ETH zürich

Institute for Theoretical Physics

Can Hilbert space fragmentation be explained with generalized symmetries?

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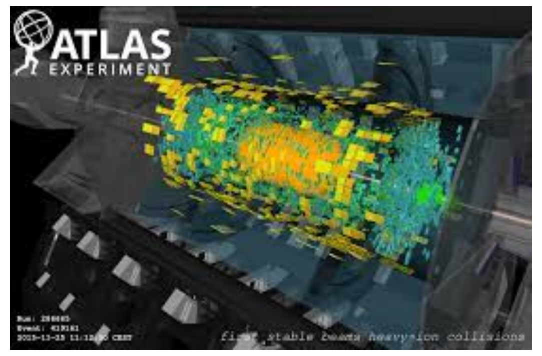
in collab. with **Thea Budde** and **Joao Pinto Barros**

QuantHEP 2026, 14 July 2026, Queen Mary University of London

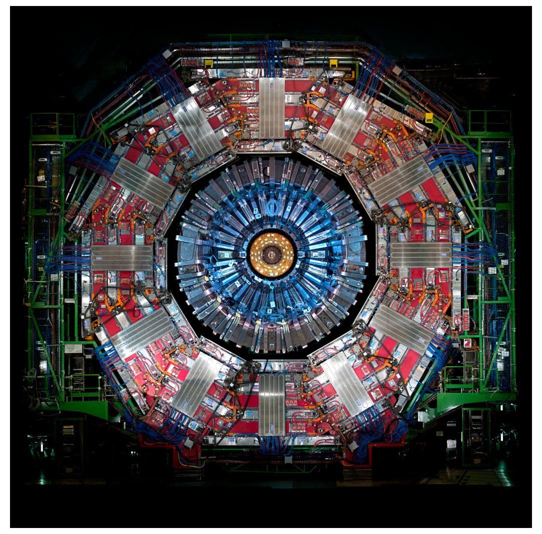
Strong interaction: limiting factor in precision calculations

phase structure, real time dynamics, θ-terms …

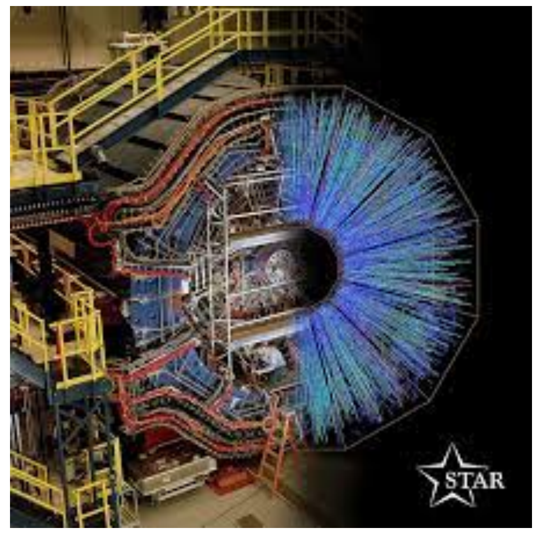
Experiments probing nonzero baryonic density



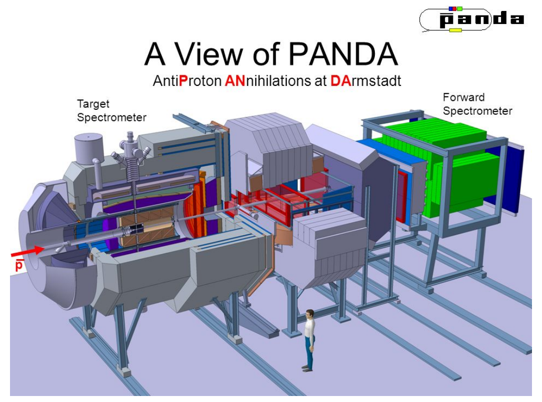
LHC: ATLAS



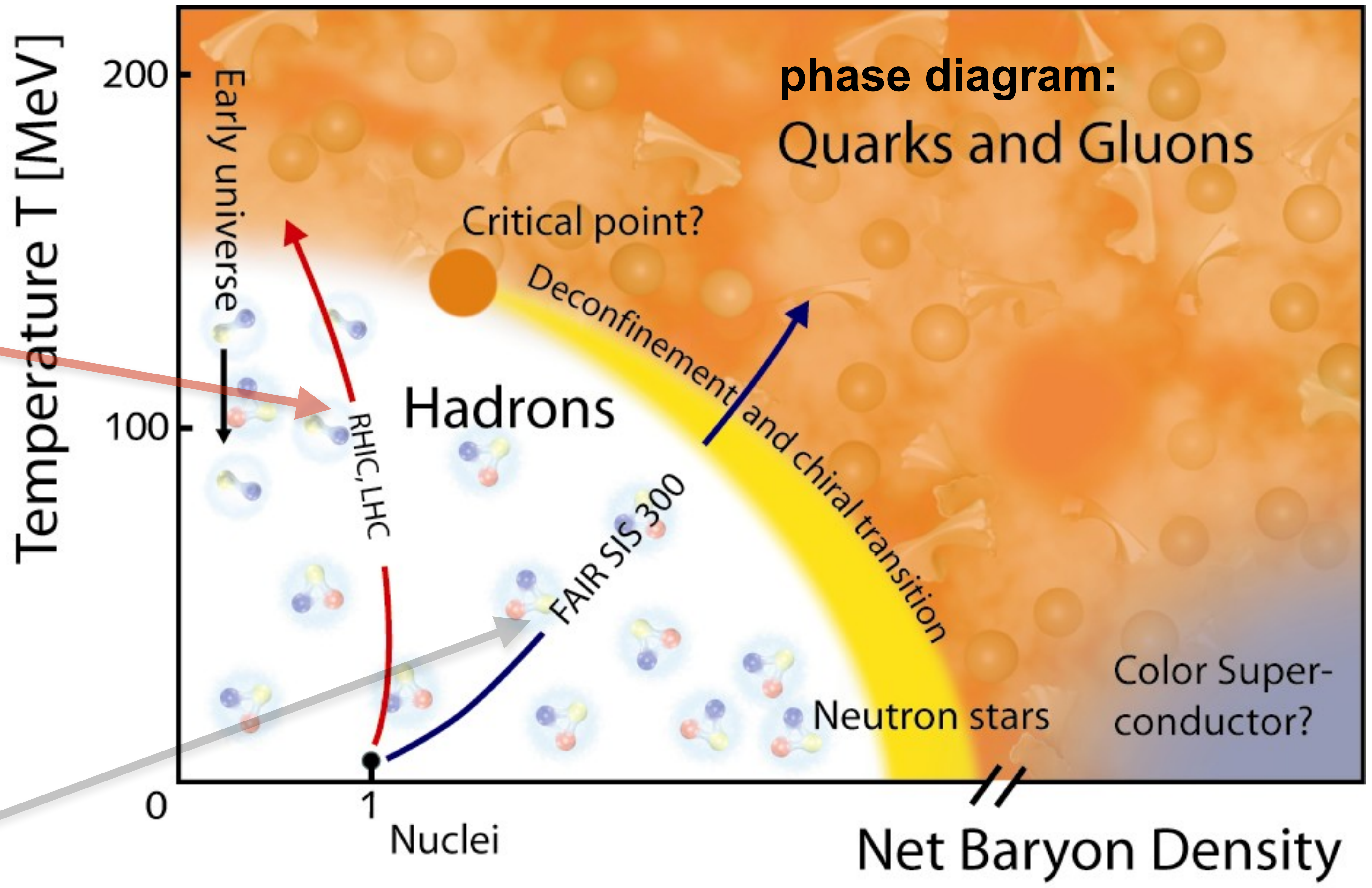
LHC: ALICE



RHIC: STAR

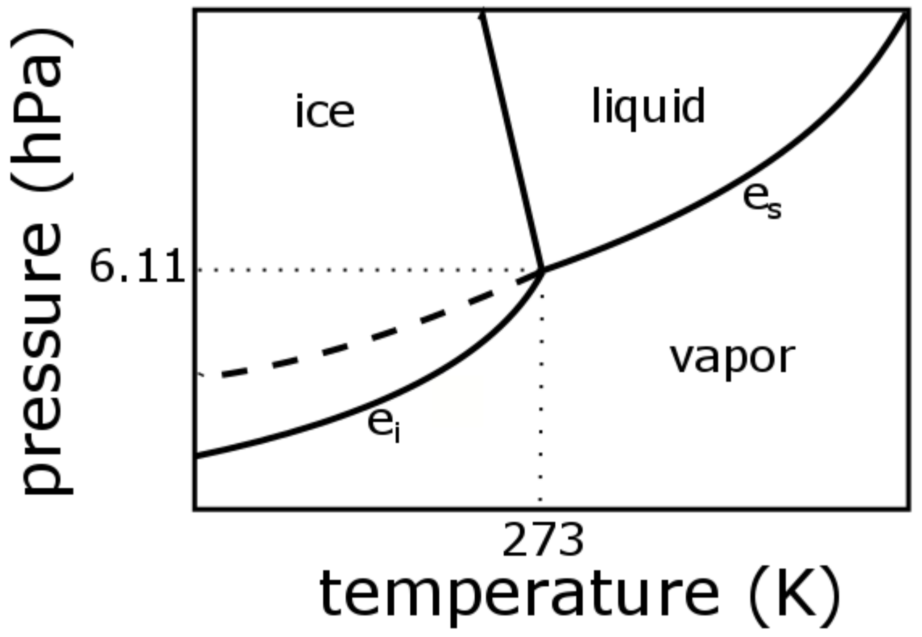


FAIR: PANDA

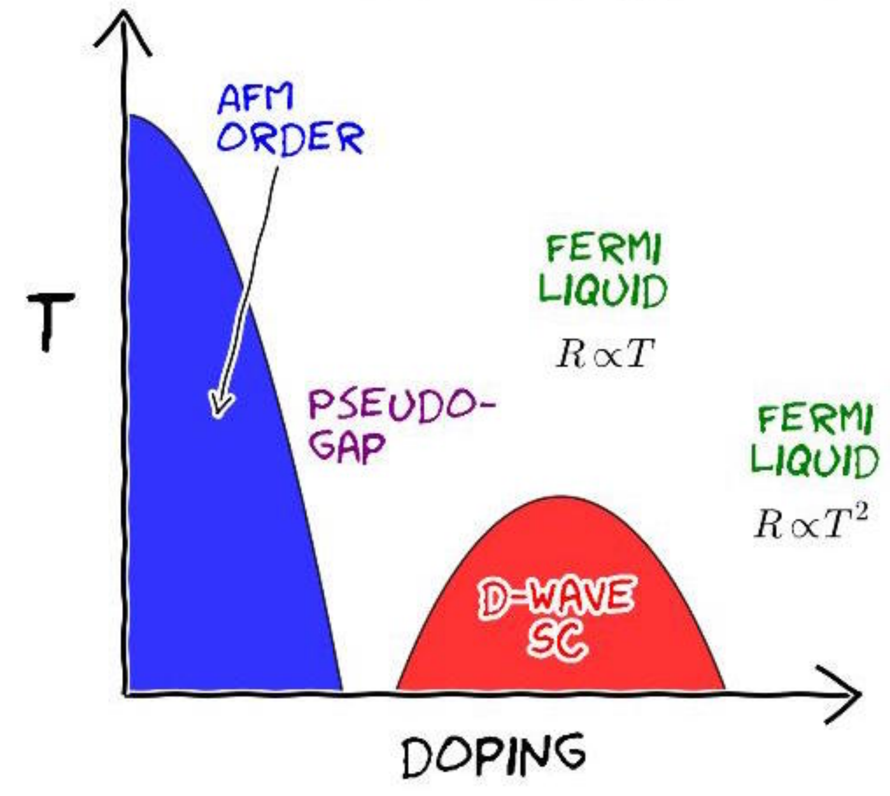


[Credit: Peter Senger (p.senger@gsi.de)]

