From encoding fermions to generating particle showers: Parity methods for HEP

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QuantHEP 2026

parityqc

parityqc

Introduction

Generating particle showers:

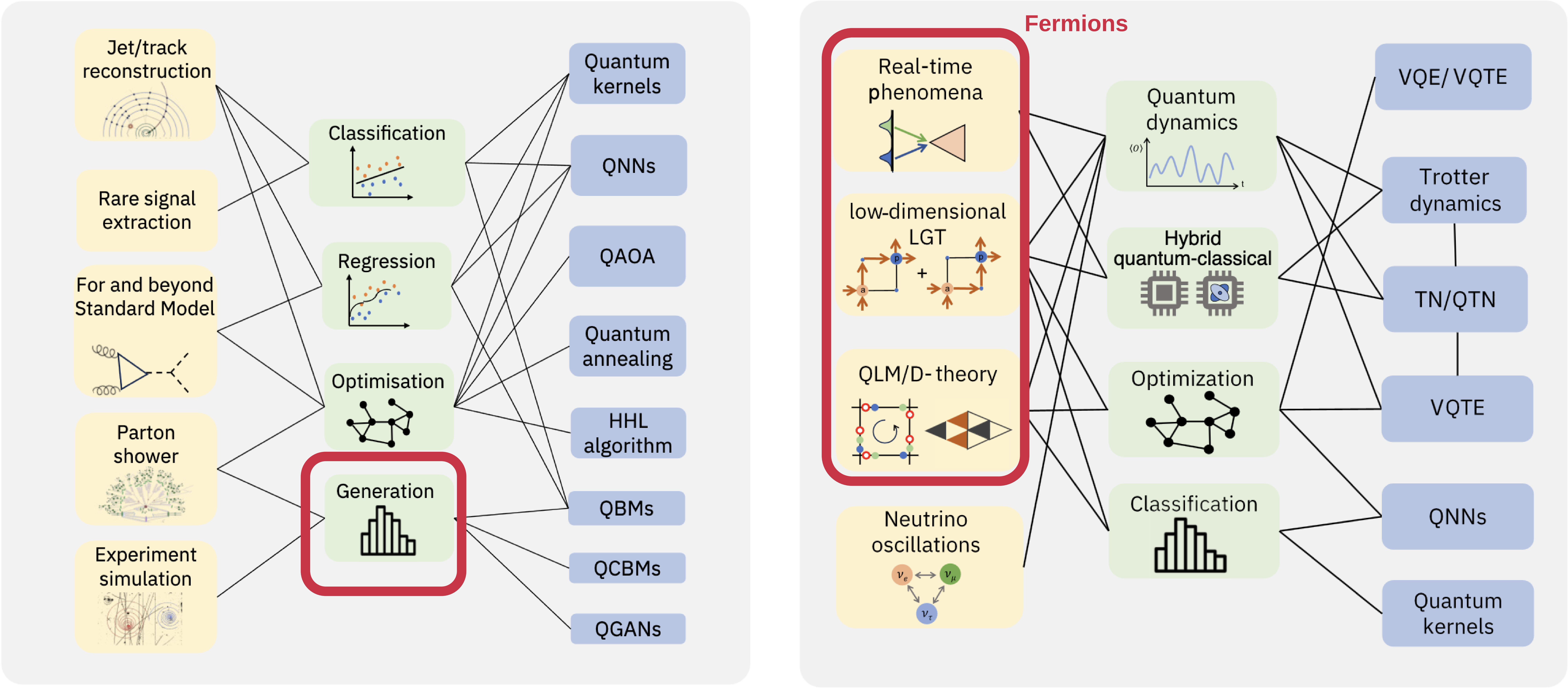
- Instantaneous Quantum Polynomial circuits
- Results of calorimeter images
- Incorporating symmetries
- Parity Twine implementation

Encoding fermions

- Geometrically local fermionic interactions
- Non-local fermionic interactions

Conclusion

Experimental challenges and theoretical physical models with corresponding approaches and quantum algorithms.



Di Meglio, Alberto, et al. *PRX Quantum* 5.3 (2024): 037001.

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- **Primary use case:** Accelerate processing of large **HEP** image datasets**.**

- **Technical approach:** Combine quantum computing, novel algorithms, and ion-trap hardware (*QSea I*).

Particle detector

❖ Detection of particles through total absorption in a block of matter.

❖ We consider dataset of energy deposition in the electromagnetic calorimeter.

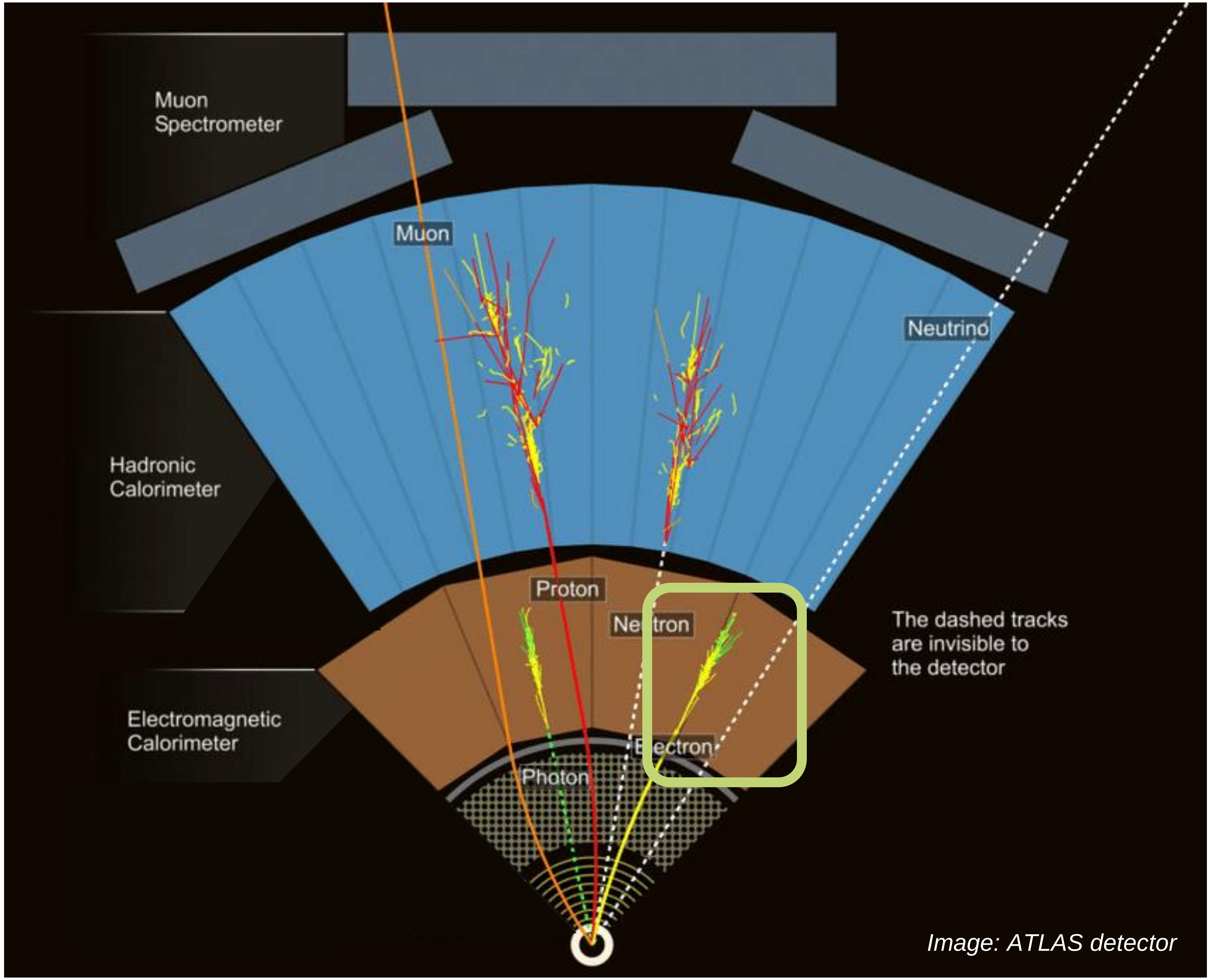
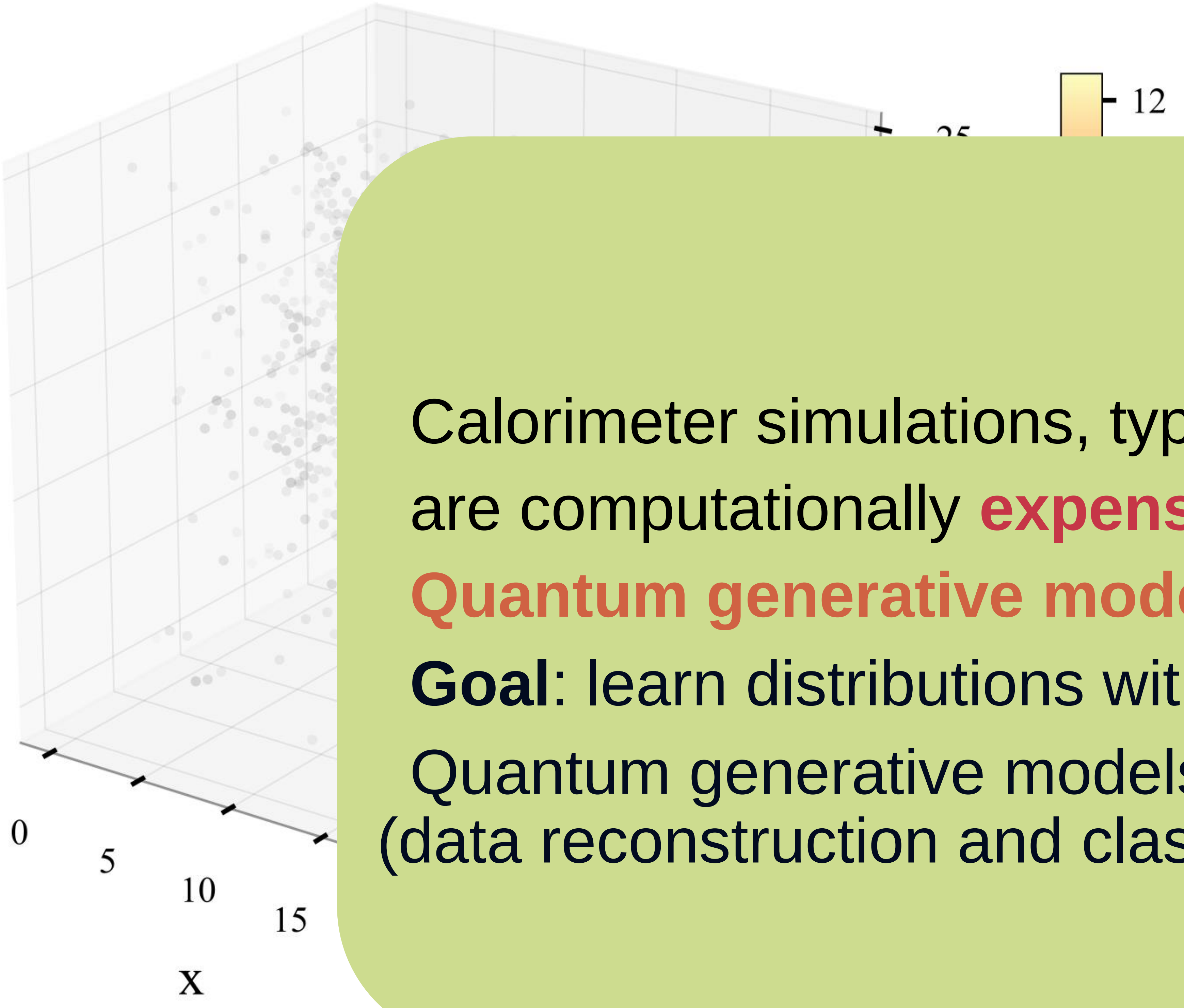


