Efficient Quantum Computing for Lattice QED and Error Correction Codes for Pure SU(2) LGT

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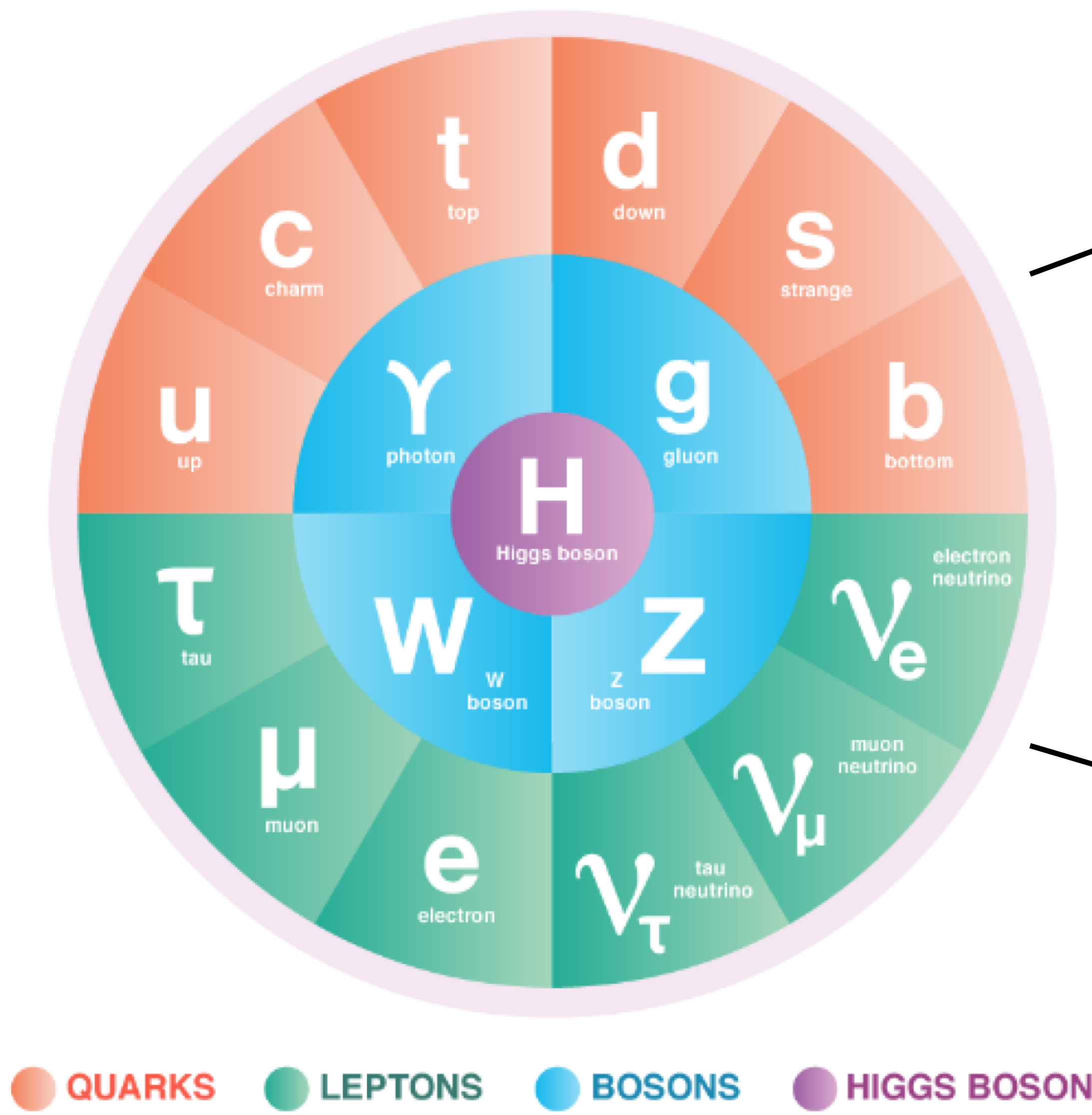
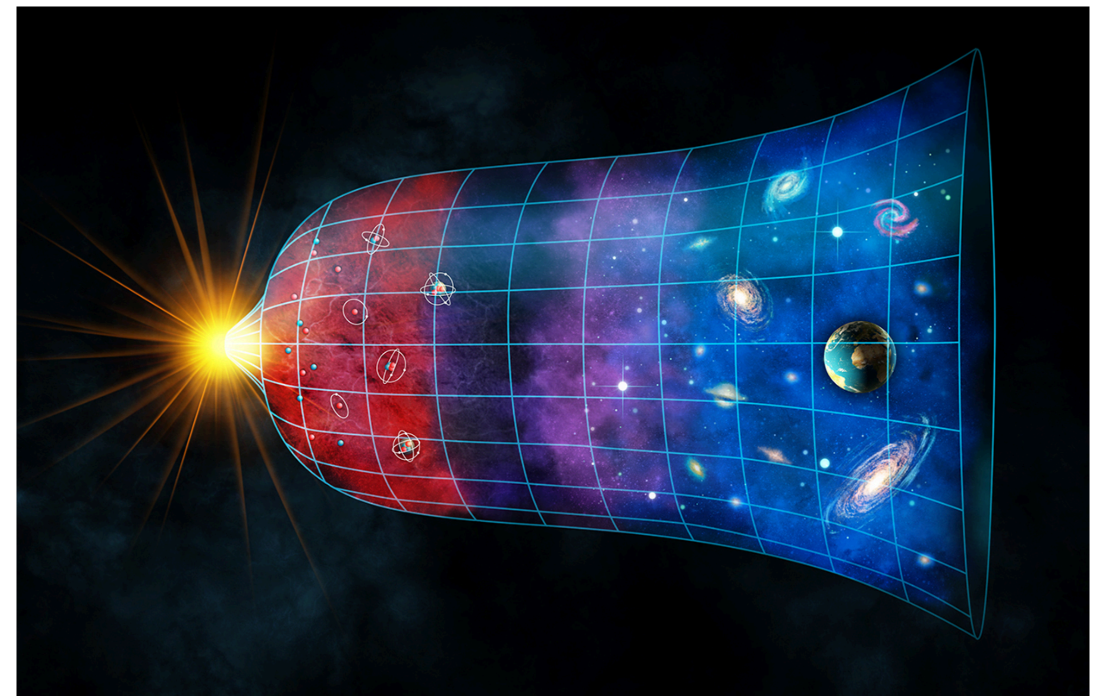
Standard Model

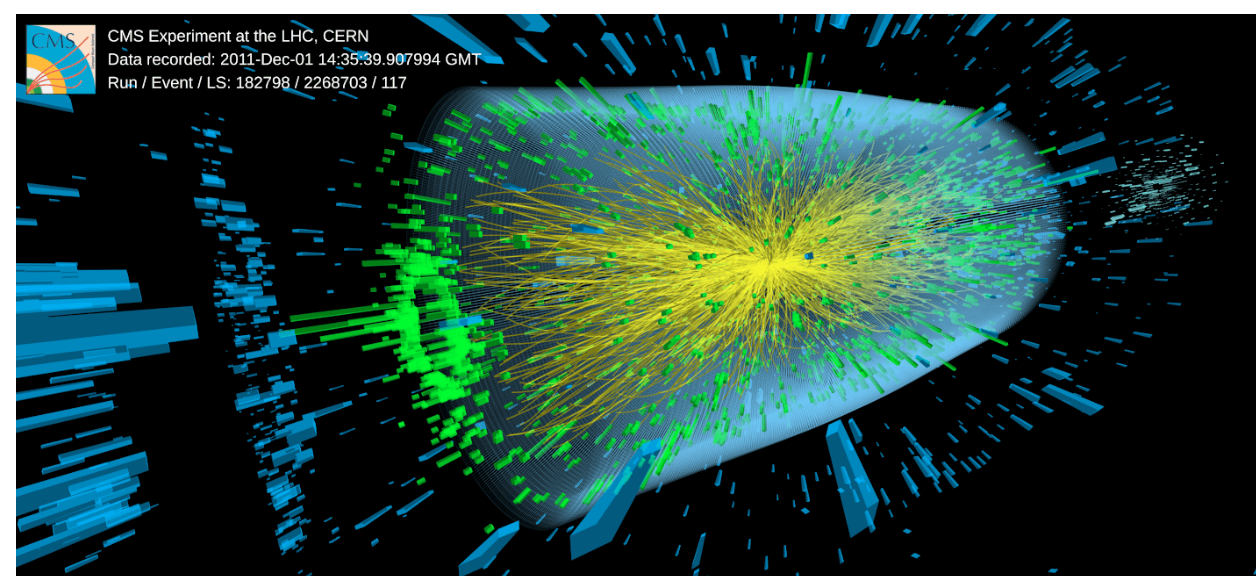
- **EM, weak, strong forces formulated as gauge theories**



