Sample-Based Krylov Quantum Diagonalization for Matrix Quantum Mechanics

Ground state properties of a holographic matrix model via sampled Krylov subspaces

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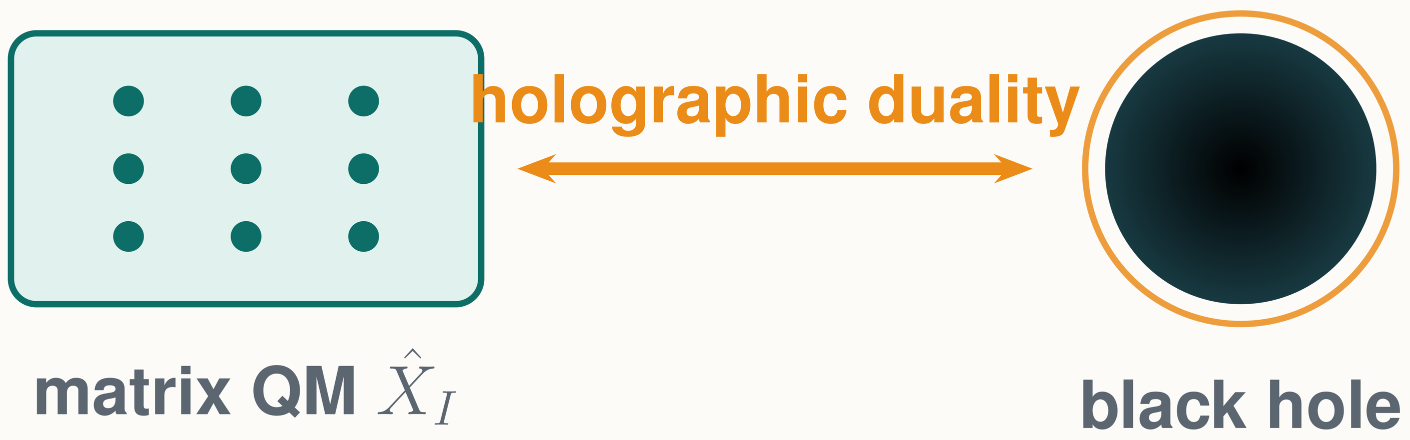
THE UPSHOT. The **first sample-based Krylov quantum diagonalization (SKQD) for matrix quantum mechanics**: an **optimization-free** eigensolver reaching **machine-precision ground states** of a gauge/gravity-relevant model — 10^{+} **orders** beyond VQE on the same emulator. On **IBM Heron hardware** it recovers **exact** E_0 **from a partial, noisy subspace** (as few as **20 of 64** basis states, ≈50% noise-induced), and it yields the **full wavefunction**, so entanglement inherits the same precision.

Why quantum computers for matrix models?

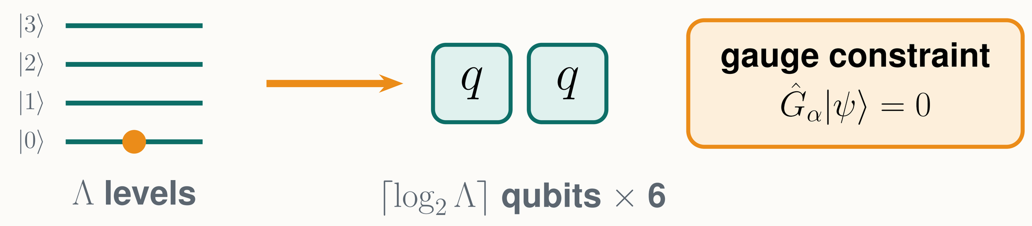
- **BFSS / BMN matrix models = holography in 0+1d** [Banks+'97; Berenstein+'02] — quantum states encode black holes, D-branes, and M-theory.
- **Sign problem** blocks Monte Carlo; **exact diagonalization (ED) explodes** as $\dim\mathcal{H} = \Lambda^{d(N^2-1)}$; **VQE degrades**: errors $10^{-3} \to 10^{-1}$, growing with coupling [Rinaldi+'22].
- **This work: first SKQD for matrix QM** — optimization-free, demonstrated on a real QPU [Yu+ 2501.09702; Rosanowski+ 2510.26951].



