Background electric field effects in $(2+1)$-QED from digital quantum simulation

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Contents

Introduction and motivation

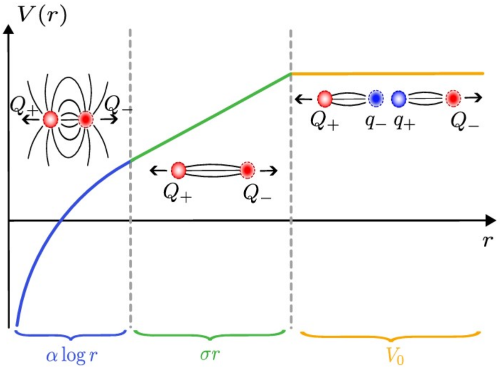
- External-field probes are a natural way to study the response of strongly interacting quantum systems – a background electric field (BEF) can drive a non-trivial reorganization of the vacuum
- $(2+1)$-dimensional QED is an Abelian, confining theory with chiral symmetry breaking - a tractable stepping stone towards QCD
- A BEF makes the Euclidean action complex $\implies$ sign problem. Background *magnetic* fields have been studied on the lattice for QCD, but the *electric* case is largely unexplored
- Hamiltonian (quantum-simulation) methods sidestep the sign problem entirely
- **This work:** the first study of BEF-induced screening and confinement in this formulation. Notably, Chernodub et al. (10.1103/PhysRevD.64.114502) see *no* shift in confinement from a real background flux, whereas the c-number BEF here *does* shift it

$(2+1)$-QED

- Confining theory with *static potential* given by

$$V(r) = \alpha \log r + \sigma r + V_0,$$

  where r is the distance between charges, α is the Coulomb coefficient, σ is the *string tension*, and V_0 is a constant
- This model has *chiral symmetry breaking* - there is a discontinuous jump in the chiral condensate at $m = 0$
- Picture taken from Crippa, et al. (2411.05628)



The Hamiltonian

- Using the staggered Hamiltonian from Crippa (10.3204/PUBDB-2025-02390)

$$
\begin{aligned}
H = {} & \frac{g^2}{2} \sum_{\vec{n}} \left[\left(E_{\vec{n},\vec{x}} + \mathcal{E}_{\vec{x}}\right)^2 + \left(E_{\vec{n},\vec{y}} + \mathcal{E}_{\vec{y}}\right)^2 \right] - \frac{1}{2a^2 g^2} \sum_{\vec{n}} \left[P_{\vec{n}} + P^{\dagger}_{\vec{n}} \right] \\
& + \frac{i}{2a} \sum_{\vec{n}} \left[\phi^{\dagger}_{\vec{n}} U^{\dagger}_{\vec{n},\vec{x}} \phi_{\vec{n}+\vec{x}} - \text{h.c.} \right] - \frac{1}{2a} \sum_{\vec{n}} \left[(-1)^{n_x+n_y} \phi^{\dagger}_{\vec{n}} U^{\dagger}_{\vec{n},\vec{y}} \phi_{\vec{n}+\vec{y}} + \text{h.c.} \right] \\
& + m \sum_{\vec{n}} \left[(-1)^{n_x+n_y} \phi^{\dagger}_{\vec{n}} \phi_{\vec{n}} \right] \\
= {} & H_E(\mathcal{E}) + H_B + H_K + H_M,
\end{aligned}
$$

using open boundary conditions. Here $\mathcal{E}_{\vec{\mu}}$ is the constant BEF. This is defined as a c-number shift

$$
E^2_{\vec{n},\vec{\mu}} \to \left(E_{\vec{n},\vec{\mu}} + \mathcal{E}_{\vec{\mu}}\right)^2
$$

- Dimensionless quantities: $\lambda = \frac{m}{g^2}$ and $x = ag^2$

Sign problem

- Writing the partition function in terms of the Hamiltonian

$$Z = \mathrm{Tr}\left[e^{-\beta H}\right],$$

- From here, consider just the electric term

$$H_E = \frac{g^2}{2}\sum_{\vec{n}}\left[(E_{\vec{n},\vec{x}} + \mathcal{E}_{\vec{x}})^2 + (E_{\vec{n},\vec{y}} + \mathcal{E}_{\vec{y}})^2\right],$$

moreover, a single link

$$H_{\vec{n},\vec{\mu}} = \frac{g^2}{2}(E_{\vec{n},\vec{\mu}} + \mathcal{E}_{\vec{\mu}})^2$$

$$\mathcal{T}_{\vec{n},\vec{\mu}} = \langle A_{\tau+\epsilon}|e^{-\epsilon H_{\vec{n},\vec{\mu}}}|A_\tau\rangle$$