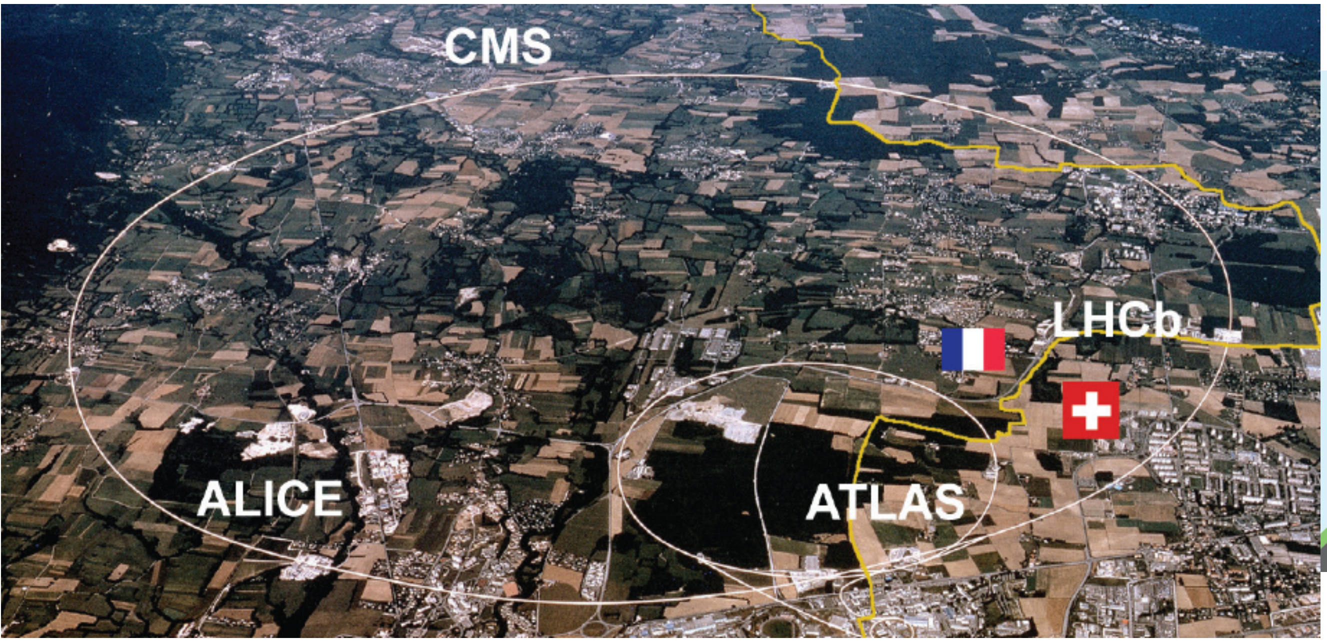
https://www.energy.gov/science/doe-explainsthe-standard-model-particle-physics

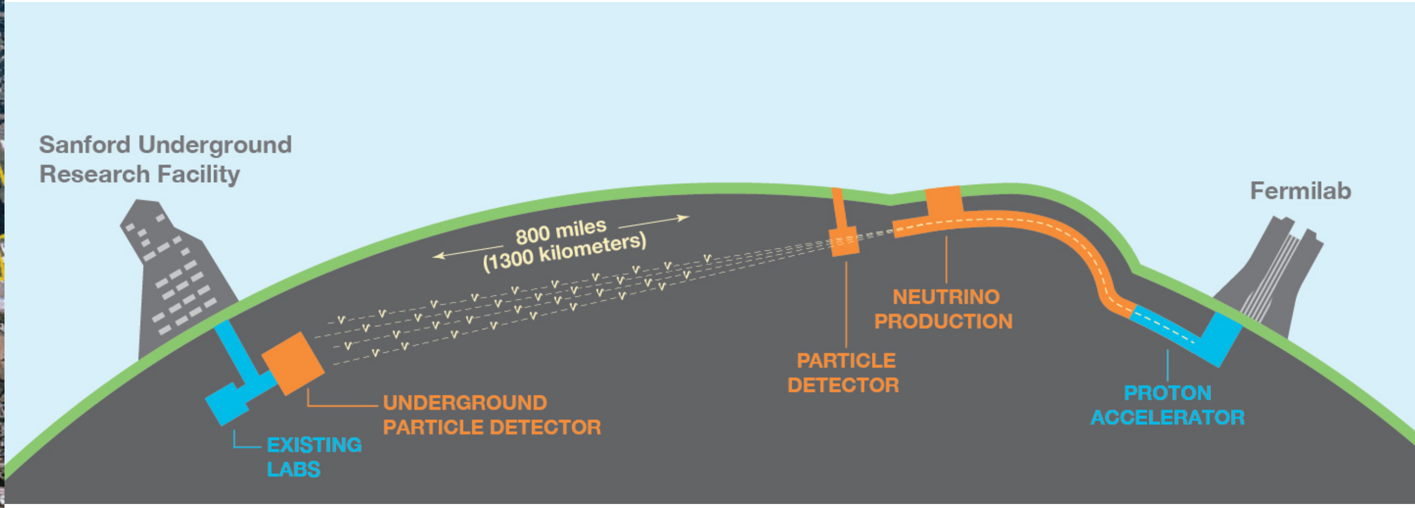


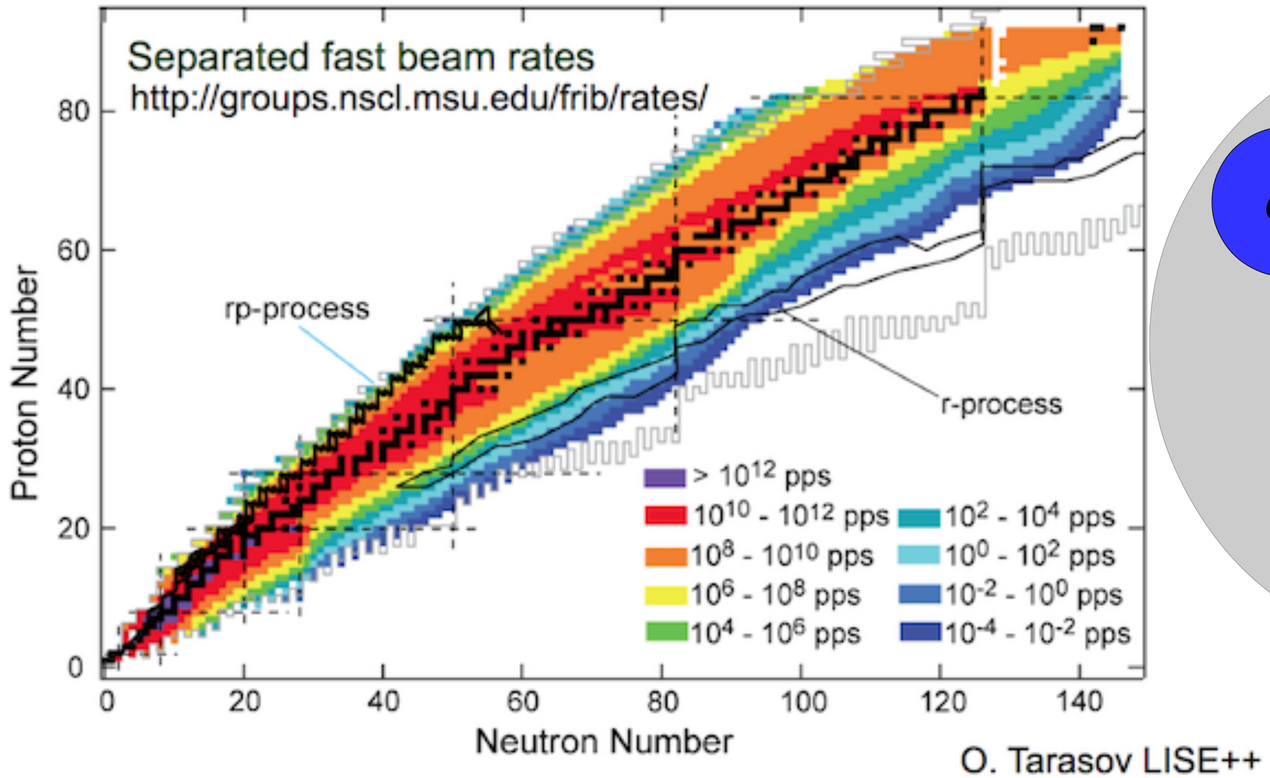
https://www.skyatnightmagazine.com/space-science/expansion-universe

