

Quantum Simulation of Integrable Models

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QuantHEP @



Queen Mary
University of London

Introduction



We are now in the "Utility Era"
with $\Theta(100)$ qubits

Quantum computations
hard to be simulated classically

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Quantum computations
hard to be simulated classically

How do you make sure
quantum computation is correct?



OK

Hmm, classical simulations are
by definition not available here :..

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What happens if the system
can be solved analytically?

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What happens if the system
can be solved analytically?

Why quantum computer if solved already?

OK



integrable models as stepping stones

Integrable Models: "exactly solvable"

- some observables can be computed exactly
- many conserved charges $\{Q_n\}$: $\frac{dQ_n}{dt} = 0$

Idea: Measure Q_n on quantum hardware

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benchmark quantum
devices

- noise
- error mitigation
/ correction

Study the physics of
integrability (breaking)

- ~~integrability~~ perturbation
- thermalization
non-equilibrium dynamics

Why quantum computer if solved already?



OK — We can

- * verify results
- * benchmark devices
- * study unsolved observables
- * break solvability by perturbation
- * study thermalization (ETH breaking)
- * compare with chaotic systems

Quantum Simulation of

Integrable Modes

I started working on this in 2021!

arXiv > quant-ph > arXiv:2208.00576

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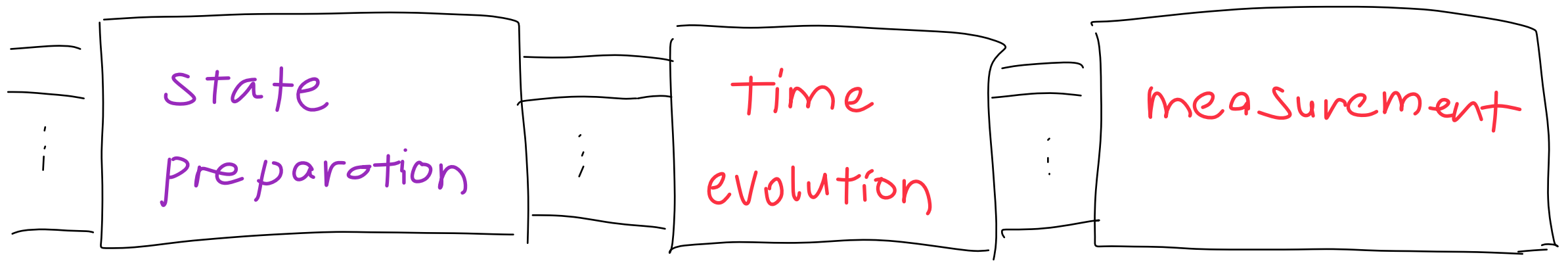
Quantum Physics

[Submitted on 1 Aug 2022 (v1), last revised 3 Apr 2023 (this version, v2)]

Conserved charges in the quantum simulation of integrable spin chains

Kazunobu Maruyoshi, Takuya Okuda, Juan William Pedersen, Ryo Suzuki, Masahito Yamazaki, Yutaka Yoshida

When simulating the time evolution of quantum many-body systems on a digital quantum computer, one faces the challenges of quantum noise and of the Trotter error due to time discretization. The Trotter error in integrable spin chains can be under control if the discrete time evolution preserves integrability. In this work we implement, on a real quantum computer and on classical simulators, the integrable Trotterization of the spin-1/2 Heisenberg XXX spin chain. We study how quantum noise affects the time evolution of several conserved charges, and observe the decay of the expectation values. We in addition study the early time behaviors of the time evolution, which can potentially be used to benchmark quantum devices and algorithms in the future. We also provide an efficient method to generate the conserved charges at higher orders.



Time Evolution

A hand-drawn wavy underline consisting of a single continuous line that starts on the left, dips slightly, rises to a small peak, dips again, and then rises to the right.

e.g. XXX spin- $\frac{1}{2}$ spin chain

$$H = \sum_{i=1}^N (X_i X_{i+1} + Y_i Y_{i+1} + Z_i Z_{i+1})$$

$$= \sum_{i=1}^N (2 \underbrace{P_{i,i+1}}_{\text{permutation}} - I_i I_{i+1})$$

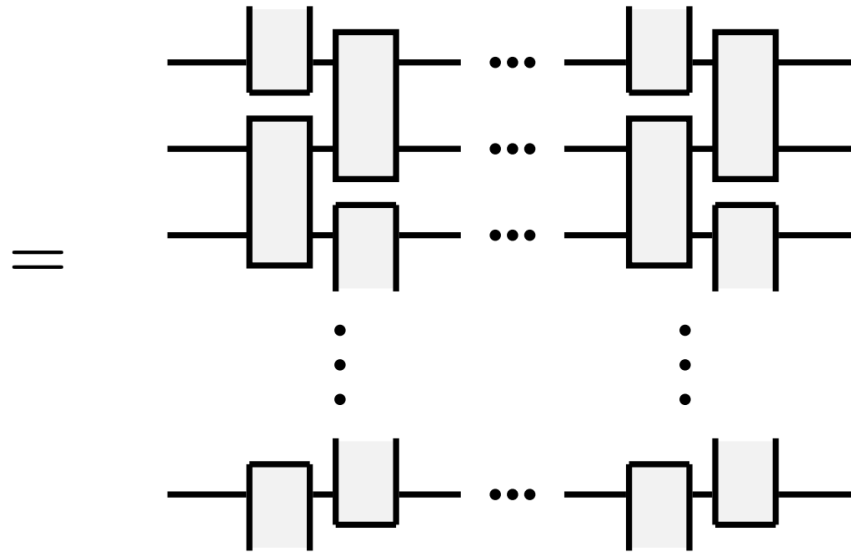
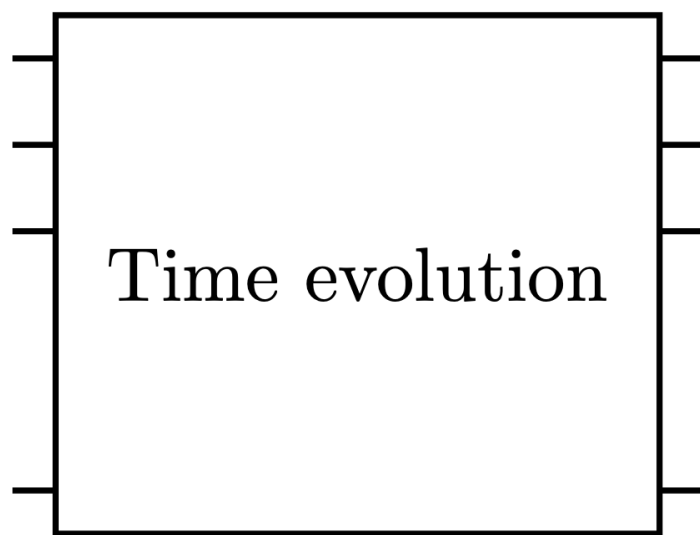
$$= \sum_{i: \text{even}} (2 P_{i,i+1} - I_i I_{i+1}) + \sum_{i: \text{odd}} (2 P_{i,i+1} - I_i I_{i+1})$$

H_{even}

H_{odd}

Standard Trotterization

$$e^{iHt} \sim \left(e^{i \frac{H_{\text{even}}}{N} t} e^{i \frac{H_{\text{odd}}}{N} t} \right)^N$$



brick-wall
circuit

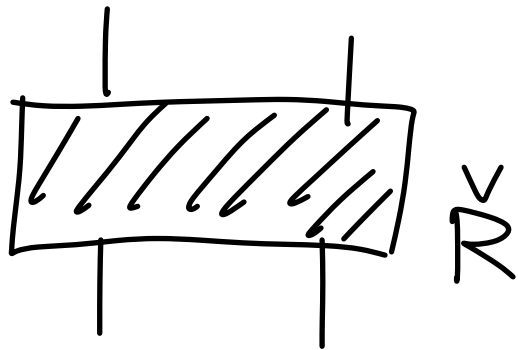
This, however, breaks integrability

Integrable Trotterization

[Destri - de Vega ('88, '89)
Vanicat - Zadnik - Prosen ('17)]