- Via Poisson resummation

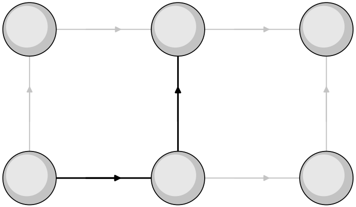
$$\mathcal{T}_{\vec{n},\vec{\mu}} \propto \sum_{\omega_{\vec{n},\vec{\mu}}\in\mathbb{Z}} \exp\left[-\frac{(\Delta_\tau A_{\vec{n},\vec{\mu}} + 2\pi\omega_{\vec{n},\vec{\mu}})^2}{2g^2\epsilon} - i\mathcal{E}_{\vec{\mu}}(\Delta_\tau A_{\vec{n},\vec{\mu}} + 2\pi\omega_{\vec{n},\vec{\mu}})\right]$$

Gauss law and link elimination

- Gauss law is defined by

$$G_{\vec{n}} = [E_{\vec{n}-\vec{x},\vec{x}} + E_{\vec{n}-\vec{y},\vec{y}} - E_{\vec{n},\vec{x}} - E_{\vec{n},\vec{y}}] - q_{\vec{n}} - Q_{\vec{n}}$$
$$G_{\vec{n}}|\phi\rangle = 0 \ \forall \vec{n} \iff |\phi\rangle \in \mathbb{H}_{\text{phys}}$$

- This is used, alongside zero charge, to reduce the number of *dynamical links*. Example shown here:



- Only requires $\tilde{N}_g = N_g - (N-1)$ dynamical links

Truncations and digitizations

- The $U(1)$ gauge symmetry is *truncated* to a $\mathbb{Z}_L$, $\mathbb{Z}_L \to U(1)$ as $L \to \infty$
- This limits the eigenvalues of $E_{\vec{n},\vec{\mu}}$ to be in $[-L, \ldots, \; L]$, achieved with $n_g = \lceil \log_2(2L+1) \rceil$ qubits per link
- Examples of different truncations:

$$n_g = 2 \leftrightarrow [-1, 0, 1], \quad n_g = 3 \leftrightarrow [-3, \ldots, 3], \quad n_g = 4 \leftrightarrow [-7, \ldots, 7]$$

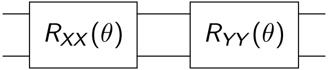
- Gauge links: Gray-encoding; fermions: Jordan-Wigner (one qubit per site)

$$\phi_{\vec{n}} = \left[\prod_{\vec{k}<\vec{n}} -iZ^{\vec{k}} \right] (X^{\vec{n}} + iY^{\vec{n}})$$

- Total qubits $N_{\text{total}} = N + n_g \cdot \tilde{N}_g$

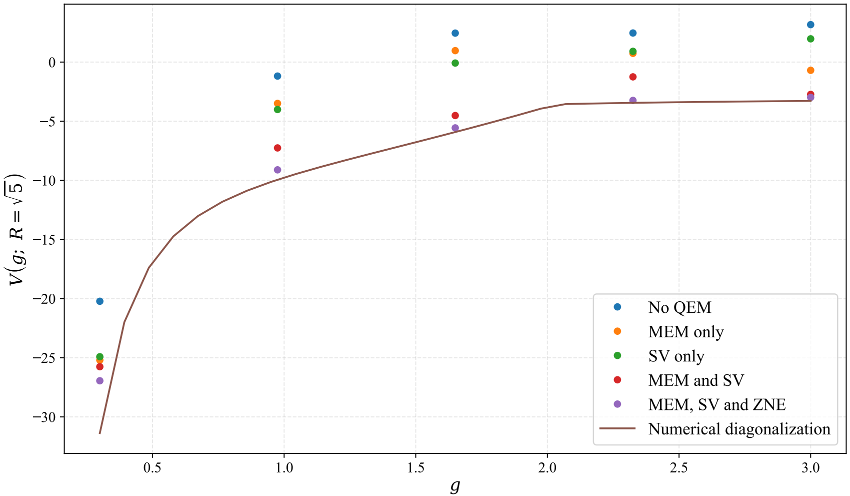
VQE strategy

- $i\mathrm{SWAP}(\theta)$ gates chosen to maintain zero charge.