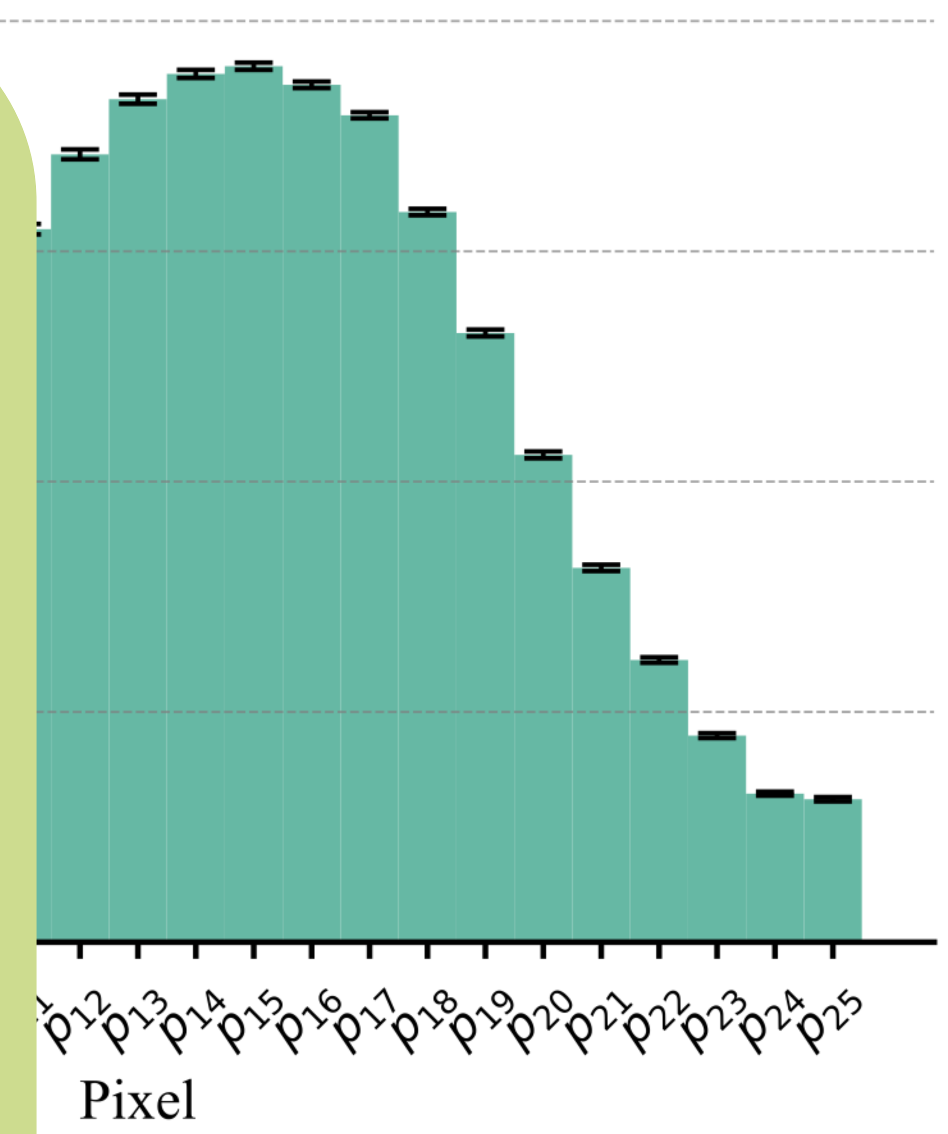
Image: ATLAS detector

Calorimeter data

Energy deposition 3D images

Projection onto z-axis





Calorimeter simulations, typically based on MC methods, are computationally **expensive**.

Quantum generative models alternative ?

Goal: learn distributions with QC.

Quantum generative models could also serve as simulators (data reconstruction and classification).

D. Belayneh et al., arXiv: 1912.06794

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Train on classical, deploy on quantum: scaling generative quantum machine learning to a thousand qubits

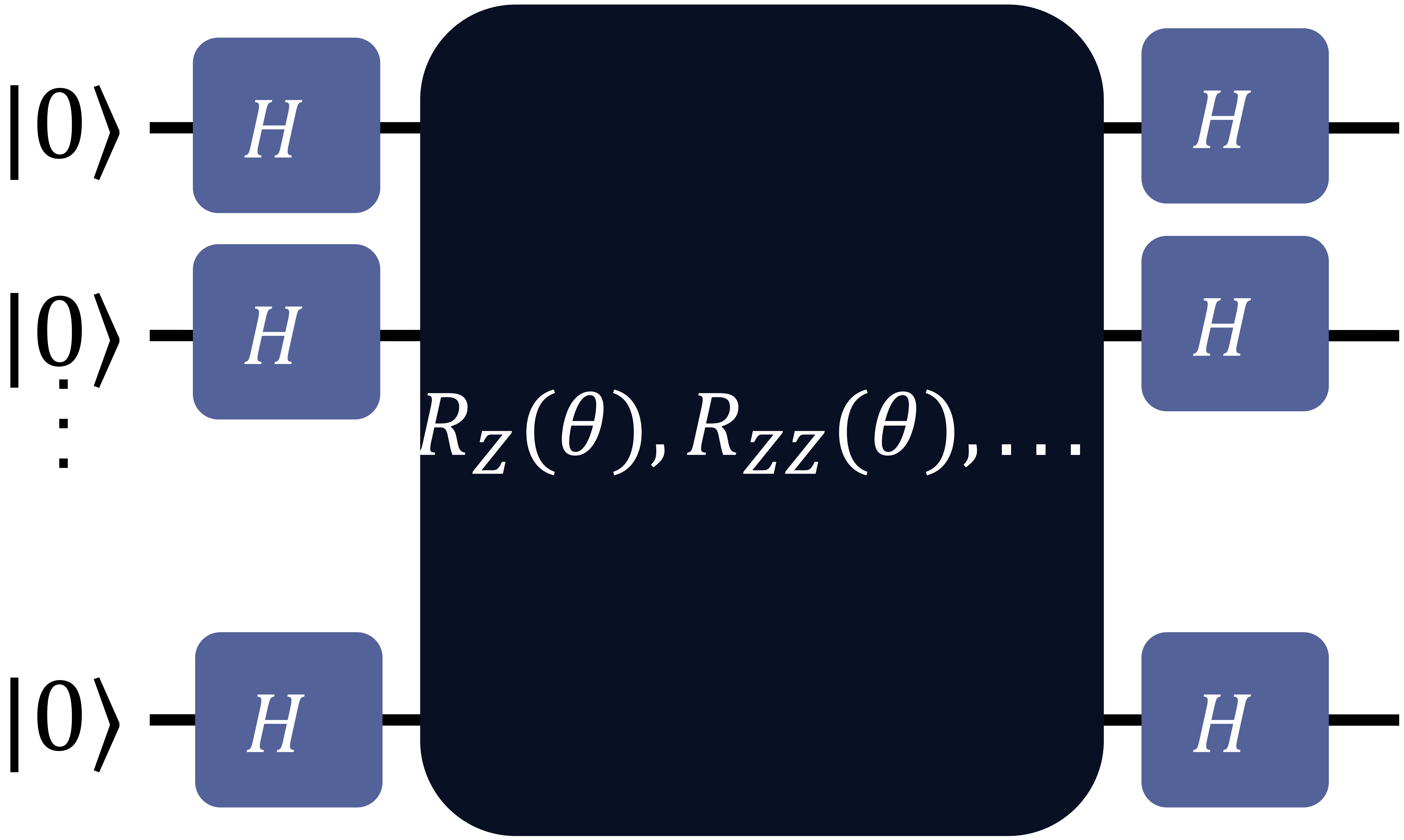
Erik Recio-Armengol*[1,2,3], Shahnawaz Ahmed[1], and Joseph Bowles†[1]

[1]Xanadu, Toronto, ON, M5G 2C8, Canada
[2]ICFO-Institut de Ciencies Fotoniques, The Barcelona Institute of Science and Technology, 08860, Castelldefels (Barcelona), Spain
[3]Eurecat, Centre Tecnològic de Catalunya, Multimedia Technologies, 08005 Barcelona, Spain

arXiv:2503.02934

- **Expectation values** from **instantaneous quantum polynomial circuits** (IQP) is classically **tractable**.
- **Sampling** from generated probability distribution is classically **intractable** in the average case.
- Natural application: **generative modeling** (task requires sampling from probability distributions).