The model & its symbols

$$\hat{H} = \mathrm{Tr}\Big(\tfrac{1}{2}\hat{P}_I^2 + \tfrac{m^2}{2}\hat{X}_I^2 - \tfrac{g^2}{4}[\hat{X}_I, \hat{X}_J]^2\Big), \qquad \lambda = g^2 N \text{ [Rinaldi+'22]}$$

$$\hat{a}\,|n\rangle = \sqrt{n}\,|n-1\rangle, \quad n = 0,\dots,\Lambda-1; \qquad \dim\mathcal{H} = \Lambda^{d(N^2-1)}, \quad n_q = d(N^2-1)\lceil\log_2\Lambda\rceil$$



$N=2$, $d=2$ matrices ⇒ 6 modes; Λ: Fock levels/mode; λ: 't Hooft coupling; $\hat{G}_\alpha$: gauge (Gauss-law) generators, singlet ⇔ $\langle\hat{G}^2\rangle=0$.

emulator: $\Lambda = 2, 3, 4$ (6–12 qubits) **hardware: $\Lambda = 2$ (6 qubits)**

- **Verified** vs. published exact diagonalization to 10^{-9}.
- **Gauge singlets** isolated by $\langle\hat{G}^2\rangle$ + penalty: **4/64**, **2/729**, **16/4096** at $\Lambda = 2, 3, 4$; E_0 independent of penalty strength.

The SKQD algorithm

QUANTUM — sampling only (cheap, noise-tolerant) **CLASSICAL — exact inside subspace $\mathcal{B}$**

$|0\cdots0\rangle$ (1. vacuum) → $U^k = e^{-ik\hat{H}\Delta t}$ (2. Trotter, $k < D$) → **measure** (3. bitstrings) → $\mathcal{B} = \bigcup_k\{b\}$ (3. union) → $V^\top\hat{H}V$ (4. project) → $E_0,\ |\psi_0\rangle$ (4. diagonalize)

1. **Initialize** $|\psi_0\rangle = |0\cdots0\rangle$ — Fock vacuum (free ground state, a gauge singlet).
2. **Krylov evolve** $|\psi_k\rangle = U^k|\psi_0\rangle$, $U = e^{-i\hat{H}\Delta t}$, $\Delta t = \pi/\|\hat{H}\|$ (Trotter on QPU).
3. **Sample** each $|\psi_k\rangle$; keep the union $\mathcal{B} = \bigcup_{k<D}\{b\}$ of distinct bitstrings.
4. **Project & diagonalize** $H_{\mathrm{proj}} = V^\top\hat{H}V$ on $\mathrm{span}(\mathcal{B})$, exactly → E_0, $|\psi_0\rangle$.

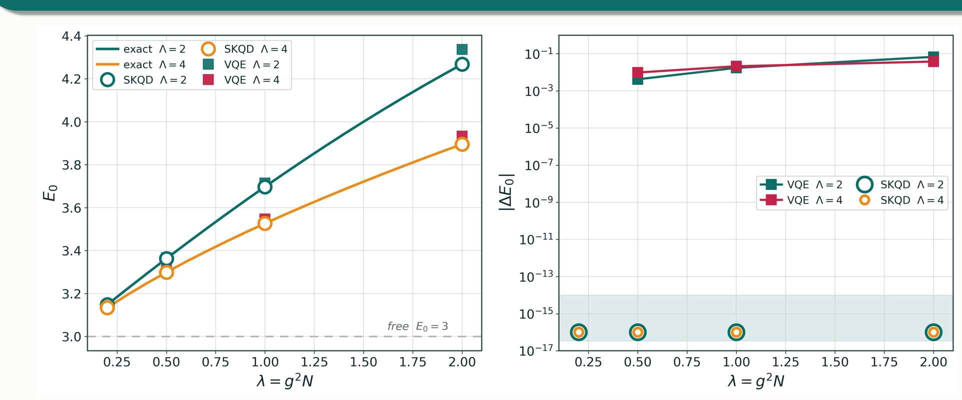
D: Krylov dimension; V: isometry whose columns are the sampled basis states ($V^\top V = I$, so overlap $S = I$).