R-matrix $\searrow$ spectral parameter $\downarrow$ permutation $\swarrow$

$$\check{R}_{ij}(\lambda) = (1 + i\lambda P_{ij}) / (1 + i\lambda)$$



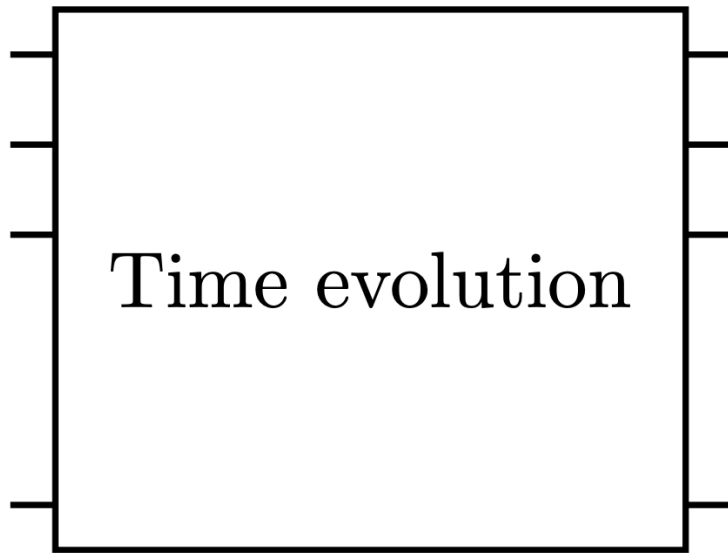
$$= \frac{1}{1 + i\lambda} \begin{matrix} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{matrix} \begin{pmatrix} 1 & & & \\ & 1 & i\lambda & \\ & i\lambda & 1 & \\ & & & 1 \end{pmatrix}$$

Integrable Trotterization

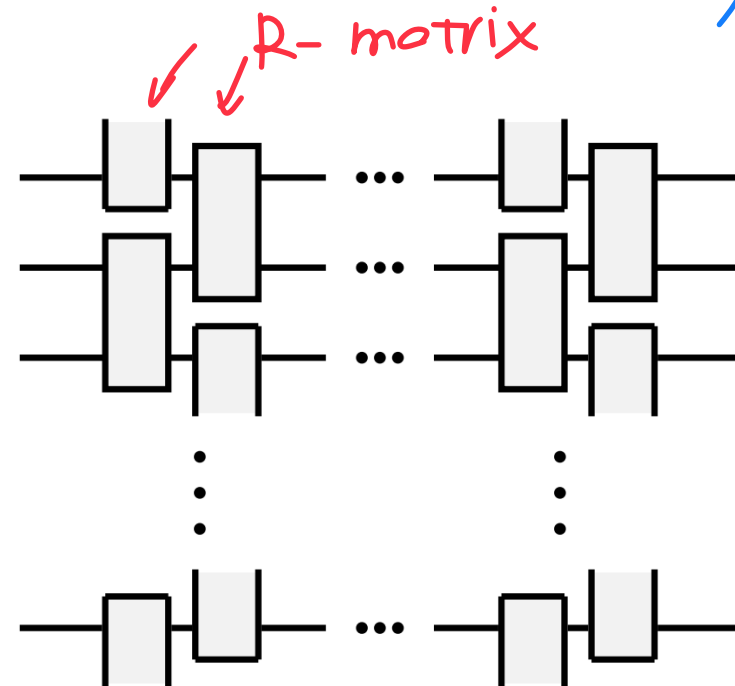
[Destri - de Vega ('88, '89)
Vanicat - Zadnik - Prosen ('17)]

$$\mathcal{U}(\delta) = \left(\prod_{j=1}^{N/2} \overset{v}{R}_{2j-1, 2j}(\delta) \right) \left(\prod_{j=1}^{N/2} \overset{v}{R}_{2j, 2j+1}(\delta) \right)$$

deformation parameter

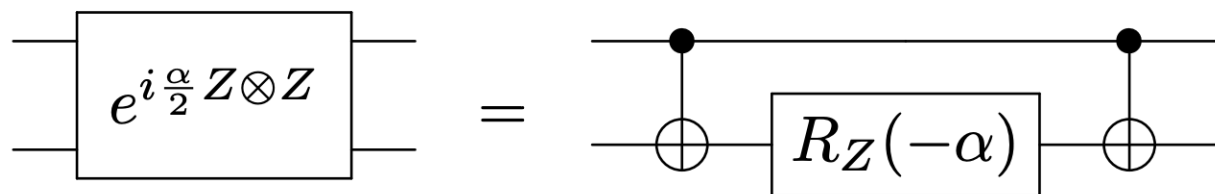
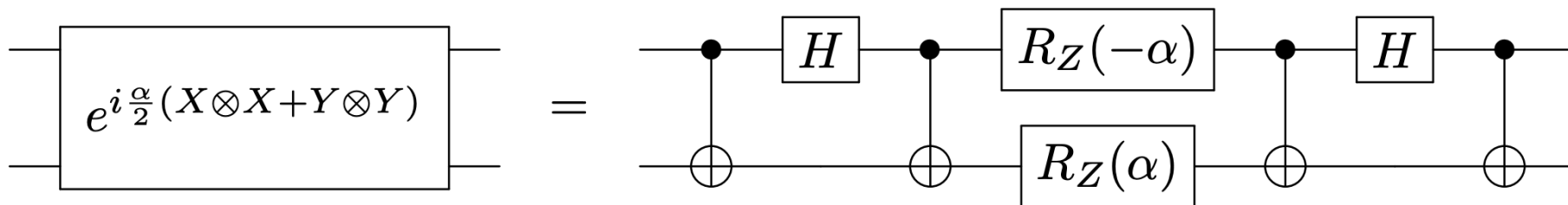


=



$$R_{j,j+1}(\delta) = e^{-i\frac{\alpha}{2}} e^{\frac{i\alpha}{2}(X_j X_{j+1} + Y_j Y_{j+1})} e^{i\frac{\alpha}{2} Z_j Z_{j+1}}$$

($\delta = \tan \theta$)



Measurement of Conserved Charges

- $T(\lambda) := \text{tr}_0 \left[\prod_{1 \leq j \leq N} R_{0j} \left(\lambda - \underbrace{(-1)^j \frac{\delta}{2}}_{\begin{array}{c} \downarrow \\ \downarrow \\ \downarrow \\ \text{xxx} \rightarrow \text{xxx}_\delta \end{array}} \right) \right]$
 transfer matrix

- conserved charges

$$Q_n^\pm = \frac{d^n}{d\lambda^n} \log T(\lambda) \Big|_{\lambda = \pm \delta/2}$$

- recursion by boost operator

$$Q_{n+1}^\pm = [B, Q_n^\pm]$$

B

↑

known from
R-matrix

$$Q_n^+(\delta) = \sum_{j=1}^{N/2} q_{2j-2, 2j-1, \dots, 2j+2n-2}^{[n, +]}(\delta),$$

$$Q_n^-(\delta) = \sum_{j=1}^{N/2} q_{2j-1, 2j, \dots, 2j+2n-1}^{[n, -]}(\delta),$$

$$q_{1,2,3}^{[1, \pm]}(\delta) = \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + \boldsymbol{\sigma}_2 \cdot \boldsymbol{\sigma}_3 \mp \delta \boldsymbol{\sigma}_1 \cdot (\boldsymbol{\sigma}_2 \times \boldsymbol{\sigma}_3) + \delta^2 \boldsymbol{\sigma}_2 \cdot \boldsymbol{\sigma}_3,$$