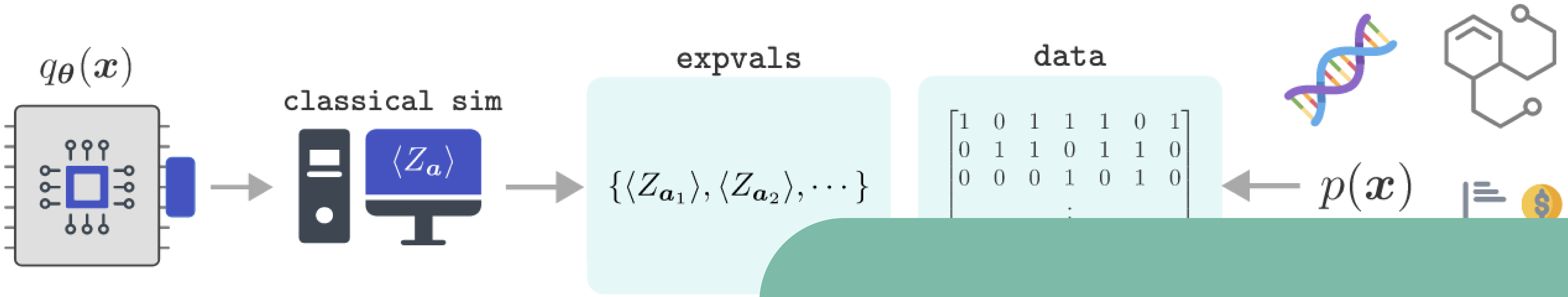
Quantum circuit

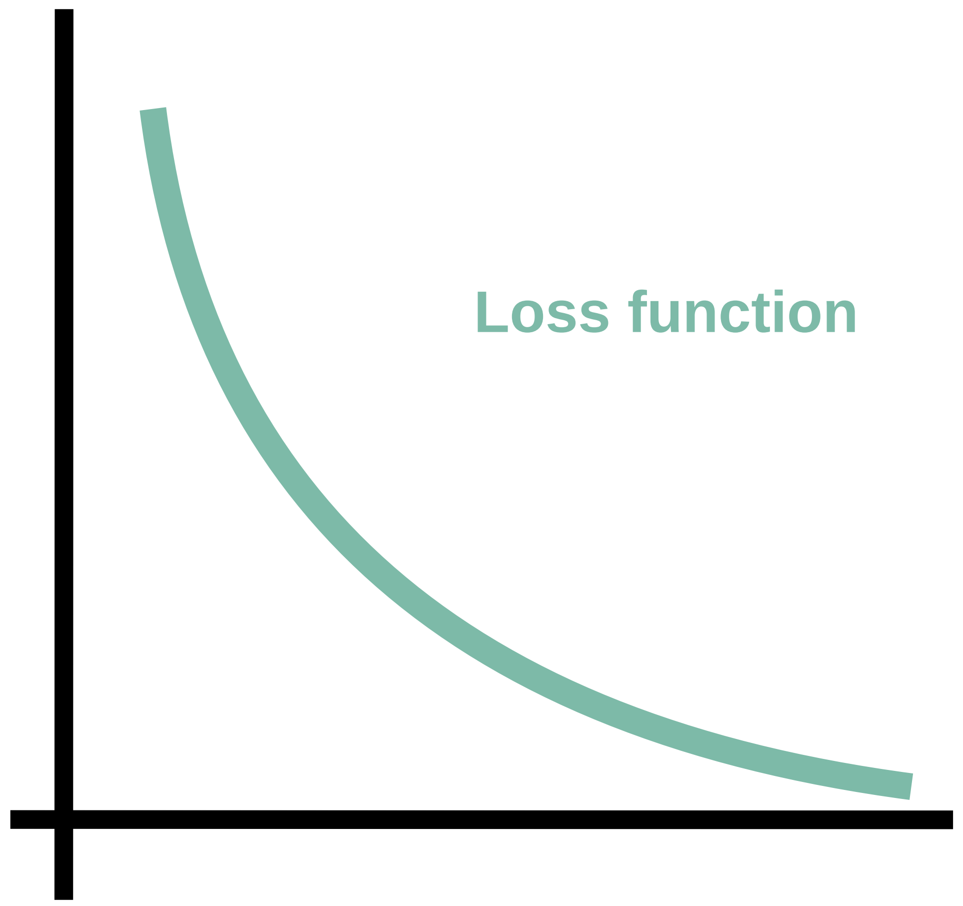


Circuit acts as follows:

- Prepares qubits in the $|+\rangle$ state using Hadamard gates.
- Applies a layer of commuting diagonal gates in the computational basis.
- Applies a final layer of Hadamards.

Method for generative machine learning





Maximum Mean Discrepancy

$$MMD^2(p, q_\theta) = \mathbb{E}_{\mathbf{a}\sim P_\sigma}\left[\left|\langle Z_{\mathbf{a}}\rangle_p - \langle Z_{\mathbf{a}}\rangle_{q_\theta}\right|^2\right]$$

MMD quantifies the distance between two distributions by comparing their samples through a kernel function, such as the Gaussian kernel.

Recio-Armengol, E., Ahmed, S. & Bowles, J. (2025) *arXiv:2503.02934*

Limitations for non-binary distributions

❖ IQP circuits work with binary distributions.
8 —> 1000
0 —> 0000

❖ IQP MMD loss function measures distance between two distributions.

Distance binary: 1000 and 0000 is only one unit apart. **SMALL**

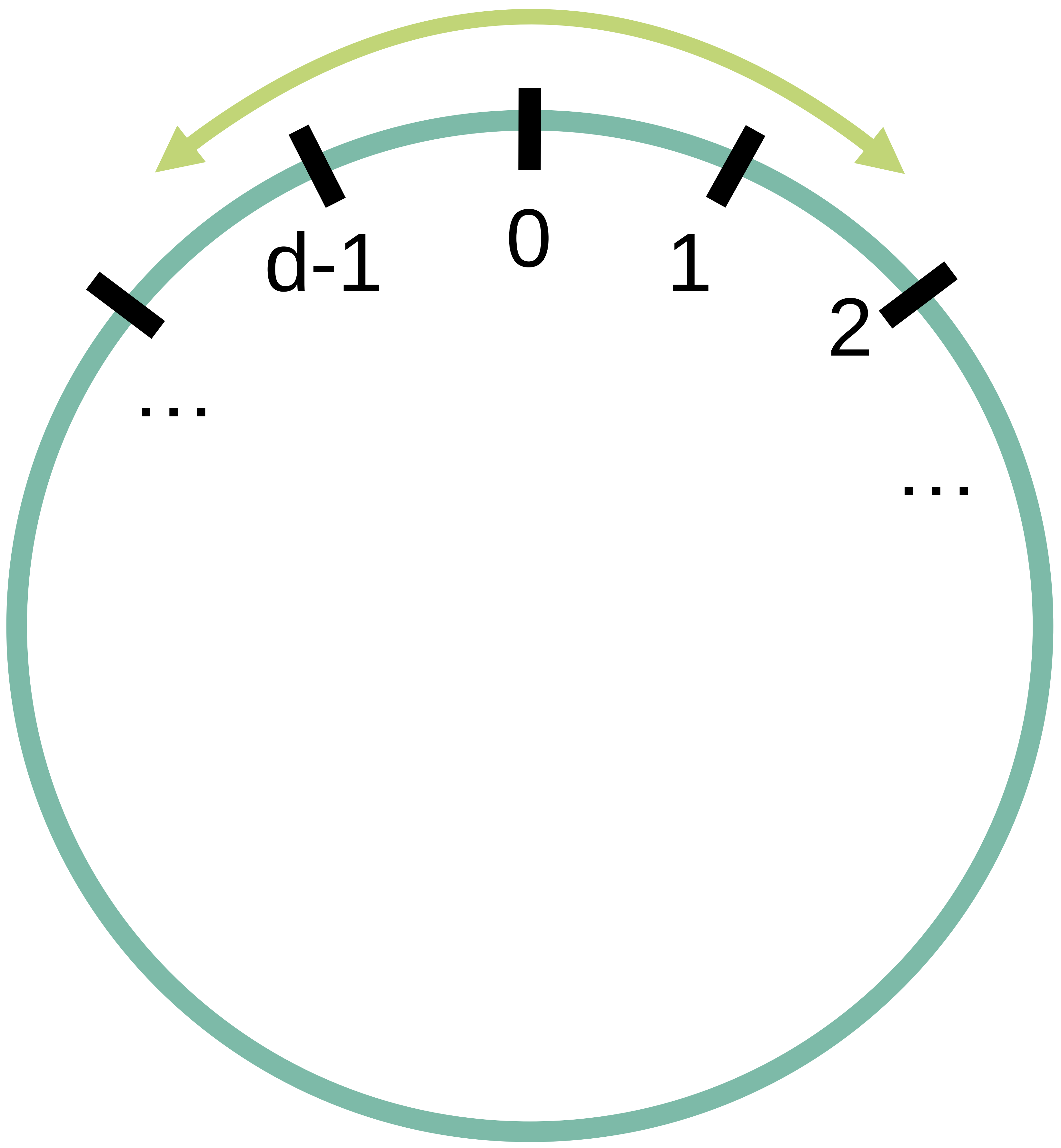
Distance integer: 8 and 0 is 8 units apart. **LARGE**