Why it works. Rayleigh–Ritz: the lowest eigenvalue of $V^\top\hat{H}V$ obeys $\theta_0 \geq E_0$ and can only *decrease* as $\mathcal{B}$ grows. Noise adds extra bitstrings ⇒ larger $\mathcal{B}$, never an upward bias; the only failure mode is *missing* the dominant bitstrings.

no variational loop **no barren plateaus**

QPU: ~50 two-qubit gates $D=4$, **1 Trotter step**, $N_s = 100$–4096

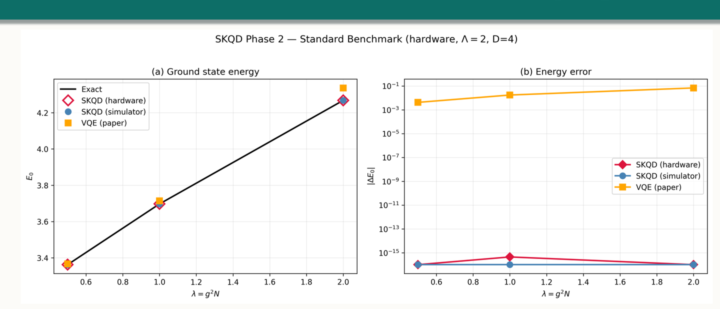
SKQD vs. VQE : Emulator Results



	$\Lambda = 2$		$\Lambda = 4$	
λ	**SKQD**	VQE	**SKQD**	VQE
0.5	$< 10^{-15}$	4.2×10^{-3}	$< 10^{-13}$	9.8×10^{-3}
1.0	$< 10^{-15}$	1.7×10^{-2}	$< 10^{-13}$	2.1×10^{-2}
2.0	$< 10^{-15}$	6.8×10^{-2}	$< 10^{-13}$	3.8×10^{-2}