phase diagram: water

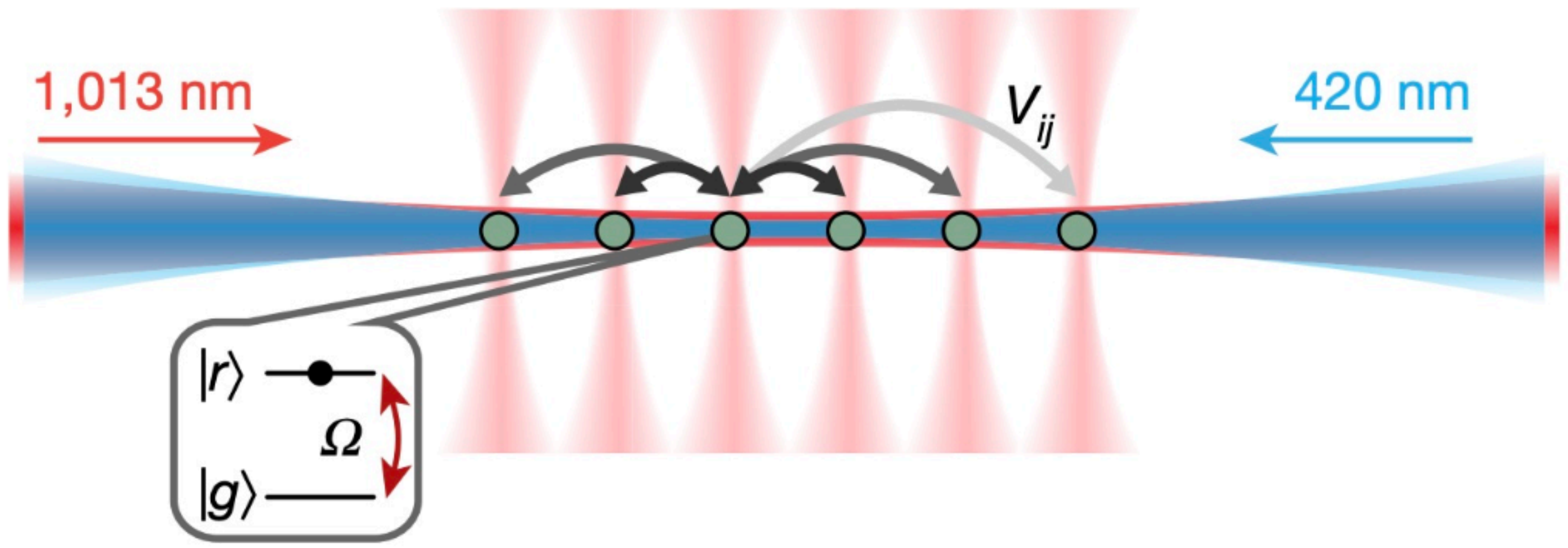


phase diagram: high Tc superconductors



[Credit: http://atomcool.rice.edu/]

Co-design of quantum simulations of LGTs



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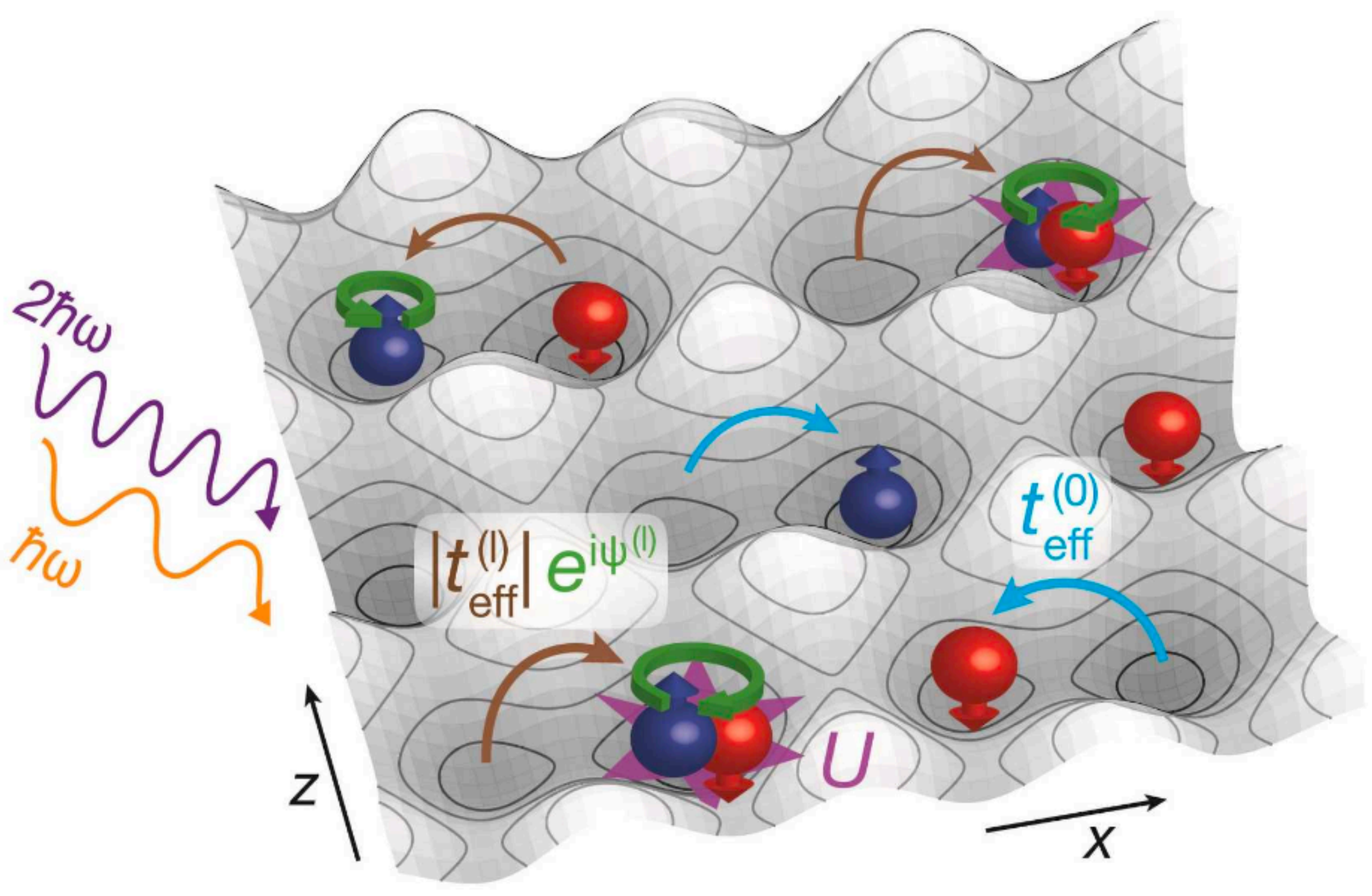

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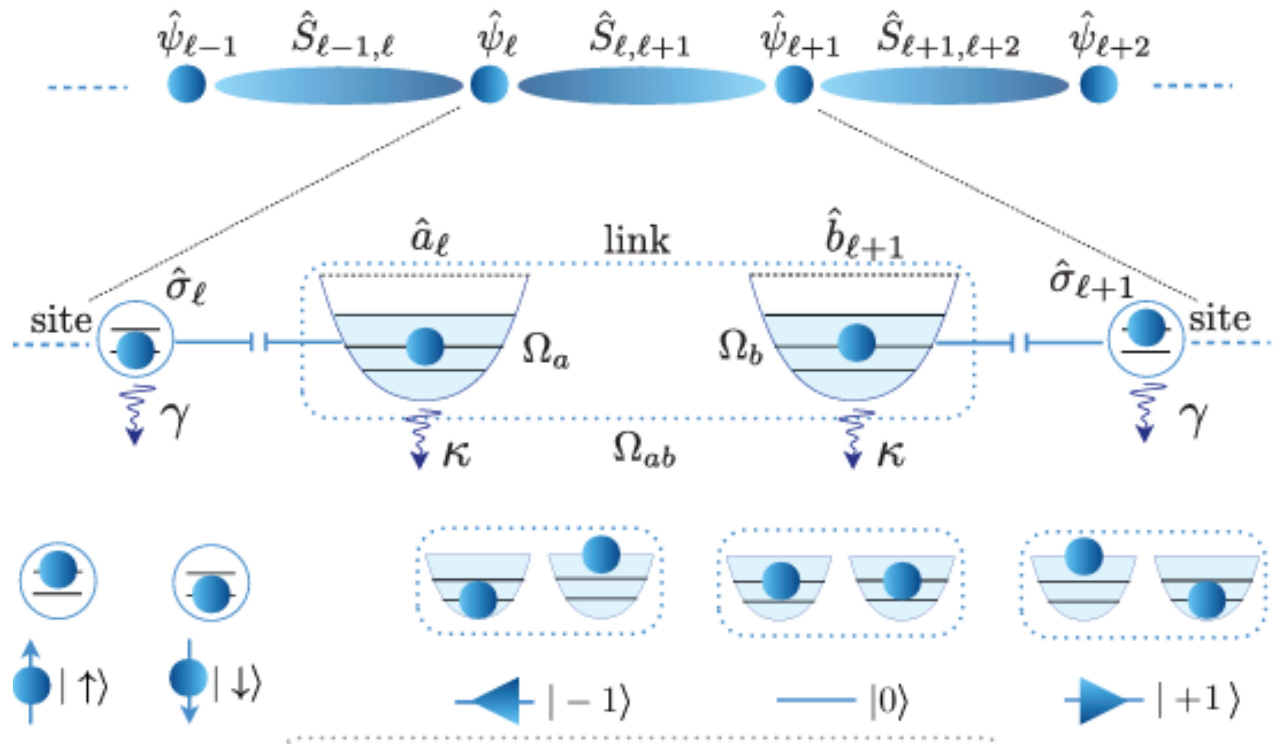


3

[[1]F. Görg, T. Esslinger et al., Nature Physics 15, p. 1161–1167 (2019)]

[[2]H. Bernien et al., Nature volume 551, p. 579–584 (2017)]

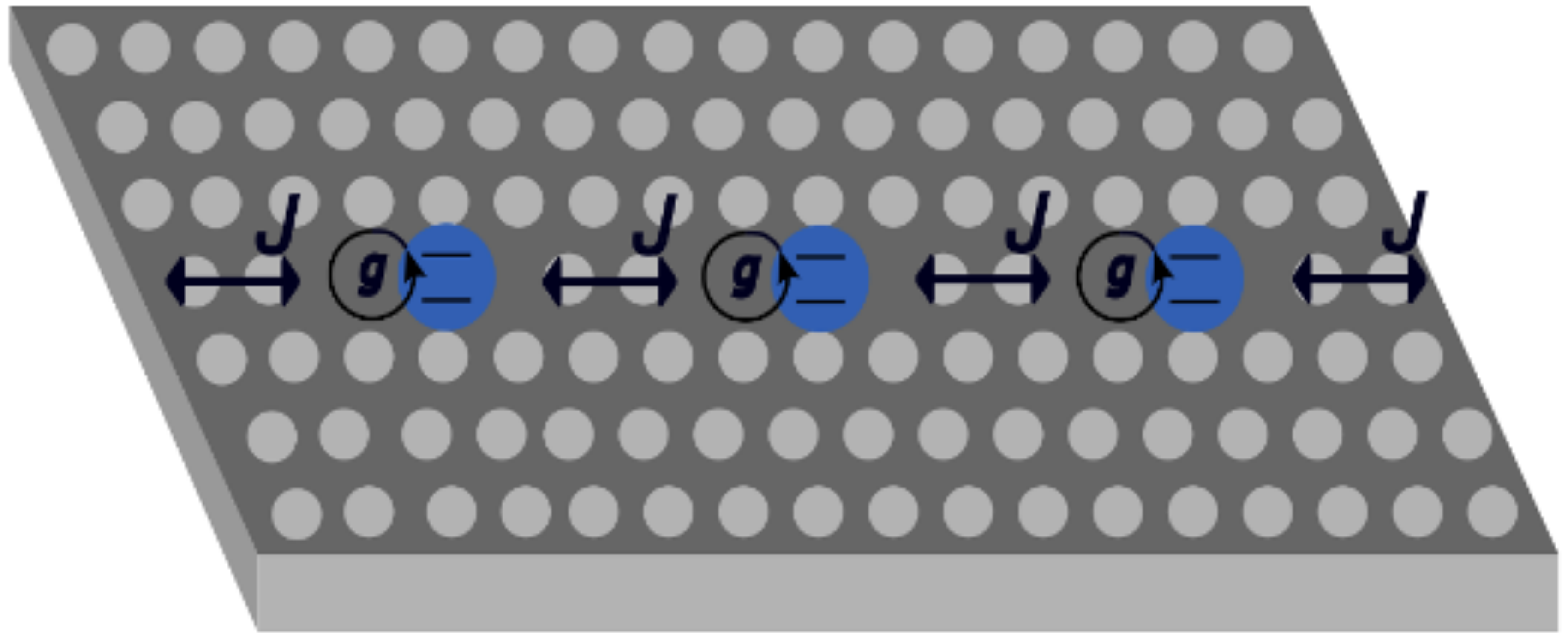
[[3]D. Marcos et al., Phys.Rev.Lett. 111 (2013) 11]

...