Redesigning the loss-function

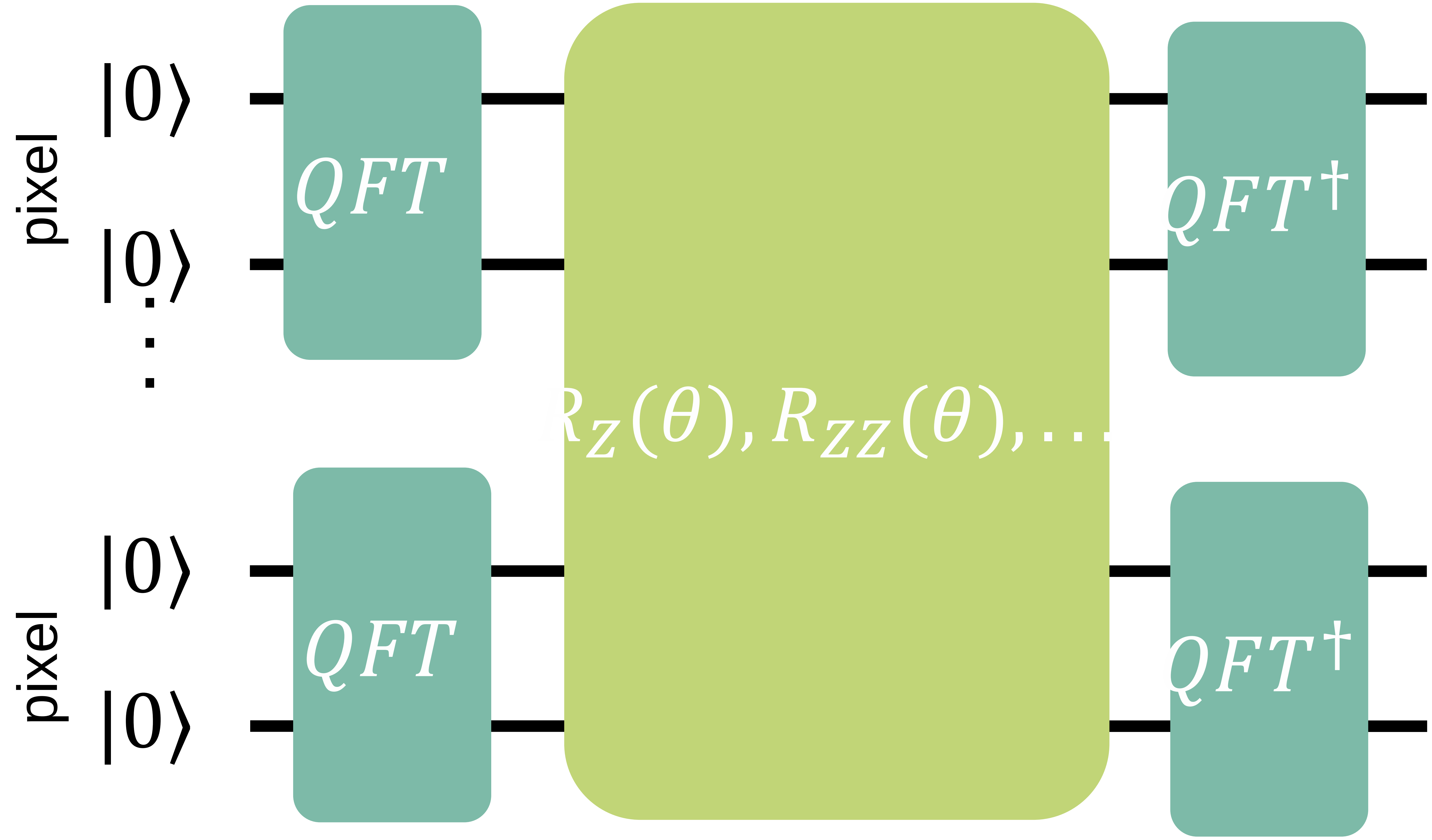
- Training data are mapped to an integer between 0 and d-1.

- Define the distance as the **cyclic distance**.

- To train classically we need to change the quantum circuit.



Redesigning the circuit



- For each of the n pixels, $\log_2 d$ qubits are used.
- These qubits are initialized in the zero state $|0\rangle$.
- A layer of QFTs is followed by a sequence of multi-body diagonal rotations.
- A final layer of inverse QFT gates is then applied.

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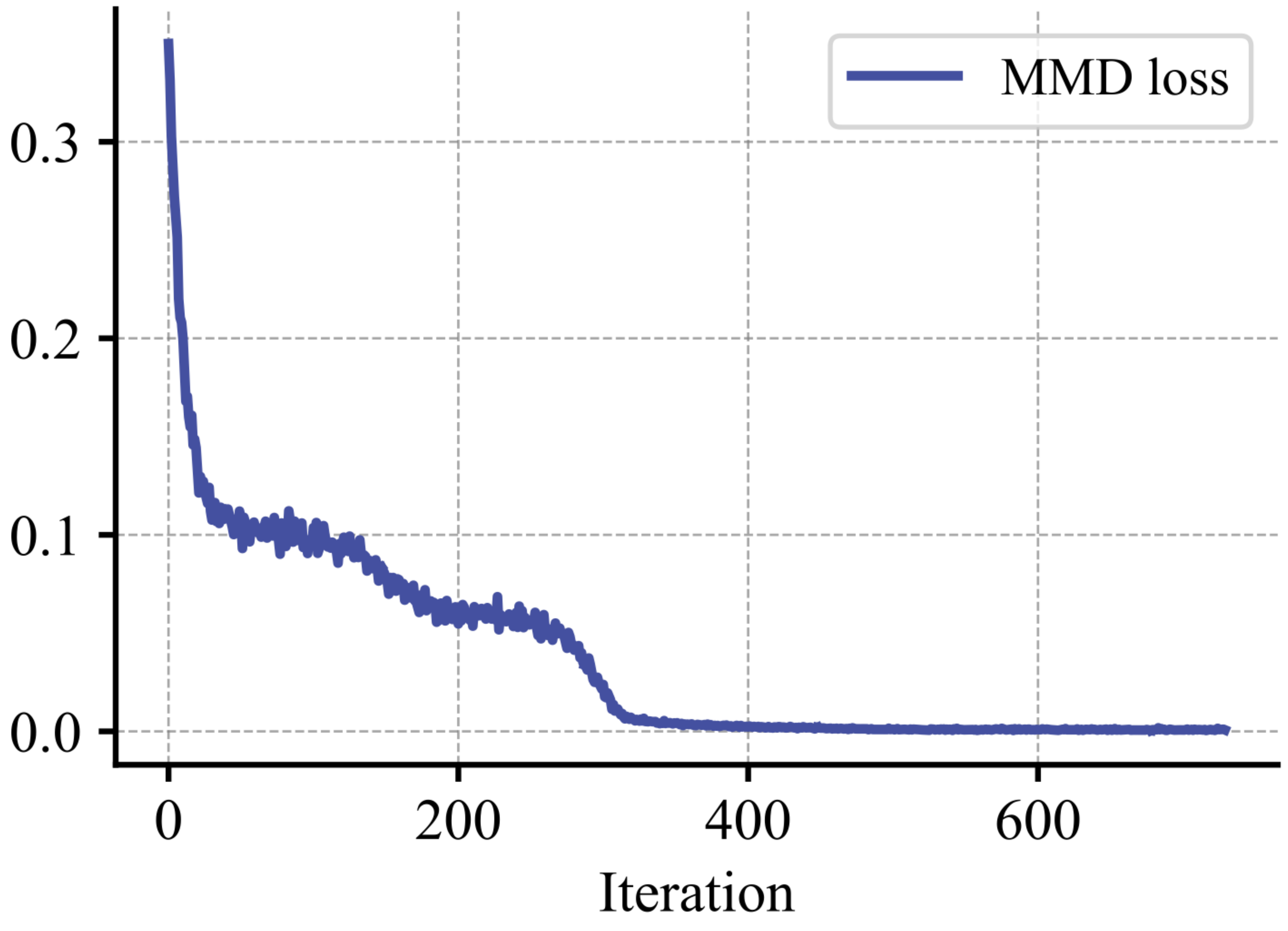
Reduction to 6 pixels: simulation feasible



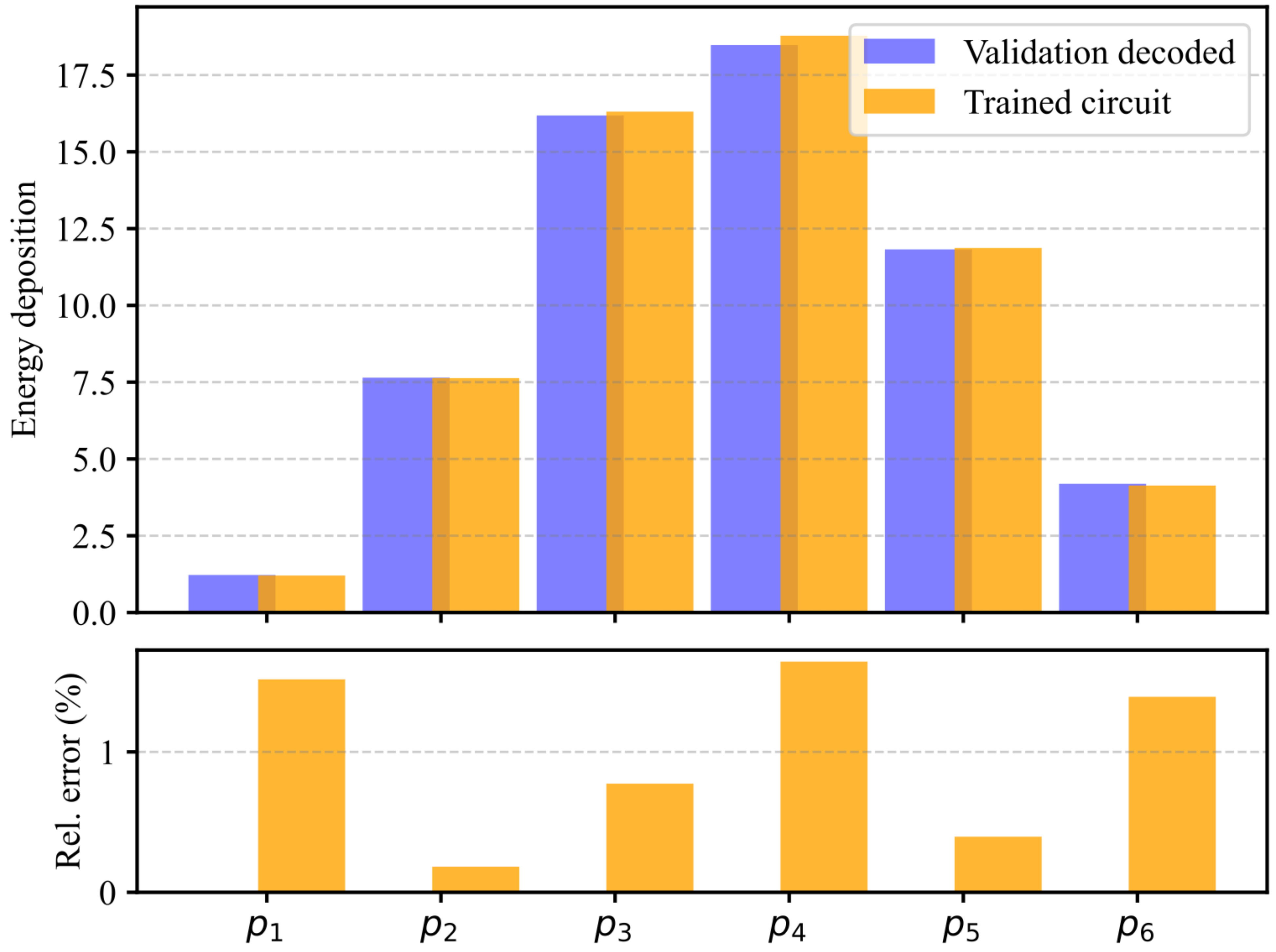
Training with MMD loss function:

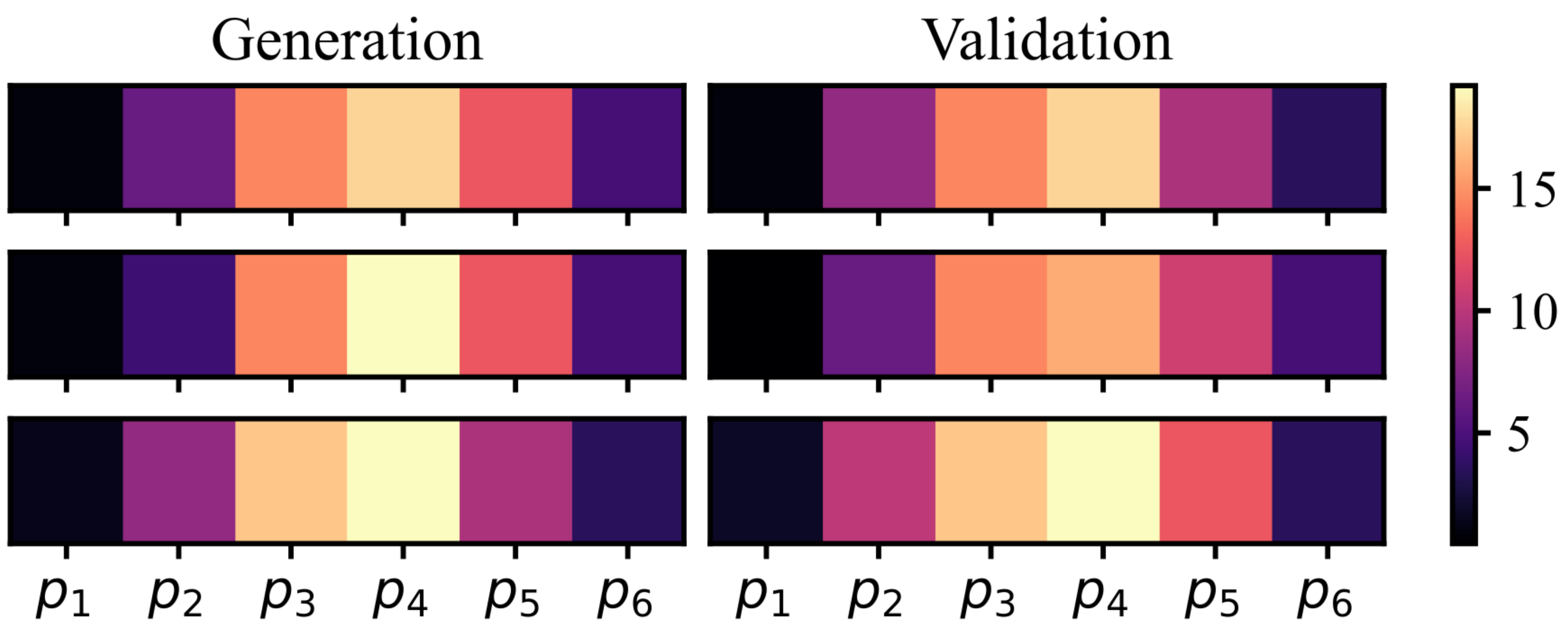
- ❖ Circuit **parameters** initialized (*warm start*).
- ❖ Training stops when **convergence** is reached.

Hyperparameter optimization (*e.g. learning rate*)

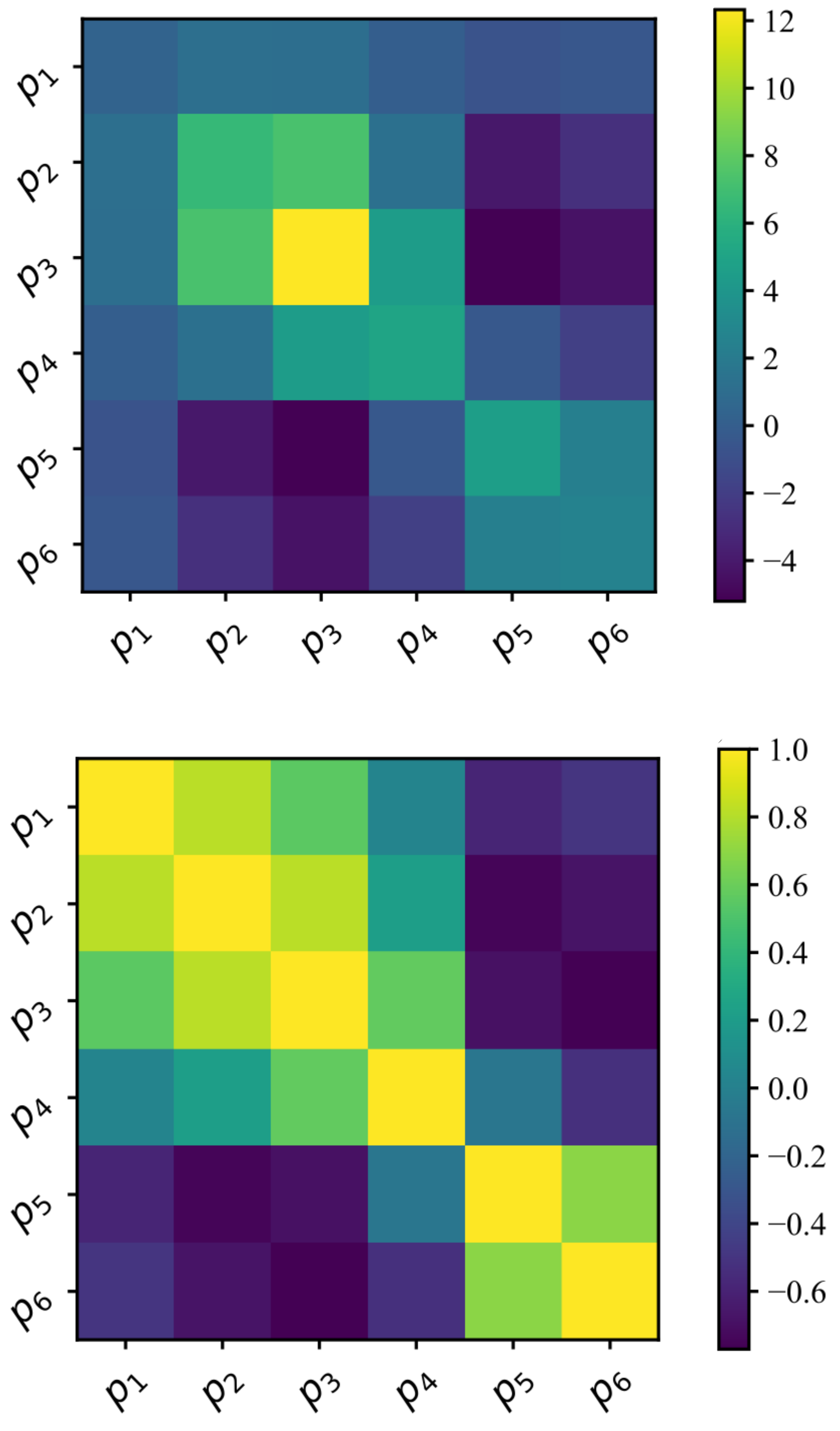
Reduction to 6 pixels: simulation feasible

Total number of qubits 24, sampling is possible using a simulator (*Pennylane*).





