Entanglement transitions in random tensor networks and shallow quantum circuits

Max McGinley

TCM Group, University of Cambridge

QuantHEP 2026, QMUL

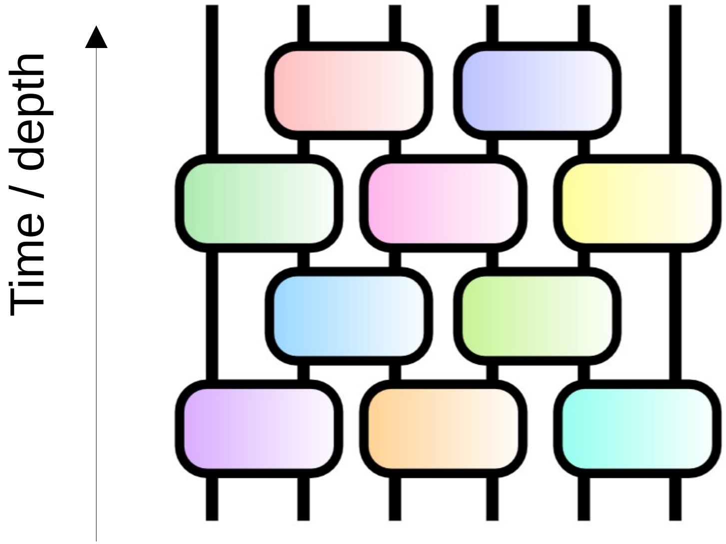
16th July 2026

*QIP 2025; PRX **15**, 021059 (2025)*

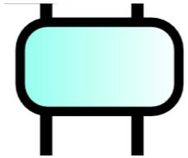
Background: random quantum circuits

Background: random quantum circuits

Image: Schuster *et al.* (2025)



e.g. 1D brickwork

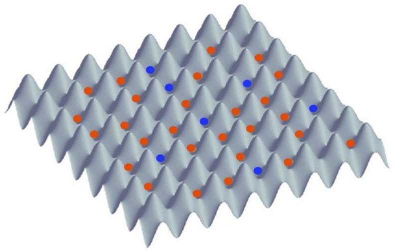


= a random 2-site gate

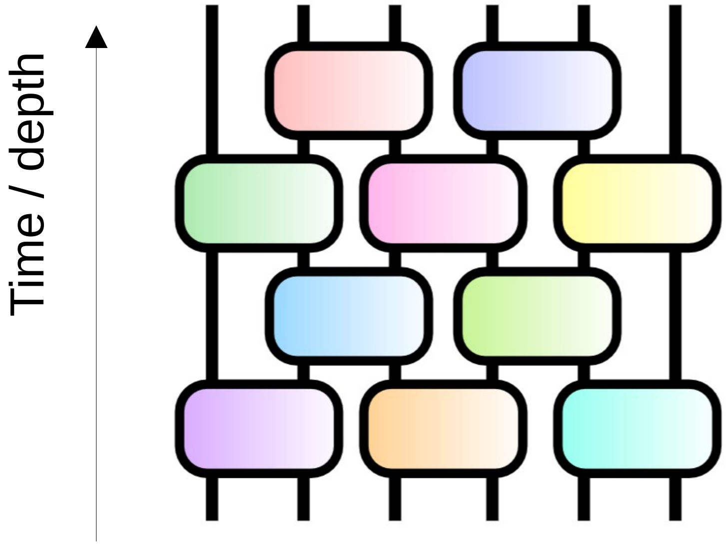
Background: random quantum circuits

Image: Schuster *et al.* (2025)

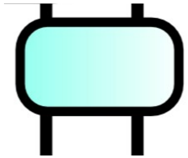
- A testbed for understanding:
  - Many-body quantum dynamics



$$e^{-iHt} \leftrightarrow U_{\mathrm{RQC}}$$



e.g. 1D brickwork

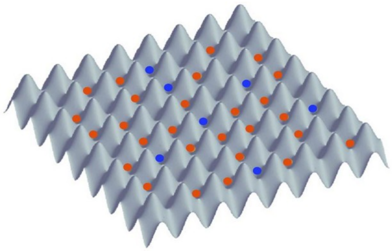


= a random 2-site gate

Background: random quantum circuits

Image: Schuster *et al.* (2025)

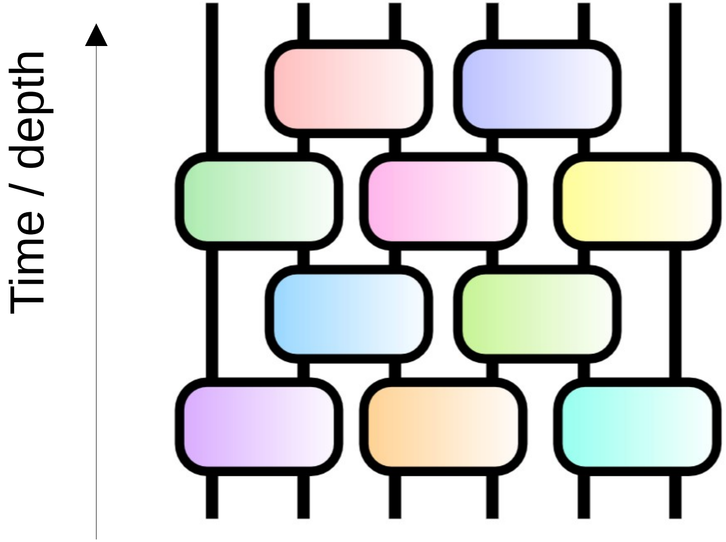
- A testbed for understanding:
  - Many-body quantum dynamics



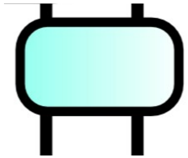
$$e^{-iHt} \leftrightarrow U_{\mathrm{RQC}}$$

  - Quantum computational advantage…

Image: *quantum insider*



e.g. 1D brickwork



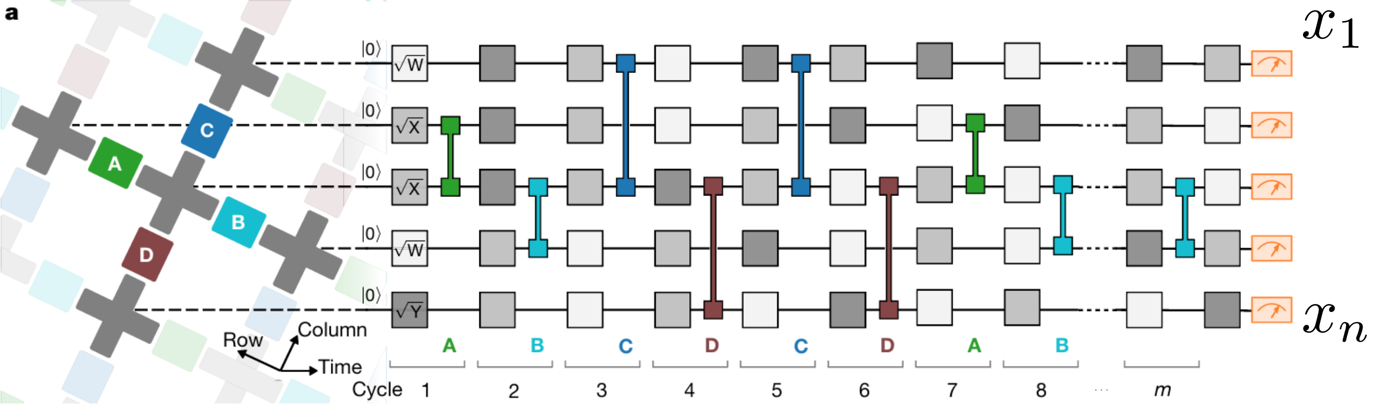
= a random 2-site gate

A hard (but useless!) task

- Given a random circuit U, can we predict the statistics of its output distribution?