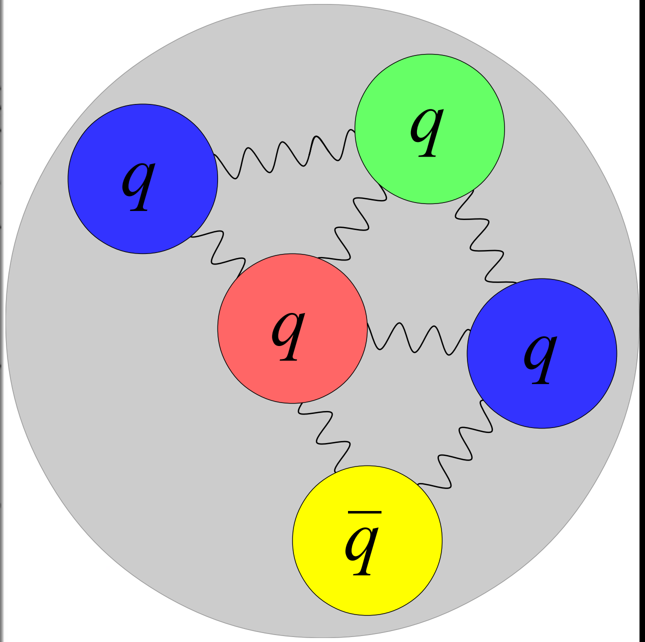


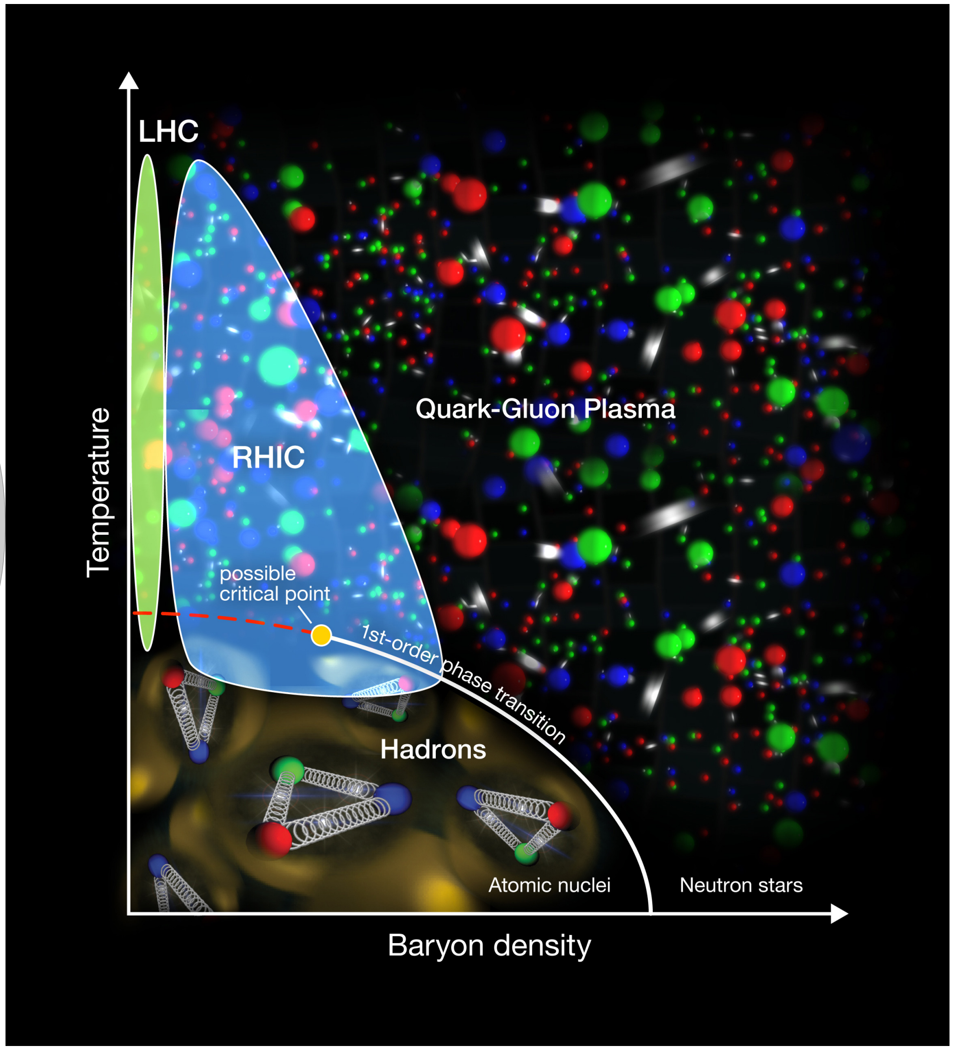
Nuclear and Particle Physics











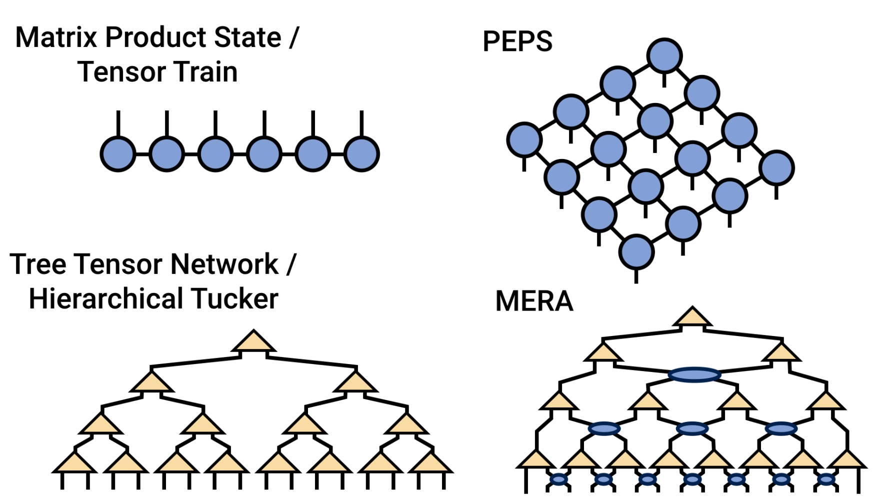
Why Hamiltonian Lattice Gauge Theory?

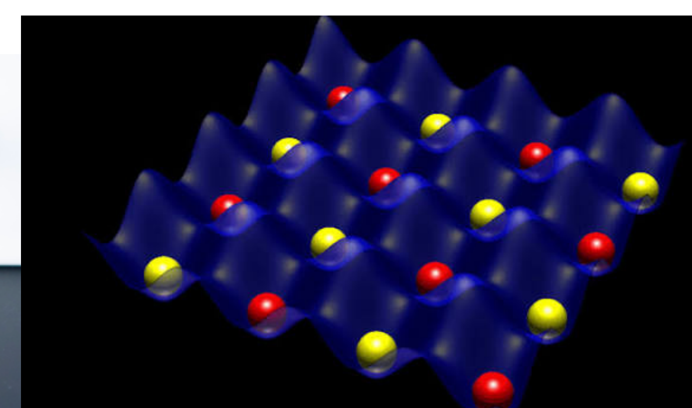
- **QCD is nonperturbative at hadronic scale: need nonperturbative input for precision comparison with experimental data**

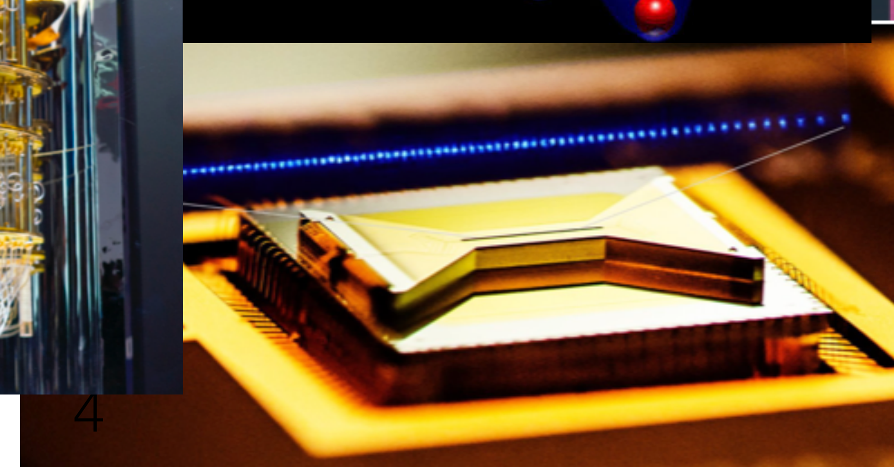
- **Euclidean lattice QCD is very successful:** hadron mass and structure, muon g-2, transport coefficients such as heavy quark diffusion

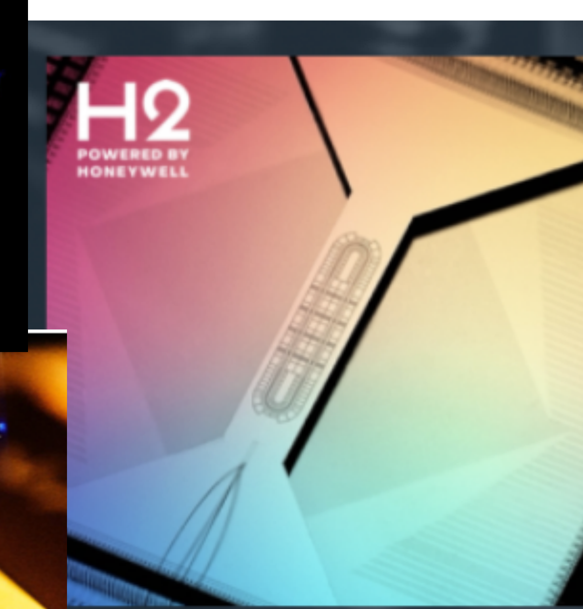
- **But faces sign problems for real-time and finite baryon density:** QCD critical point, real-time non-equilibrium dynamics

- **Hamiltonian lattice gauge theory attracts a lot of interest due to advancement of computing tools**