$$\begin{aligned} q_{1,2,3,4,5}^{[2, \pm]}(\delta) = & \mp 2\delta(\boldsymbol{\sigma}_3 \cdot \boldsymbol{\sigma}_4 + \boldsymbol{\sigma}_4 \cdot \boldsymbol{\sigma}_5 - \boldsymbol{\sigma}_3 \cdot \boldsymbol{\sigma}_5) - (1 - \delta^2)\boldsymbol{\sigma}_3 \cdot (\boldsymbol{\sigma}_4 \times \boldsymbol{\sigma}_5) - \boldsymbol{\sigma}_2 \cdot (\boldsymbol{\sigma}_3 \times \boldsymbol{\sigma}_4) - \delta^2 \boldsymbol{\sigma}_2 \cdot (\boldsymbol{\sigma}_3 \times \boldsymbol{\sigma}_5) \\ & - \delta^2 \boldsymbol{\sigma}_1 \cdot (\boldsymbol{\sigma}_3 \times \boldsymbol{\sigma}_4) - \delta^4 \boldsymbol{\sigma}_1 \cdot (\boldsymbol{\sigma}_3 \times \boldsymbol{\sigma}_5) \pm \delta \boldsymbol{\sigma}_2 \cdot (\boldsymbol{\sigma}_3 \times \boldsymbol{\sigma}_4 \times \boldsymbol{\sigma}_5) \pm \delta \boldsymbol{\sigma}_1 \cdot (\boldsymbol{\sigma}_2 \times \boldsymbol{\sigma}_3 \times \boldsymbol{\sigma}_4) \\ & \pm \delta^3 \boldsymbol{\sigma}_1 \cdot (\boldsymbol{\sigma}_3 \times \boldsymbol{\sigma}_4 \times \boldsymbol{\sigma}_5) \pm \delta^3 \boldsymbol{\sigma}_1 \cdot (\boldsymbol{\sigma}_2 \times \boldsymbol{\sigma}_3 \times \boldsymbol{\sigma}_5) - \delta^2 \boldsymbol{\sigma}_1 \cdot (\boldsymbol{\sigma}_2 \times \boldsymbol{\sigma}_3 \times \boldsymbol{\sigma}_4 \times \boldsymbol{\sigma}_5). \end{aligned}$$

$$\begin{aligned}
q_{1,2,3,4,5,6,7}^{[3,+]} = & -4\sigma_6 \cdot \sigma_7 + 2\sigma_5 \cdot \sigma_7 - 4\sigma_5 \cdot \sigma_6 + 2\sigma_4 \cdot \sigma_6 + 2\sigma_4 \cdot (\sigma_5 \times \sigma_6 \times \sigma_7) + 2\sigma_3 \cdot (\sigma_4 \times \sigma_5 \times \sigma_6) \\
& + \delta \left(10\sigma_5 \cdot (\sigma_6 \times \sigma_7) - 2\sigma_4 \cdot (\sigma_6 \times \sigma_7) - 4\sigma_4 \cdot (\sigma_5 \times \sigma_7) + 8\sigma_4 \cdot (\sigma_5 \times \sigma_6) - 4\sigma_3 \cdot (\sigma_5 \times \sigma_6) \right. \\
& \left. - 2\sigma_3 \cdot (\sigma_4 \times \sigma_6) - 4\sigma_3 \cdot (\sigma_4 \times \sigma_5 \times \sigma_6 \times \sigma_7) - 2\sigma_2 \cdot (\sigma_3 \times \sigma_4 \times \sigma_5 \times \sigma_6) \right) \\
& + \delta^2 \left(2\sigma_6 \cdot \sigma_7 - 10\sigma_5 \cdot \sigma_7 + 2\sigma_5 \cdot \sigma_6 + 2\sigma_4 \cdot \sigma_7 + 2\sigma_4 \cdot \sigma_6 + 2\sigma_3 \cdot \sigma_6 \right. \\
& - 6\sigma_4 \cdot (\sigma_5 \times \sigma_6 \times \sigma_7) + 6\sigma_3 \cdot (\sigma_5 \times \sigma_6 \times \sigma_7) + 2\sigma_3 \cdot (\sigma_4 \times \sigma_6 \times \sigma_7) + 6\sigma_3 \cdot (\sigma_4 \times \sigma_5 \times \sigma_7) \\
& - 6\sigma_3 \cdot (\sigma_4 \times \sigma_5 \times \sigma_6) + 2\sigma_2 \cdot (\sigma_3 \times \sigma_5 \times \sigma_6) \\
& \left. + 2\sigma_2 \cdot (\sigma_3 \times \sigma_4 \times \sigma_5 \times \sigma_6 \times \sigma_7) + 2\sigma_1 \cdot (\sigma_2 \times \sigma_3 \times \sigma_4 \times \sigma_5 \times \sigma_6) \right) \\
& + \delta^3 \left(6\sigma_5 \cdot (\sigma_6 \times \sigma_7) - 2\sigma_4 \cdot (\sigma_6 \times \sigma_7) + 4\sigma_4 \cdot (\sigma_5 \times \sigma_7) - 2\sigma_3 \cdot (\sigma_6 \times \sigma_7) - 8\sigma_3 \cdot (\sigma_5 \times \sigma_7) \right. \\
& - 2\sigma_3 \cdot (\sigma_4 \times \sigma_6) + 4\sigma_3 \cdot (\sigma_5 \times \sigma_6) - 2\sigma_3 \cdot (\sigma_4 \times \sigma_7) + 4\sigma_3 \cdot (\sigma_4 \times \sigma_5 \times \sigma_6 \times \sigma_7) \\
& - 2\sigma_2 \cdot (\sigma_3 \times \sigma_5 \times \sigma_6 \times \sigma_7) - 2\sigma_2 \cdot (\sigma_3 \times \sigma_4 \times \sigma_5 \times \sigma_7) - 2\sigma_1 \cdot (\sigma_3 \times \sigma_4 \times \sigma_5 \times \sigma_6) \\
& \left. - 2\sigma_1 \cdot (\sigma_2 \times \sigma_3 \times \sigma_5 \times \sigma_6) - 2\sigma_1 \cdot (\sigma_2 \times \sigma_3 \times \sigma_4 \times \sigma_5 \times \sigma_6 \times \sigma_7) \right) \\
& + \delta^4 \left(-2\sigma_6 \cdot \sigma_7 - 8\sigma_5 \cdot \sigma_7 - 2\sigma_5 \cdot \sigma_6 + 2\sigma_4 \cdot \sigma_7 + 2\sigma_3 \cdot \sigma_6 + 2\sigma_3 \cdot \sigma_7 - 2\sigma_3 \cdot (\sigma_5 \times \sigma_6 \times \sigma_7) \right. \\
& + 2\sigma_3 \cdot (\sigma_4 \times \sigma_6 \times \sigma_7) - 2\sigma_3 \cdot (\sigma_4 \times \sigma_5 \times \sigma_7) + 2\sigma_2 \cdot (\sigma_3 \times \sigma_5 \times \sigma_7) + 2\sigma_1 \cdot (\sigma_3 \times \sigma_5 \times \sigma_6) \\
& \left. + 2\sigma_1 \cdot (\sigma_3 \times \sigma_4 \times \sigma_5 \times \sigma_6 \times \sigma_7) + 2\sigma_1 \cdot (\sigma_2 \times \sigma_3 \times \sigma_5 \times \sigma_6 \times \sigma_7) + 2\sigma_1 \cdot (\sigma_2 \times \sigma_3 \times \sigma_4 \times \sigma_5 \times \sigma_7) \right) \\
& + \delta^5 \left(4\sigma_5 \cdot (\sigma_6 \times \sigma_7) - 2\sigma_3 \cdot (\sigma_6 \times \sigma_7) - 2\sigma_3 \cdot (\sigma_4 \times \sigma_7) - 2\sigma_1 \cdot (\sigma_3 \times \sigma_5 \times \sigma_6 \times \sigma_7) \right. \\
& \left. - 2\sigma_1 \cdot (\sigma_3 \times \sigma_4 \times \sigma_5 \times \sigma_7) - 2\sigma_1 \cdot (\sigma_2 \times \sigma_3 \times \sigma_5 \times \sigma_7) \right) \\
& + \delta^6 \left(-4\sigma_5 \cdot \sigma_7 + 2\sigma_3 \cdot \sigma_7 + 2\sigma_1 \cdot (\sigma_3 \times \sigma_5 \times \sigma_7) \right).
\end{aligned}$$