A hard (but useless!) task

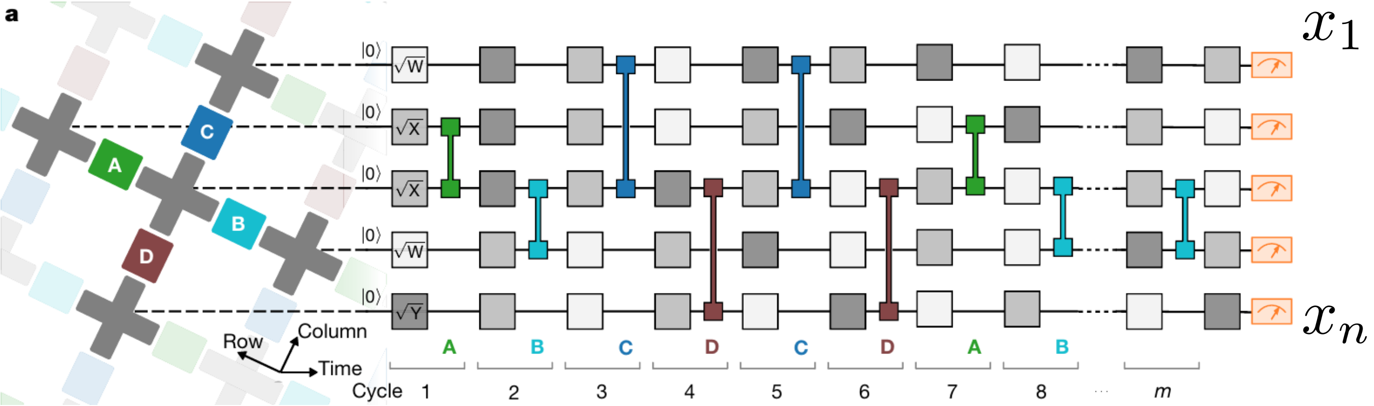
- Given a random circuit U, can we predict the statistics of its output distribution?



[Arute *et al.*, *Nature* (2019)]

A hard (but useless!) task

- Given a random circuit U, can we predict the statistics of its output distribution?



[Arute *et al.*, *Nature* (2019)]

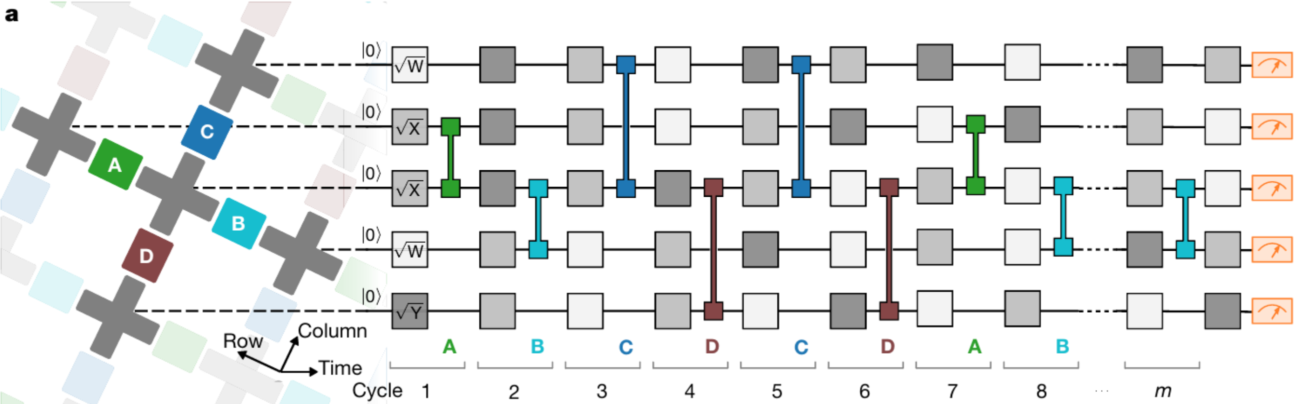
Output $x = x_1, \ldots, x_n$

w/ prob.

$$p(x) = |\langle x|U|0^n\rangle|^2$$

A hard (but useless!) task

- Given a random circuit U, can we predict the statistics of its output distribution?



x_1

x_n

[Arute *et al.*, *Nature* (2019)]

Output $x = x_1, \dots, x_n$

w/ prob.

$$p(x) = |\langle x|U|0^n\rangle|^2$$

Aaronson & Arkhipov (2010)
Bremner, Jozsa, Shepherd (2011)
Hangleiter & Eisert (2022)

Experiments
Google
USTC
Quantinuum
IBM
QuEra
...

- Conjecture: for sufficiently **deep** RQCs, there is no efficient classical algorithm that can (approximately) sample from $p(x)$

Our interest

Our interest

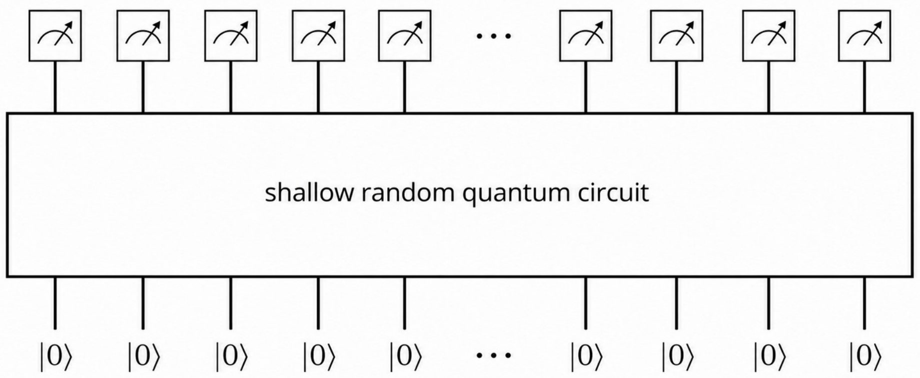
- Does conjectured quantum advantage persist in different regimes of:
  - Circuit depth d
  - Noise
  - Spatial dimension…

Our interest

- Does conjectured quantum advantage persist in different regimes of:
  - Circuit depth d
  - Noise
  - Spatial dimension…

- In particular, interested in **shallow quantum circuits**

$$d = \text{const.}, n \to \infty$$

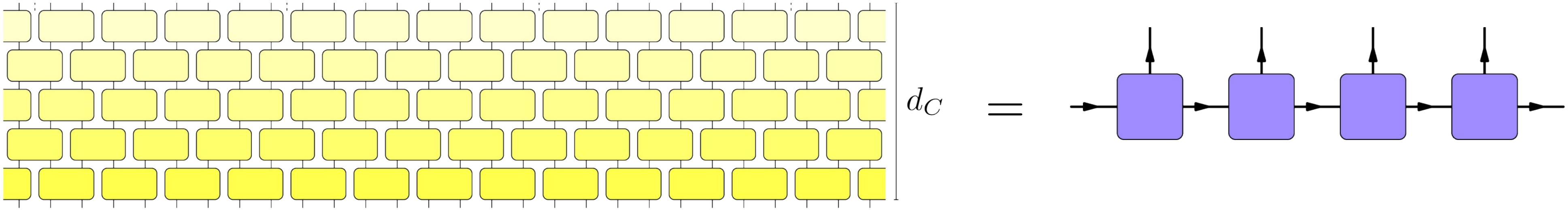


Shallow circuits

- Output state has short-range correlations and low entanglement

Shallow circuits

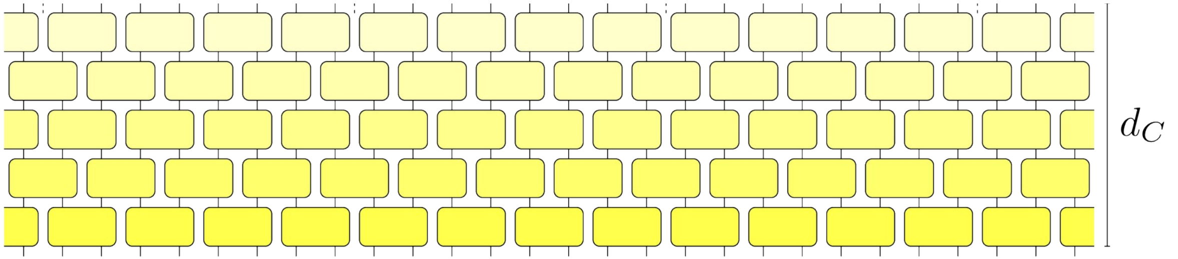
- Output state has short-range correlations and low entanglement

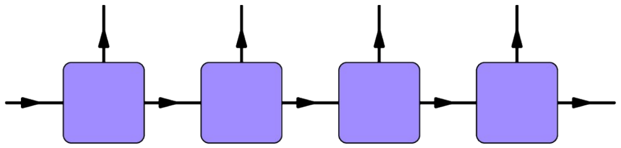


  - Efficient tensor-network representation (χ =const.)