Co-design of quantum simulations of LGTs

[N. R. Gonzalez, T. Budde, Z. Steele, A. Rubin, K. Kersic, J. P. Barros,M. Radulaski, MKM arXiv:2606.02756]

- U(1) gauge theory with matter in 1+1D
- Proposals and experimental realizations in cold atoms, trapped ions, …
- S=1/2 Quantum Link Model in 1+1D
    - ➡ On a scalable and engineerable analogue quantum simulator
    - ➡ Photonic cavity QED Platform



M. Radulaski

N. Gonzalez

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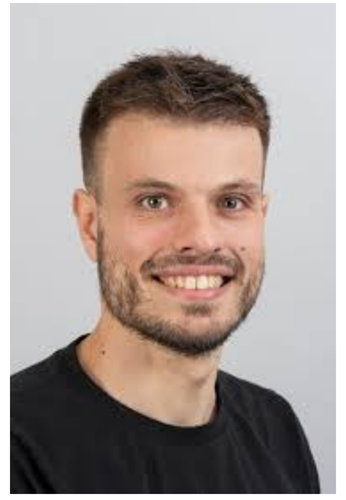

J. Pinto Barros

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Real-time dynamics of gauge theories

- Ground state properties

- Real-time dynamics

  ➡ Will an isolated quantum system reach equilibrium? [see Andreas', Zlatko's and Jared's talks on Mon.]

  ➡ Eigenstate Thermalization Hypothesis (ETH) [von Neumann(1929), Deutsch. PRA (1991), Srednicki PRE (1994)] local observables are expected to thermalize

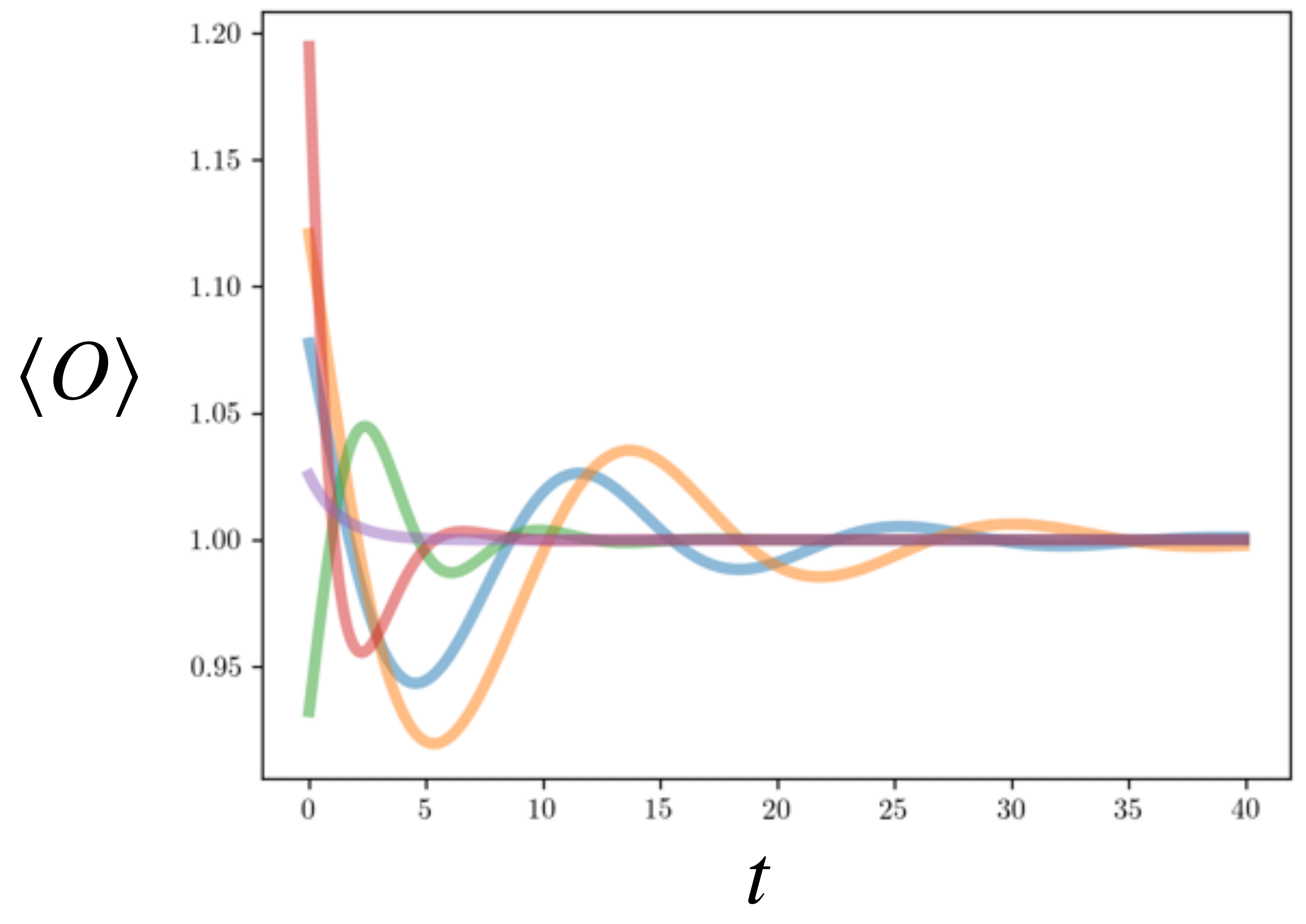
$$\mathcal{O}(t) = \langle \psi(t) | \mathcal{O} | \psi(t) \rangle = \sum_n |c_n|^2 \mathcal{O}_{nn} + \sum_{m \neq n} c_m^* c_n \, e^{i(E_m - E_n)t} \, \mathcal{O}_{mn}$$

$$\mathcal{O}_{mn} = \mathcal{O}(\overline{E}) \, \delta_{mn} + e^{-S(\overline{E})/2} f_{\mathcal{O}}(\overline{E}, \omega) \, R_{mn}$$

$\overline{E} = (E_m + E_n)/2$; $\omega = E_m - E_n$; $f_{\mathcal{O}}$ — smooth function; R_{mn} — random variable

  ➡ ETH bounds the fluctuations

Thermalization of isolated quantum systems



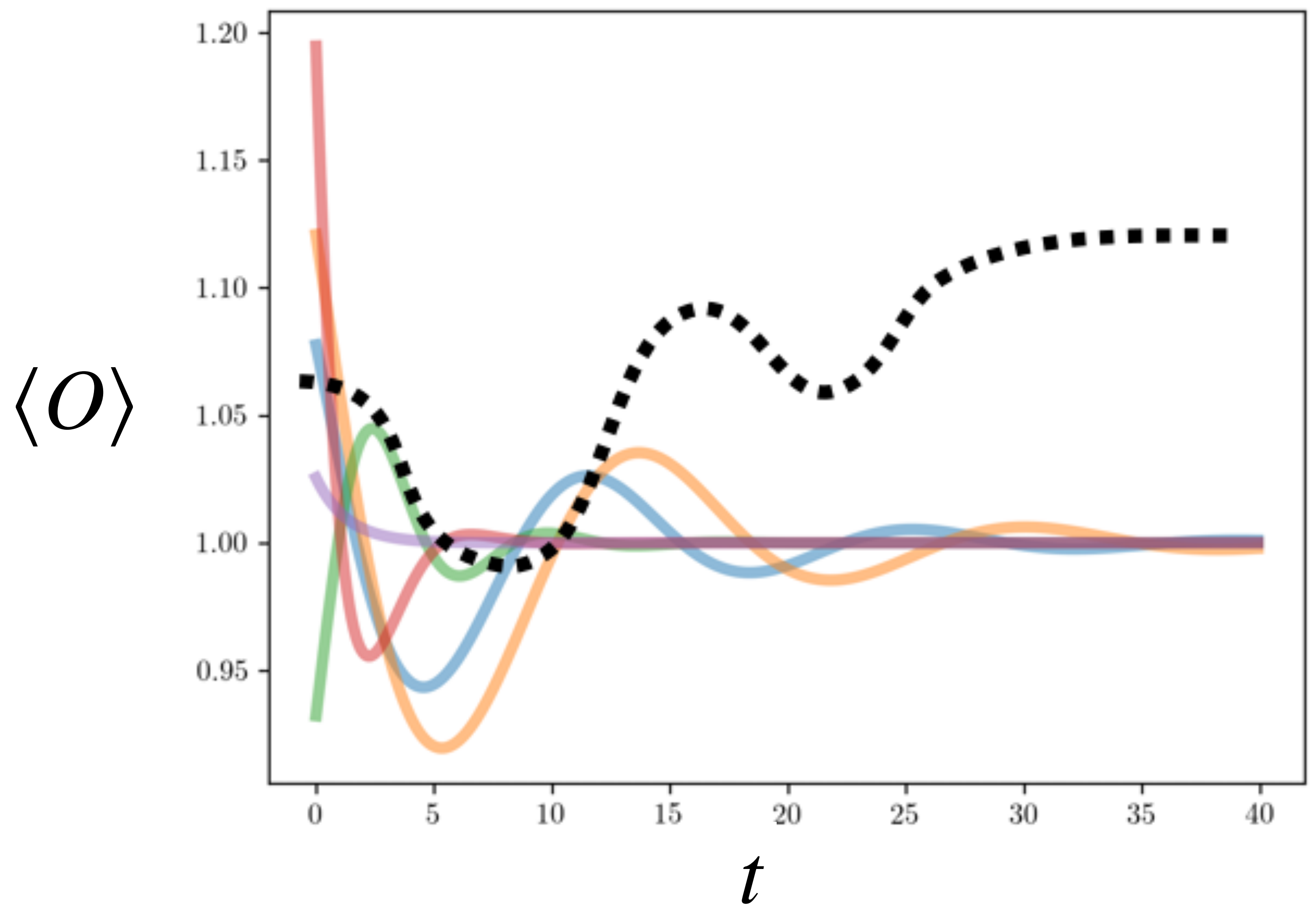
Exceptions to thermalization:

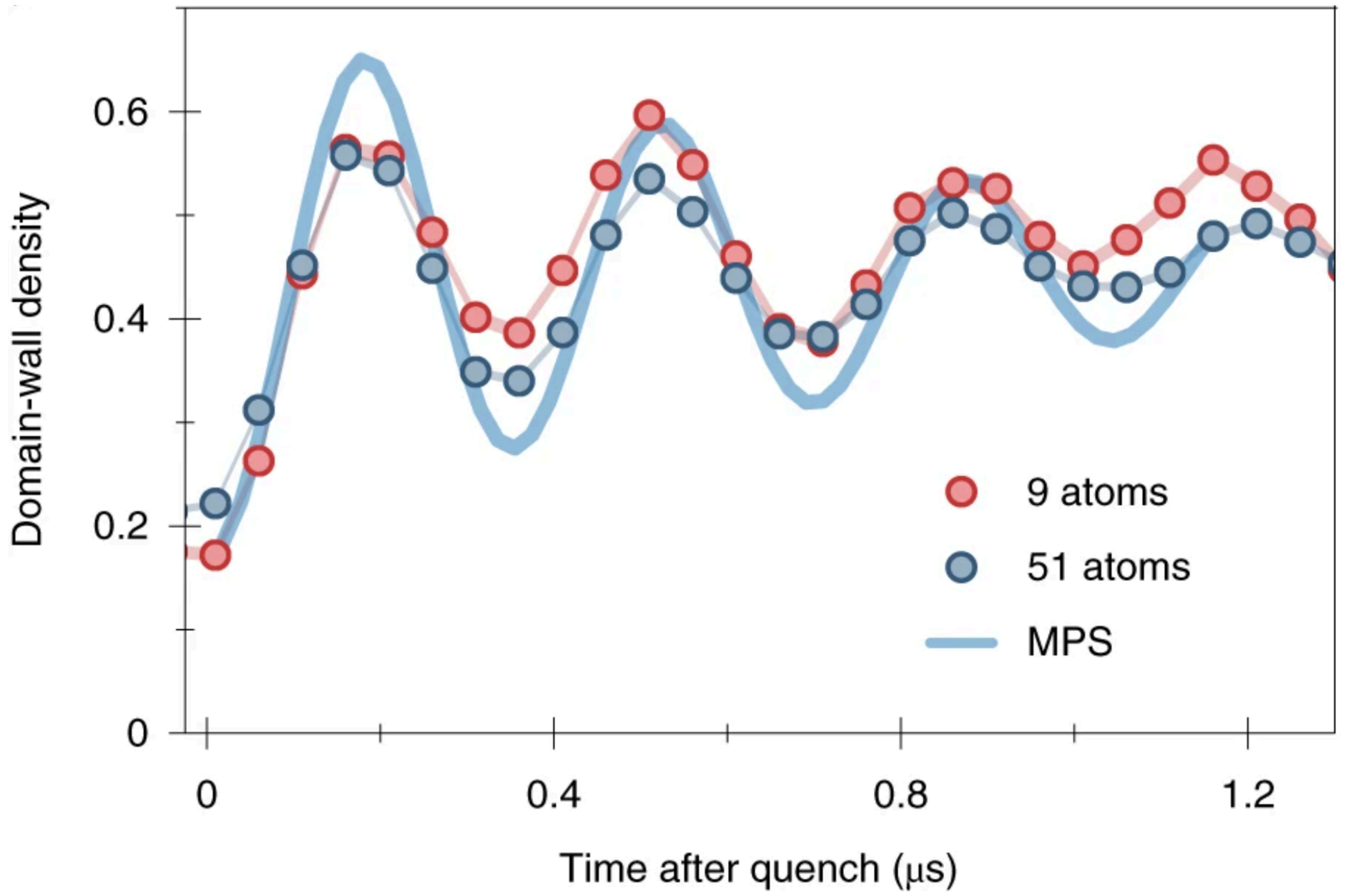
- ❖ Integrable models
- ❖ *Quantum Many-Body Scars*: special initial conditions avoid thermalization [Turner et al.,Nature Physics 14, 745–749 (2018)]
- ❖ Many Body Localization
- ❖ Fragmentation

- Ergodicity: single fragment Hilbert space; Hamiltonian connects all basis states
- Strong breaking of ETH: integrable models, disordered systems; exponentially many fragments; $\dfrac{d_{accessible}}{N_{tot}} \sim e^{-V} \xrightarrow{V\to\infty} 0$
- QMBS: weak breaking of ETH, (many) anomalous high-energy states; $\dfrac{n_{\text{QMBS states}}}{N_{\text{tot}}} \to 1$ as $V \to \infty$.

h

Thermalization of isolated quantum systems

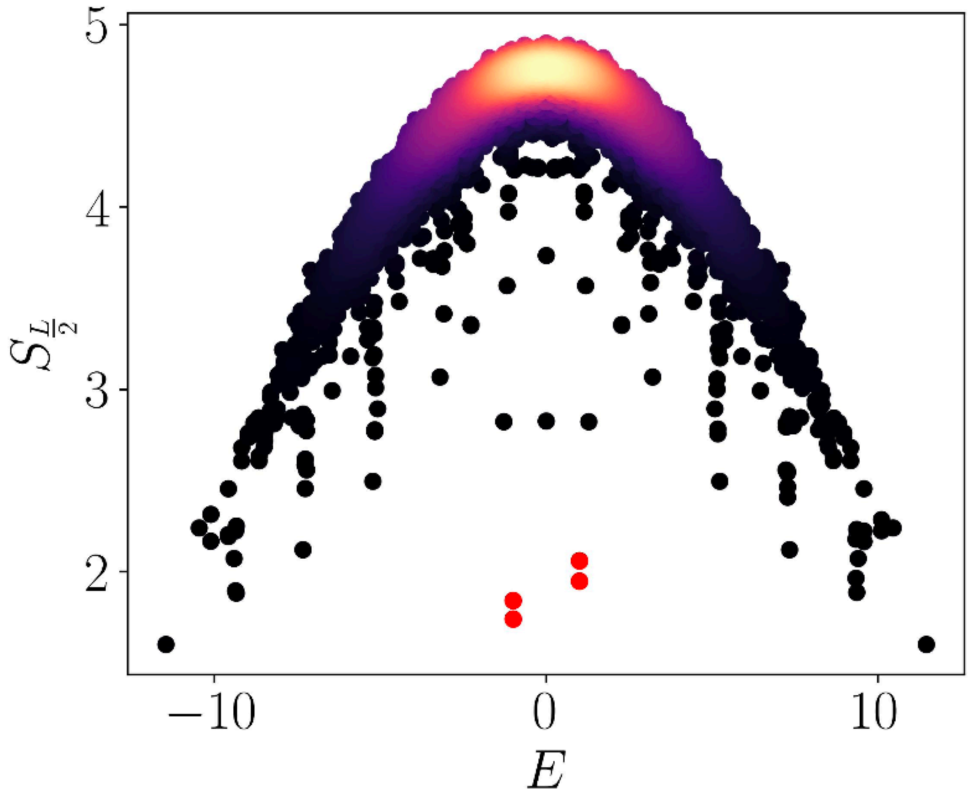




[Bernien et al. Nature 551 (2017)]

- Quantum Many-Body Scars: special initial conditions avoid thermalization [Turner et al.,Nature Physics 14, 745–749 (2018)]
- Fundamental questions that might be answered on quantum simulators [see Zlatko's talk on Mon.]
- Real-time evolution; severe sign problems [Troyer, Wiese PRL 94 170201(2005)]
- Signatures found in analog quantum simulations with Rydberg atom arrays [Bernien et al. Nature 551 (2017)] etc.

Analytic construction of quantum scars in U(1) gauge theory

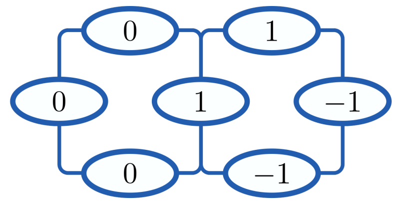
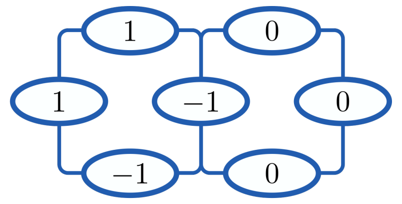


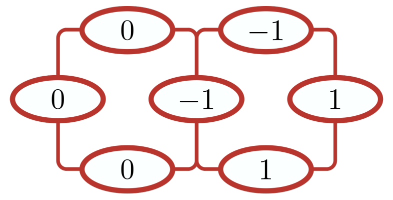
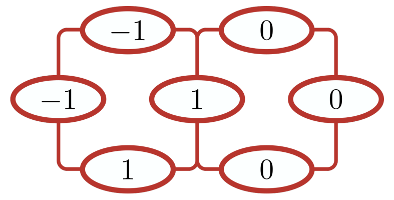
[Budde et al. PRD 110, 094506 (2024)]

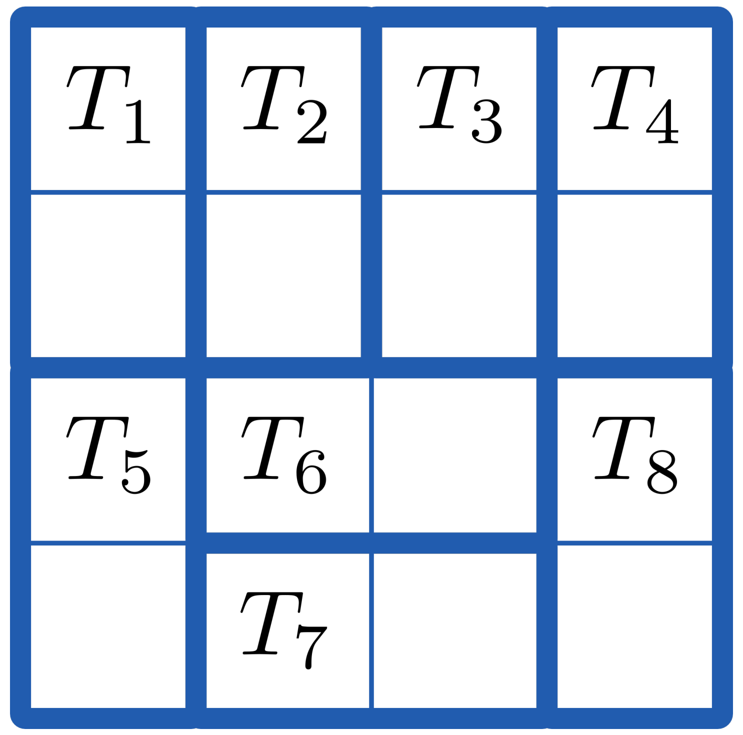
- Analytically construct scars for 2+1D *arbitrary integer spin* truncated link models [Budde, MKM, Pinto Barros, PRD 110, 094506 (2024), arXiv:2403.08892]; see also [Banerjee, Sen PRL 126 (2021)]
- Construct many zero-mode states by tensoring appropriate zero-modes for $\square\square$ lattice

$$\sum_n \left(U_\square + U_\square^\dagger\right) |\Psi_1\rangle = \sum_n \left(U_\square + U_\square^\dagger\right) |\Psi_2\rangle = 0$$

- Now possible to study quantum scars in the continuum limit of gauge theories in 2+1D
- Benchmarks of medium- and long-time dynamics in quantum simulations of LGTs
- “Stabilizer scars” [Hartse, Fidkowski, Mueller PRL 135, 060402 (2025) arXiv:2411.12797]