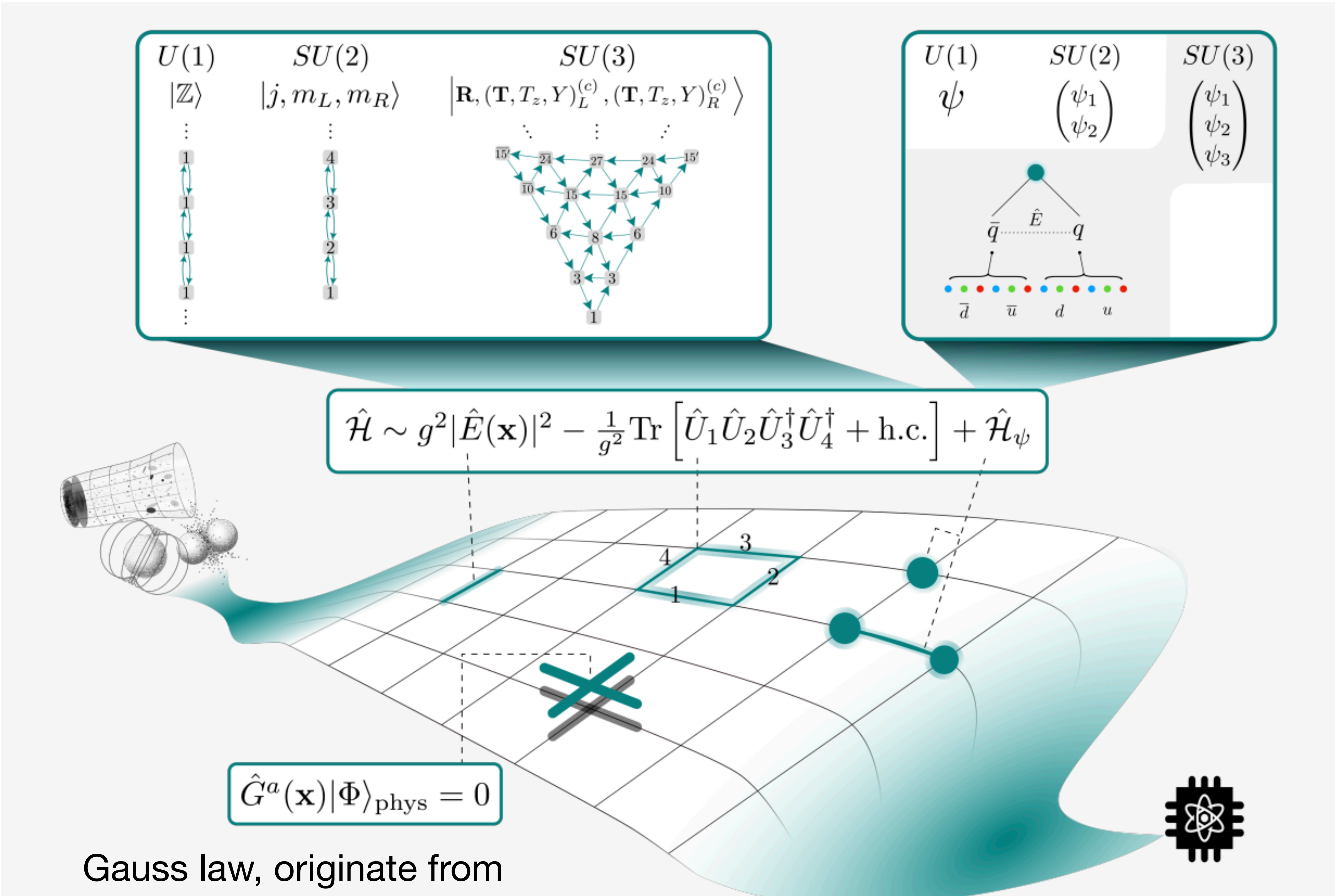




Contents

- Review of Hamiltonian lattice gauge theory (LGT): Kogut-Susskind (KS) Hamiltonian in the electric basis
- Efficient setup for lattice QED using a different Hamiltonian
- Convert Gauss law in KS into error correction codes

Kogut-Susskind Hamiltonian $(A_0 = 0)$



Gauss law, originate from secondary first-class constraint or integrating A_0 in path-integral

Bauer, Davoudi, Klco, Savage, 2404.06298

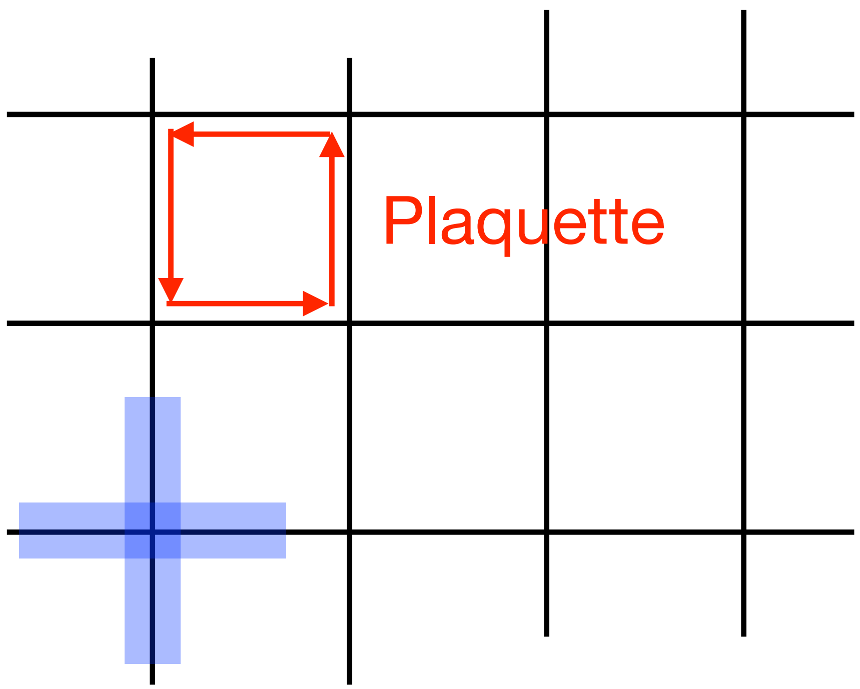
Electric Basis and Gauss Law

- **Electric basis on links: irrep and weight** $|R, m\rangle$

Electric energy is diagonal $(E_i^a)^2 |R, m\rangle = C_1 |R, m\rangle$

Total Hilbert space $\bigotimes_{\text{links}} |R, m\rangle$

Byrnes, Yamamoto, quant-ph/0510027



- **Work in the subspace of gauge invariant states**

Impose Gauss law: construct **singlets** at each vertex, which is given by a linear combination

$$\sum_{m_1,m_2,m_3,m_4} c_{m_1m_2m_3m_4} |R_1, m_1\rangle \otimes |R_2, m_2\rangle \otimes |R_3, m_3\rangle \otimes |R_4, m_4\rangle$$

Klco, Stryker, Savage, 1908.06935; Rahman, Lewis, Mendicelli, Powell, 2103.08661; Zache, González-Cuadra, Zoller, 2304.02527; Hayata, Hidaka, 2305.05950; Müller, XY, 2307.00045

- **Express plaquette operator in 6j symbols**
- **On quantum computer, need to work out controlled gates**

Lattice QED in Coulomb Gauge

Motivation: Efficient Simulation for ϕ^4 Theory

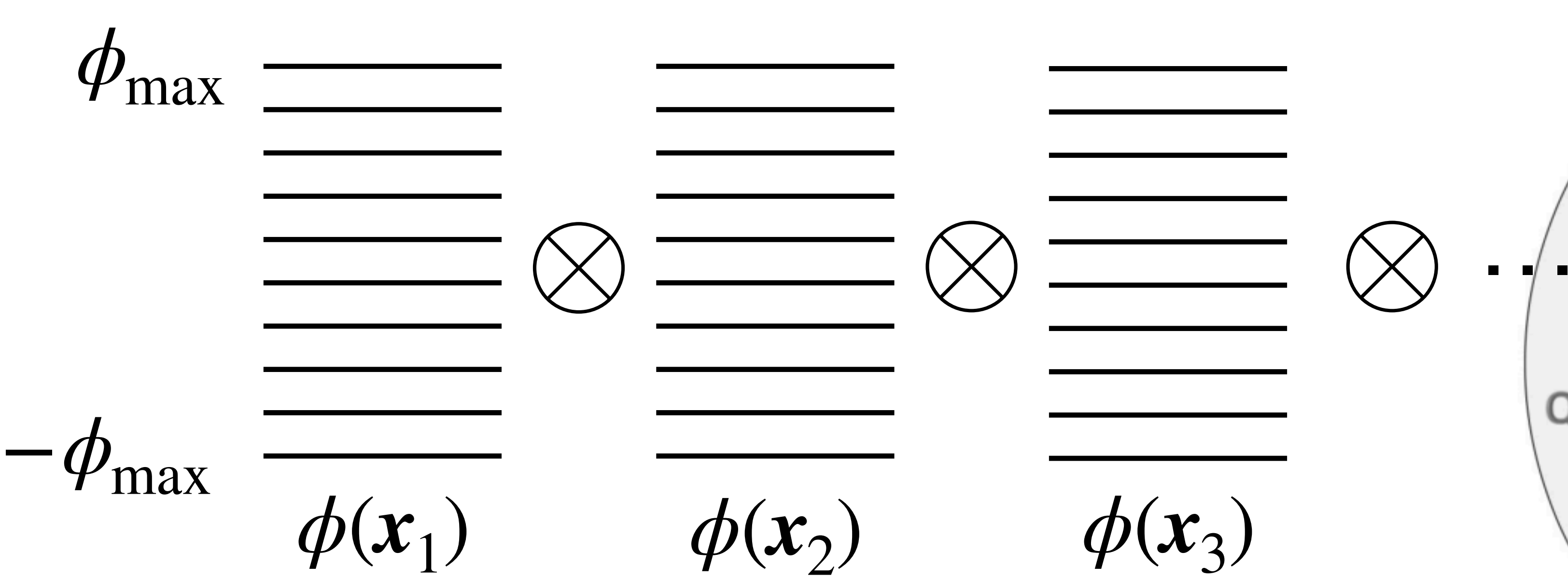
- **Quantum Fourier transform to efficiently convert between A and Π local spaces**