Shallow circuits

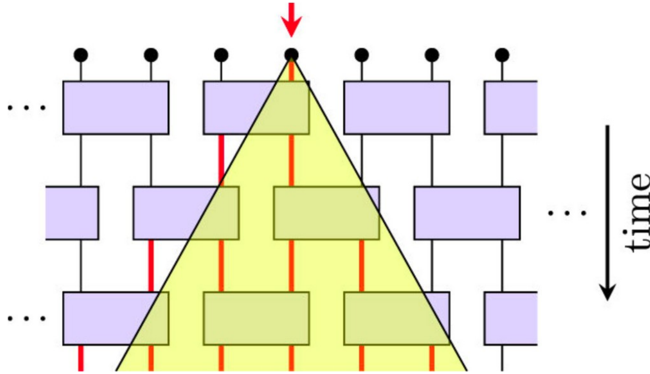
- Output state has short-range correlations and low entanglement





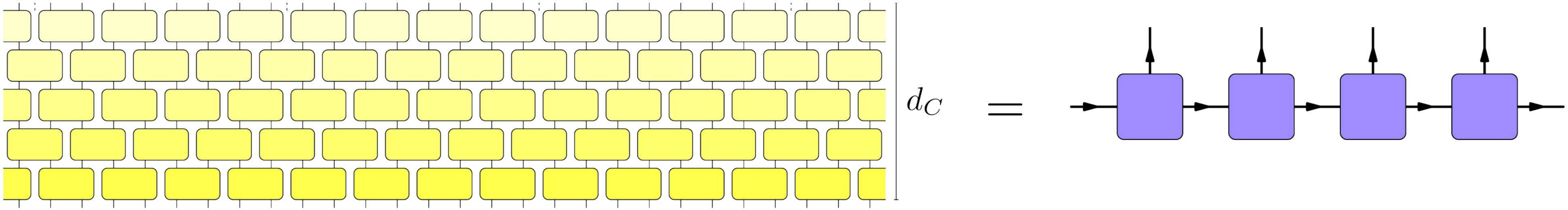
  - Efficient tensor-network representation (χ =const.)

- Local observables can be efficiently computed classically (lightcone)



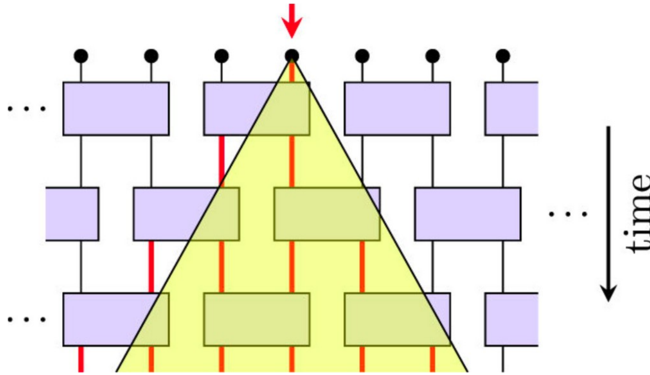
Shallow circuits

- Output state has short-range correlations and low entanglement



  - Efficient tensor-network representation (χ =const.)

- Local observables can be efficiently computed classically (lightcone)



- What about full output distribution?

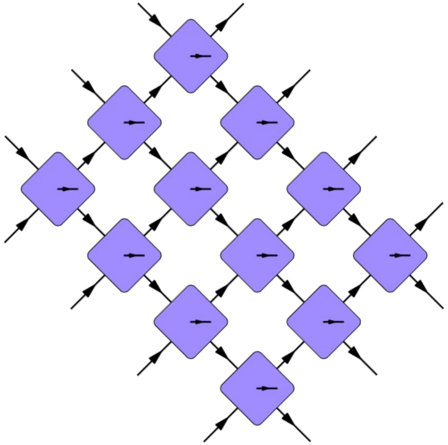
A tensor network contraction problem

- Compute probability of some measurement outcome $x = x_1 \dots x_n$
  $p(x) = |\langle x|\Psi\rangle|^2$, with $|\Psi\rangle = U|0^n\rangle$

A tensor network contraction problem

- Compute probability of some measurement outcome $x = x_1 \dots x_n$
  $p(x) = |\langle x|\Psi\rangle|^2$, with $|\Psi\rangle = U|0^n\rangle$

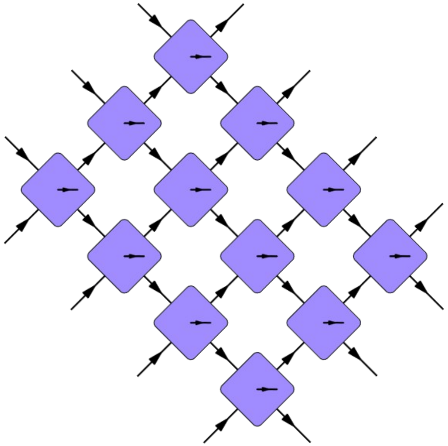
TNS rep. of $|\Psi\rangle$



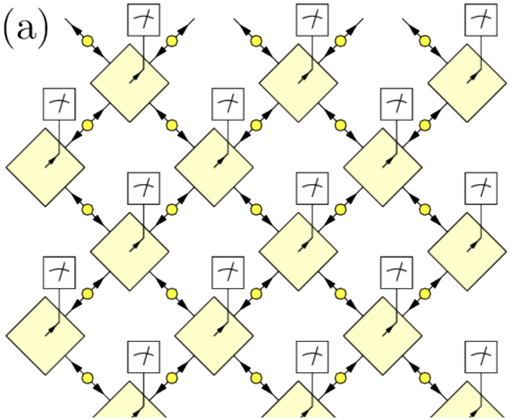
A tensor network contraction problem

- Compute probability of some measurement outcome $x = x_1 \dots x_n$
  $p(x) = |\langle x|\Psi\rangle|^2$, with $|\Psi\rangle = U|0^n\rangle$

TNS rep. of $|\Psi\rangle$



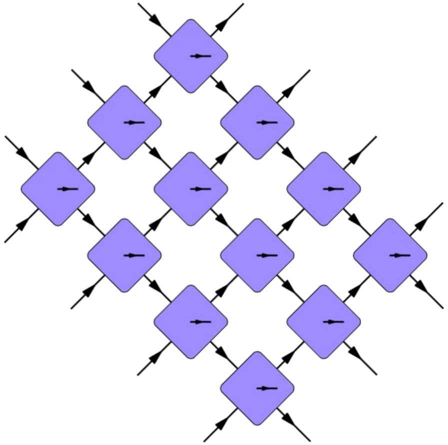
Project each leg onto state $|x_i\rangle$



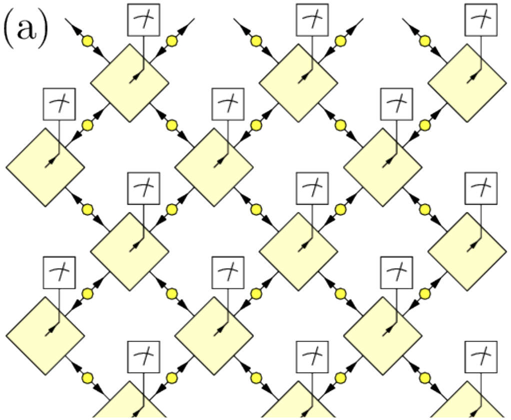
A tensor network contraction problem

- Compute probability of some measurement outcome $x = x_1 \ldots x_n$
  $p(x) = |\langle x|\Psi\rangle|^2$, with $|\Psi\rangle = U|0^n\rangle$