Reduction to 6 pixels: simulation feasible

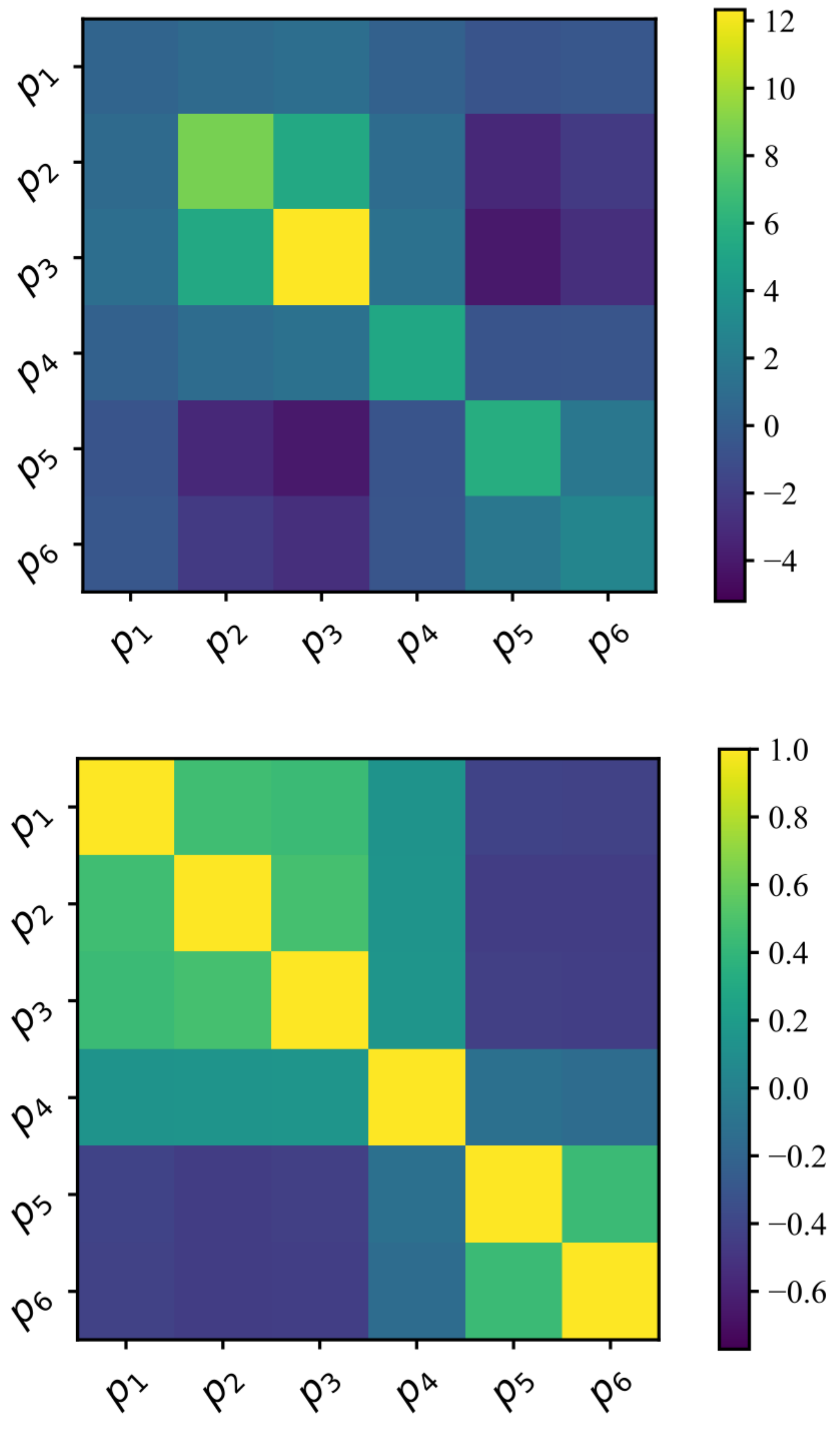


Validation data

We compute the covariance (and correlation) between each pixel.

The covariance matrix for the IQP circuit is obtained from sampled distribution with trained parameters.

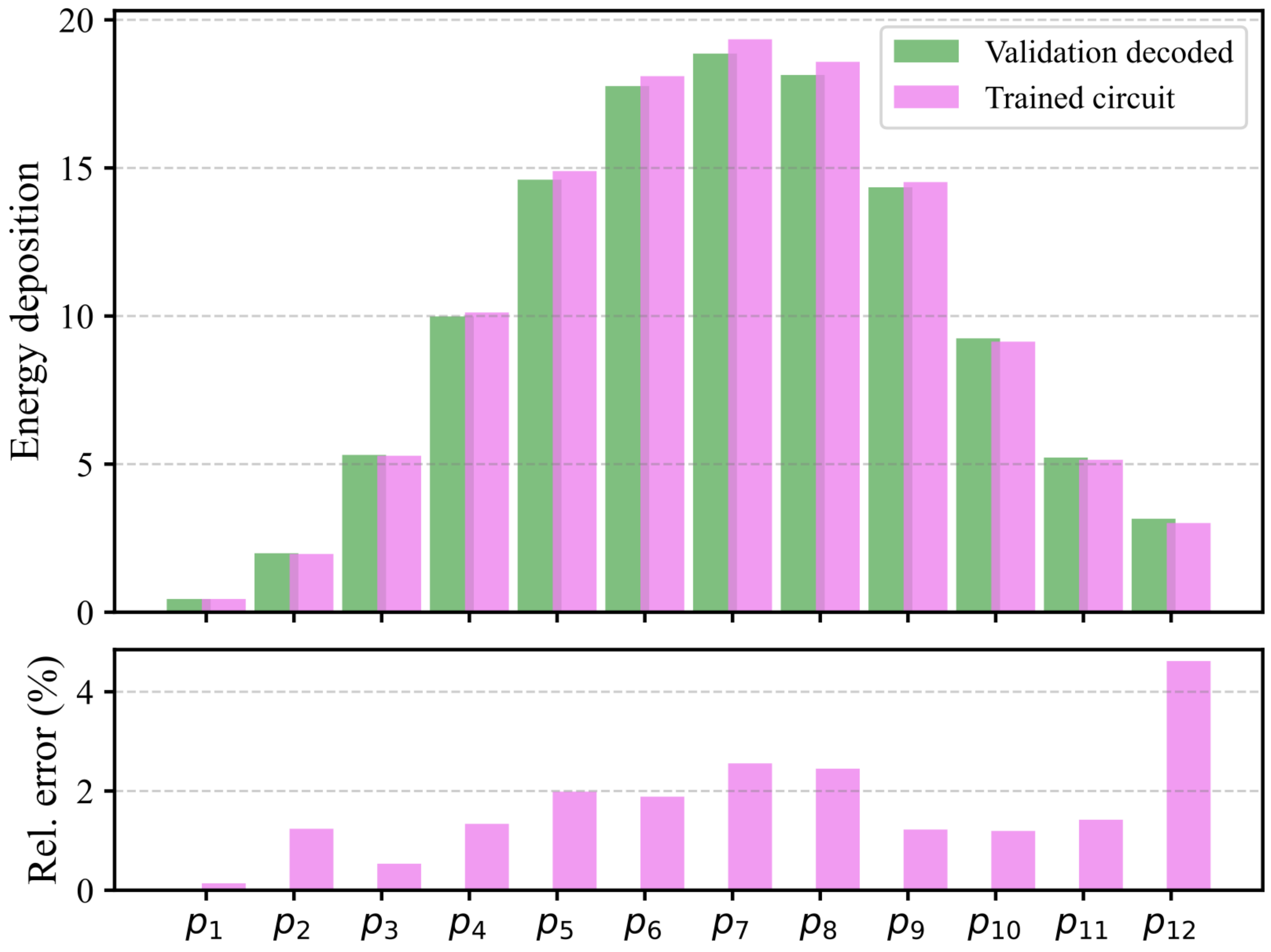
Comparison with the validation dataset.

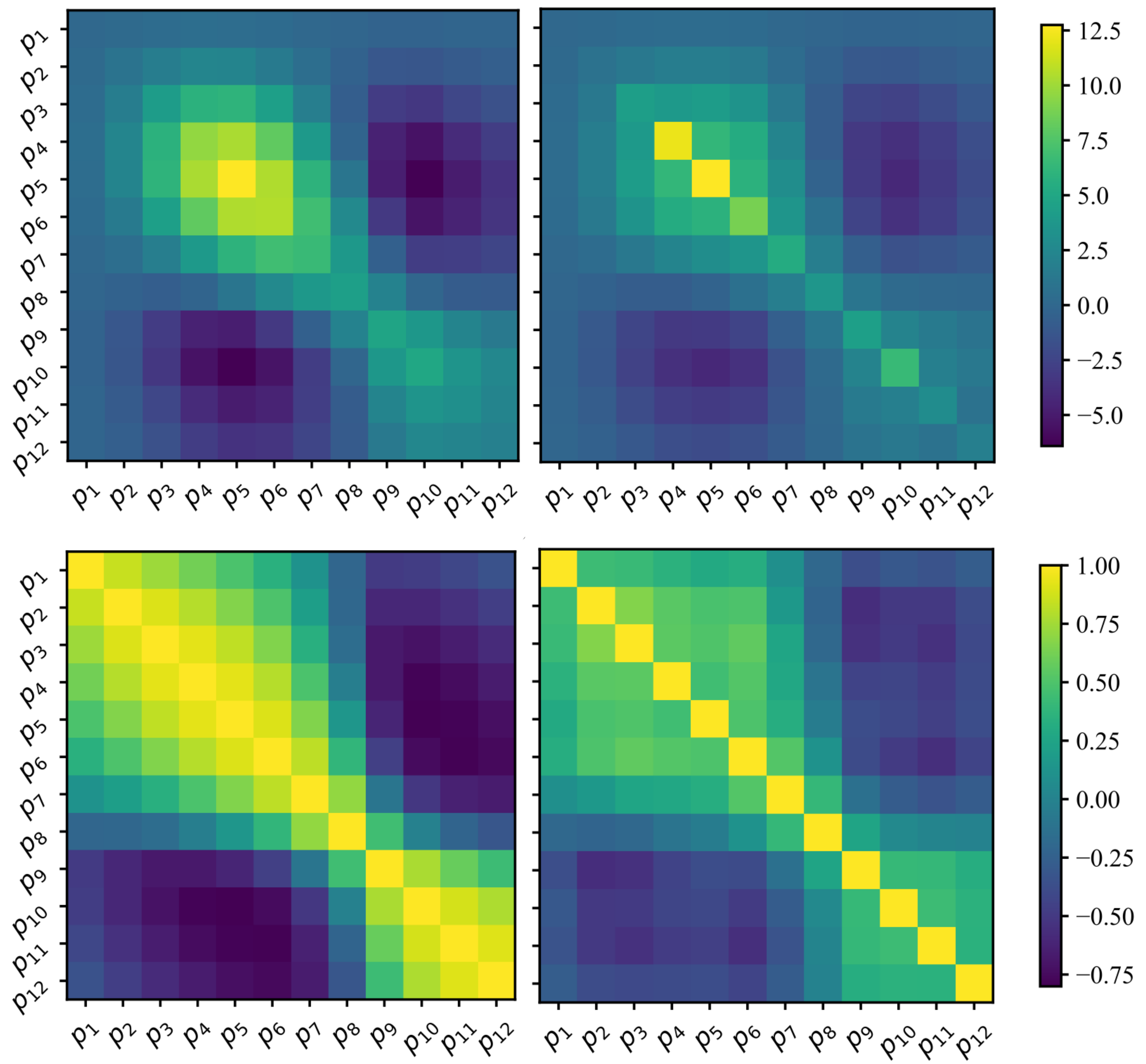


Generation data

Results 12 pixels

Total number of qubits 60, need a quantum computer.