$|\Psi_1\rangle = \frac{1}{\sqrt{2}}$  - 

$|\Psi_2\rangle = \frac{1}{\sqrt{2}}$  - 



Example: non-integrability of a 2+1D system with ladder geometry

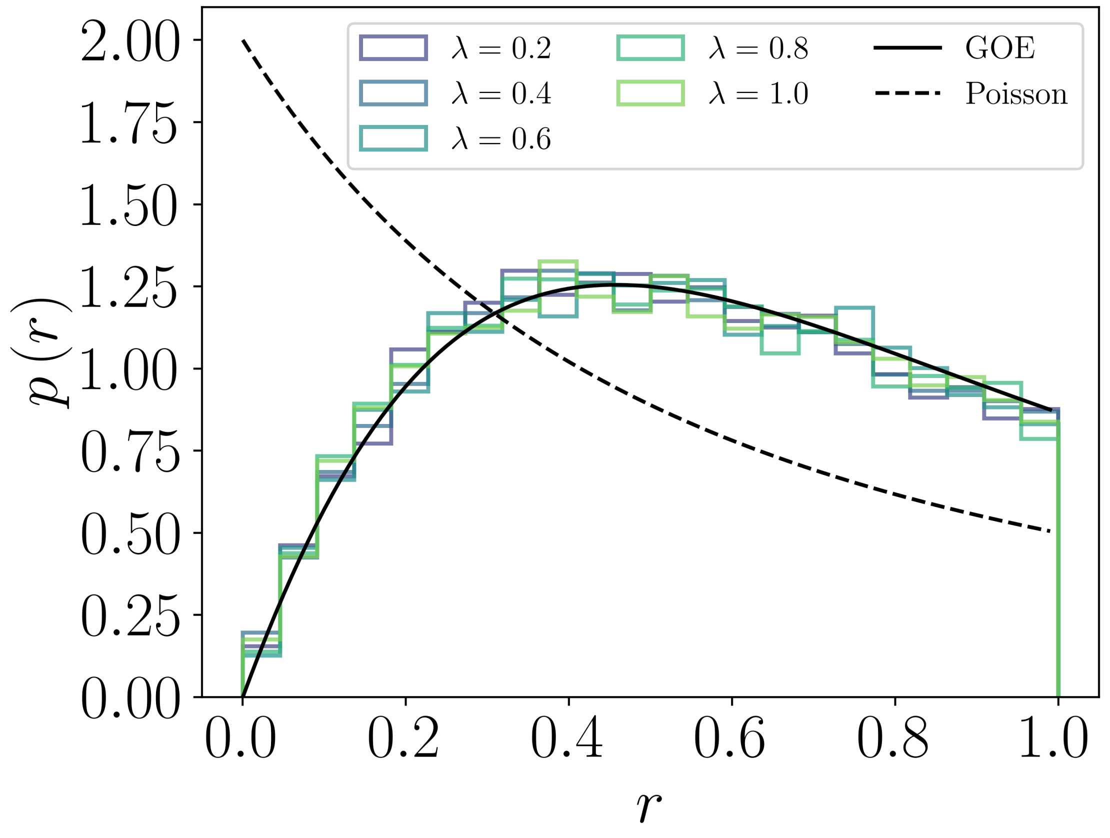
[Budde, MKM, Pinto Barros, PRD 110, 094506 (2024), arXiv:2403.08892]

- Planar quantum (truncated) link models are strongly interacting, non-integrable physical systems

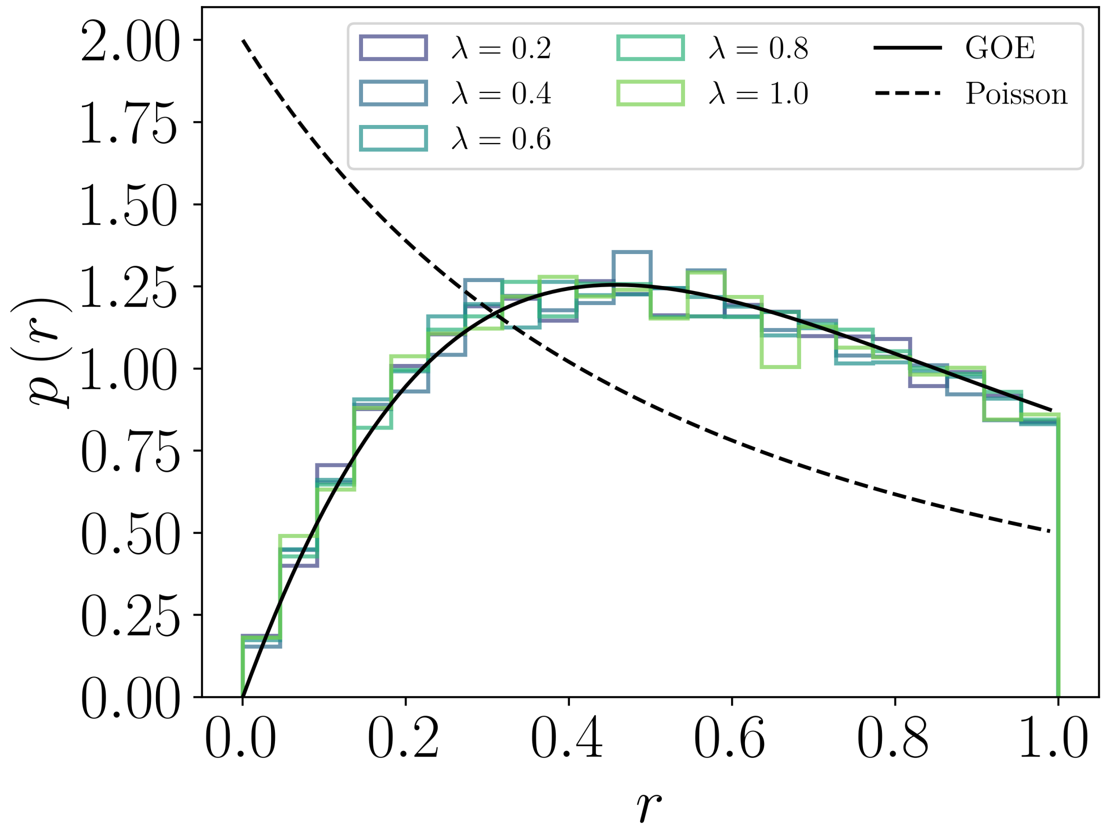
- Demonstrate non-integrability: resolve symmetries and compute level space distribution

$$\hat{H} = \sum_{n} \left(U_{\square} + U^{\dagger}_{\square} \right) + \lambda \sum_{m \text{ top row}} E_m$$

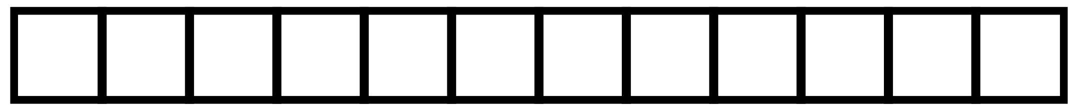
$-$ Parity Symmetry Sector



$+$ Parity Symmetry Sector



12×1 **lattice:**



$$p(r), \quad r_n = \min\left\{ \frac{E_{n+1} - E_n}{E_n - E_{n-1}}, \frac{E_n - E_n - 1}{E_{n+1} - E_n} \right\}$$

- Integrable systems: Poisson $\quad p_{Poisson}(r) = \dfrac{1}{(1+r)^2}$

- Non-integrable: Gaussian Orthogonal Ensemble (GOE) $\quad p_{GOE}(r) = \dfrac{27}{4} \dfrac{r + r^2}{(1 + r + r^2)^{5/2}}$

Ergodicity breaking in lattice field theories

- Ergodicity: single fragment Hilbert space; Hamiltonian connects all basis states

- QMBS: weak breaking of ETH, (many) anomalous high-energy states; $\frac{n_{thermal}}{N_{tot}} \to 1$ as $V \to \infty$ $(\frac{n_{QMBS}}{N_{tot}} \to 0)$

- Strong breaking of ETH: integrable models, disordered systems; exponentially many fragments; $\frac{d_{accessible}}{N_{tot}} \sim e^{-V} \xrightarrow{V \to \infty} 0$

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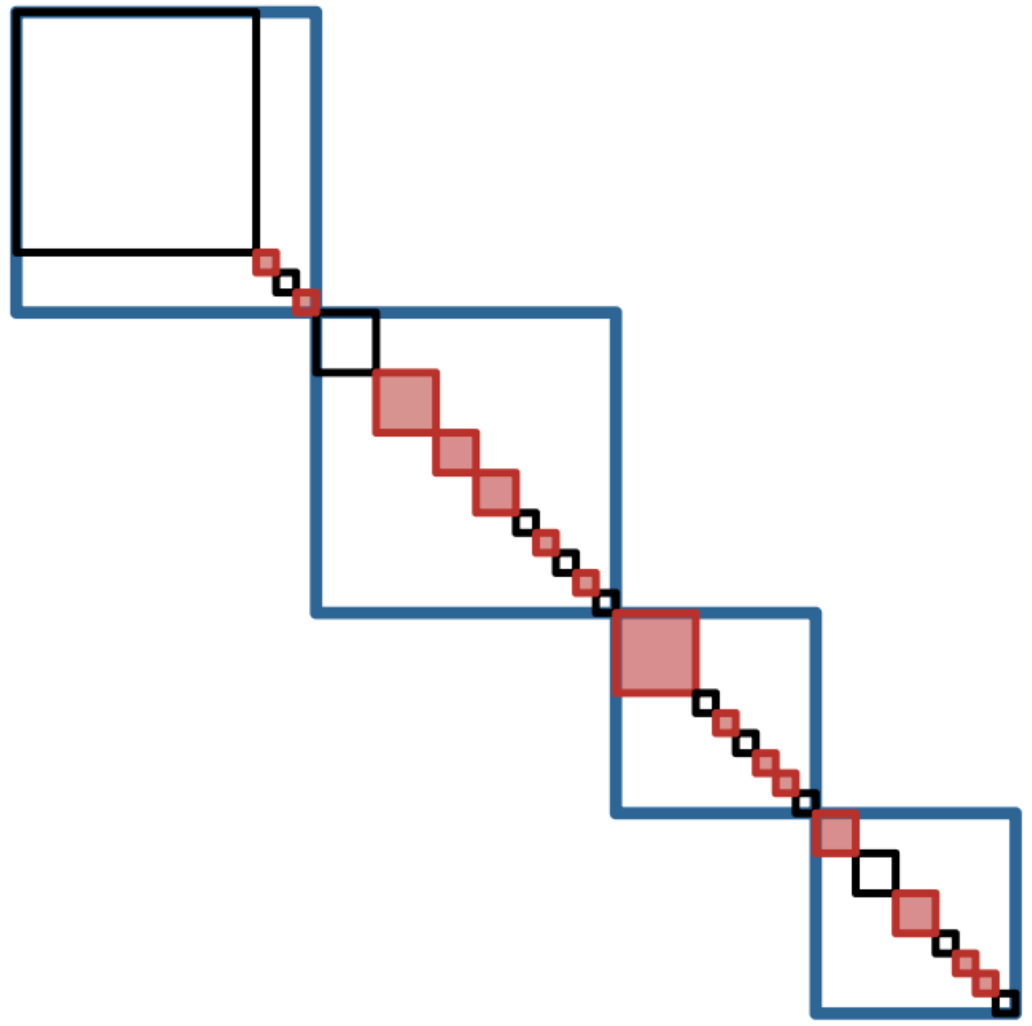
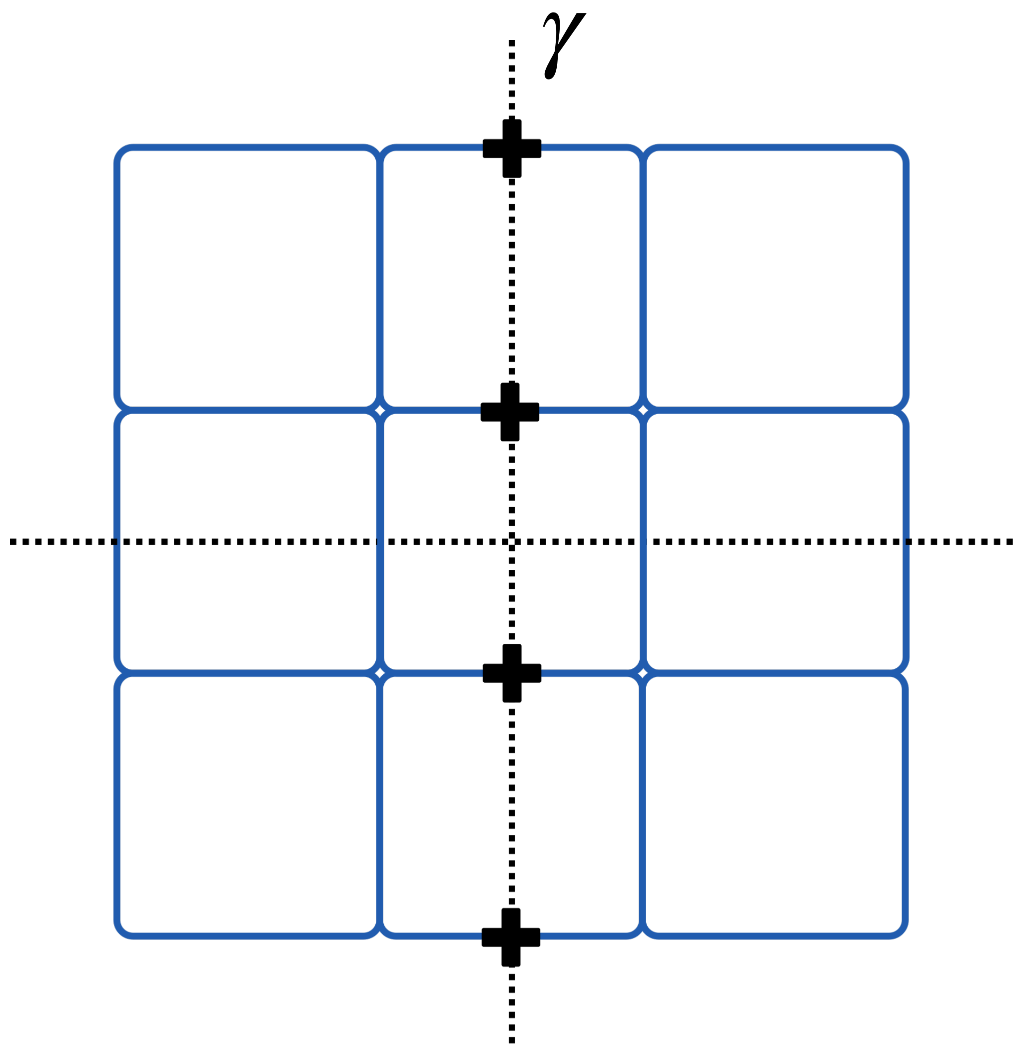