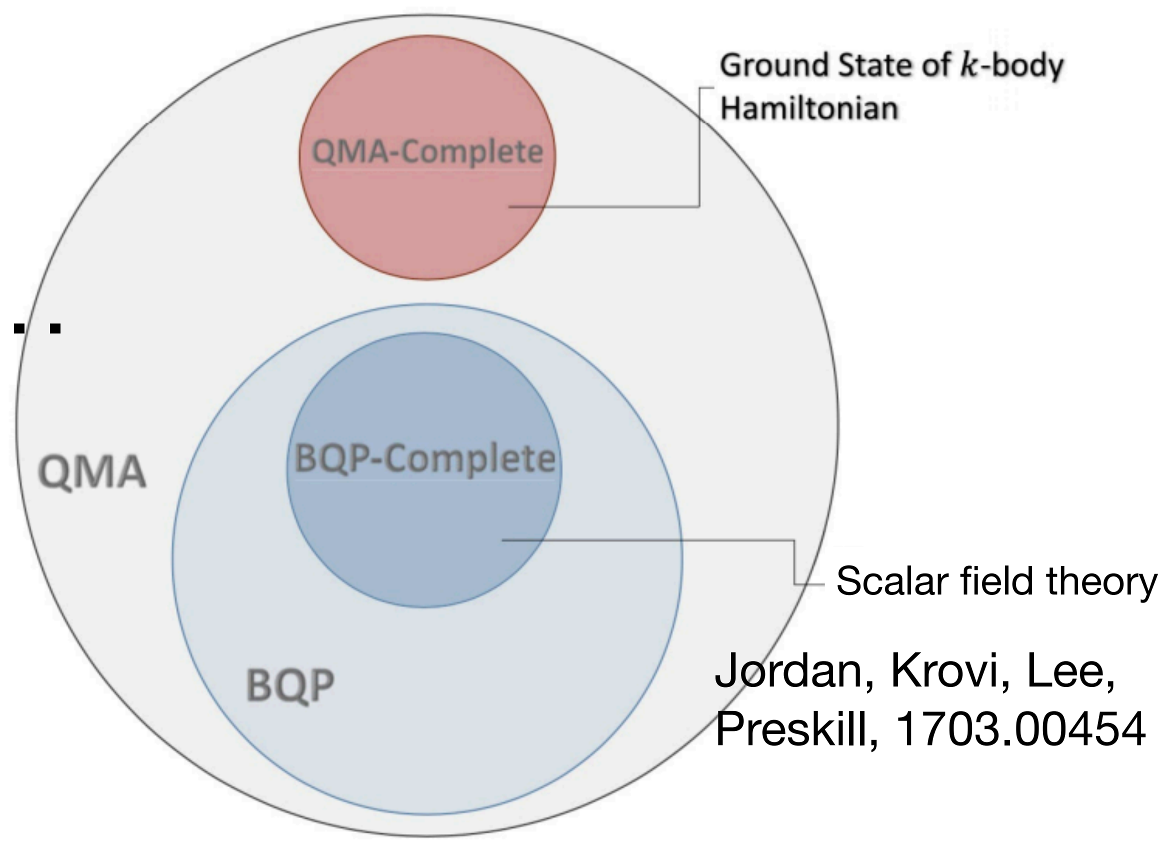
$$H = a^d \sum_{\boldsymbol{x}} \left[\frac{1}{2}\Pi^2 + \frac{1}{2}(\nabla_i \phi)^2 + \frac{1}{2} m_0^2 \phi^2 + \frac{\lambda}{4!}\phi^4 \right]$$

$$[\phi(\boldsymbol{x}), \Pi(\boldsymbol{y})] = i a^{-d} \delta_{\boldsymbol{x}\boldsymbol{y}}$$

Jordan, Lee, Preskill, 1112.4833

Trotterized Hamiltonian pieces H_ϕ and H_Π are diagonal in each space, also efficient to construct Pauli gates





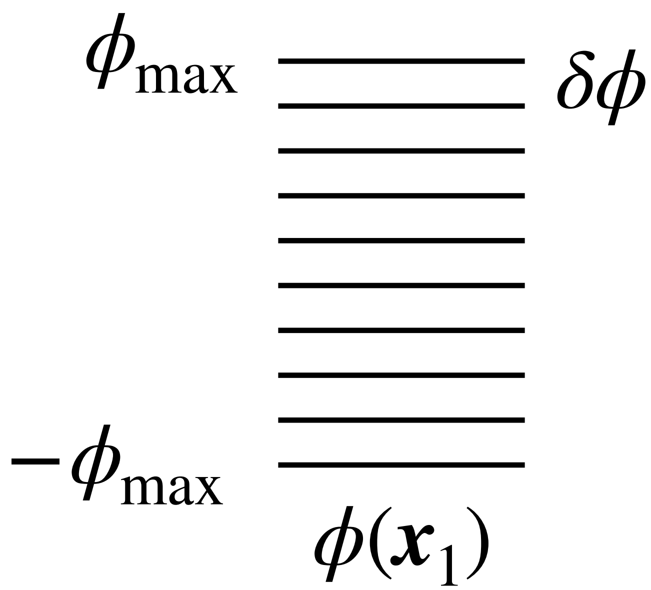
Jordan, Krovi, Lee, Preskill, 1703.00454

Credit: Klco, Roggero, Savage

Motivation: Proved Bounds of Resources for ϕ^4

- Easy to prove resources at given volume, field digitization $\delta\phi$ and truncation $\phi_{\max}$
- **Key to prove resources for describing all states up to E with accuracy ϵ**

Desired QFT is only low-energy EFT of the LGT



Use Chebyshev's inequality

$$\phi_{\max} = O\left(\max_{\mathbf{x}\in\Omega}\sqrt{\frac{\mathcal{V}}{\epsilon_{\mathrm{trunc}}}\langle\psi|\phi(\mathbf{x})^2|\psi\rangle}\right),$$

$$\pi_{\max} = O\left(\max_{\mathbf{x}\in\Omega}\sqrt{\frac{\mathcal{V}}{\epsilon_{\mathrm{trunc}}}\langle\psi|\pi(\mathbf{x})^2|\psi\rangle}\right),$$

Number of qubits per site

$$n_b = \lceil \log_2\left(1 + 2\phi_{\max}/\delta_\phi\right)\rceil$$

$$\pi_{\max} = \frac{\pi}{a^d \delta_{\phi(\mathbf{x})}}$$

$$\langle\psi|\phi(\mathbf{x})^2|\psi\rangle \leq \frac{2E}{a^d m_0^2},$$

$$\langle\psi|\pi(\mathbf{x})^2|\psi\rangle \leq \frac{2E}{a^d}.$$

Jordan, Lee, Preskill, 1112.4833

Dirac Quantization of QED in Coulomb Gauge

- **Motivation: straightforward circuit construction, rigorous resource estimate**
- **Choosing a gauge turns first-class constraints to second-class**

$$\Pi^0 = 0,\ \partial_i \Pi^i = J^0$$

In Coulomb gauge:

$$\mathcal{H}(\boldsymbol{x}) = \frac{1}{2}\Pi^2_{\perp i}(\boldsymbol{x}) + \frac{1}{2}[\varepsilon_{ijk}\partial_j A_k(\boldsymbol{x})]^2 - J^i(\boldsymbol{x})A_i(\boldsymbol{x}) - \frac{1}{2}J^0(\boldsymbol{x})A_0(\boldsymbol{x}) - \bar{\psi}(\boldsymbol{x})(i\gamma^i\partial_i - m)\psi(\boldsymbol{x})$$

Dirac quantization: $[A_i(\boldsymbol{x}), A_j(\boldsymbol{y})] = 0\,,\ [\Pi_{\perp i}(\boldsymbol{x}), \Pi_{\perp j}(\boldsymbol{y})] = 0\,,$

$$[A_i(\boldsymbol{x}), \Pi_{\perp j}(\boldsymbol{y})] = i\left(\delta_{ij} - \frac{\partial_i\partial_j}{\nabla^2}\right)\delta^{(3)}(\boldsymbol{x}-\boldsymbol{y})$$

Choose dynamical variable to be Π_j $\quad \Pi_{\perp i}(\boldsymbol{x}) = \left(\delta_{ij} - \frac{\partial_i\partial_j}{\nabla^2}\right)\Pi_j(\boldsymbol{x})$

$$[\Pi_{\perp i}(\boldsymbol{x})]^2 \to \Pi_i(\boldsymbol{x})\left(\delta_{ij} - \frac{\partial_i\partial_j}{\nabla^2}\right)\Pi_j(\boldsymbol{x})$$

$[A_i(\boldsymbol{x}), \Pi_j(\boldsymbol{y})] = i\delta_{ij}\delta^{(3)}(\boldsymbol{x}-\boldsymbol{y})$ Like scalar field theory now