machine precision, every λ & cutoff **10^{+} orders beyond VQE**

Hardware – exact E_0 from a noisy partial subspace



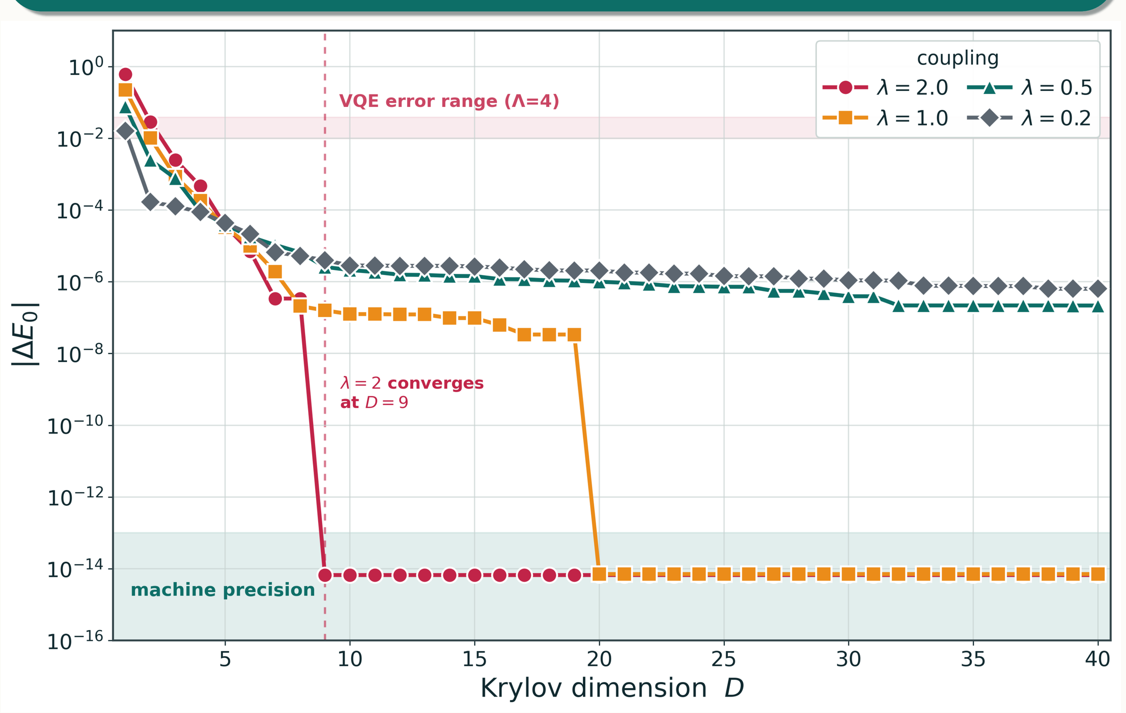
λ	$\lvert\mathcal{B}\rvert$ of 64	ρ_S	$\lvert\Delta E_0\rvert$ **hardware**
0.5	27	0.41	4.4×10^{-3}
1.0	20	0.50	$< 10^{-15}$
2.0	35	0.51	$< 10^{-15}$

$N_s=100$, $D=4$, 1 Trotter step (`ibm_kingston`). At $N_s=500$: exact for *all* λ ($|\mathcal{B}|=41$–49).

20 of 64 states, ≈50% noise-induced — E_0 still exact

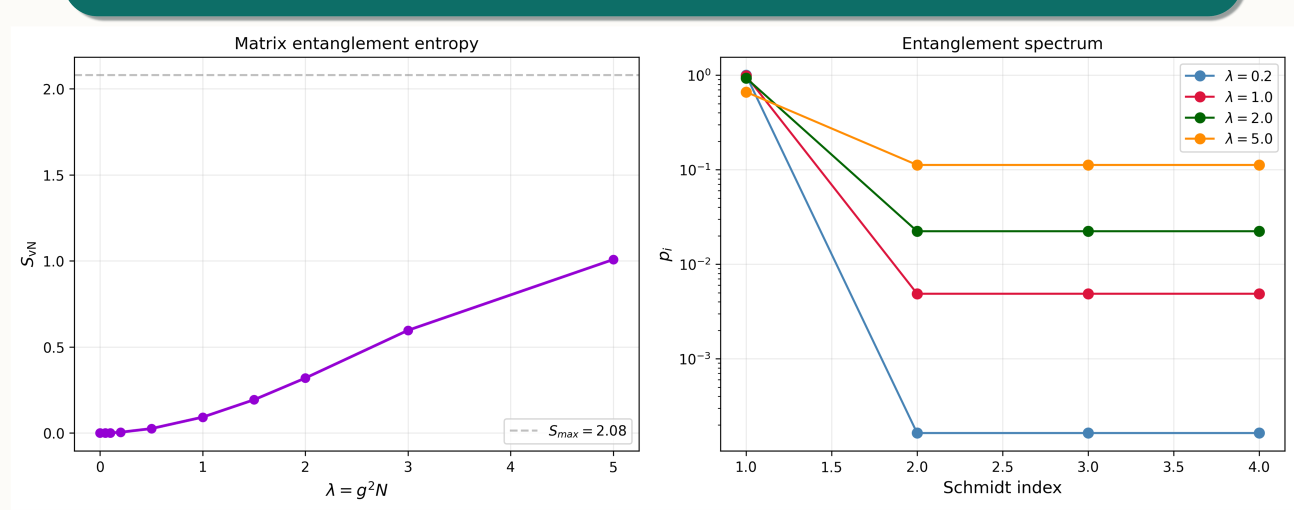
$\rho_S = |\mathcal{B}\cap\mathcal{S}|/|\mathcal{B}|$; $\mathcal{S}$: bitstrings supporting the low-lying singlets (post-hoc diagnostic). Only failure: $\lambda=0.5$, one dominant bitstring missed. *Caveat:* at $N_s=4096$, $|\mathcal{B}|\to 63$–$64 \approx$ full ED — a sharp test needs $\Lambda \geq 3$.