$$Q = \sum_{P \in \{I, X, Y, Z\}^{\otimes n}}$$

$\mathbb{C}$ Pauli word

$c_{Q,P}$ P



measure in

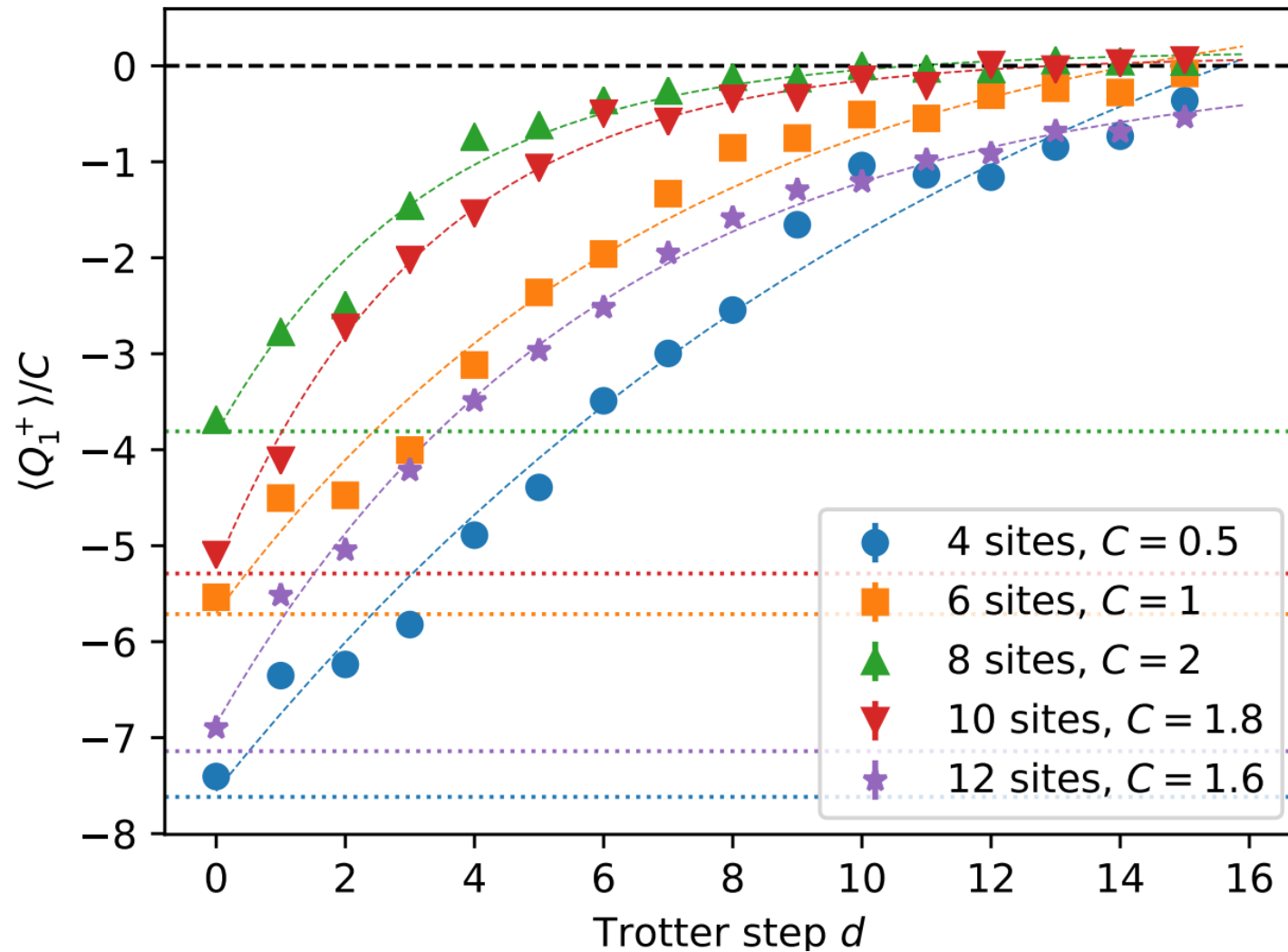
$$\left(\begin{array}{l} Z: |0\rangle, |1\rangle \\ X: |H0\rangle, |H1\rangle \\ Y: |SH0\rangle, |SH1\rangle \end{array} \right)$$

Results

[Maruyoshi - Okuda - Pedersen - Suzuki - Yamazaki - Okuda]
(22)

[MY + Alberton - Lindner - Scheider (to appear)]

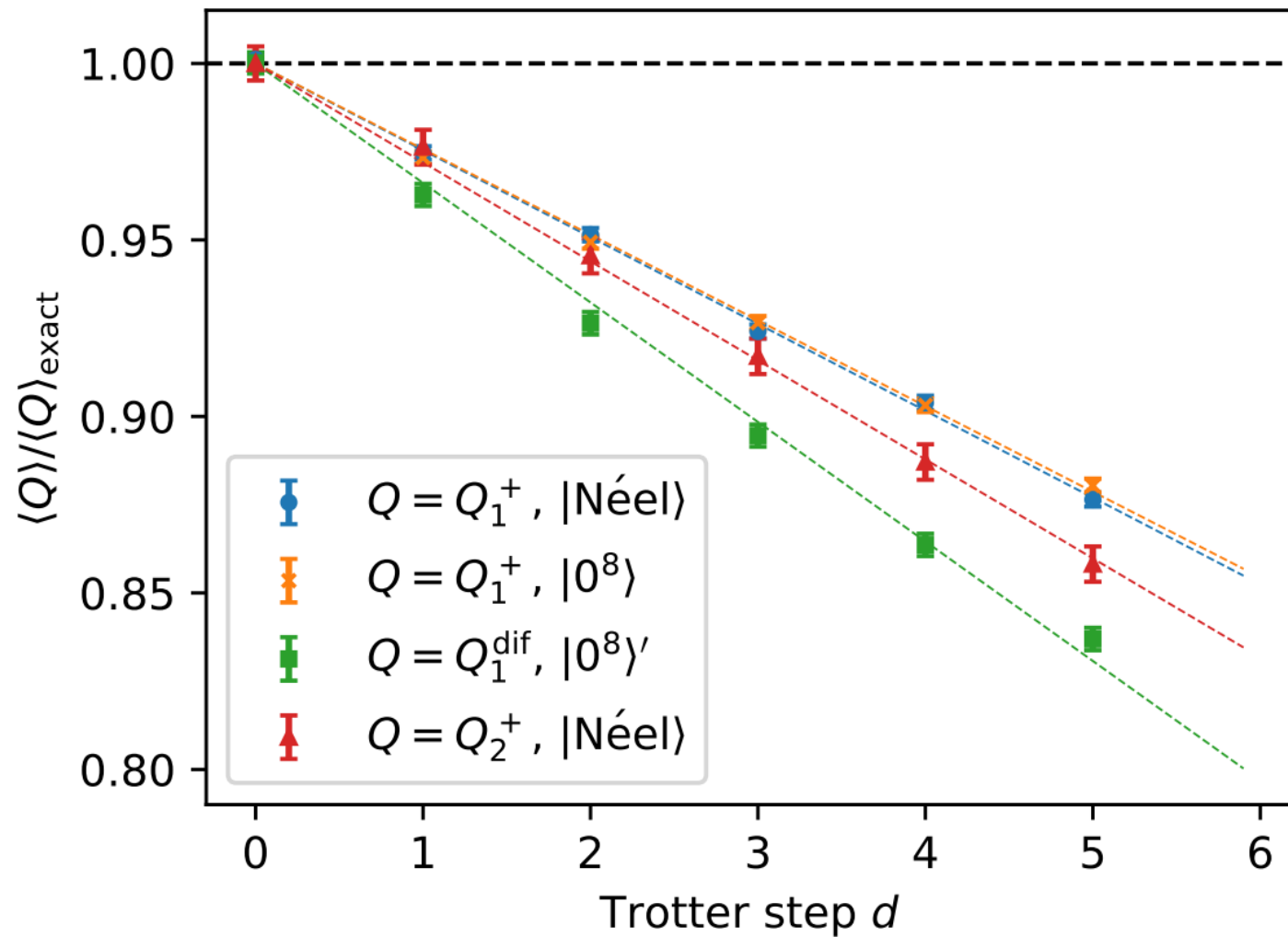
[Maruyoshi - Okuda - Pedersen - Suzuki - Yamazaki - Okuda]
(22)



Superconducting
qubits

(ibm kawasaki)
 10^5 shots

also ion traps



classical sim..

(2Q error $p=0.0013$)
 (1Q error $p=0.00013$)

benchmark quantum devices?

After ~ 5 years,

things are "better than expected" !