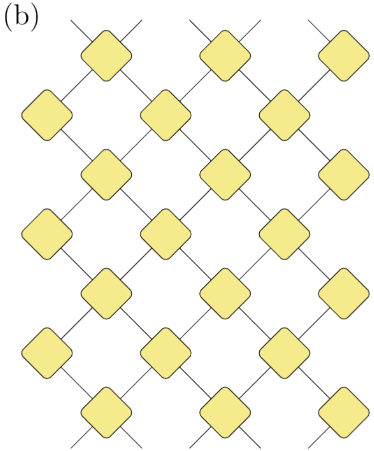
TNS rep. of $|\Psi\rangle$



Project each leg onto state $|x_i\rangle$



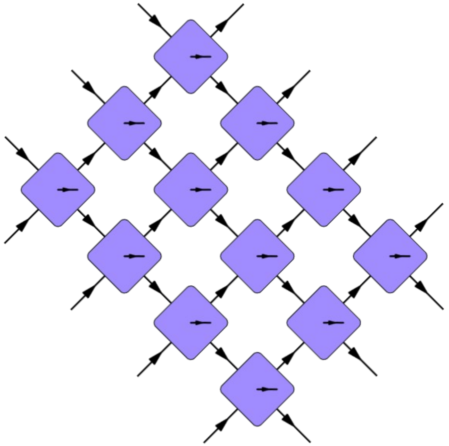
Tensor network



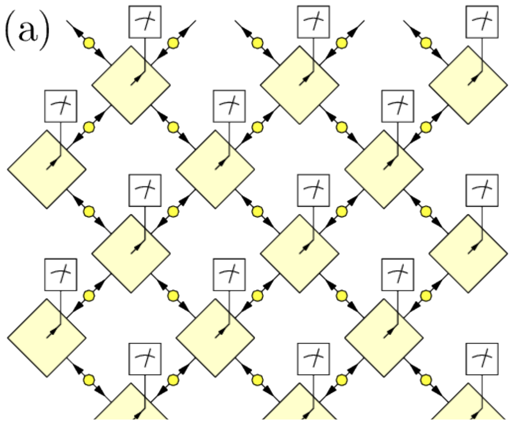
A tensor network contraction problem

- Compute probability of some measurement outcome $x = x_1 \ldots x_n$
  $p(x) = |\langle x|\Psi\rangle|^2$, with $|\Psi\rangle = U|0^n\rangle$

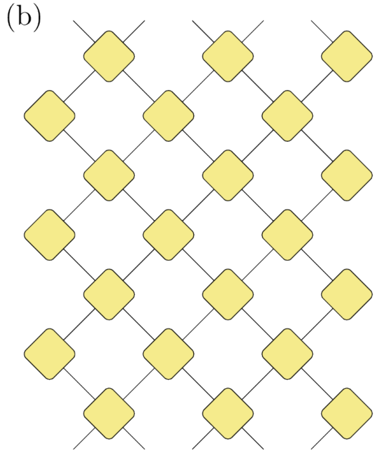
TNS rep. of $|\Psi\rangle$



Project each leg onto state $|x_i\rangle$



Tensor network



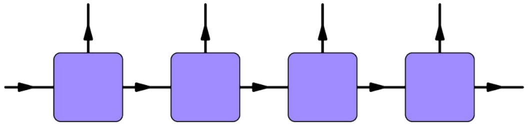
Complexity of simulating random shallow circuit ↔ complexity of contracting random tensor network

Known results

- Complexity of simulating circuit / tensor network contraction

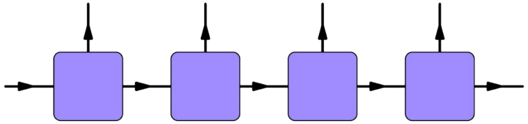
Known results

- Complexity of simulating circuit / tensor network contraction
- One dimension:
  - Shallow circuit $d = O(\log n)$ or tensor network $\chi = \mathrm{poly}(n)$ → $\mathrm{poly}(n)$-time classical algorithm for exact evaluation (MPS)



Known results

- Complexity of simulating circuit / tensor network contraction
- One dimension:
  - Shallow circuit $d = O(\log n)$ or tensor network $\chi = \mathrm{poly}(n)$ → $\mathrm{poly}(n)$-time classical algorithm for exact evaluation (MPS)



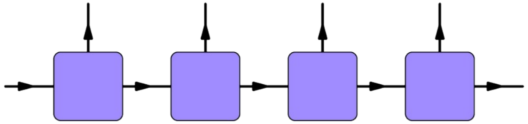
- Two dimensions:
  - *Worst case* instances provably hard* for $d \geq 3$ (MBQC)

Terhal & DiVincenzo (2004)

*under standard complexity-theoretic assumptions

Known results

- Complexity of simulating circuit / tensor network contraction
- One dimension:
  - Shallow circuit $d = O(\log n)$ or tensor network $\chi = \mathrm{poly}(n)$ → $\mathrm{poly}(n)$-time classical algorithm for exact evaluation (MPS)



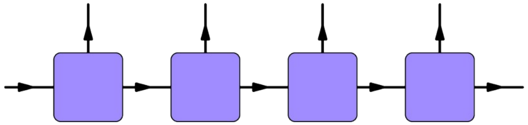
- Two dimensions:
  - *Worst case* instances provably hard* for $d \geq 3$ (MBQC)
  - **Conjecture** (Napp *et al.* 2021): For *random* SQCs

Terhal & DiVincenzo (2004)

*under standard complexity-theoretic assumptions

Known results

- Complexity of simulating circuit / tensor network contraction
- One dimension:
  - Shallow circuit $d = O(\log n)$ or tensor network $\chi = \mathrm{poly}(n)$ $\rightarrow$ $\mathrm{poly}(n)$-time classical algorithm for exact evaluation (MPS)



Terhal & DiVincenzo (2004)

- Two dimensions:
  - *Worst case* instances provably hard* for $d \geq 3$ (MBQC)
  - **Conjecture** (Napp *et al.* 2021): For *random* SQCs

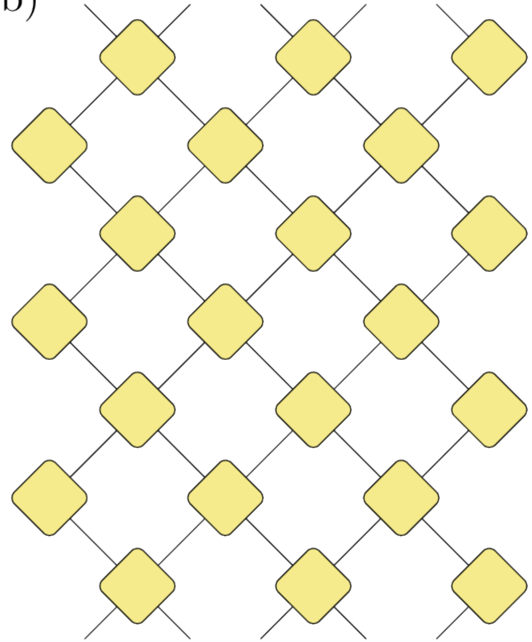
$d_{\mathrm{crit}} = O(1)$

Classically simulable | Hard to simulate classically

*under standard complexity-theoretic assumptions

Phase transition in complexity

(b)