Convergence – strong coupling wins



Interaction-driven mixing populates the relevant Fock states, so **strong coupling converges fastest** ($\lambda=2$: $D=9$) — the inverse of VQE. Shown at $\Lambda=4$, $N_s=2\times10^5$, $\lambda \in \{0.2, 0.5, 1.0, 2.0\}$; sampled subspace stays small: **10–54×** compression ($|\mathcal{B}| \ll \dim\mathcal{H}$).

Beyond energy – entanglement entropy



$$S_{\mathrm{vN}} = -\mathrm{Tr}(\rho_A \ln \rho_A) = -\sum_i p_i \ln p_i, \qquad \rho_A = \mathrm{Tr}_{X_2}|\psi_0\rangle\langle\psi_0|, \qquad S_{\max} = 3\ln\Lambda$$

Fock-mode entropy at $\Lambda=2$, partition $\hat{X}_1 \,|\, \hat{X}_2$ (p_i: Schmidt spectrum) in the *extended* (ungauged) space — distinct from gauge-invariant target-space entanglement [Gautam–Hanada–Jevicki'25]. SKQD reproduces S_{vN} and the full state to $|\Delta S|,\ 1-F < 10^{-15}$ ($F = |\langle\psi_{\mathrm{exact}}|\psi_{\mathrm{SKQD}}\rangle|^2$).

Why SKQD wins

- **vs classical ED:** solve a tiny $|\mathcal{B}|\times|\mathcal{B}|$ block, not Λ^6 — **10–54×** compression where ED explodes.
- **vs VQE:** no optimization ⇒ no barren plateaus, local minima, or ansatz ceiling.
- **vs KQD:** sampling only (no Hadamard tests); orthonormal basis ⇒ overlap $S=I$, no ill-conditioning.
- **vs SQD:** Krylov systematically explores beyond a single reference state.
- **Ground-state properties:** full wavefunction ⇒ observables beyond energy at the same precision as E_0.

Next steps

- algebraic Pauli decomposition → 12-qubit hardware ($\Lambda = 3, 4$) — also the sharp noise-resilience test
- gauge-singlet post-selection (via $\hat{G}_\alpha$) as error mitigation
- **variational reference mass** to tame weak coupling (in prep.):

$$\frac{m^2}{2}\hat{X}^2 = \underbrace{\frac{\mu^2}{2}\hat{X}^2}_{\text{reference } |\psi_0\rangle} + \underbrace{\frac{m^2-\mu^2}{2}\hat{X}^2}_{\text{into interaction}}, \qquad \mu \neq m$$

Tune μ so the vacuum matches the true ground-state width ⇒ shorter Fock tails, easier weak-λ sampling.

- gauge-invariant target-space entanglement; minimal-BMN & fermions
- **QAOA state preparation:** combine QAOA [Farhi+'14] with an efficient bosonic quantum-simulation protocol for bosonic ground states.

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Key refs: Yu+ 2501.09702 · Rosanowski+ 2510.26951 · Rinaldi+ PRX Quantum **3**, 010324 (2022) · HLR 2604.14094 · BFSS (1997) · BMN (2002) · Farhi+ 1411.4028.

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