- Gates on gauge qubits are chosen to not allow the state $|10\ldots0\rangle$ for a given number of qubits per gauge link
- I use natural gradient simultaneous perturbation stochastic approximation (NG-SPSA) as the optimization routine.
- Quantum error mitigation methods used: MEM, SV, ZNE

Effectiveness of QEM



- Recreating the work by Crippa, et al. (2411.05628)

Screening phase structure of $\mathbb{Z}_3$ model

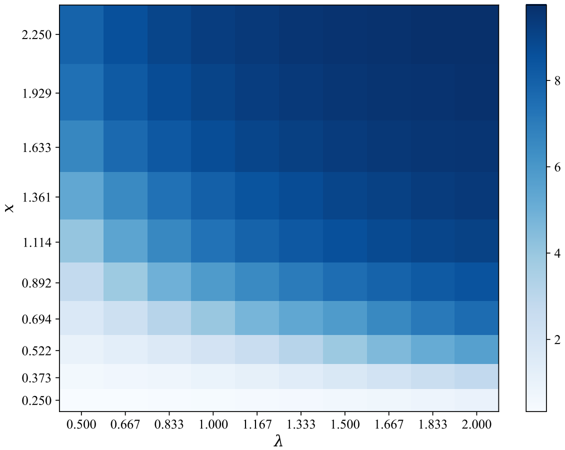
- Using $n_g = 2$, I investigated phase structure of BEF-induced screening
- Tested the value of

$$\chi_P(\mathcal{E}_{\vec{\mu}}) = \frac{d\langle P\rangle}{d\mathcal{E}_{\vec{\mu}}},$$

  where

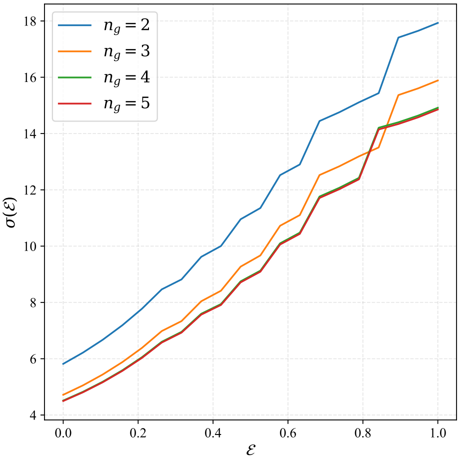
$$P = \sum_{\vec{n}} P_{\vec{n}} = \frac{1}{2}\sum_{\vec{n}}\left[1 - (-1)^{n_x+n_y} Z^{\vec{n}}\right].$$

- On the right, I show the maximum value this takes for a given $(\lambda,\ x)$
- The located screening field values ($\mathcal{E}^* \lesssim 7$) lie well outside the representable dynamical range $\mathrm{eig}(E_{\vec{n},\vec{\mu}}) \in \{-1, 0, 1\}$



Effect on confinement properties

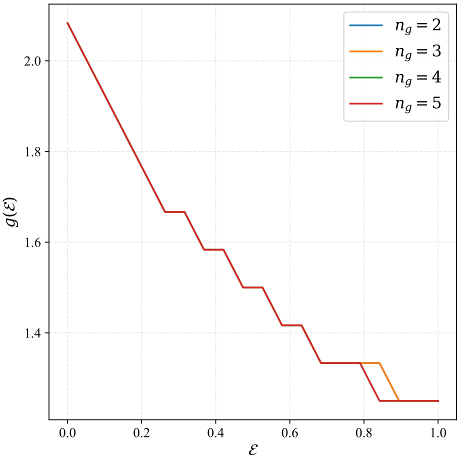
- $\mathcal{E} = \mathcal{E}_{\vec{x}+\vec{y}}$ is a bidirectional field
- Studied the change in *string tension* and *location of string breaking*
- Increases string tension; makes sense as it provides flux strings with energy to produce dynamical fermions
- Can see that the truncation shifts the value of this, but qualitatively is the same across them all
- Signals to the values being not exact, but a qualitatively physical result



Effect on confinement properties

- $g^*(\mathcal{E})$ marks where the confining linear region ends - where screening occurs and particle number jumps
- As $\mathcal{E}$ increases, this corner shifts to lower g: the BEF drives screening at weaker coupling
- Essentially n_g-independent - curves for $n_g = 2, \ldots, 5$ overlap, so this is not a truncation artefact
- Consistent with the string-tension picture: the field feeds the flux string, reaching the pair-production threshold sooner

Steps are from discrete g-sampling, not physical plateaus



Chiral symmetry breaking (preliminary)

- Have also explored chiral symmetry breaking properties via

$$\chi_{\mathcal{C}} = \frac{\partial \langle \mathcal{C} \rangle}{\partial m},$$

