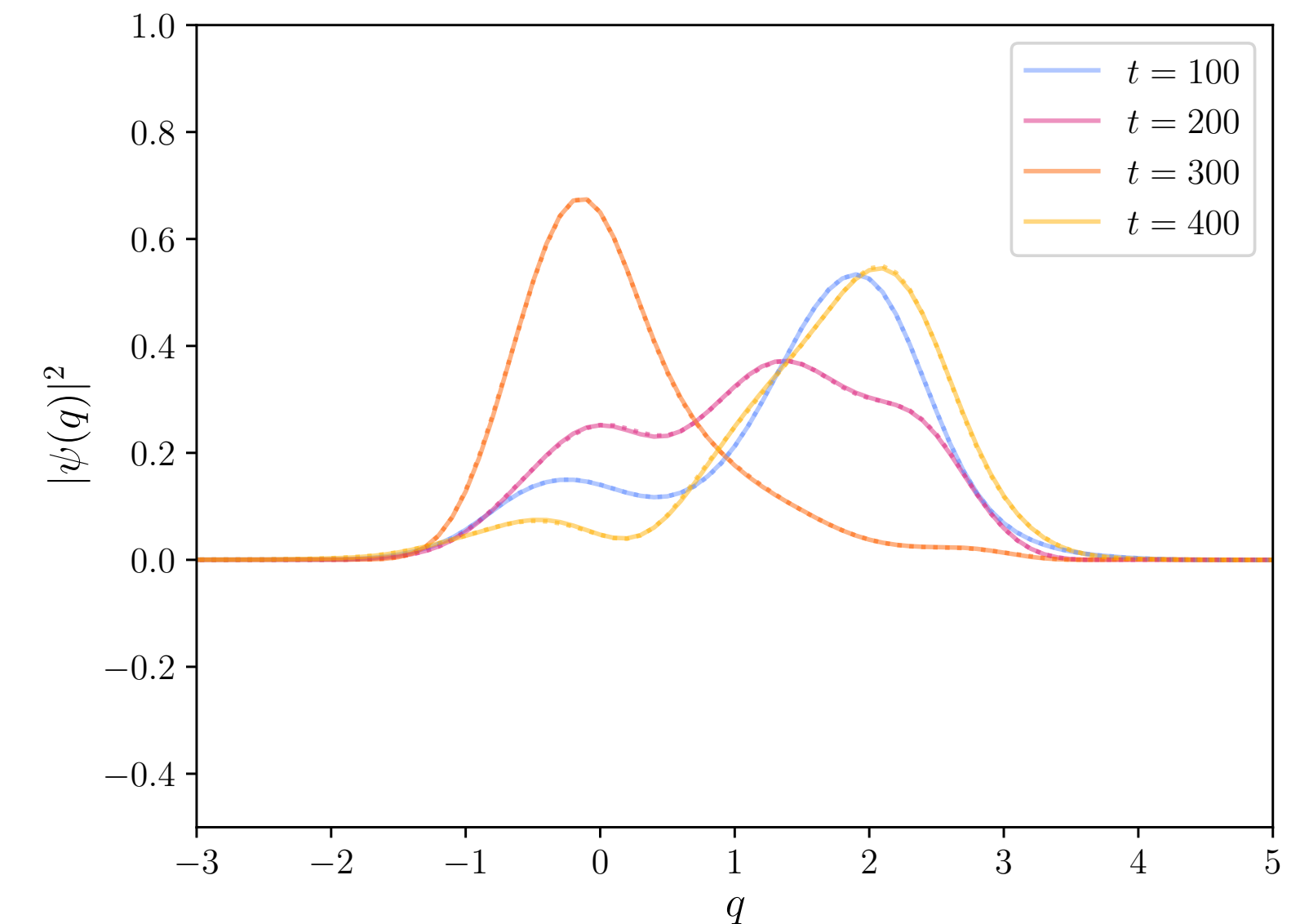
With the overall lattice wavefunction:

$$|\Psi\rangle = \bigotimes_n |\psi_n\rangle \quad ; \quad |\mathbf{q}\rangle = \bigotimes_n |q_n\rangle$$

Single qumode dynamics

Element-wise simulation:

$$m = 200$$
$$dt = 0.01$$



Test the evolution with effective potential:

$$V_I = -\frac{(1 + \epsilon/4)}{2} q^3 + \frac{1}{8} q^4$$

Initialising the Vacuum

QFT Vacuum

To obtain the QFT vacuum we solve $\hat{a}_\alpha |\mathbf{0}\rangle = 0$:

$$\langle \mathbf{q} | \mathbf{0} \rangle_{\text{QFT}} = \prod_{\alpha=0}^{N-1} \langle q_\alpha | 0_{\omega_\alpha} \rangle$$

which is a Gaussian form which is **not factorised** and instead proportional to

$$\langle \mathbf{q} | \mathbf{0} \rangle_{\text{QFT}} \propto \exp \left(- \sum_{\alpha} \omega_{\alpha} \sum_{nm} U_{\alpha n} U_{m\alpha}^{\dagger} q_n q_m / 2 \right)$$

The QFT vacuum **maximally entangles** the qumode lattice if we excite high momentum modes

Qumode Tensor Network Vacuum

Consider the vacuum of the qumode lattice in which every qumode is in the SHO ground state:

$$\langle \mathbf{q} | \mathbf{0} \rangle = \prod_{n=1}^N \langle q_n | 0_{\omega} \rangle$$

This is a product of Gaussian SHO wavefunctions:

$$\langle \mathbf{q} | \mathbf{0} \rangle \propto \exp \left(-\omega \sum_n q_n^2 \right)$$

This **does not** correspond to the vacuum of the QFT!

Introducing the hopping term

Truncating the hopping term

In the classical simulation we must approximate the hopping term by performing an entanglement truncation by projecting the state after hopping onto the quadratures

$$|\Psi'\rangle = \exp(-i \sum_n q_n q_{n+1} \delta t / a^2) |\Psi\rangle$$

The tensor-network time evolution becomes:

$$|\psi_n\rangle \rightarrow \sum_m (\mathcal{U}_R \mathcal{U}_V \tilde{\mathcal{U}}_{\text{hop}})_{nm} |\psi_m\rangle$$

To facilitate classical benchmarks, we therefore restrict ourselves to **non-relativistic dynamics** and apply an **entanglement truncation**