Game - Changers

* upgrade of IBM machines

Falcone → Eagle → Heron

* Advancement of error mitigation

Qiskit - function QESem

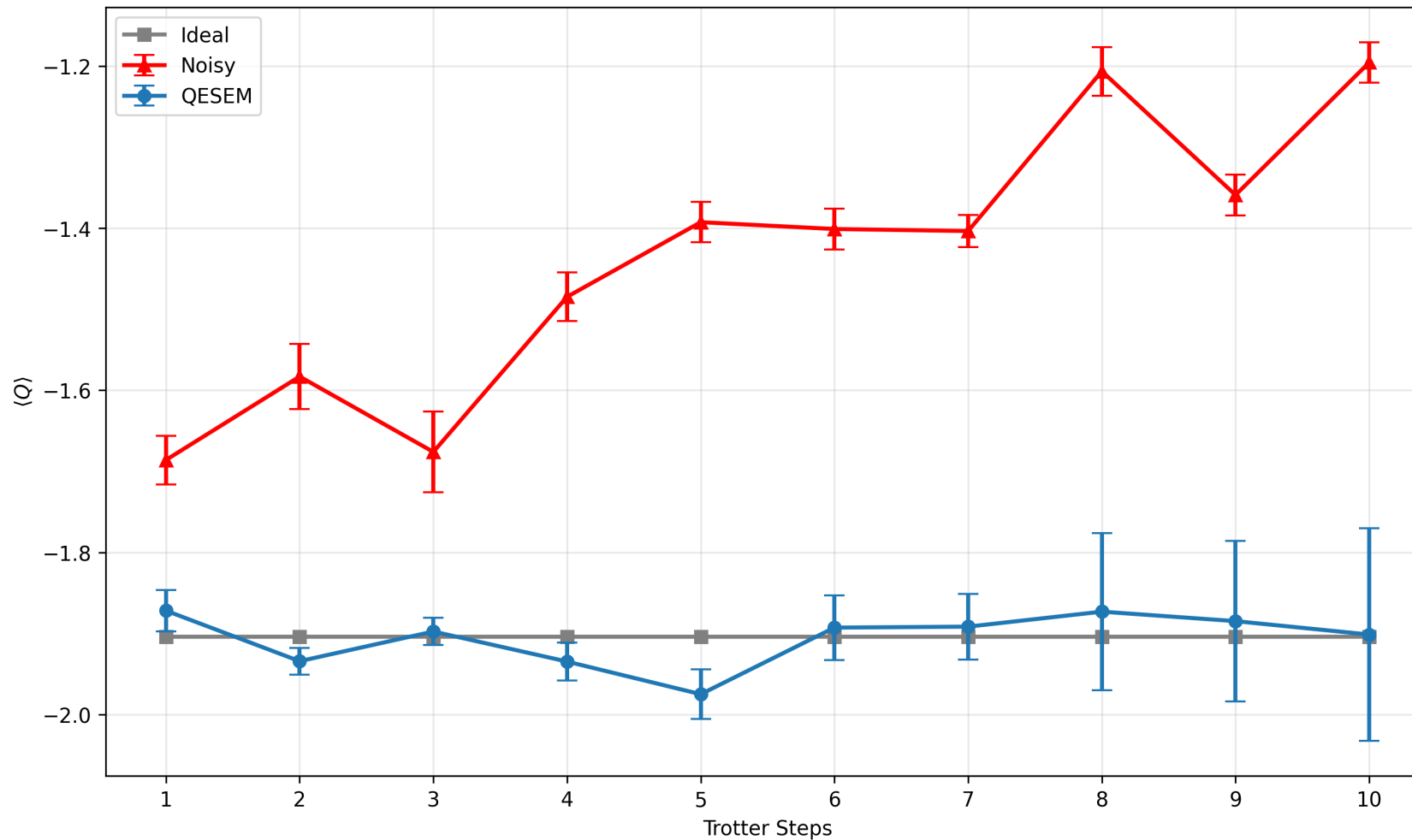
arXiv: 2508.10997

collaboration
w/

T. Schneider, O. Alberton, N. Lidner



Preliminary Results



QPU: ibm_kingston
Number of qubits: 12
Trotter steps: 10
Number of 2 qubit gates: 480

almost constant after error mitigation

Quantum Simulation of

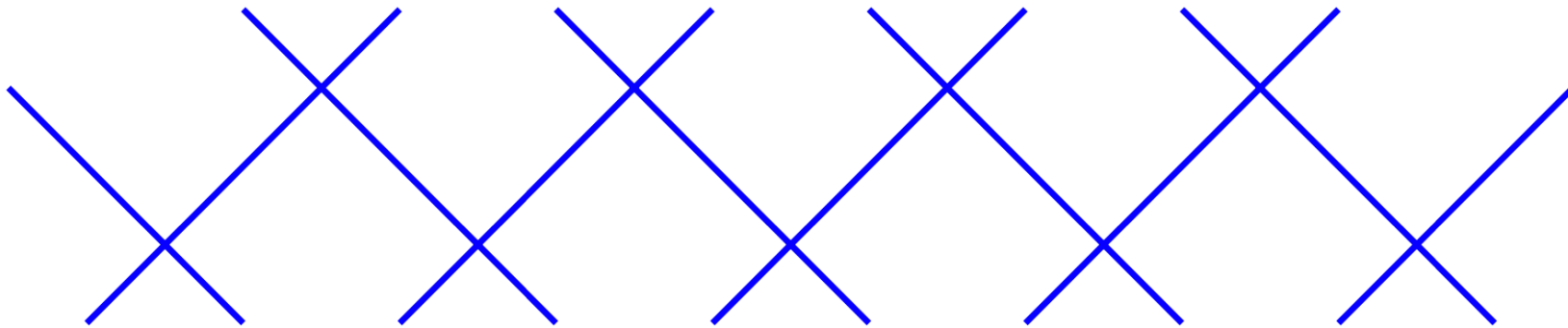
Integrable Quantum Field Theories

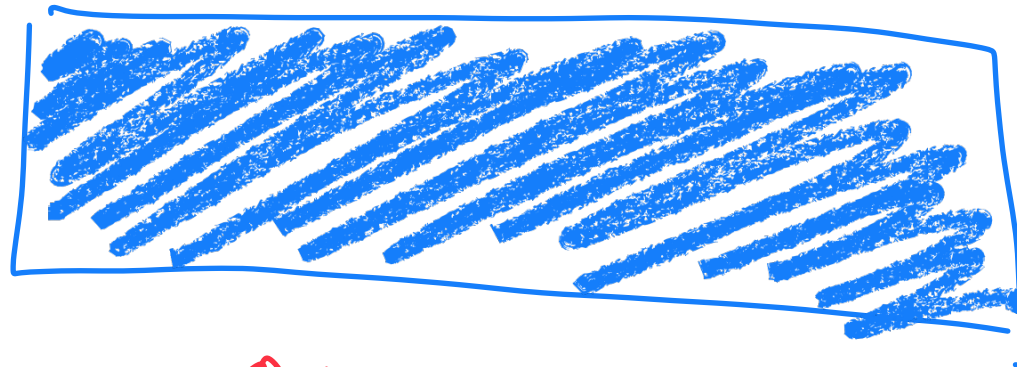
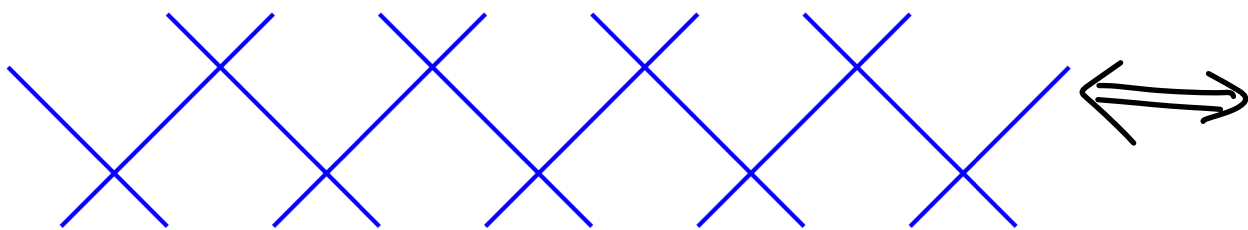
Historically, one of the motivations of

integrable models is to solve QFT

[Faddeev-Reshetikhin ('86), Faddeev-Volkov ('92, '93)]