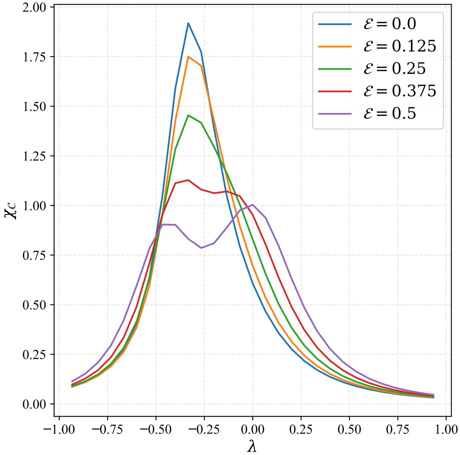
where

$$\langle \mathcal{C} \rangle = \langle \bar{\psi} \psi \rangle,$$

and

$$\mathcal{C} = \sum_{\vec{n}} \left[(-1)^{n_x + n_y} \phi^{\dagger}_{\vec{n}} \phi_{\vec{n}} \right]$$

- The BEF induces non-trivial behaviour
- These results are currently still under investigation



Conclusion

- Even these small lattices show clear signals
- Truncation has a quantitative effect, but contains many of the useful features
- Truncation doesn't limit too badly in moderate coupling
- QEM is very effective for these symmetry-predictable systems
- Much larger lattices are required to extrapolate to true phase structure
- In the medium-future, non-variational groundstate preparation would reduce the classical overhead

Bonus: circuit requirements

Lattice size	n_g	Qubit count	Rotational gauge gates	Rotational fermion gates	Total rotational gates
3×2	2	10	4	7	11
3×2	3	12	8	7	15
3×2	4	14	12	7	19
3×2	5	16	16	7	23
4×2	2	14	6	10	16
4×2	3	17	12	10	22
4×2	4	20	18	10	28
4×2	5	23	24	10	34
4×3	2	24	12	17	29
4×3	3	30	24	17	41
4×3	4	36	36	17	53
4×3	5	42	48	17	65

Table: Quantum circuit requirements for $(2+1)$-QED lattice models.

Bonus: kinetic term convention

- In the Hamiltonian, the kinetic term is defined as

$$H_K = \sum_{\vec{\mu}} \sum_{\vec{n}} p(\vec{n}, \vec{\mu}) \phi^\dagger_{\vec{n}} A_{\vec{n},\vec{\mu}} \phi_{\vec{n}+\vec{\mu}},$$

- If the Gauss law is defined as

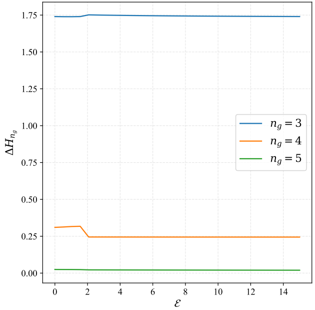
$$G_{\vec{n}} = [E_{\vec{n}-\vec{x},\vec{x}} + E_{\vec{n}-\vec{y},\vec{y}} - E_{\vec{n},\vec{x}} - E_{\vec{n},\vec{y}}] - q_{\vec{n}} - Q_{\vec{n}} \implies A_{\vec{n},\vec{\mu}} = U^\dagger_{\vec{n},\vec{\mu}}$$

- Or

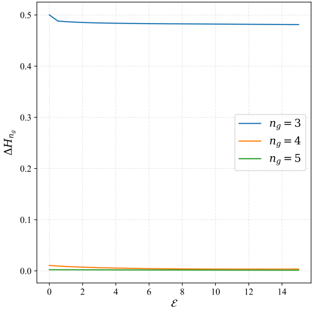
$$G_{\vec{n}} = [E_{\vec{n},\vec{x}} + E_{\vec{n},\vec{y}} - E_{\vec{n}-\vec{x},\vec{x}} - E_{\vec{n}-\vec{y},\vec{y}}] + q_{\vec{n}} + Q_{\vec{n}} \implies A_{\vec{n},\vec{\mu}} = U_{\vec{n},\vec{\mu}}$$

Bonus: energy difference for truncations

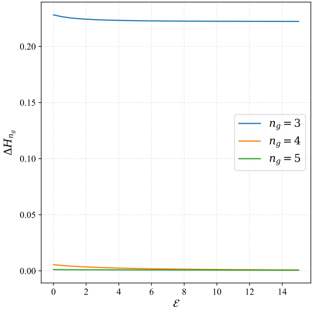
$$\Delta H_{n_g} = H(n_g) - H(n_g - 1)$$



(a) $g = 1.0$.