Validation data

Generation data

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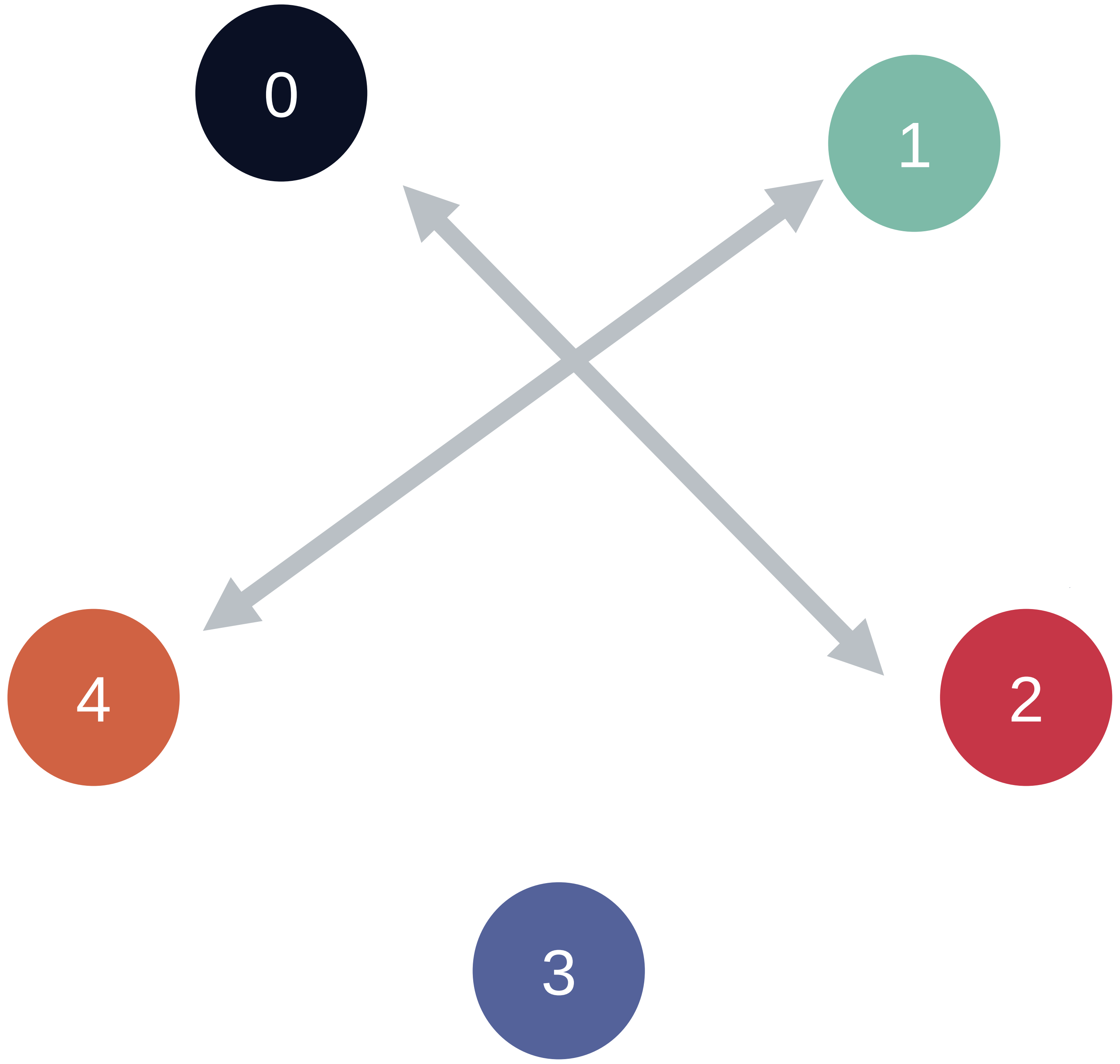
Incorporating symmetries

- If a dataset presents symmetries, one should include them in the generative model.
- For IQP models, this can be achieved by changing the initial state.
  - Example: **symmetry under permutations of labels**.
  - Change initial state from $|0\rangle$ to:

$$|\psi_i\rangle = \frac{1}{\sqrt{d}} \sum_{i=0}^{d-1} |i\rangle^{\otimes n}$$



Potts model

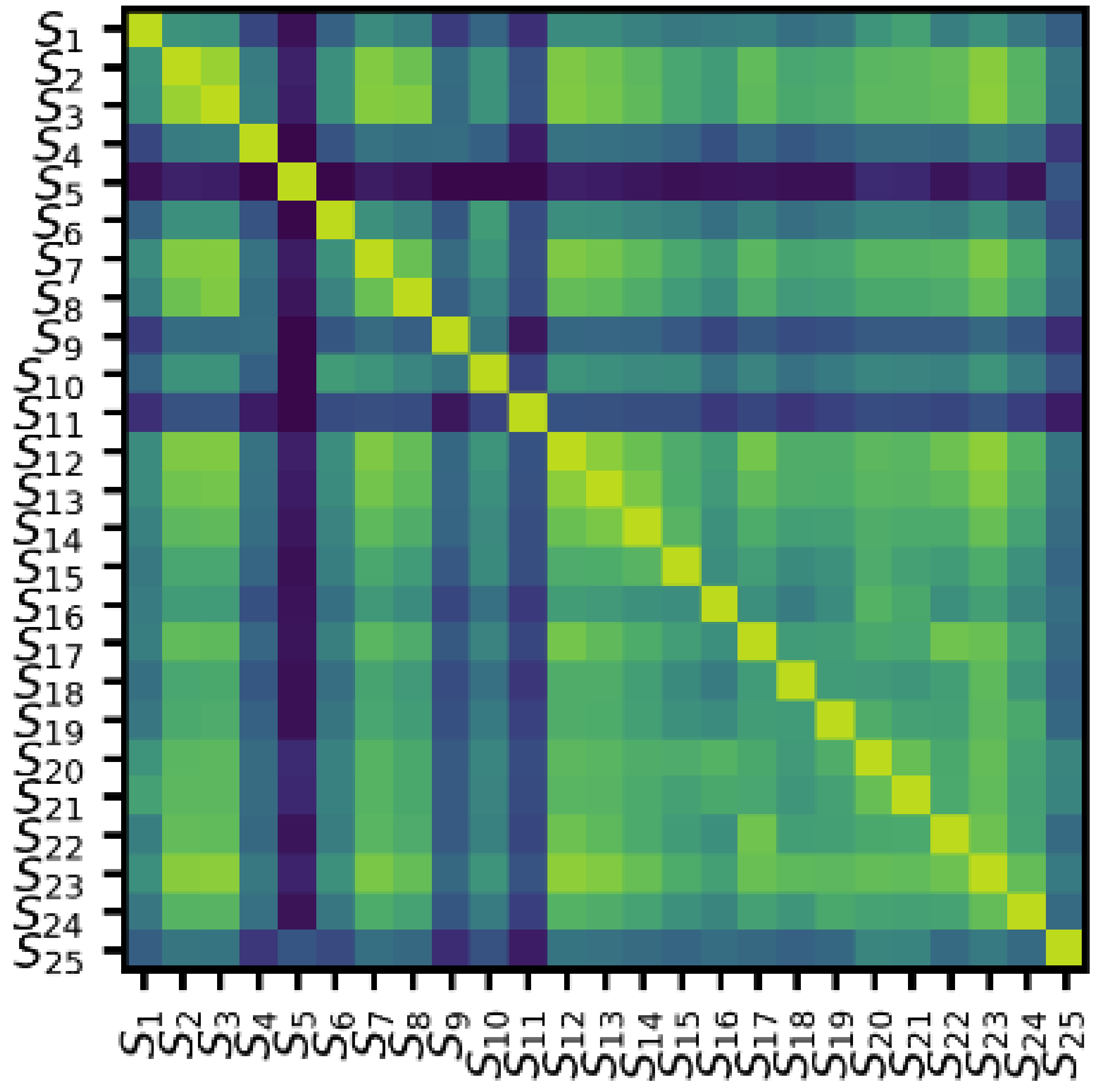
On a square lattice of size 5x5 and at finite temperature.

❖ Hamiltonian:

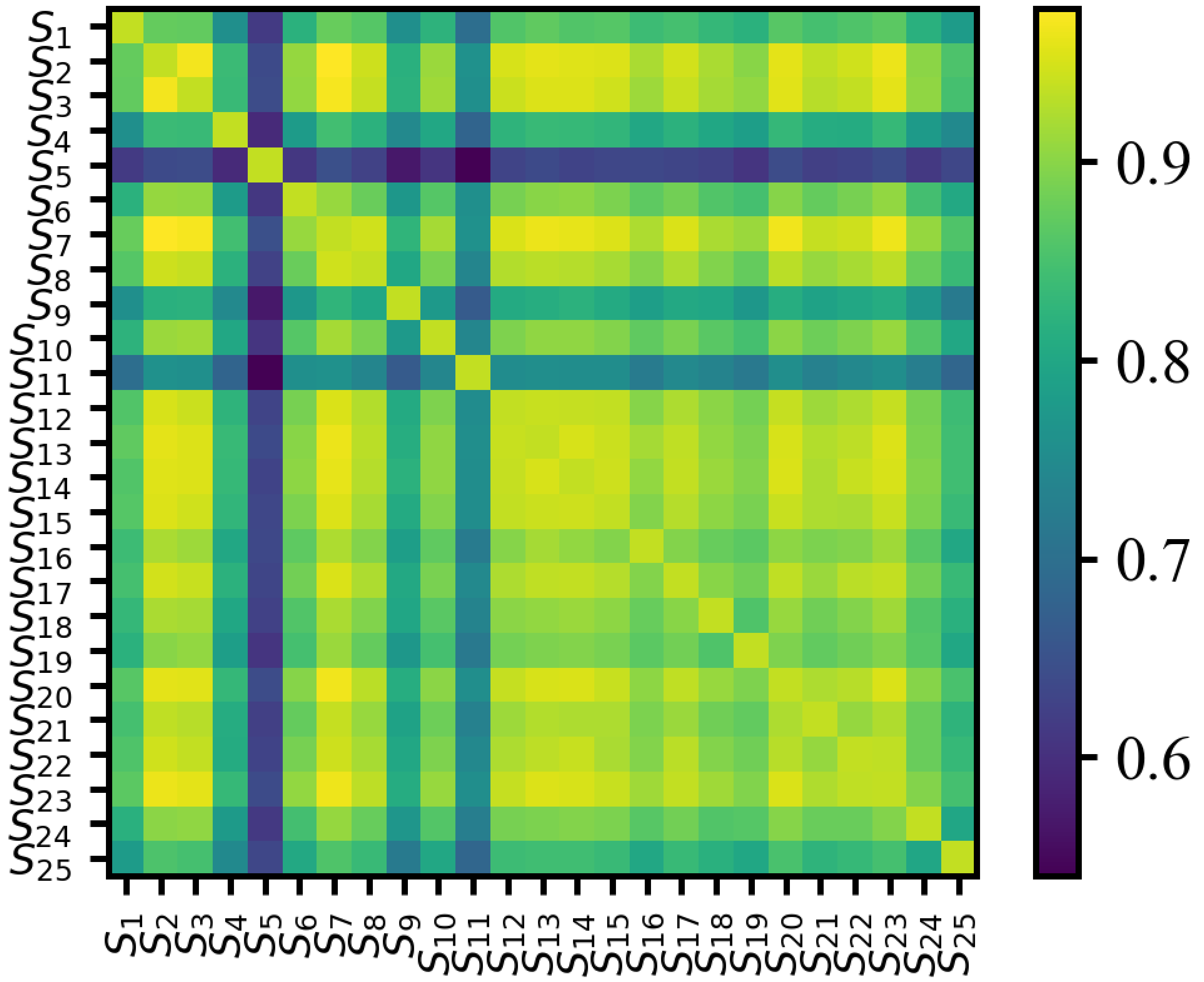
$$H_{Potts} = -\sum_{\langle i,j \rangle} J_{i,j} \delta_{s_i, s_j}$$