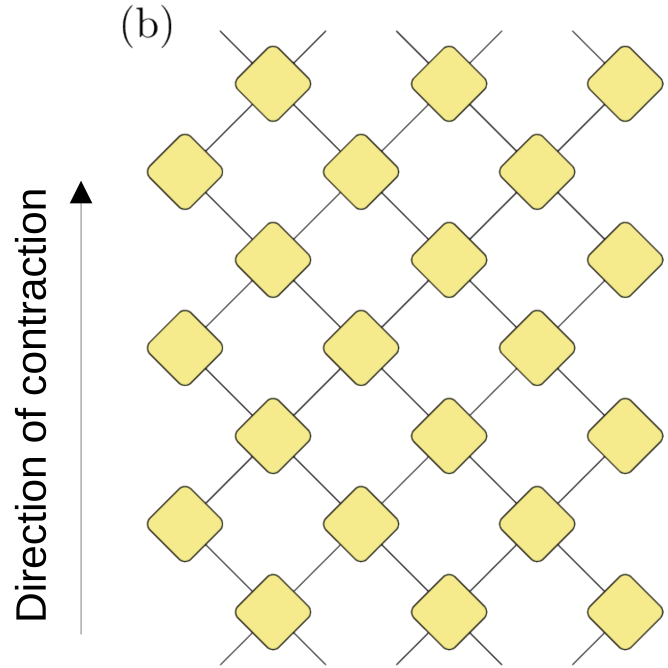
$d_{\mathrm{crit}} = O(1)$

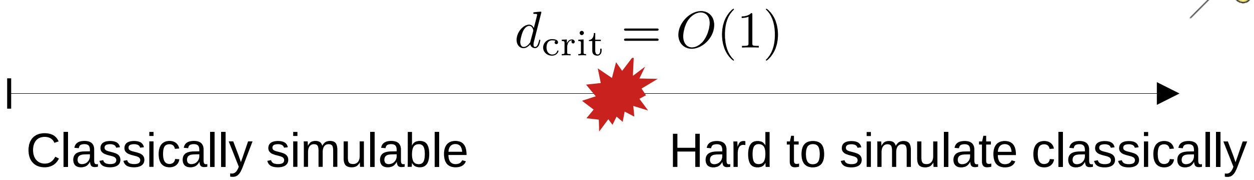
Classically simulable

Hard to simulate classically

Phase transition in complexity

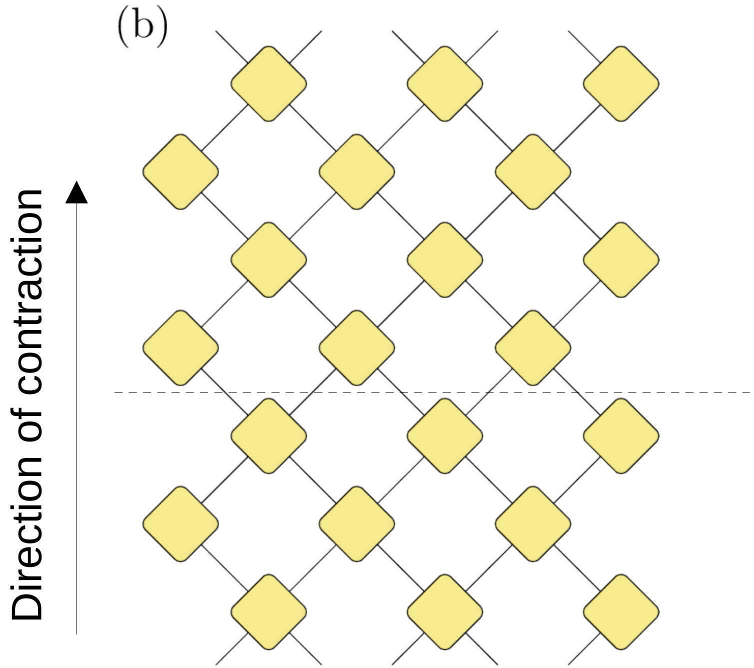
- Standard method for contracting tensor network ”sweeps” across system





Phase transition in complexity

- Standard method for contracting tensor network ”sweeps” across system
- Have to store a 1D ‘holographic’ state



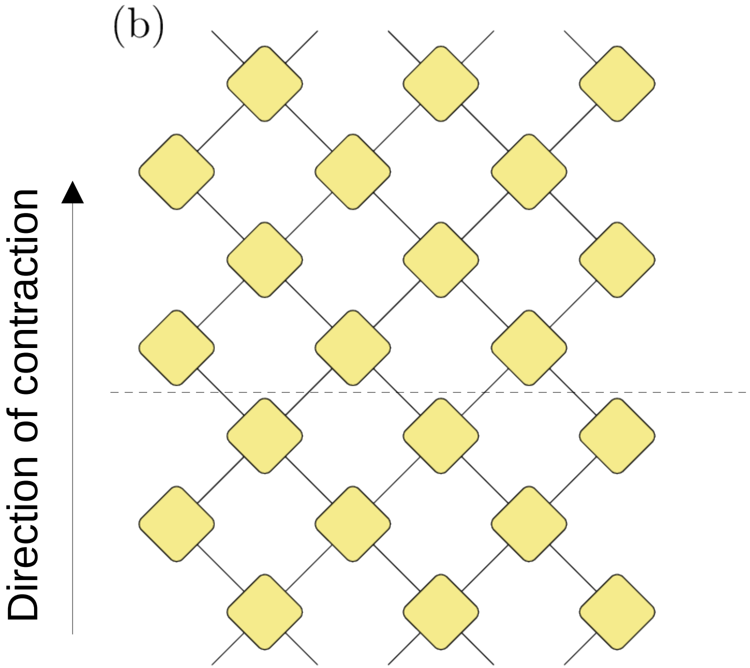
$d_{\mathrm{crit}} = O(1)$

Classically simulable

Hard to simulate classically

Phase transition in complexity

- Standard method for contracting tensor network ”sweeps” across system
- Have to store a 1D ‘holographic’ state
  - How much **entanglement?**



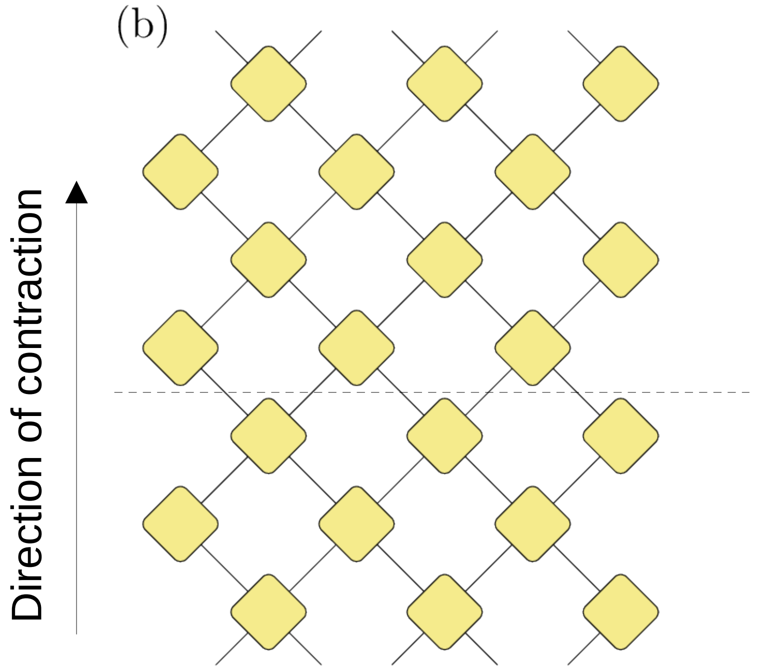
$$d_{\text{crit}} = O(1)$$

Classically simulable — Hard to simulate classically

Phase transition in complexity

- Standard method for contracting tensor network ”sweeps” across system
- Have to store a 1D ‘holographic’ state
  - How much **entanglement?**

    “Measurement-induced entanglement”



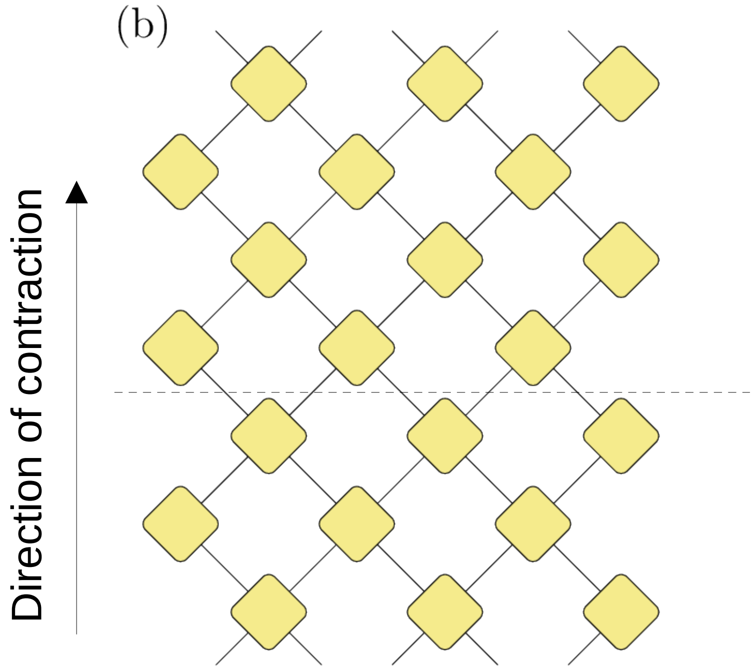
$$d_{\mathrm{crit}} = O(1)$$

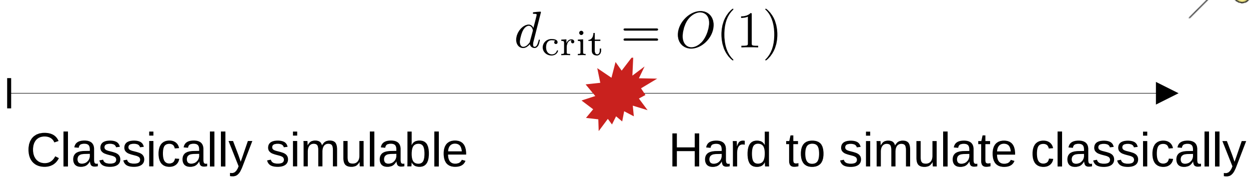
Classically simulable

Hard to simulate classically

Phase transition in complexity

- Standard method for contracting tensor network ”sweeps” across system
- Have to store a 1D ‘holographic’ state
  - How much **entanglement?**
    “Measurement-induced entanglement”
- Low/high entanglement $\leftrightarrow$ efficient/ inefficient classical representation