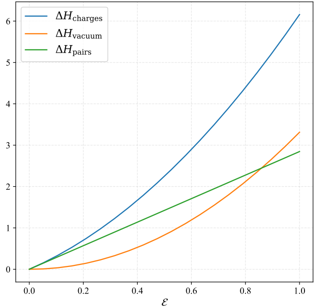
(b) $g \approx 2$.



(c) $g = 3.0$.

Bonus: energy difference for different sectors

$$\Delta H_{\text{pairs}} = [H(g,\mathcal{E};\text{charges}) - H(g;\mathcal{E}=0.0,\text{charges})] - [H(g,\mathcal{E};\text{no charges}) - H(g;\mathcal{E}=0.0,\text{no charges})]$$
$$= \Delta H_{\text{charges}} - \Delta H_{\text{vacuum}}$$



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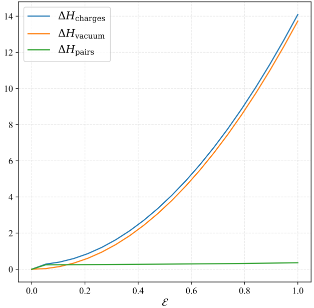

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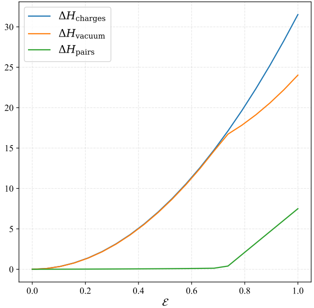
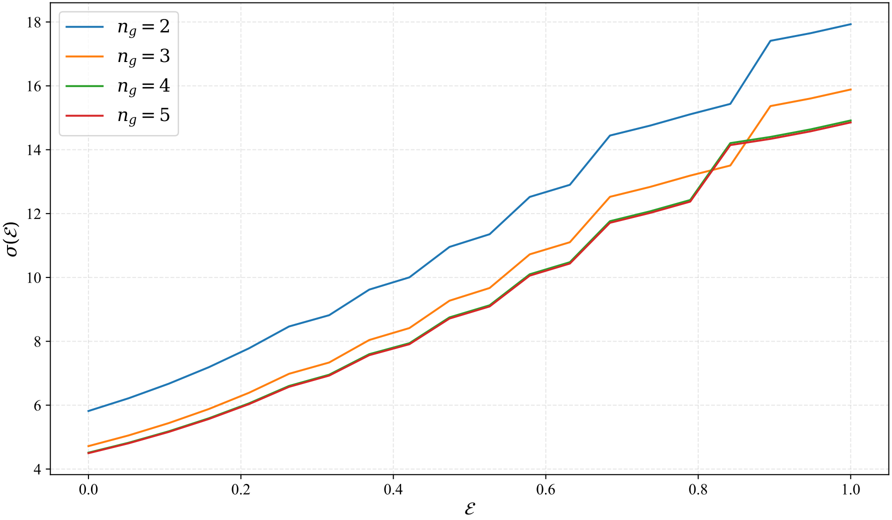
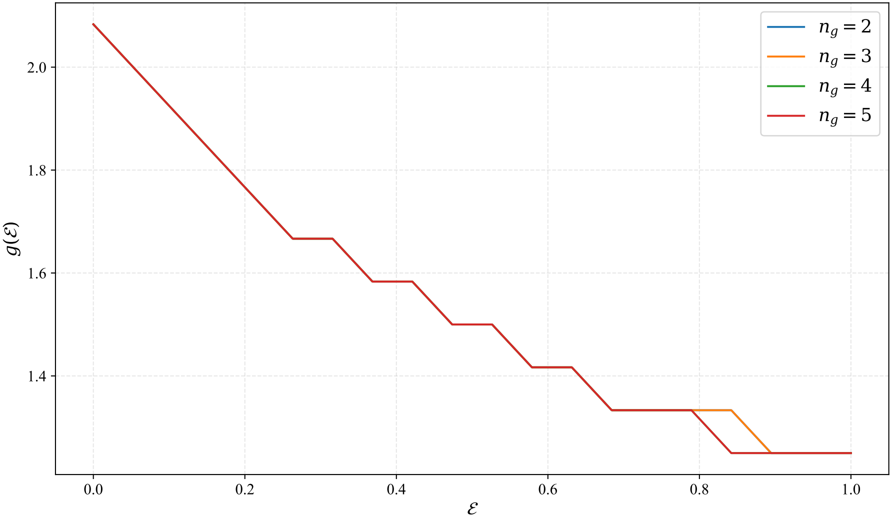


(c) $g \approx 3.0$.

Bonus: big $\sigma(\mathcal{E})$ plot



Bonus: big $g(\mathcal{E})$ plot



Bonus: big $\chi_{\mathcal{C}}$ plot