A Universal, Scalable Framework for Quantum Simulation of Lattice Gauge Theories

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Our work in a nutshell

The Orbifold lattice uses Cartesian coordinates to overcome the limitations of compact variables, enabling scalable quantum simulations of SU(N) lattice gauge theories in arbitrary dimensions. Quantum encoding strategies and Kogut-Susskind (KS) extrapolations are presented.

The advantage of the Orbifold lattice

Why do we like the Orbifold Lattice for QCD and Yang-Mills?

- Explicit Hamiltonian $\forall$ gauge groups, dimensions, truncations
- Quantum circuit can be written explicitly
- Gate counts grows polynomially with qubits
- Natural connection to KS in large mass limit
- Shallow circuits via negligible-term omission arXiv:2604.15132
- Universal framework with fermions arXiv:2506.18966
- Gauge symmetry by Orbifold arXiv:2512.22932

The Orbifold link variable

Compact Variables $\longrightarrow$ Cartesian Coordinate:

- $\mathrm{SU}(N) \subset \mathbb{C}^{N^2} \cong \mathbb{R}^{2N^2}$

$$\hat{Z}_{j,\vec{n}} = \sqrt{\frac{a^{d-2}}{2g_d^2}} W_{j,\vec{n}} U_{j,\vec{n}}$$

j $\qquad$ $j+\vec{n}$

- $W_{j,\vec{n}} = \exp\left(a g_d \phi_{j,\vec{n}}\right)$ positive Hermitian $\longrightarrow \phi_j$ adjoint scalar field
  $W \neq 1$ –> effective shift of lattice spacing
- $U_{j,\vec{n}} = \exp(\mathrm{i} a g_d A_{j,\vec{n}})$, unitary link variable $\longrightarrow A_j$ gauge field

Representing a Yang-Mills theory coupled to scalars

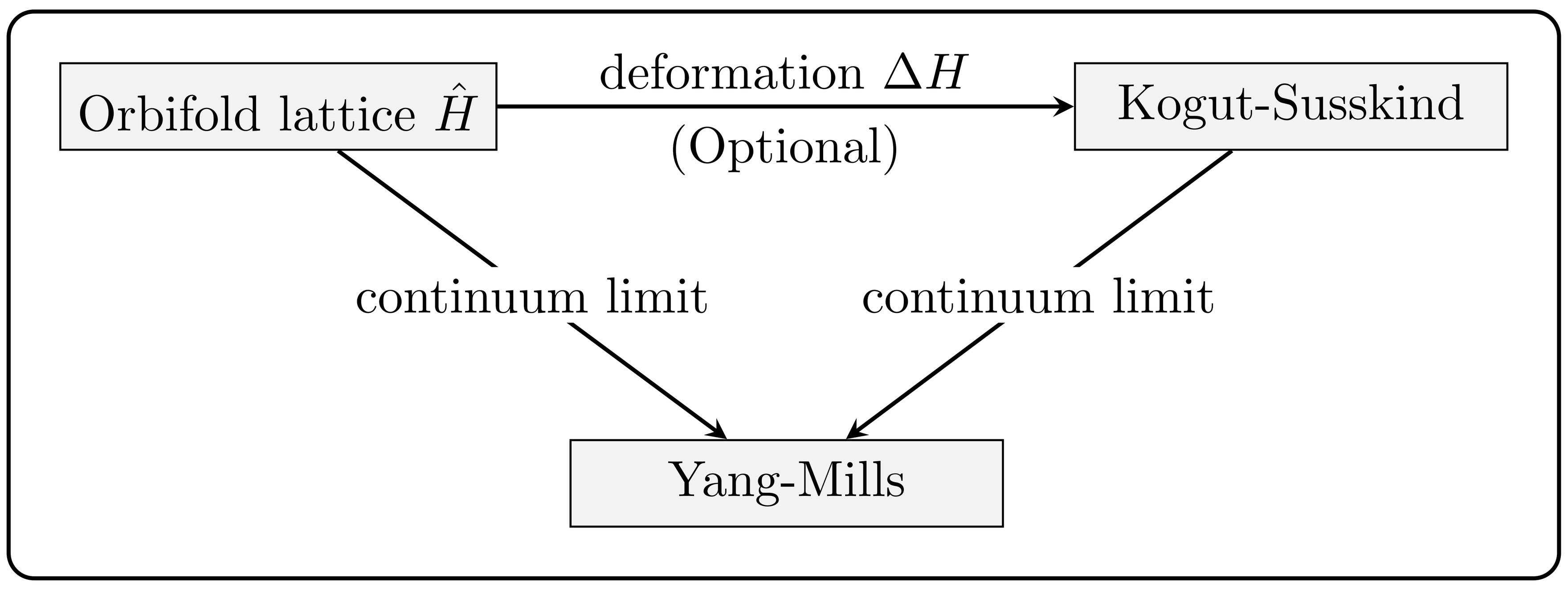
The SU(N) Orbifold lattice Hamiltonian for quantum simulations