Hilbert space fragmentation in lattice field theories

- Conventional ("0-form") symmetries:
  - ➡ Product of unitary local operators $U(g) = \prod_n u_n(g)$
  - ➡ Group structure $U(g)U(h) = U(gh)$
- "1-form" symmetries
  - ➡ Product of unitary local operators on a closed codimension-1 manifold
  - ➡ Act on line operators e.g. Wilson lines which carry 1-form charge: $U_\alpha(\gamma) = \prod_n e^{i\alpha E_n}$
- Generalized symmetries: higher-form symmetries [Gaiotto et al. JHEP 02 (2015) 172 arXiv:1412.5148], non-invertible symmetries, …



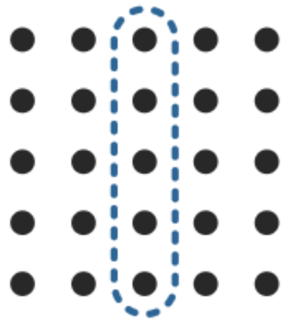
Hilbert space fragmentation in lattice field theories

- ➡ "0-form" symmetries $\left[\sum_n G_n, H\right] = 0$  $\Rightarrow \mathcal{O}\left(\mathrm{poly}(V)\right)$ fragments

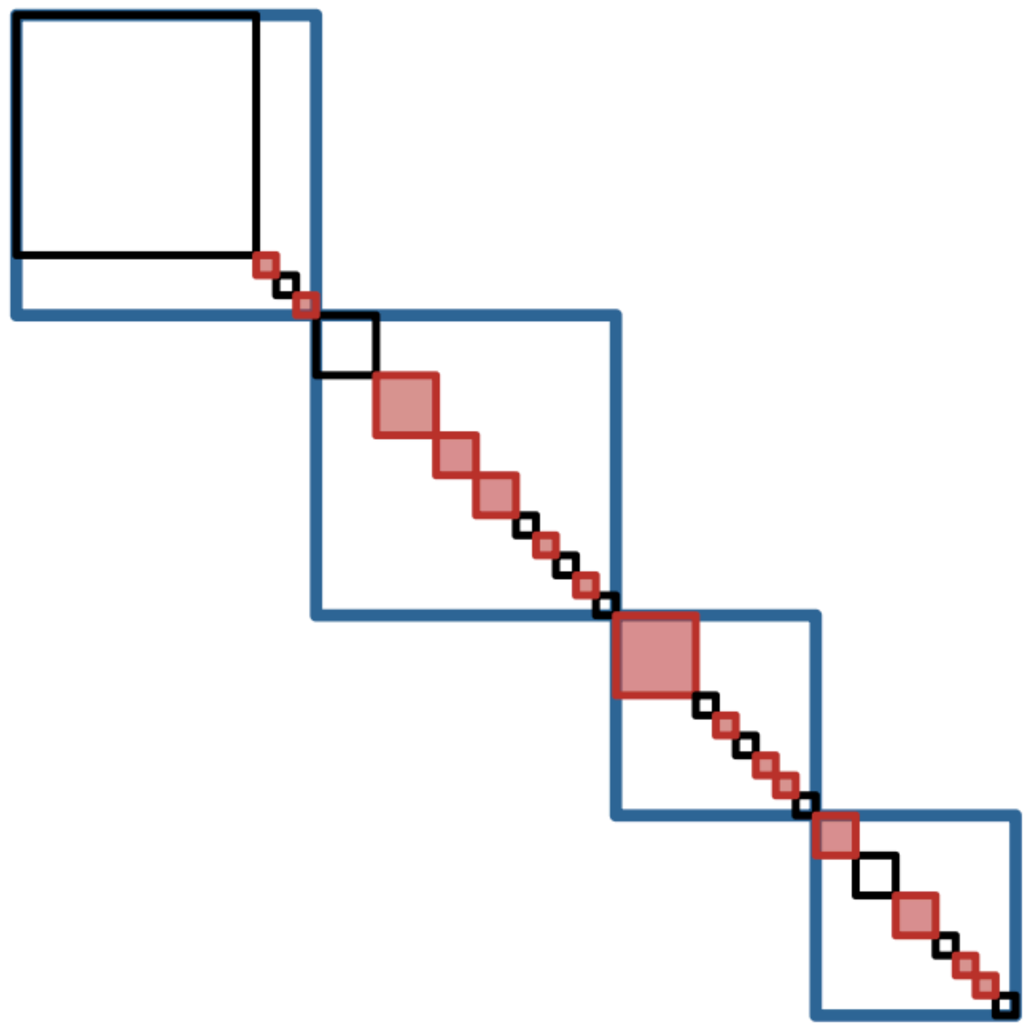
  [Moudgalya et al. Rept.Prog.Phys. 85 (2022) 8, 086501arXiv:2109.00548]

- ➡ Gauge symmetries $\left[G_n, H\right] = 0$  $\Rightarrow \mathcal{O}\left(\exp V\right)$ fragments

  [Steinegger et al., Phys. Rev. D 112, 114512 (2025) arXiv:2508.03802]

- ➡ Subsystem symmetries $\left[\sum_{\mathrm{column}} G_n, H\right]$  $\Rightarrow \mathcal{O}\left(\exp L\right)$ fragments

  [Steinegger et al., Phys. Rev. D 112, 114512 (2025) arXiv:2508.03802]



Hilbert space fragmentation in lattice field theories

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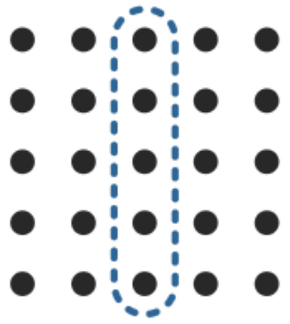
➡ Subsystem symmetries $[\sum_{\mathrm{column}} G_n, H]$  $\Rightarrow \mathcal{O}\left(\exp L\right)$ fragments

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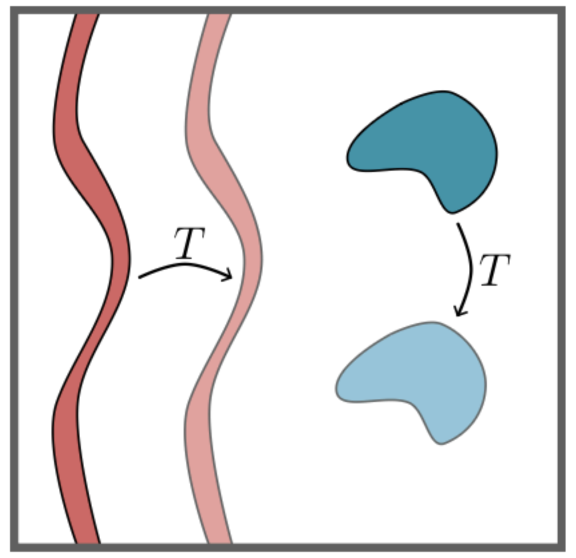

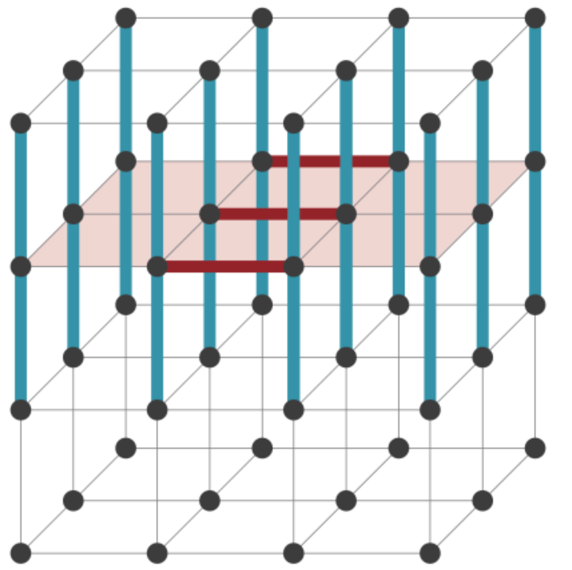

Generalized symmetries explain Hilbert space fragmentation [1]

[T. Budde, MKM, J. Pinto Barros, arXiv:2604.12907]

- Origin of fragmentation explained with previously unidentified generalized symmetries

- PXP model [see Zlatko's talk on Mon.]

$$\hat{H} = \sum_{n=1}^{L} \hat{P}_n \hat{X}_{n+1} \hat{P}_{n+2} \qquad \hat{P}_n = | \downarrow \rangle\langle \downarrow |_n$$

➡ when restricted to the a physical sector of the Hilbert space, excludes states with $| \uparrow\uparrow \rangle_{n,n+1}$ "Rydberg blockade regime"