Discretized Green's Function for Transversality

- **Discretization on lattice**

Green's function of discrete Laplacian, analytically known

$$\hat{H}_\Pi = \frac{1}{2}\sum_{\hat{\boldsymbol{x}},i}\hat{\Pi}_i^2(\hat{\boldsymbol{x}}) + \frac{1}{2}\sum_{\hat{\boldsymbol{x}},\hat{\boldsymbol{y}},i,j}\hat{\Pi}_i(\hat{\boldsymbol{x}})\hat{\Pi}_j(\hat{\boldsymbol{y}})\partial^{(L+)}_{\hat{x}_i}\partial^{(L+)}_{\hat{y}_j}G(\hat{\boldsymbol{x}},\hat{\boldsymbol{y}})$$

$$\hat{H}_A = -\frac{1}{2}\sum_{\hat{\boldsymbol{x}},i}\hat{A}_i(\hat{\boldsymbol{x}})\left[\nabla^{(L)}_{\hat{\boldsymbol{x}}}\right]^2\hat{A}_i(\hat{\boldsymbol{x}}) + \frac{1}{2}\sum_{\hat{\boldsymbol{x}},i,j}\hat{A}_i(\hat{\boldsymbol{x}})\partial^{(L+)}_{\hat{x}_i}\partial^{(L-)}_{\hat{x}_j}\hat{A}_j(\hat{\boldsymbol{x}})$$

$$[\hat{A}_i(\hat{\boldsymbol{x}}), \hat{\Pi}_j(\hat{\boldsymbol{y}})] = i\delta_{ij}\delta_{\hat{\boldsymbol{x}}\hat{\boldsymbol{y}}}$$

$$\partial^{(L+)}_{\hat{x}_i}f(\hat{\boldsymbol{x}}) \equiv f(\hat{\boldsymbol{x}}+\hat{i}) - f(\hat{\boldsymbol{x}})\,,$$
$$\partial^{(L-)}_{\hat{x}_i}f(\hat{\boldsymbol{x}}) \equiv f(\hat{\boldsymbol{x}}) - f(\hat{\boldsymbol{x}}-\hat{i})$$

$$\left[\nabla^{(L)}_{\hat{\boldsymbol{x}}}\right]^2 \equiv \sum_i \partial^{(L+)}_{\hat{x}_i}\partial^{(L-)}_{\hat{x}_i} = \sum_i \partial^{(L-)}_{\hat{x}_i}\partial^{(L+)}_{\hat{x}_i}$$

- **Use the lattice inverse of Laplacian guarantees decoupling of longitudinal fields**

$$\hat{A}_{//\,i}(\hat{\boldsymbol{x}}) = \sum_{\hat{\boldsymbol{y}},j}\partial^{(L+)}_{\hat{x}_i}G(\hat{\boldsymbol{x}},\hat{\boldsymbol{y}})\partial^{(L-)}_{\hat{y}_j}A_j(\boldsymbol{y})$$ gives vanishing $\hat{H}_A$

$$\left[\sum_i \partial^{(L-)}_{\hat{x}_i}\hat{A}_i(\hat{\boldsymbol{x}}), \hat{H}\right] = 0\,,\ \forall\hat{\boldsymbol{x}}$$

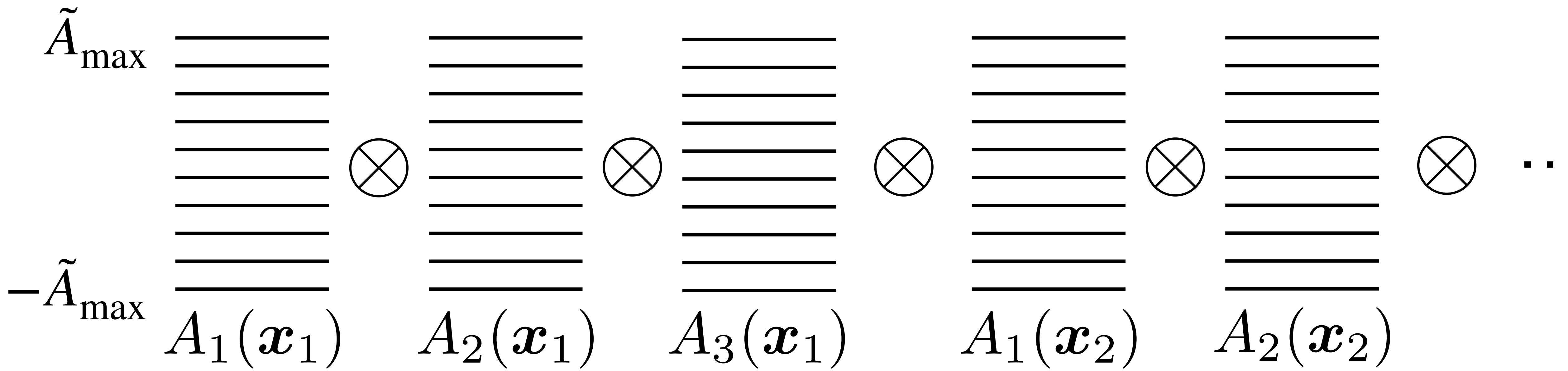
Longitudinal field is never generated

Similarly for $\hat{\Pi}_{//}$

Efficient Simulation for LQED in Coulomb Gauge

- **Quantum Fourier transform to efficiently convert between A and Π local spaces**

$$[\hat{A}_i(\hat{\boldsymbol{x}}), \hat{\Pi}_j(\hat{\boldsymbol{y}})] = i\delta_{ij}\delta_{\hat{\boldsymbol{x}}\hat{\boldsymbol{y}}}$$



Qubit# per site per direction $\quad n_A = \lceil \log_2(2\tilde{A}_{\max}/\delta\tilde{A} + 1) \rceil$

Use Chebyshev's inequality, take care of coupling w/ fermion and **Dirac fermion sea**

$$\tilde{A}_{\max} = \sqrt{\frac{\hat{E}'\hat{V}^{5/3}}{2\pi^2\epsilon} + \frac{g\hat{V}^{2/3}}{6\pi^2}} \qquad \frac{\pi}{\delta\tilde{A}} = \tilde{\Pi}_{\max} = \sqrt{\frac{6\hat{E}'\hat{V}}{\epsilon}}$$

XY, 2507.01089

Single- and two-qubit gate costs $\quad O(n_A^2\hat{V}^2) \longleftarrow \hat{A}(\hat{\boldsymbol{x}}) = -\frac{1}{2}\delta\tilde{A}\sum_{j=0}^{n_A-1} 2^j\sigma_j^z(\hat{\boldsymbol{x}})$

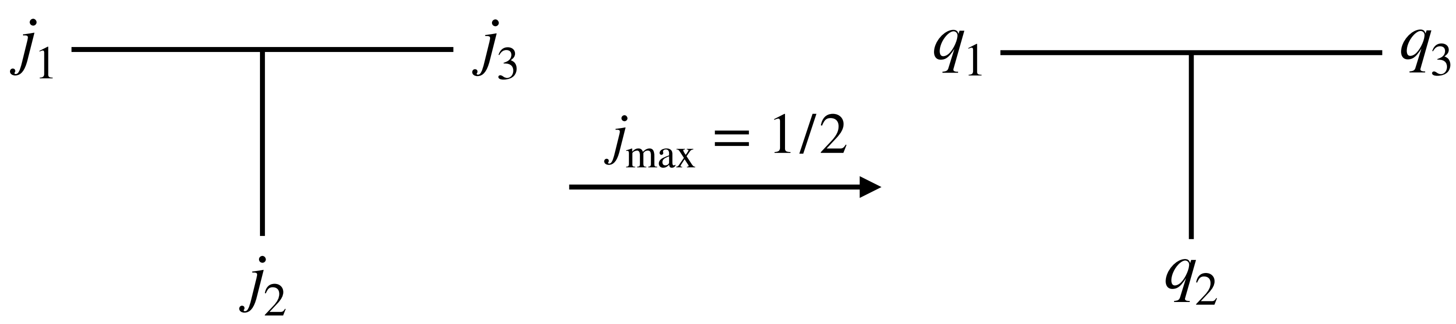
Quantum Error Correction Codes from Gauss Law in Truncated SU(2)

Example 1

Gauss Law Error Correction Code in Truncated SU(2)