brick-wall circuit = light-cone discretization
of spacetime



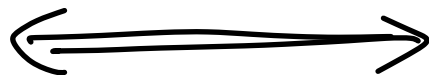


quantum circuit

polarization

QFT

XXZ



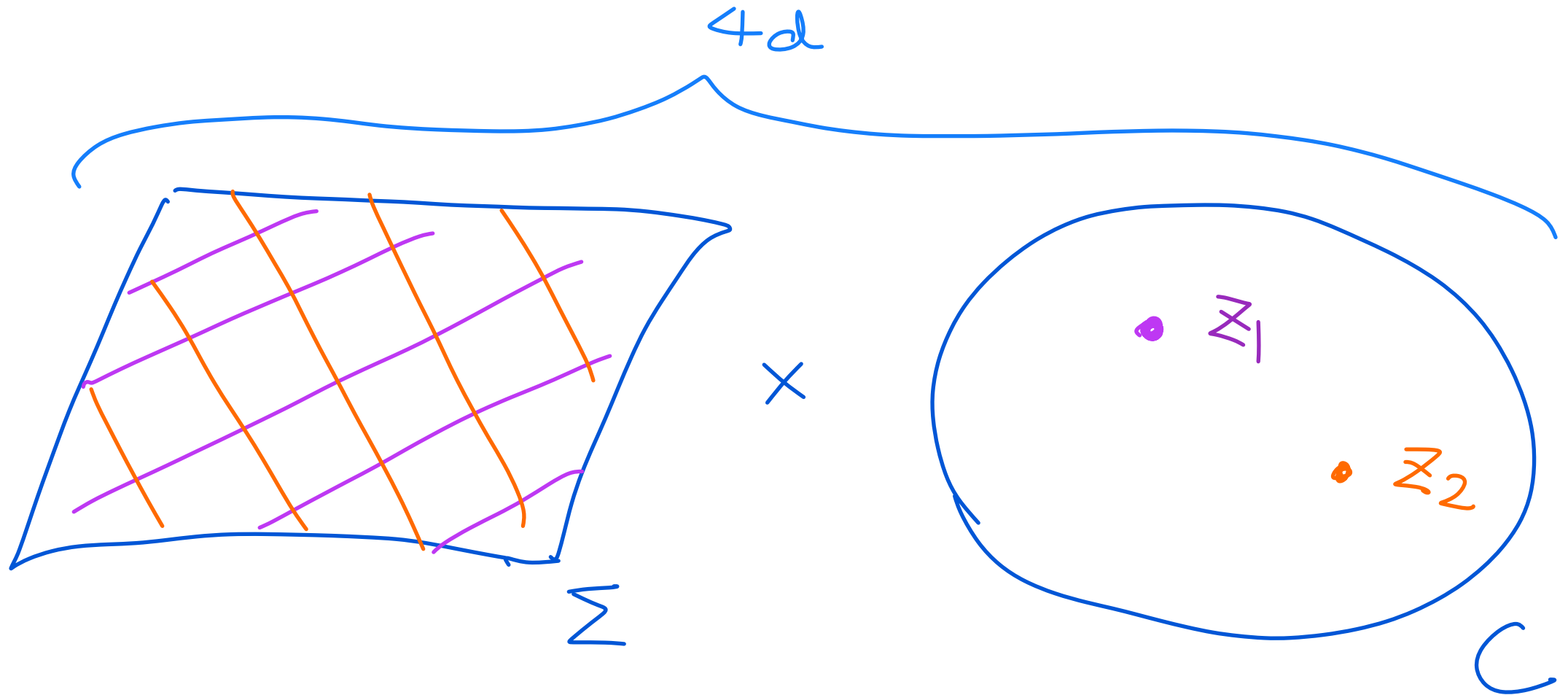
Sine-Gordon
/ massive-Thirring

[Luther-Peschel ('75)
Destri-de Vega ('90s)]

Surprisingly, no general prescription existed

for (lattice circuit) $\iff$ QFT until 2023!