➡ on the full computational basis has local $\mathbb{Z}_2$ symmetry

➡ $\tilde{G}_n = 2| \uparrow\uparrow \rangle\langle \uparrow\uparrow |_{n,n+1} - 1 \qquad [\hat{H}, | \uparrow\uparrow \rangle\langle \uparrow\uparrow |_{n,n+1}] = 0$

➡ full system: fragmented, exponentially many sectors, but explained by local $\mathbb{Z}_2$ symmetry

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- Revisit what do we call ergodicity breaking: 'Hilbert space fragmentation' for cases where it cannot be explained by generalized symmetries

Generalized symmetries explain Hilbert space fragmentation [2]

[T. Budde, MKM, J. Pinto Barros, arXiv:2604.12907]

- Partial isometries: individual symmetry sectors have additional symmetry

- Conserved operator

$$\hat{D} = \hat{U}\hat{P} \qquad\qquad [\hat{H}, \hat{D}] = 0$$

➡ $\hat{U}$ is unitary, but $[\hat{H}, \hat{U}] \neq 0$

➡ $\hat{P}$ projects onto a sector of $\hat{H}$

- $\hat{D}$ and other partial isometries are compatible with a generalized Wigner's theorem [G. Ortiz et al., Phys.Rev.B 113 (2026) 16, 165126 arXiv:2509.25327]

Generalized symmetries explain Hilbert space fragmentation [2]

[T. Budde, MKM, J. Pinto Barros, arXiv:2604.12907]

- Partial isometries: individual symmetry sectors have additional symmetry

- Critical Ising model

$$\hat{H} = \sum_{n=1}^{L} \left(\hat{X}_n + \hat{Z}_n \hat{Z}_{n+1} \right)$$

➡ is symmetric under $\hat{X}_n \rightsquigarrow \hat{Z}_{n-1}\hat{Z}_n$ and $\hat{Z}_n\hat{Z}_{n+1} \rightsquigarrow \hat{X}_{n-1}$

➡ This symmetry cannot be implemented by a unitary operator

➡ $\hat{U}\eta\hat{U}^{-1} = \hat{U}\prod_n \hat{X}_n \hat{U}^{-1} = \prod_n \hat{Z}_{n-1}\hat{Z}_n = 1 \quad \Rightarrow \quad \eta = 1 \quad \perp$

- This critical Ising model has a categorical symmetry: $\eta^2 = 1,\qquad \hat{D}\eta = \eta\hat{D} = D,\qquad \hat{D}^2 = (1+\eta)\hat{T}$

- Non-invertible translations, anomalies, and emanant symmetries [N. Seiberg, S.H. Shao, SciPost Phys. 16, 064 (2024) arXiv:2307.02534]

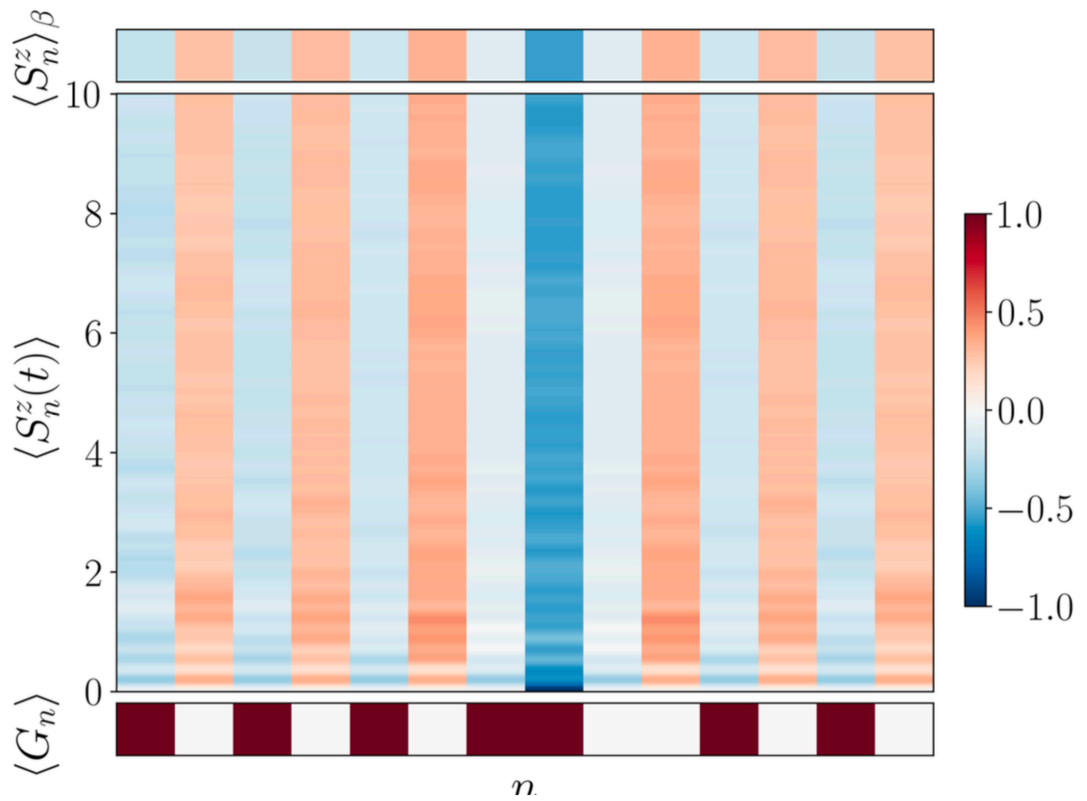
Generalized symmetries explain Hilbert space fragmentation [3]

[T. Budde, MKM, J. Pinto Barros, arXiv:2604.12907]

- Origin of fragmentation explained with previously unidentified generalized symmetries

- $U(1)$ Quantum Link Model in 1+1D, $S = 1$

$$\hat{H} = \sum_{n=1}^{L} \frac{\kappa}{2a} \left(\hat{c}_n^\dagger \hat{S}_n^+ \hat{c}_{n+1} + h.c. \right) + \frac{a}{2} \sum_{n=1}^{L} \left(\hat{S}_n^z \right)^2$$



(a)

Generalized symmetries explain Hilbert space fragmentation [3]

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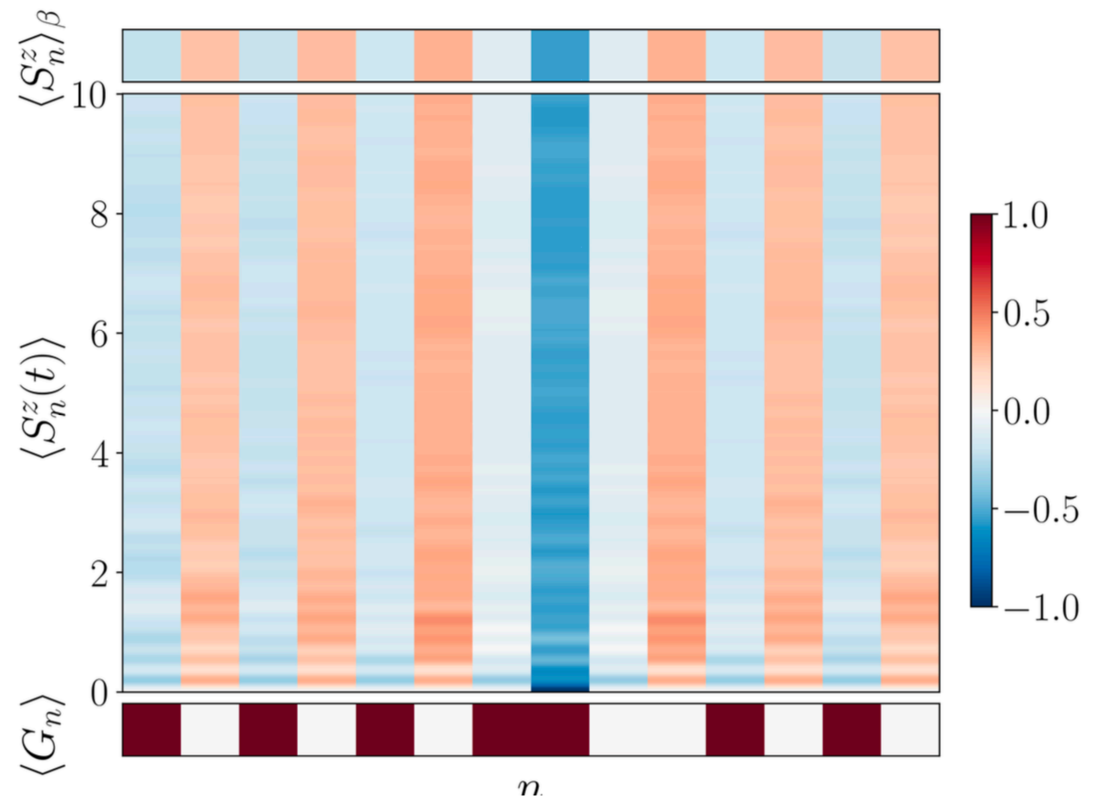
➡ Local generators $\hat{G}_n = \hat{c}_n^\dagger \hat{c}_n - \hat{S}_n^z + \hat{S}_{n-1}^z, \quad [\hat{G}_n, \hat{H}] = 0$



(a)

➡ Thermalizes into a state that breaks translation invariance but does not break ergodicity (a) L=14

➡ Possible thermalization within distinct (fragmented) Krylov sectors

Generalized symmetries explain Hilbert space fragmentation [3]

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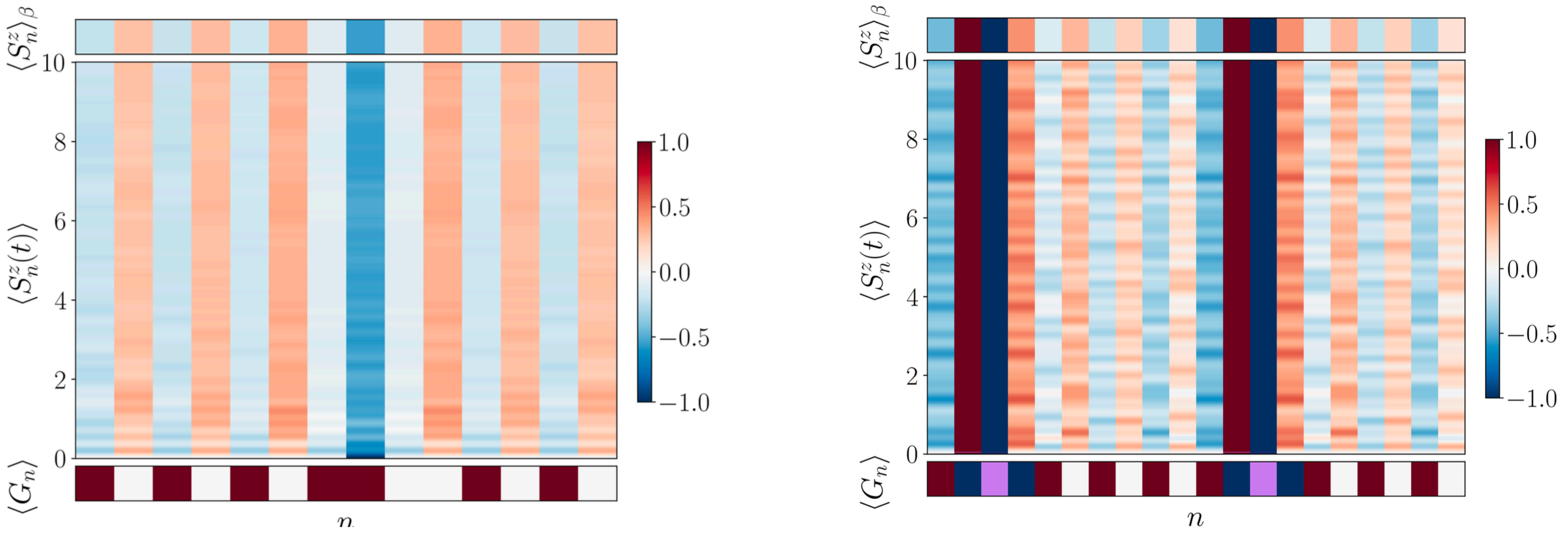