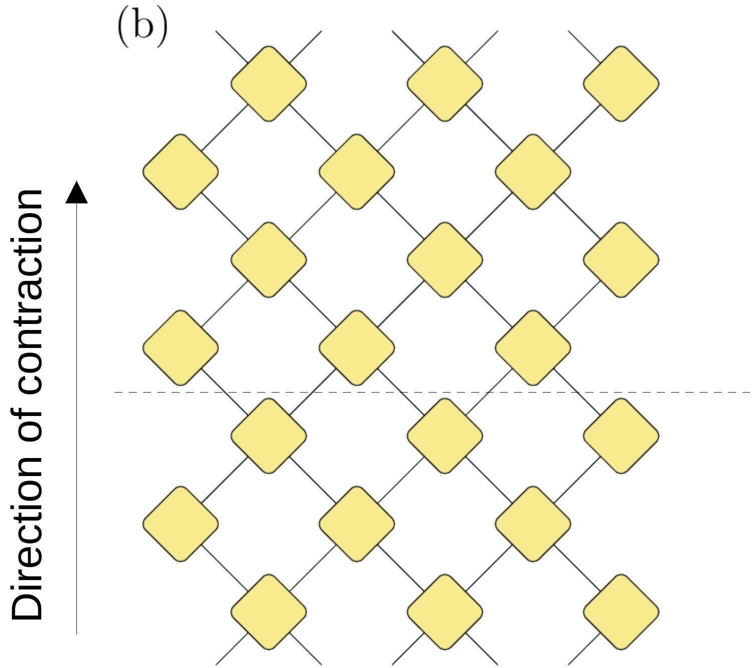




Phase transition in complexity

- Standard method for contracting tensor network ”sweeps” across system
- Have to store a 1D ‘holographic’ state
  - How much **entanglement?**

    “Measurement-induced entanglement”
- Low/high entanglement ↔ efficient/ inefficient classical representation



$d_{\mathrm{crit}} = O(1)$

Classically simulable | Hard to simulate classically

Low MIE | High MIE

Our contribution

- First rigorous results on entanglement transitions in random shallow circuits / random tensor networks:

Daniel Malz
(Basel)

Wen Wei Ho
(Singapore)

*QIP 2025; PRX **15**, 021059 (2025)*

Our contribution

- First rigorous results on entanglement transitions in random shallow circuits / random tensor networks:

Theorem (informal): *For certain natural families of random quantum circuits in 2D, there exists a depth $d = O(1)$ such that the boundary / holographic state has half-chain entanglement*

$$E(\Psi_{\mathrm{hol.}}) \geq \alpha L_x$$

Daniel Malz (Basel)

Wen Wei Ho (Singapore)

*QIP 2025; PRX **15**, 021059 (2025)*

Our contribution

- First rigorous results on entanglement transitions in random shallow circuits / random tensor networks:

Theorem (informal): *For certain natural families of random quantum circuits in 2D, there exists a depth* $d = O(1)$ *such that the boundary / holographic state has half-chain entanglement*

$$E(\Psi_{\mathrm{hol.}}) \geq \alpha L_x$$

Overcame outstanding challenge of evaluating M.I.E. (non-polynomial function of circuit U)

Daniel Malz (Basel)

Wen Wei Ho (Singapore)

QIP 2025; PRX ***15****, 021059 (2025)*

Our contribution

- First rigorous results on entanglement transitions in random shallow circuits / random tensor networks:

Theorem (informal): *For certain natural families of random quantum circuits in 2D, there exists a depth $d = O(1)$ such that the boundary / holographic state has half-chain entanglement*

$$E(\Psi_{\mathrm{hol.}}) \geq \alpha L_x$$

Daniel Malz
(Basel)

Wen Wei Ho
(Singapore)

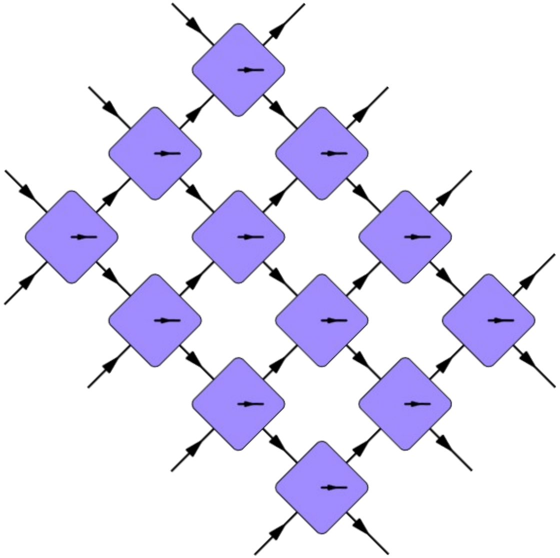
Overcame outstanding challenge of evaluating M.I.E. (non-polynomial function of circuit U)

Strong evidence in favour of complexity transition

QIP 2025; PRX ***15****, 021059 (2025)*

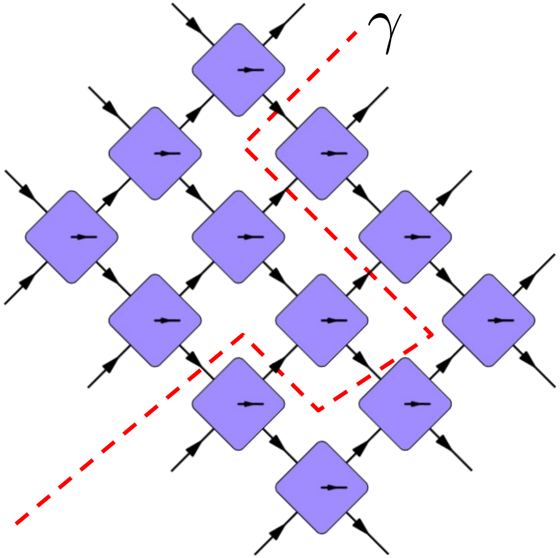
A geometric interpretation

- Relate M.I.E. to the entanglement of the pre-measurement state along different cuts:



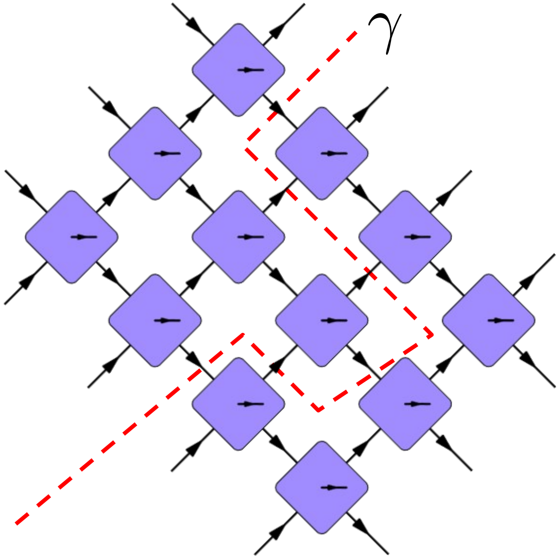
A geometric interpretation

- Relate M.I.E. to the entanglement of the pre-measurement state along different cuts:



A geometric interpretation

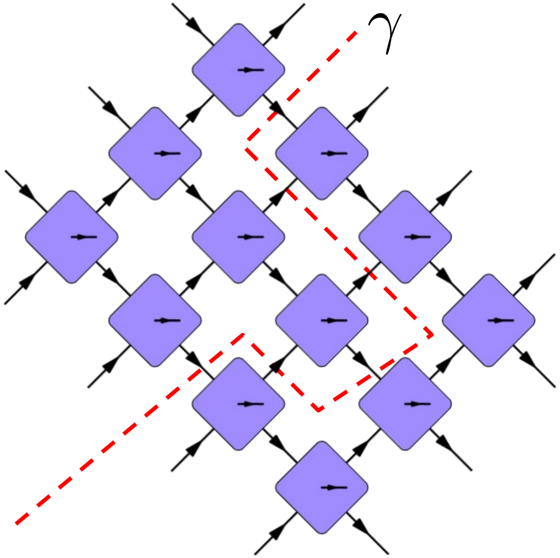
- Relate M.I.E. to the entanglement of the pre-measurement state along different cuts:



$$\mathrm{MIE} \gtrsim \sum_{\gamma} e^{-S^{(2)}(\gamma)/2}$$

A geometric interpretation

- Relate M.I.E. to the entanglement of the pre-measurement state along different cuts:

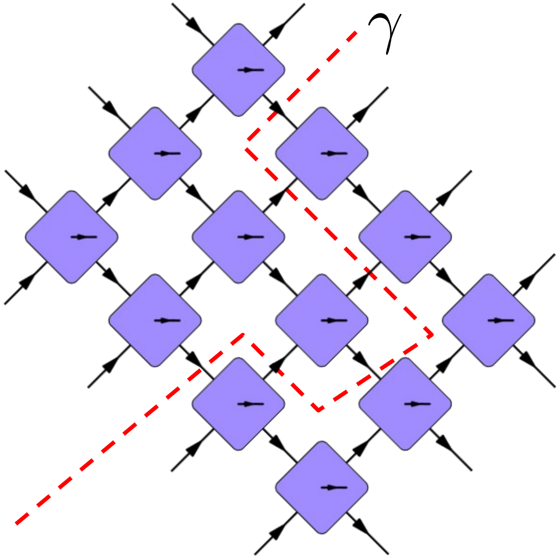


$$\mathrm{MIE} \gtrsim \sum_{\gamma} e^{-S^{(2)}(\gamma)/2}$$

Entanglement across cut γ

A geometric interpretation

- Relate M.I.E. to the entanglement of the pre-measurement state along different cuts:



$$\text{MIE} \gtrsim \sum_{\gamma} e^{-S^{(2)}(\gamma)/2}$$

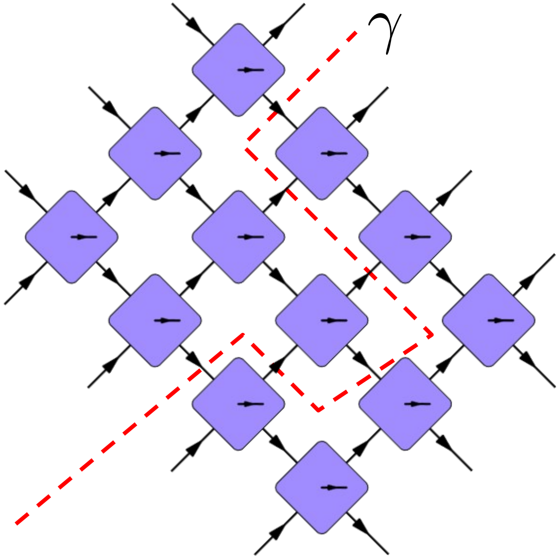
Entanglement across cut γ

$d < d_{\text{crit}}$

- Weaker pre-meas. entanglement
- Sum diverges
- Low M.I.E

A geometric interpretation

- Relate M.I.E. to the entanglement of the pre-measurement state along different cuts:



Entanglement across cut γ

$$\mathrm{MIE} \gtrsim \sum_{\gamma} e^{-S^{(2)}(\gamma)/2}$$

$d < d_{\mathrm{crit}}$

- Weaker pre-meas. entanglement
- Sum diverges
- Low M.I.E

$d > d_{\mathrm{crit}}$

- Stronger pre-meas. entanglement
- Shortest (least entangled) cuts dominate sum

Random holographic tensor networks

- Can also apply these results to entanglement of tensor networks in different geometries, e.g. 2D hyperbolic space

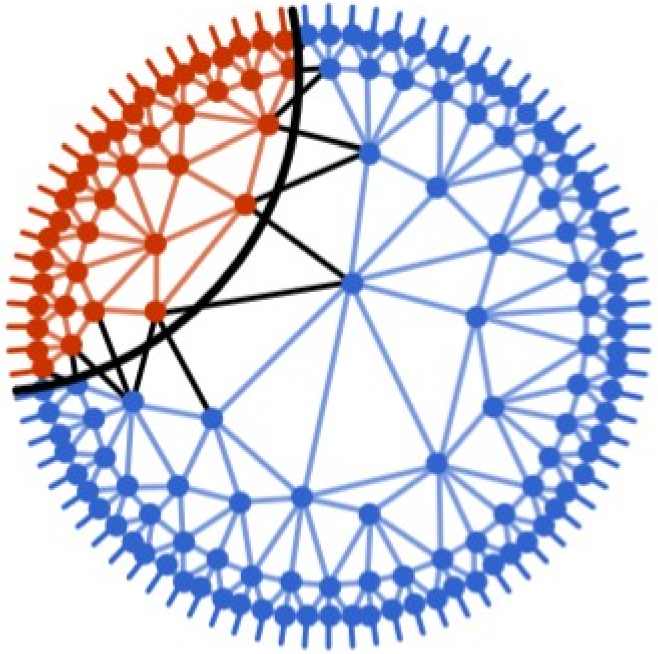


Image: Vasseur *et al.* (2019)

- Serves as a toy model for holographic dualities

Hayden *et al.* JHEP (2016)

Random holographic tensor networks

- Can also apply these results to entanglement of tensor networks in different geometries, e.g. 2D hyperbolic space

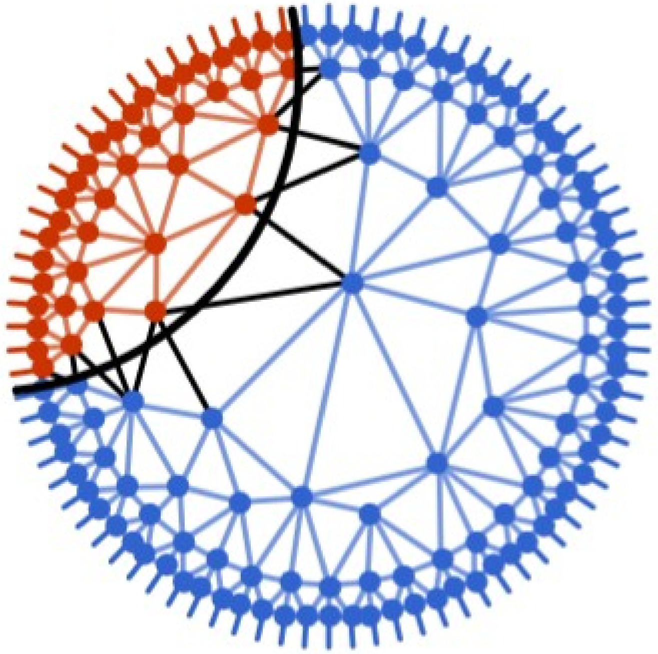


Image: Vasseur *et al.* (2019)

- Serves as a toy model for holographic dualities

Hayden *et al.* JHEP (2016)

- Ryu Takayanagi formula

$$E(A:B) \propto |\gamma_{\min}|$$

  - Shown only in limit of large bond dimension $\chi \to \infty$

Random holographic tensor networks

- Can also apply these results to entanglement of tensor networks in different geometries, e.g. 2D hyperbolic space

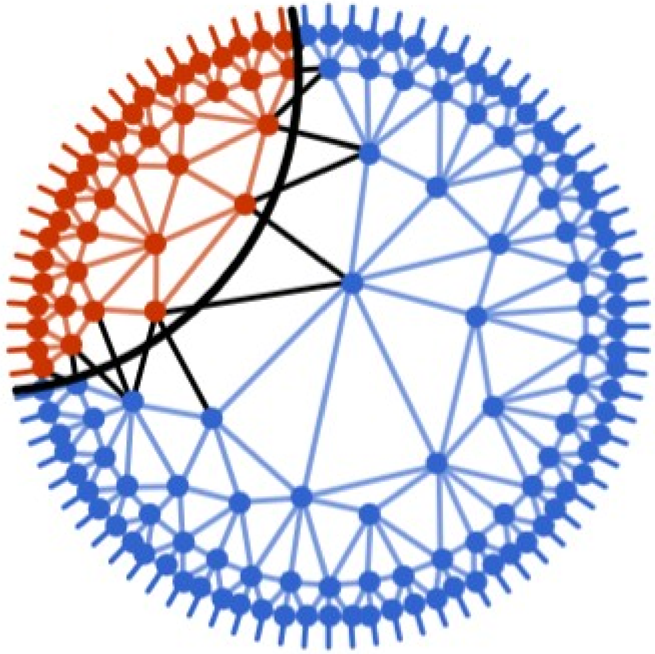


Image: Vasseur *et al.* (2019)

- Serves as a toy model for holographic dualities

Hayden *et al.* JHEP (2016)

- Ryu Takayanagi formula

$$E(A:B) \propto |\gamma_{\min}|$$

  - Shown only in limit of large bond dimension $\chi \to \infty$

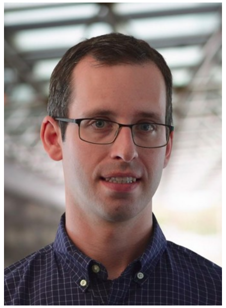
- Here: phase transition at $\chi_{crit} = O(1)$ above which RT emerges

Bonus: beyond entanglement

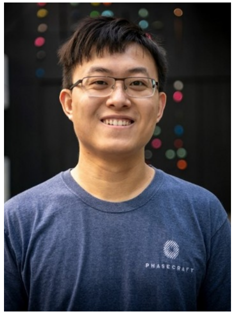
- What can we say about the distribution of boundary states that emerge in random tensor networks / random shallow circuits?