- **Consider $j_{\max} = 1/2$ and map onto qubits**

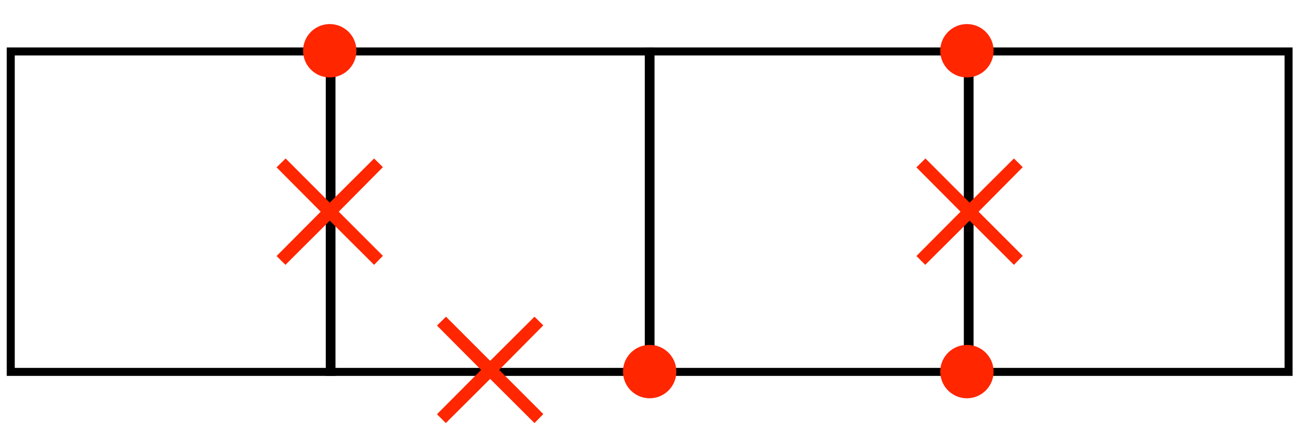
XY, 2511.13721



$\lvert j_1 j_2 j_3 \rangle$	$\lvert q_1 q_2 q_3 \rangle$
$\lvert 000 \rangle$	$\lvert 000 \rangle$
$\lvert \frac{1}{2} \frac{1}{2} 0 \rangle$	$\lvert 110 \rangle$
$\lvert \frac{1}{2} 0 \frac{1}{2} \rangle$	$\lvert 101 \rangle$
$\lvert 0 \frac{1}{2} \frac{1}{2} \rangle$	$\lvert 011 \rangle$

Stabilizer $Z_1 Z_2 Z_3$

- **Gauss law able to detect and correct any single-bit flip error (classical code)**



Detectable Correctable

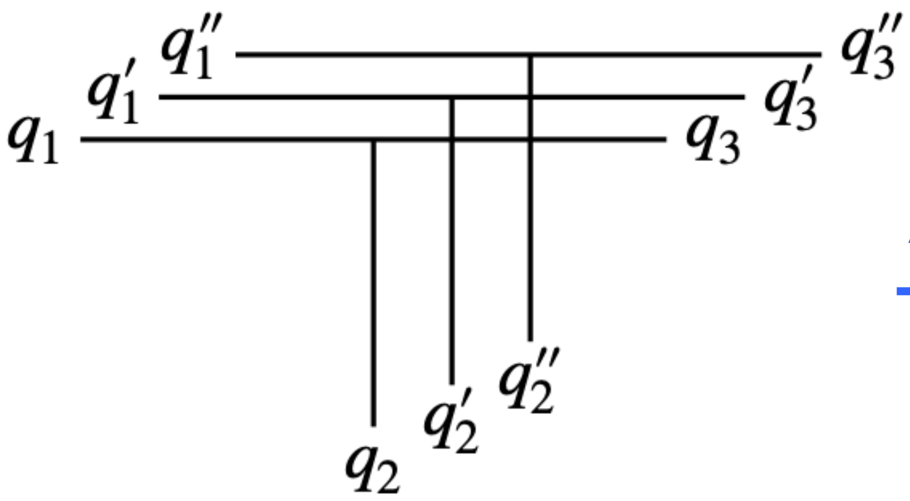
Every vertex checks the ZZZ parity

For N-plaquette w/ OBC:

$$[3N + 1, N, 4]$$

Make the Code a Quantum Code

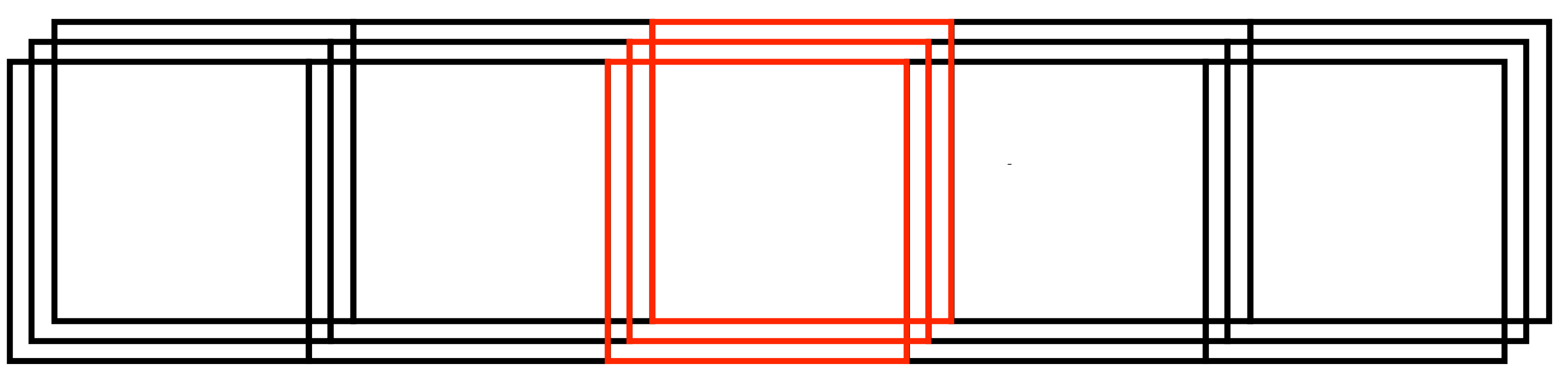
- **To correct phase error: use repetition** For Z_2, U(1), see Spagnoli, Roggero, Wiebe, 2405.19293



Add XX stabilizers

Syndromes						Error
$X_1X_{1'}$	$X_{1'}X_{1''}$	$X_2X_{2'}$	$X_{2'}X_{2''}$	$X_3X_{3'}$	$X_{3'}X_{3''}$	
1	1	1	1	1	1	No
-1	1	1	1	1	1	Z_1
-1	-1	1	1	1	1	$Z_{1'}$
1	-1	1	1	1	1	$Z_{1''}$
1	1	-1	1	1	1	Z_2
1	1	-1	-1	1	1	$Z_{2'}$
1	1	1	-1	1	1	$Z_{2''}$
1	1	1	1	-1	1	Z_3
1	1	1	1	-1	-1	$Z_{3'}$
1	1	1	1	1	-1	$Z_{3''}$

- **Able to detect and correct any single-qubit error (flip and phase)**



For N-plaquette w/ OBC:

$$[[9N+3, N, 3]]$$

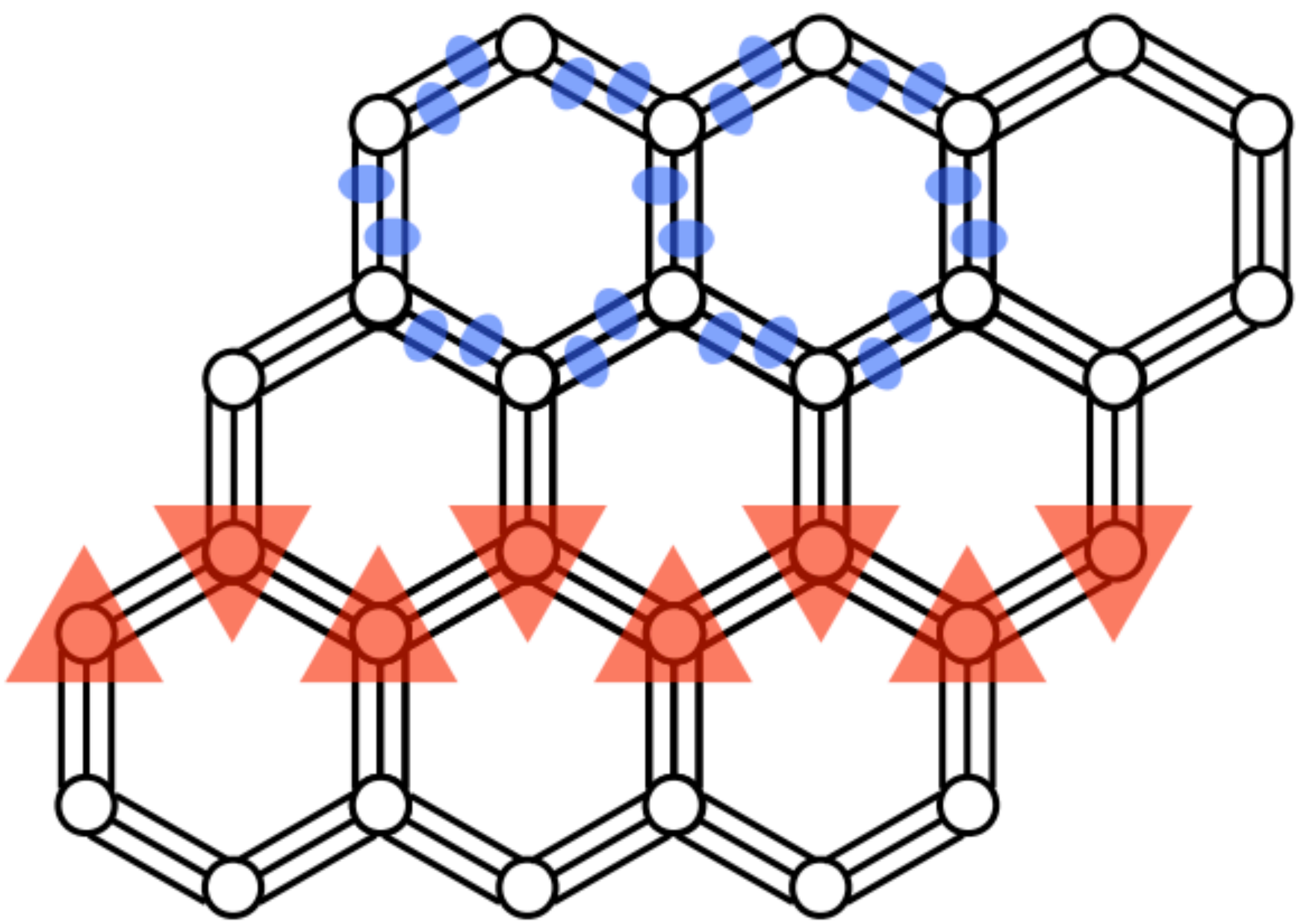
XY, 2511.13721

- **Logical operators correspond to excitations of singlet states**

$$\overline{X}(n) = X_t(n)X_r(n)X_b(n)X_l(n) \qquad \overline{Z}(n) = Z_t(n)Z_{t'}(n)Z_{t''}(n)$$