$$\hat{H} = \sum_{\vec{n}} \mathrm{Tr}\left(\sum_{j=1}^{d} \hat{P}_{j,\vec{n}} \hat{\bar{P}}_{j,\vec{n}} + \frac{g_d^2}{2a^d} \left| \sum_{j=1}^{d} \left(\hat{Z}_{j,\vec{n}} \hat{\bar{Z}}_{j,\vec{n}} - \hat{\bar{Z}}_{j,\vec{n}-\hat{j}} \hat{Z}_{j,\vec{n}-\hat{j}} \right) \right|^2 + \frac{2g_d^2}{a^d} \sum_{j<k} \left| \hat{Z}_{j,\vec{n}} \hat{Z}_{k,\vec{n}+\hat{j}} - \hat{Z}_{k,\vec{n}} \hat{Z}_{j,\vec{n}+\hat{k}} \right|^2 \right) + \Delta\hat{H}$$

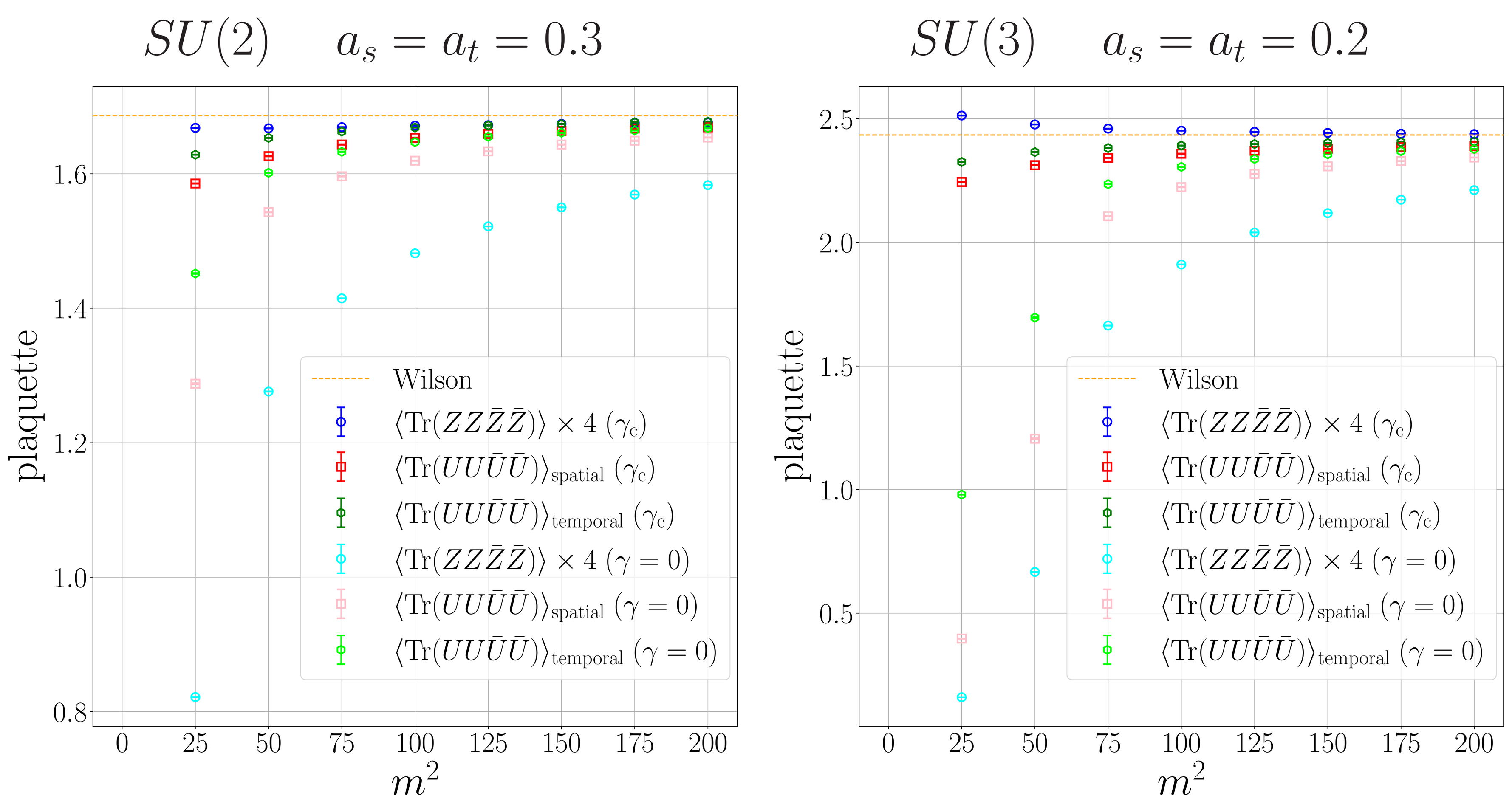
$$\Delta\hat{H} = \frac{m^2 g_d^2}{2a^{d-2}} \sum_{\vec{n}} \sum_{j=1}^{d} \mathrm{Tr} \left| \hat{Z}_{j,\vec{n}} \hat{\bar{Z}}_{j,\vec{n}} - \frac{a^{d-2}}{2g_d^2} \right|^2 + \frac{m_{\mathrm{U}(1)}^2 a^{d-2}}{2g_d^2} \sum_{\vec{n}} \sum_{j=1}^{d} \left| \left(\frac{a^{d-2}}{2g_d^2} \right)^{-N/2} \det(\hat{Z}_{j,\vec{n}}) - 1 \right|^2 - \gamma \sum_{\vec{n}} \sum_{j=1}^{d} \mathrm{Tr}\left(\hat{Z}_{j,\vec{n}} \hat{\bar{Z}}_{j,\vec{n}} \right)$$