(a) (b)

➡ Thermalizes into a state that breaks translation invariance but does not break ergodicity (a) L=14

➡ Possible thermalization within distinct (fragmented) Krylov sectors, decoupling to subsystems survives in large-V limit (b) L=20

➡ Since it can be explained by symmetries, dynamics will not necessarily lead to exotic/athermal behaviour

Generalized symmetries explain Hilbert space fragmentation [3]

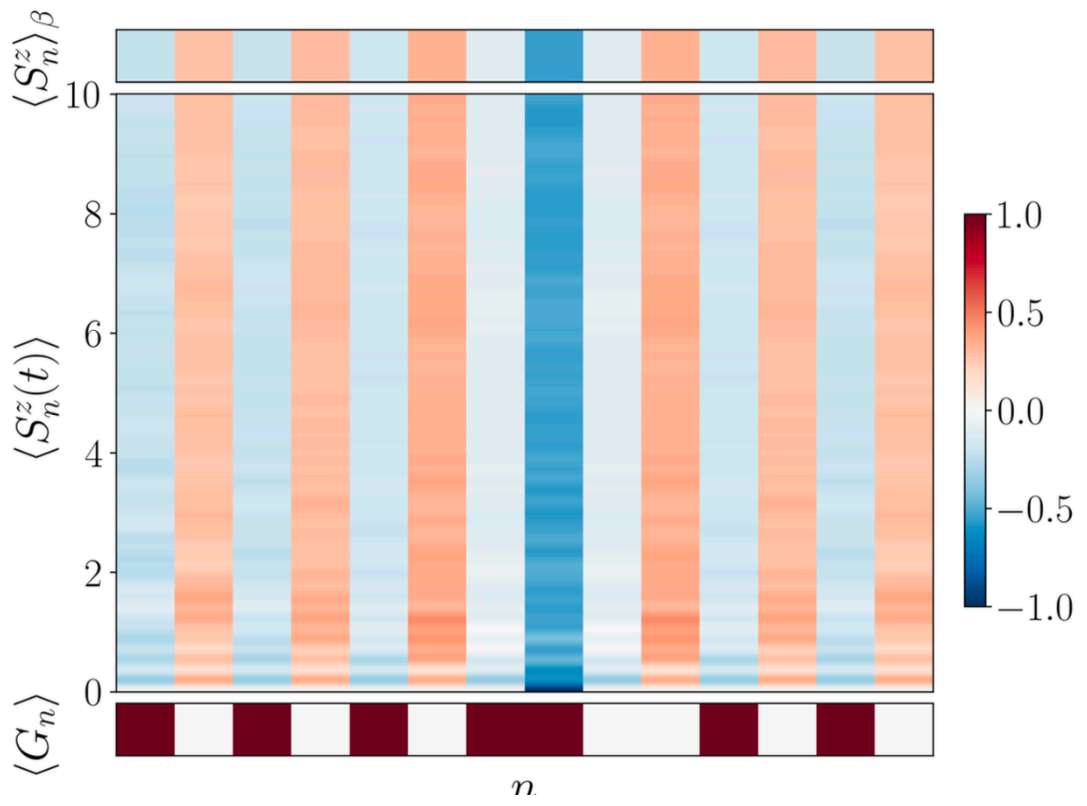
[T. Budde, MKM, J. Pinto Barros, arXiv:2604.12907]

- Origin of fragmentation explained with previously unidentified generalized symmetries

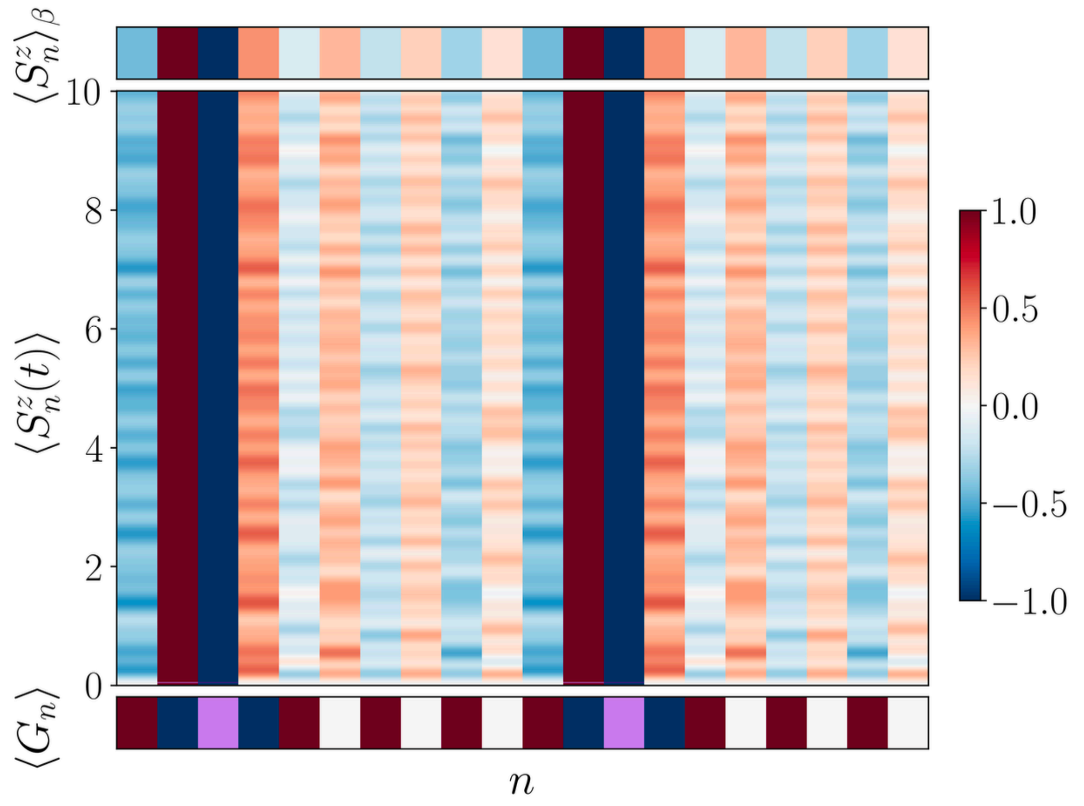
- $U(1)$ Quantum Link Model in 1+1D, $S = 1$

$$\hat{H} = \sum_{n=1}^{L} \frac{\kappa}{2a} \left(\hat{c}_n^\dagger \hat{S}_n^+ \hat{c}_{n+1} + h.c. \right) + \frac{a}{2} \sum_{n=1}^{L} \left(\hat{S}_n^z \right)^2$$

➡ Local generators $\hat{G}_n = \hat{c}_n^\dagger \hat{c}_n - \hat{S}_n^z + \hat{S}_{n-1}^z, \quad [\hat{G}_n, \hat{H}] = 0$



(a)



(b)

➡ Thermalizes into a state that breaks translation invariance but does not break ergodicity (a) L=14

➡ Possible thermalization within distinct (fragmented) Krylov sectors, decoupling to subsystems survives in large-V limit (b) L=20

➡ Since it can be explained by symmetries, dynamics will not necessarily lead to exotic/athermal behaviour

- Weak ergodicity breaking could be present in the model

Summary & Outlook

- Conventional MC approaches to LGTs fail at large densities, in studies of real-time dynamics, θ-terms … also anomalous thermalization

- Exceptions to thermalization: weak and strong ergodicity breaking phenomena

- Analytic construction of scars for arbitrary spin for U(1) lattice gauge theory in 2+1D [Banerjee, Sen PRL 126 (2021), Budde, MKM, Pinto Barros, PRD 110 (2024)]

- Strong ergodicity breaking such as fragmentation cannot be explained by conventional, 0-form, symmetries

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- Conventional MC approaches to LGTs fail at large densities, in studies of real-time dynamics, θ-terms … also anomalous thermalization
- Exceptions to thermalization: weak and strong ergodicity breaking phenomena
- Analytic construction of scars for arbitrary spin for U(1) lattice gauge theory in 2+1D [Banerjee, Sen PRL 126 (2021), Budde, MKM, Pinto Barros, PRD 110 (2024)]
- Strong ergodicity breaking such as fragmentation cannot be explained by conventional, 0-form, symmetries
- Resolving generalized symmetries explains the presence of an exponentially growing number of Krylov sectors in a broad class of models:
  - ➡ fragmentation in PXP model, critical Ising model, U(1) quantum link models, localization in spin-1 dipole conserving model, …
- Revisit what we call ergodicity breaking, not all fragmented systems are non-ergodic, if generalized symmetries offer an explanation
- Many models with ergodicity breaking (e. g. $t - J_z$ model) for which a generalized symmetries approach is not yet fully understood

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Additional slides

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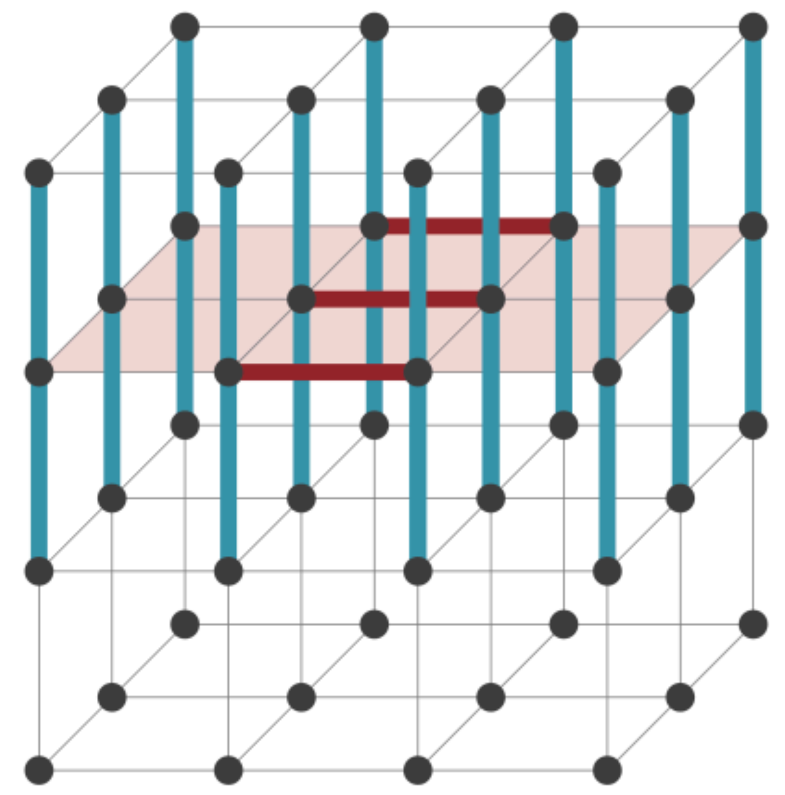

- $U(1)$ Quantum Link Model in 3+1D has 1-form symmetries over planes

$$W^{(1)}_{xy}(z) = \sum_{x,y} S^z_{r,\hat{y}}$$

➡ Only in the $\pm\frac{L^2}{2}$ sectors, the winding numbers W^{2D}_x are also conserved

➡ Exponentially many sub-fragments even though $[W^{(2)}_x, H] \neq 0$

➡ On the other hand $[\hat{H}, \hat{D}] = 0$, where $\hat{D} = W^{(2)}_x \hat{P}$ is a generalized symmetry

- Geometric fragmentation and anomalous thermalization [Steinegger et al., Phys. Rev. D 112, 114512 (2025) arXiv:2508.03802]

[Credit: T. Budde]