The problem simplifies dramatically
when we go to four dimensions

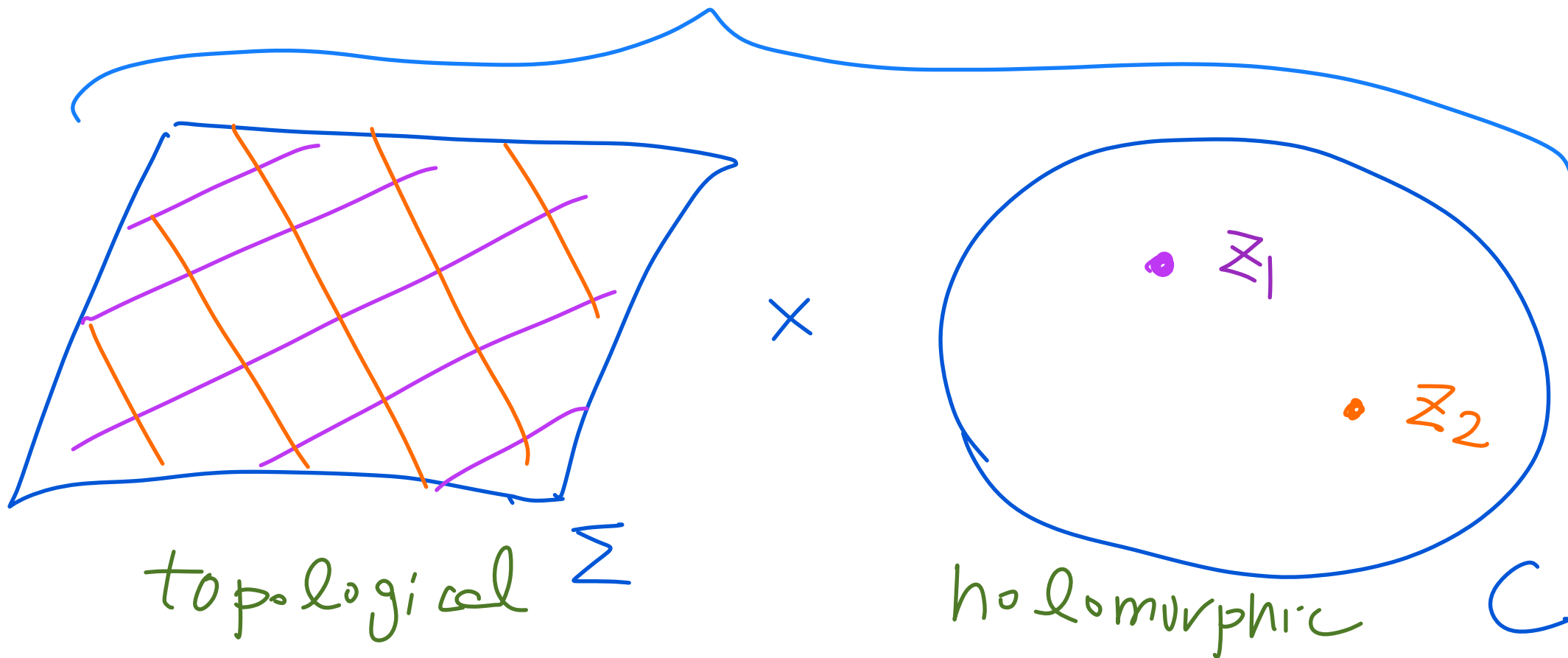


The relevant theory is the

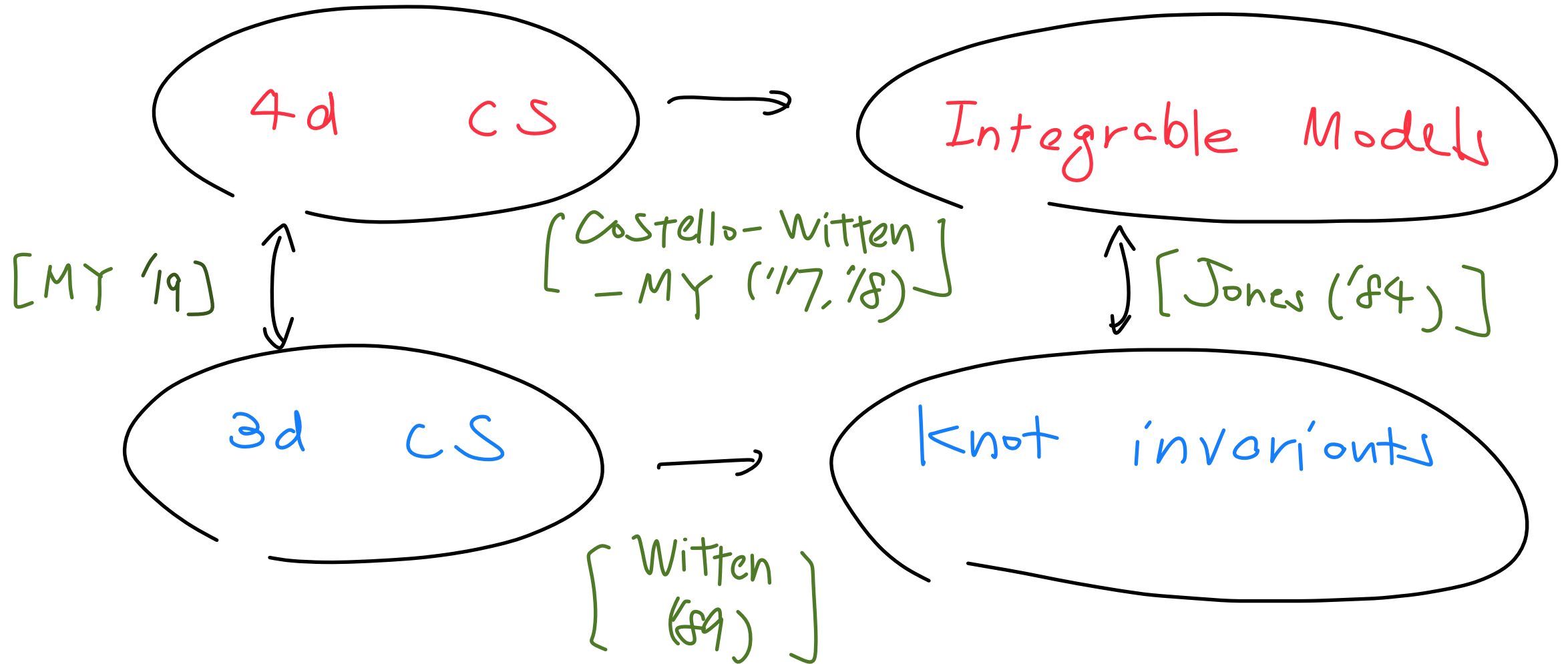
4d Chern-Simons theory

[Costello-Witten-MY
(17, 18)]

4d



This gives conceptual explanation of
integrable models

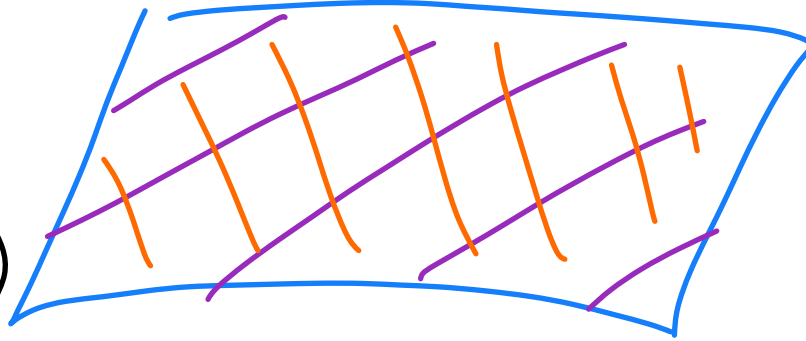


We can take the limit "defect-wise"

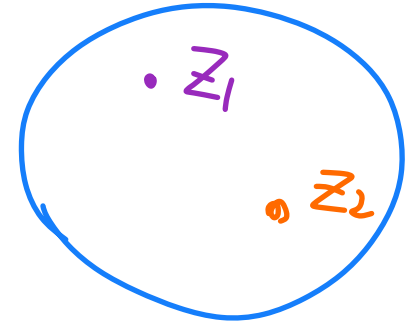
[Costello - Witten - MY ('17, '18)]

quantum circuit

($4d + 1d$ defect)



x



[Ashwinkumar - Sakamoto - MY ('23)]

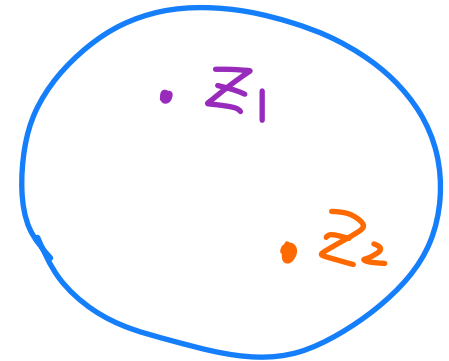
QFT

($4d + 2d$ defect)

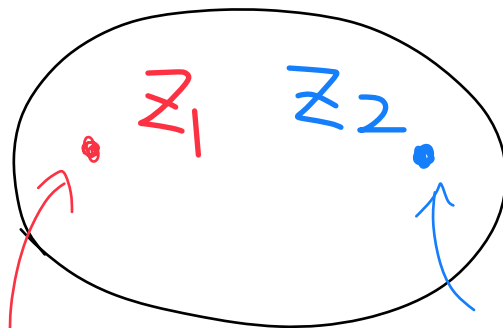
[Costello - MY ('19)]



x



e.g. 1



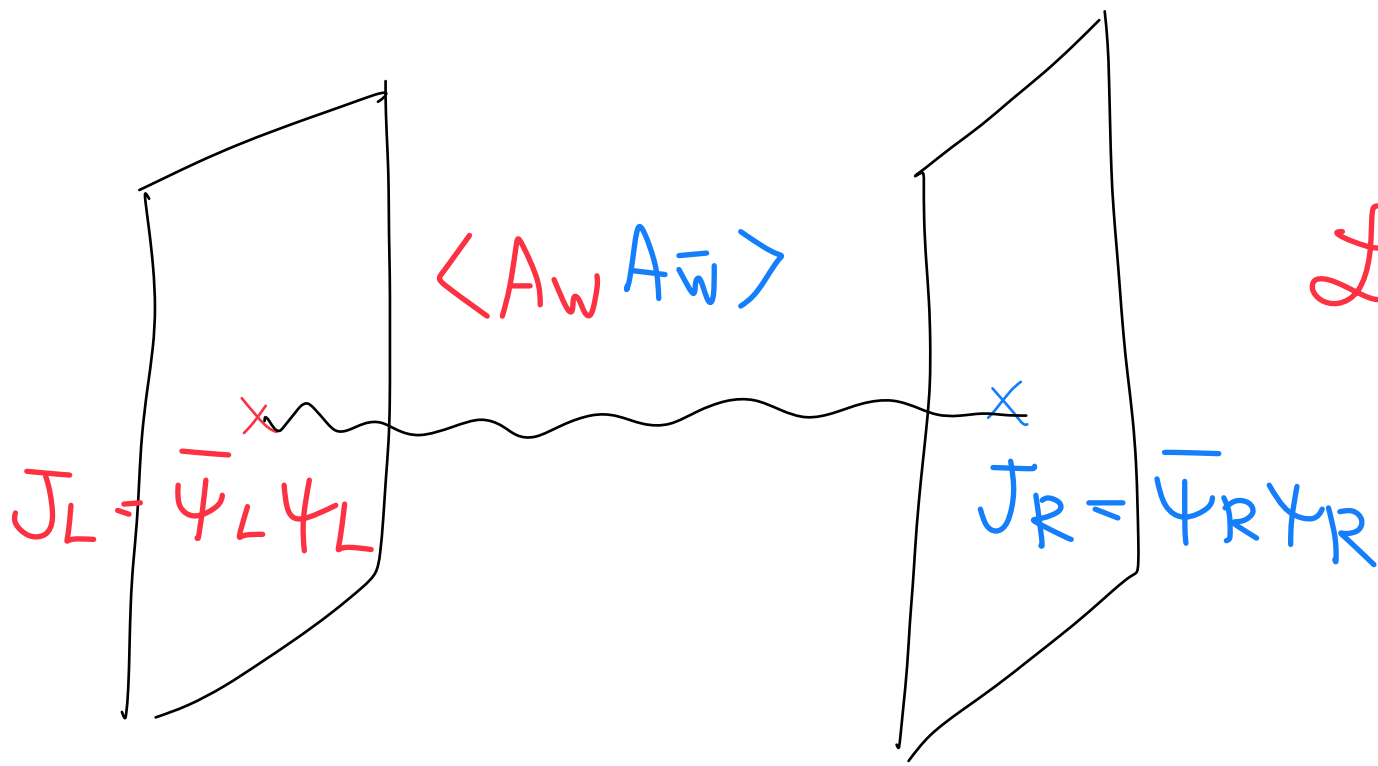
(Costello - MY '19)

chiral fermion

$$\mathcal{L}_1 = \bar{\psi}_L (\not{\partial} + A_W) \psi_L$$

anti-chiral fermion

$$\mathcal{L}_2 = \bar{\psi}_R (\not{\partial} + A_{\bar{W}}) \psi_R$$



$$\mathcal{L}_1 + \mathcal{L}_2 + \frac{1}{z_1 - z_2} (\bar{\psi}_L \psi_L) (\bar{\psi}_R \psi_R)$$