1st term forces $W \to \mathbf{1}_N$;   2nd term forces $\det(U) \to 1$;   3rd term γ tunes the effective lattice spacing, reducing anisotropy (optional)

Kogut-Susskind $\Longleftarrow$ Orbifold $\Longrightarrow$ Yang-Mills



Scalar mass dependence at fixed lattice spacing (tuning γ)

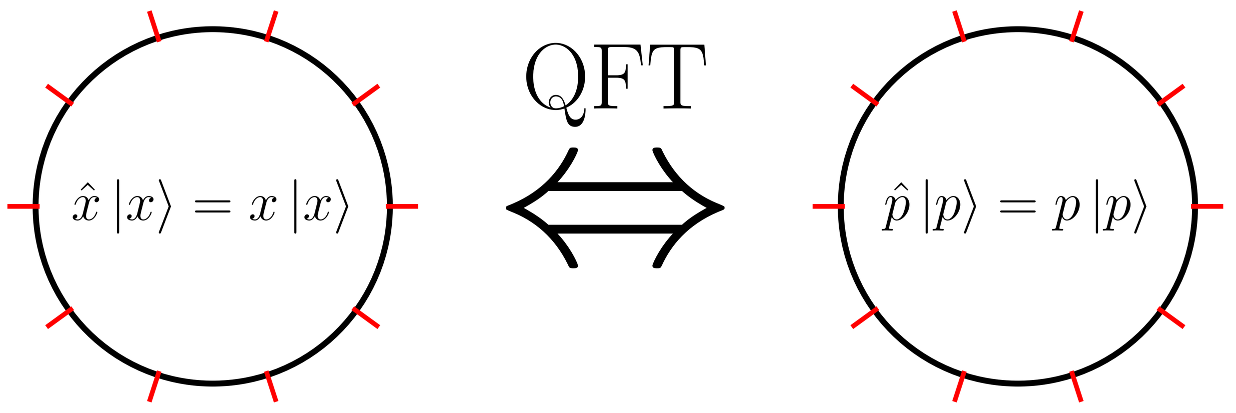


- Easier to see the KS limit when effective lattice spacing is fixed

Digitization of the model

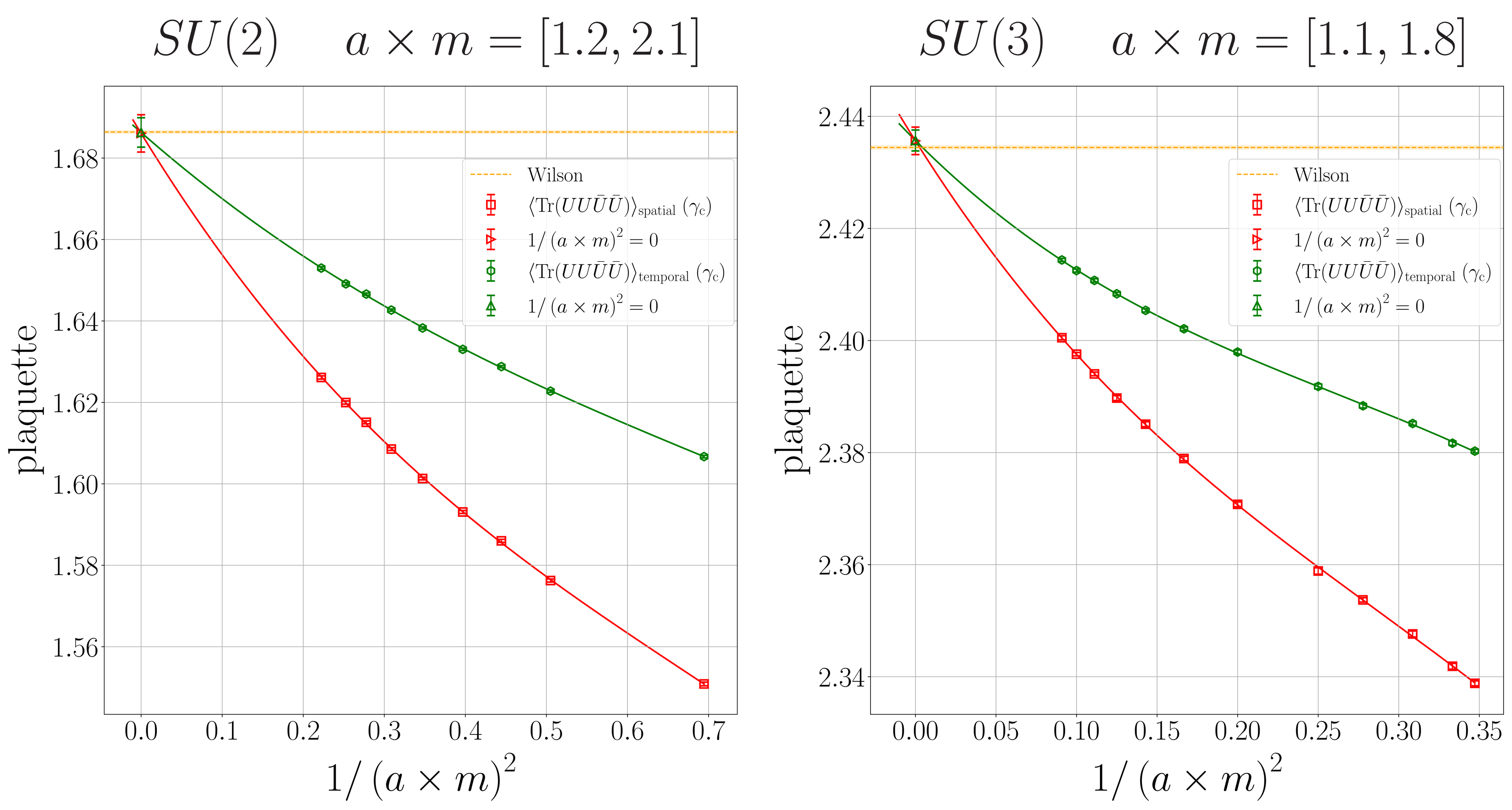
- $Z, P \in \mathbb{C}^{N^2} \longrightarrow 2N^2$ Cartesian components
  $(x_1, \dots x_{2N^2}), (p_1, \dots p_{2N^2}) \longrightarrow 2N^2$ bosons
- Q qubits per boson $\Longrightarrow$ truncation level $\Lambda = 2^Q$ grid point

Λ points on a circle of radius $2R$ in the x- and p-basis.



$$\hat{x}_a = -\frac{\delta_x}{2}\left(\hat{\sigma}_{z;a,1} + \dots + 2^{Q-1}\hat{\sigma}_{z;a,Q}\right) \quad \hat{p}_a = -\frac{\delta_p}{2}\left(\hat{\sigma}_{z;a,1} + \dots + 2^{Q-1}\hat{\sigma}_{z;a,Q}\right)$$

Improved convergence to KS limit at γ_c !



- The KS limit can be evaluated using $m \sim a^{-1}$

Quantum resources

- $\#Q_{\text{tot}} = d\, N_{\text{site}}\, 2N^2\, Q_{\text{boson}}$ $\qquad$ $\#Gates \propto N_{\text{site}}\, Q_{\text{boson}}^4 N^4$

More info and future work

- Quantify bosons truncation effects for quantum simulations
- Real-time evolution for for SU(2) and SU(3) systems

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