Easily Generalized to 2+1D and 3+1D

- **2+1D: honeycomb lattice**



Stabilizers: XX and $Z^{\otimes 9}$

XY, 2511.13721

For OBC:

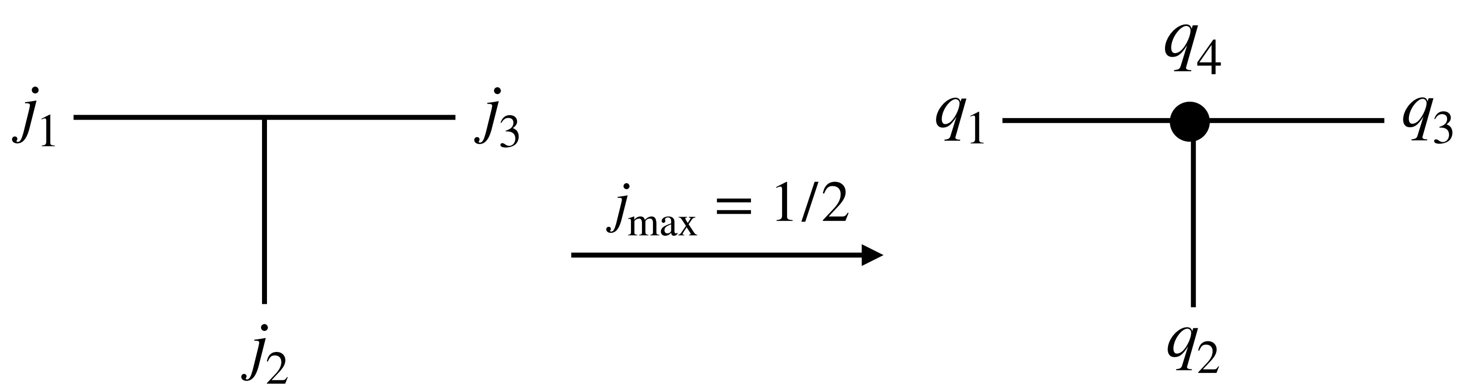
$$[[3(3N_xN_y + 2N_x + 2N_y - 1), N_xN_y, 3]]$$

Quantum Error Correction Codes from Gauss Law in Truncated SU(2)

Example 2

Carbon Code from SU(2) Gauss Law

- **Add an extra qubit at each vertex**



$\lvert j_1 j_2 j_3\rangle$	$\lvert q_1 q_2 q_3 q_4\rangle$
$\lvert 000\rangle$	$\frac{1}{\sqrt{2}}(\lvert 0000\rangle + \lvert 1111\rangle)$
$\lvert \frac{1}{2}\frac{1}{2}0\rangle$	$\frac{1}{\sqrt{2}}(\lvert 1100\rangle + \lvert 0011\rangle)$
$\lvert \frac{1}{2}0\frac{1}{2}\rangle$	$\frac{1}{\sqrt{2}}(\lvert 1010\rangle + \lvert 0101\rangle)$
$\lvert 0\frac{1}{2}\frac{1}{2}\rangle$	$\frac{1}{\sqrt{2}}(\lvert 0110\rangle + \lvert 1001\rangle)$

Stabilized by $Z_1Z_2Z_3Z_4$ and $X_1X_2X_3X_4$

Locally a [[4,2,2]] code

- **Make it a quantum error correction code: encode data qubits by another code**

Concatenate with [[6,2,2]] code and get [[12,2,4]] carbon code

$$\left| q_1 q_{1'} q_{1''} q_2 q_{2'} q_{2''} q_3 q_{3'} q_{3''} q_4 q_{4'} q_{4''} \right\rangle$$

10 stabilizers:

$$Z_1Z_2Z_3Z_4\,,\quad Z_{1'}Z_{2'}Z_{3'}Z_{4'}\,,\quad Z_{1''}Z_{2''}Z_{3''}Z_{4''}\,,$$
$$X_1X_2X_3X_4\,,\quad X_{1'}X_{2'}X_{3'}X_{4'}\,,\quad X_{1''}X_{2''}X_{3''}X_{4''}\,,$$

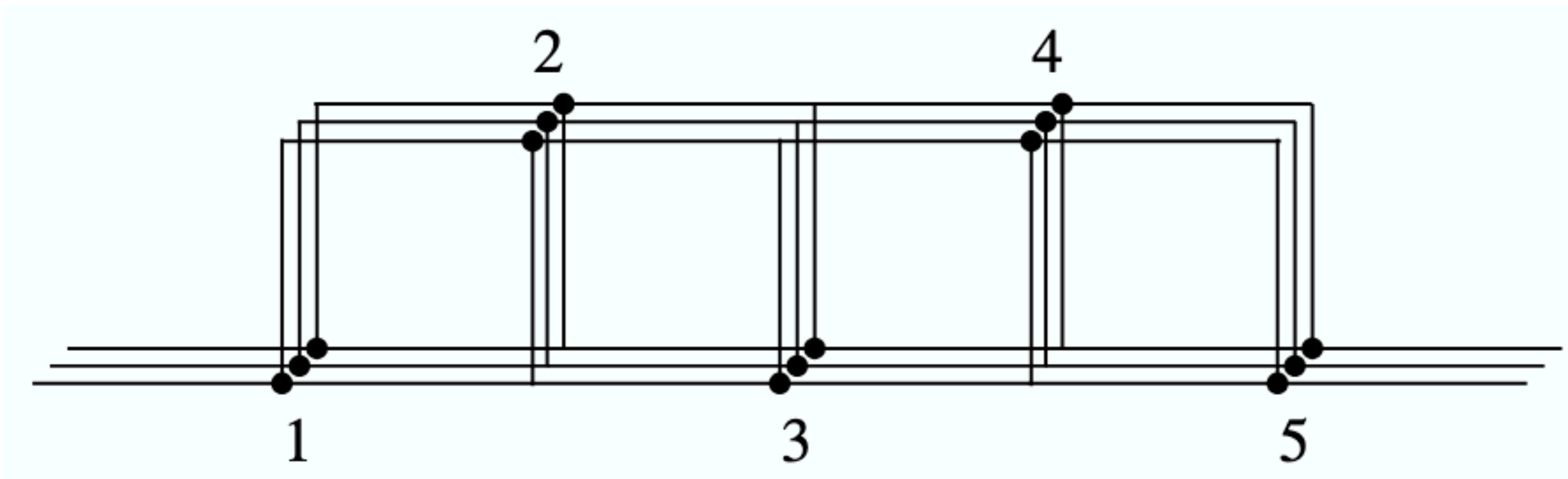
$$Z_1Z_{1'}Z_{1''}Z_2Z_{2'}Z_{2''}\,,\quad Z_2Z_{2'}Z_{2''}Z_3Z_{3'}Z_{3''}\,,$$
$$X_1X_{1'}X_{2'}X_{2''}X_{3''}X_3\,,\quad X_2X_{2'}X_{3'}X_{3''}X_{1''}X_1\,.$$

Carbon Code from SU(2) Gauss Law

- **Logical gates**

$$\overline{Z}_1 = Z_1 Z_{1'} Z_2 Z_{4'}\,, \qquad \overline{Z}_2 = Z_{1'} Z_{1''} Z_{2'} Z_{4''}\,,$$
$$\overline{X}_1 = X_3 X_{3'} X_4 X_{4'}\,, \qquad \overline{X}_2 = X_3 X_{3''} X_4 X_{4''}\,.$$

- **Hamiltonian in terms of logical gates**



$$H_E = \frac{3g^2}{16} \sum_n \left[3 - \overline{Z}_1(n) - \overline{Z}_2(n) - \overline{Z}_1(n)\overline{Z}_2(n)\right]$$

$$\Box(n) \to \overline{M}_2(n-1)\overline{M}_1(n)\overline{M}_2(n+1)\overline{M}_1(n+2)\overline{X}_2(n)\overline{X}_1(n+1)$$

$$\overline{M}_i(n) = \frac{1+\overline{Z}_i(n)}{2} + \frac{1-\overline{Z}_i(n)}{2\sqrt{2}}$$

Summary

- Efficient quantum computing for lattice QED in Coulomb gauge, proved rigorous bounds for resources to describe all states up to a given energy and accuracy

- Constructed two quantum error correction codes for minimally truncated Kogut-Susskind SU(2) LGT from Gauss law constraints