4 - Fermi

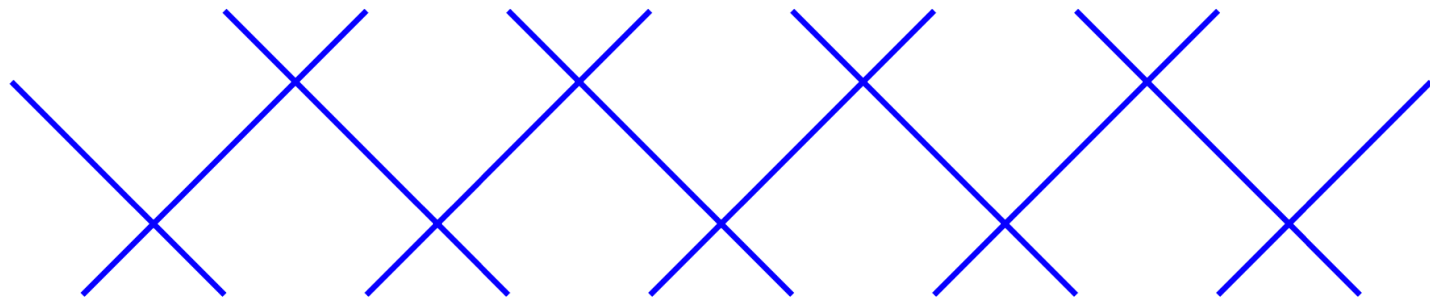
chiral defect

$$S = \int d^2\sigma \bar{\Psi} (\not{\partial}_w + A_w) \Psi \rightsquigarrow S = \sum_i \int d\sigma^- \bar{\Psi}_i (\not{\partial}_w + A_w) \Psi_i$$

anti-chiral defect

$$S = \int d^2\sigma \bar{\Psi} (\not{\partial}_{\bar{w}} + A_{\bar{w}}) \Psi \rightsquigarrow S = \sum_i \int d\sigma^+ \bar{\Psi}_i (\not{\partial}_{\bar{w}} + A_{\bar{w}}) \Psi_i$$

when combined, light-cone lattice of **1d defects**



[Borel-Weil-Bott]

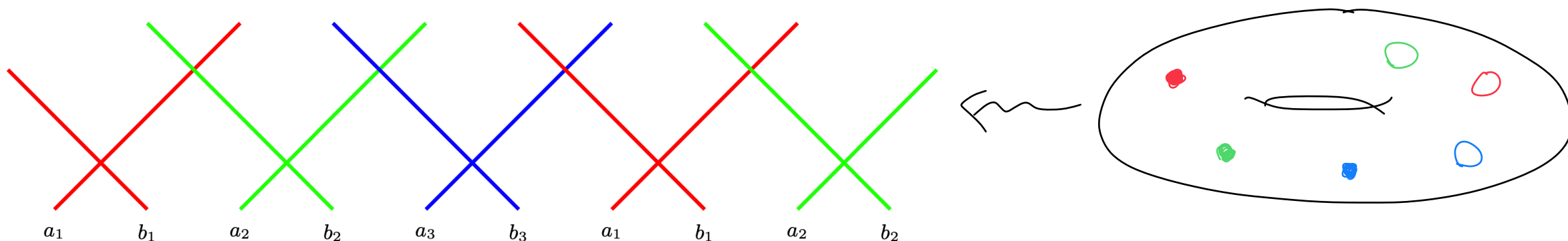
quantum circuit

This construction gives

a zoo of new integrable QFTs
[Costello - M $\mathbb{Y}$ ('19)]

& their quantum-circuit realizations
[Ashwinkumar - Sakamoto - M $\mathbb{Y}$ ('23)]

We can discuss many defects of different types

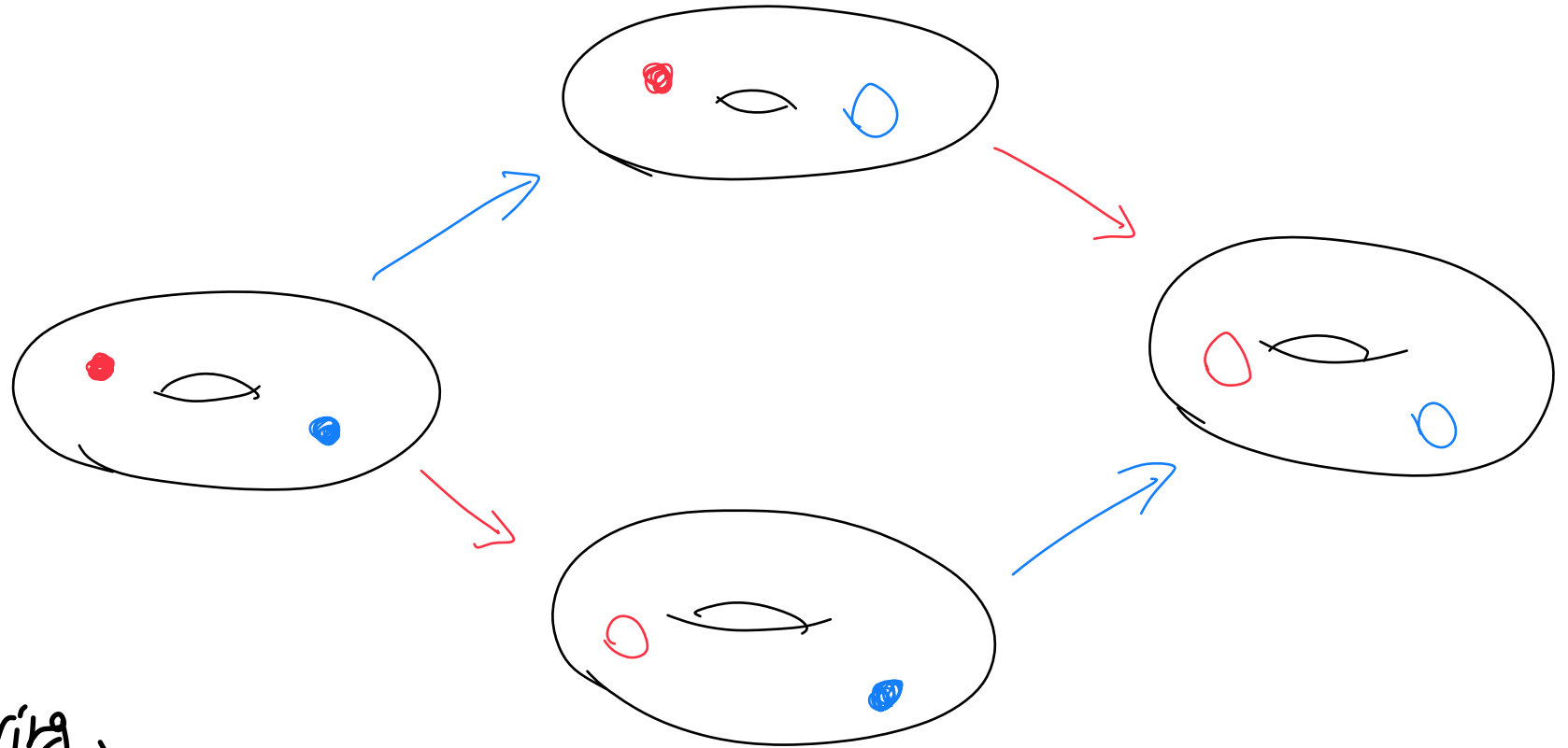


giving rise to coupling between QFTs

$$\mathcal{L} = \underbrace{\sum \mathcal{L}_i}_{\text{separately integrable}} + \underbrace{\sum \mathcal{L}_{ij}}_{\text{coupling}}$$

We can discuss dualities among defects
(e.g. bosonization)

$\bullet = \bigcirc$
one 2d defect = another 2d defect



(e.g. massless Thirring
 $\updownarrow$
coupled WZW)

"duality web"

The 4d CS framework

has recently been extended to

time-dependent integrability

[Komatsumi - Sakamoto - Wallberg - MY
I: (126), II: (to appear)]

Solvable circuits for non-equilibrium dynamics!/?

Summary

- Integrable Models: ideal playground in quantum simulations
- Quantum Circuits for Quantum Field Theories
 ↖ systematic construction via 4d CS