Thomas Schuster (Caltech)

David Gosset (Waterloo)

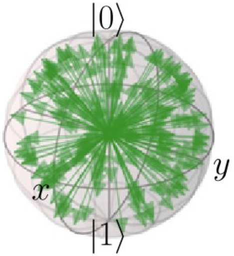


Yinchen Liu (Waterloo)



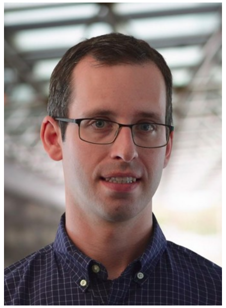
Bonus: beyond entanglement

- What can we say about the distribution of boundary states that emerge in random tensor networks / random shallow circuits?
- First guess: states are completely random (Haar distribution)

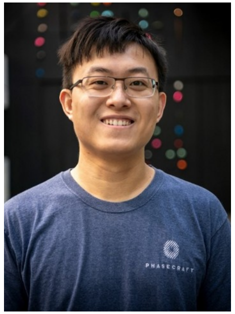


Thomas Schuster (Caltech)

David Gosset (Waterloo)

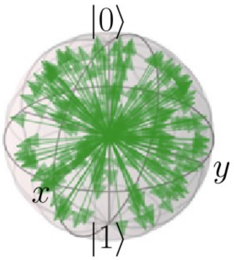


Yinchen Liu (Waterloo)



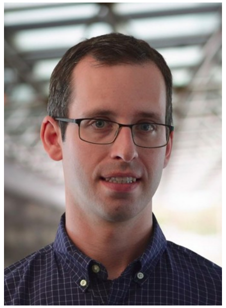
Bonus: beyond entanglement

- What can we say about the distribution of boundary states that emerge in random tensor networks / random shallow circuits?
- First guess: states are completely random (Haar distribution)
  - Incorrect—entanglement is large but not maximal (cf. Page curve)

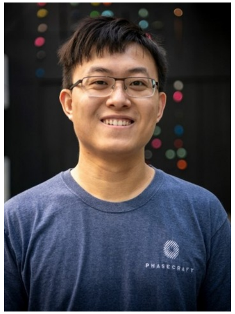


Thomas Schuster (Caltech)

David Gosset (Waterloo)

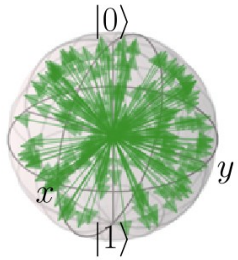


Yinchen Liu (Waterloo)



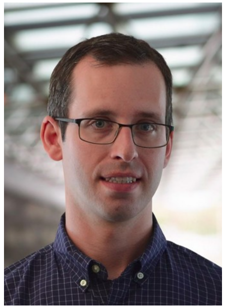
Bonus: beyond entanglement

- What can we say about the distribution of boundary states that emerge in random tensor networks / random shallow circuits?
- First guess: states are completely random (Haar distribution)
  - Incorrect—entanglement is large but not maximal (cf. Page curve)
- Hypothesis: in a random shallow circuit (random tensor network), the holographic states is that of the *Scrooge ensemble*

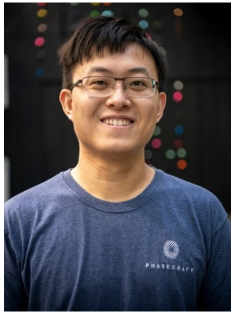


Thomas Schuster (Caltech)

David Gosset (Waterloo)

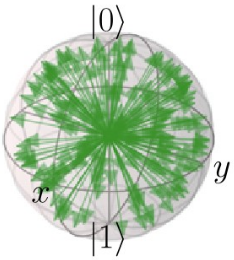


Yinchen Liu (Waterloo)



Bonus: beyond entanglement

- What can we say about the distribution of boundary states that emerge in random tensor networks / random shallow circuits?
- First guess: states are completely random (Haar distribution)
  - Incorrect—entanglement is large but not maximal (cf. Page curve)



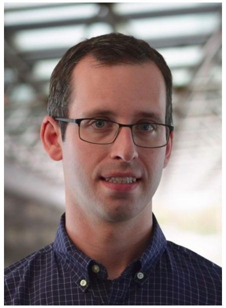
- Hypothesis: in a random shallow circuit (random tensor network), the holographic states is that of the *Scrooge ensemble*

"Distorted Haar ensemble"

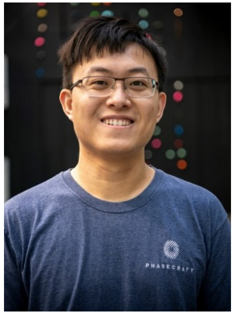
Jozsa, Robb, Wootters, PRA (1994)

Thomas Schuster (Caltech)

David Gosset (Waterloo)

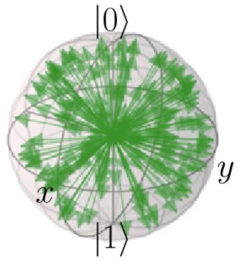


Yinchen Liu (Waterloo)



Bonus: beyond entanglement

- What can we say about the distribution of boundary states that emerge in random tensor networks / random shallow circuits?
- First guess: states are completely random (Haar distribution)
  - Incorrect—entanglement is large but not maximal (cf. Page curve)



- Hypothesis: in a random shallow circuit (random tensor network), the holographic states is that of the *Scrooge ensemble*

"Distorted Haar ensemble"

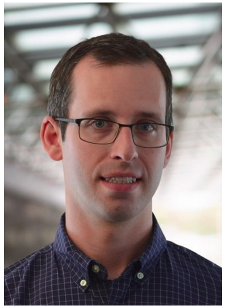
Jozsa, Robb, Wootters, PRA (1994)

- Implications for simulability with noise:

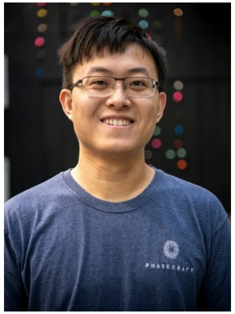
Efficient classical algorithm if noise rate $\gamma = \Omega(\log n / n)$

Thomas Schuster (Caltech)

David Gosset (Waterloo)



Yinchen Liu (Waterloo)



Conclusions

Conclusions

- Random quantum circuits and random tensor networks are useful testbeds for exploring beyond-classical computational tasks

Conclusions

- Random quantum circuits and random tensor networks are useful testbeds for exploring beyond-classical computational tasks
- Interesting landscape of complexity, depending on various physical parameters (dimensionality, circuit depth, etc.)
  - e.g. entanglement/complexity phase transition in 2D
  - Fragility of complexity to noise

Conclusions

- Random quantum circuits and random tensor networks are useful testbeds for exploring beyond-classical computational tasks
- Interesting landscape of complexity, depending on various physical parameters (dimensionality, circuit depth, etc.)
  - e.g. entanglement/complexity phase transition in 2D
  - Fragility of complexity to noise
- Establishes foundations for understanding complexity of simulating more realistic many-body dynamics
  - Energy / charge conservation
  - Local observables rather than sampling?

Conclusions

- Random quantum circuits and random tensor networks are useful testbeds for exploring beyond-classical computational tasks
- Interesting landscape of complexity, depending on various physical parameters (dimensionality, circuit depth, etc.)
  - e.g. entanglement/complexity phase transition in 2D
  - Fragility of complexity to noise
- Establishes foundations for understanding complexity of simulating more realistic many-body dynamics
  - Energy / charge conservation
  - Local observables rather than sampling?

Thank you!