where s_i is a spin variable at site i, taking values up to $d = 16$.

❖ Total number of qubits is 100.



Validation data



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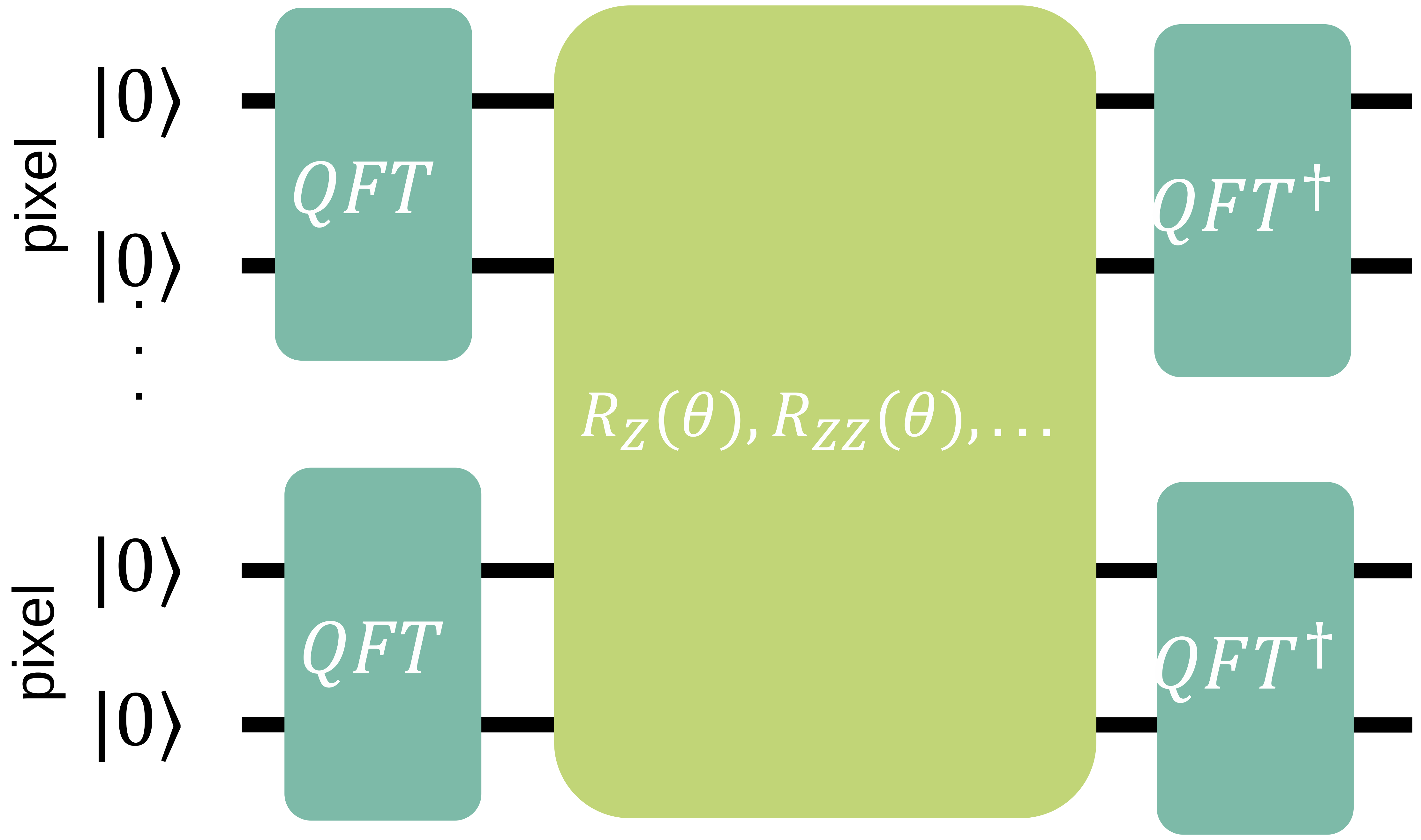
Circuit implementation



(*) *Parity Twine* method reached **new record** process fidelity for QFT for ~ 50 qubits on IBM quantum devices [2].

Circuit depth with **Parity Twine method [1]**:

- All-to-all Pauli-Z rotations scales as $\nu n \log_2 d$ (n total number of pixels).
- QFT scales $\nu \log_2 d$ **(*)**.

Device connectivity	ν
All-to-all	2
Heavy hexagon	5
Square grid	6

Take home message: depth scales linear with n and $\log$ with d .

[1] F. Dreier, *et al. arXiv:2501.14020* (2025).
[2] P. Aumann, *et al. arXiv:2604.12465* (2026).

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Dynamical fermion-to-qubit encoding

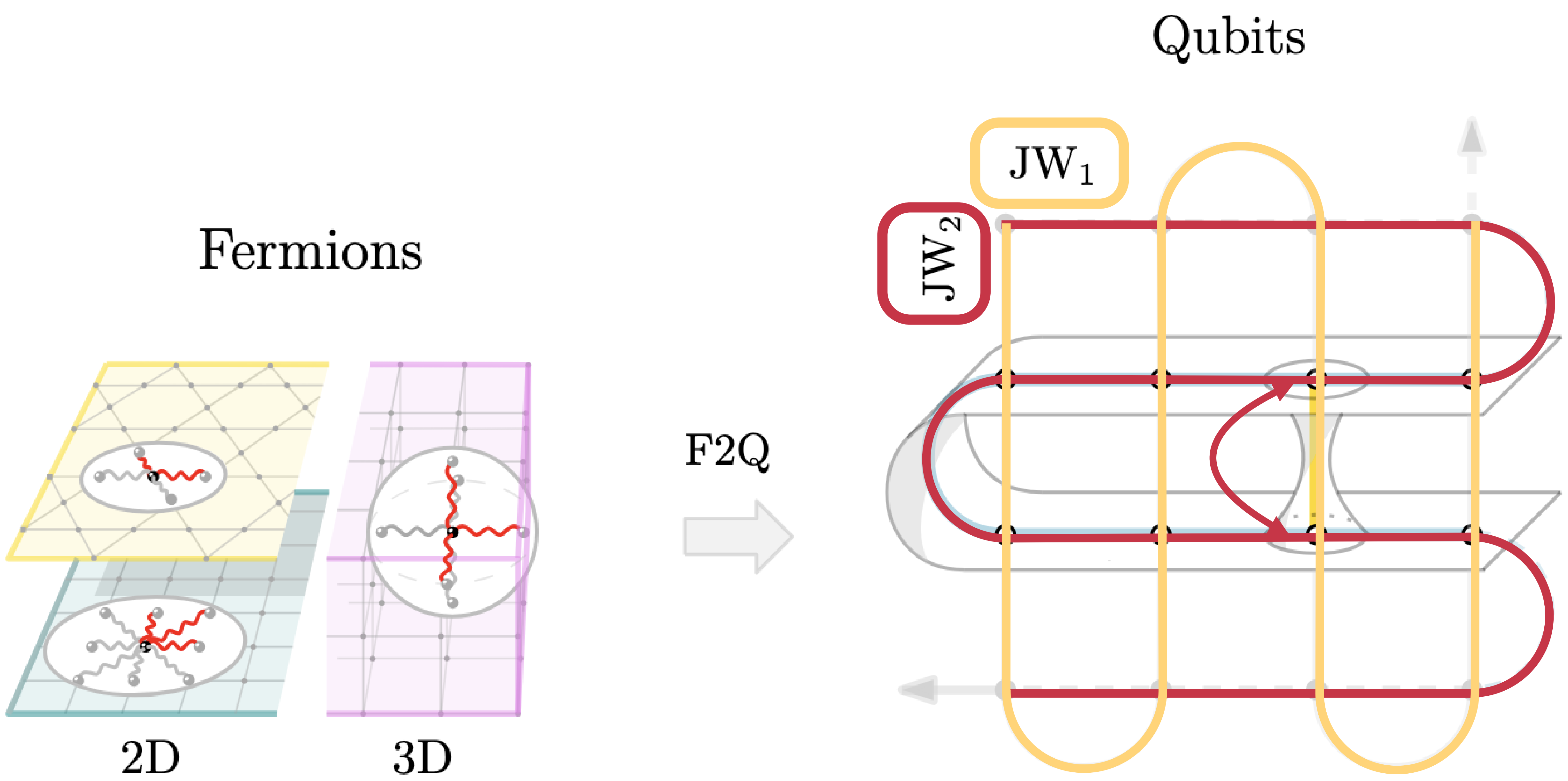
Fermionic d.o.f. on the lattice

↓

Jordan-Wigner encoding

↓

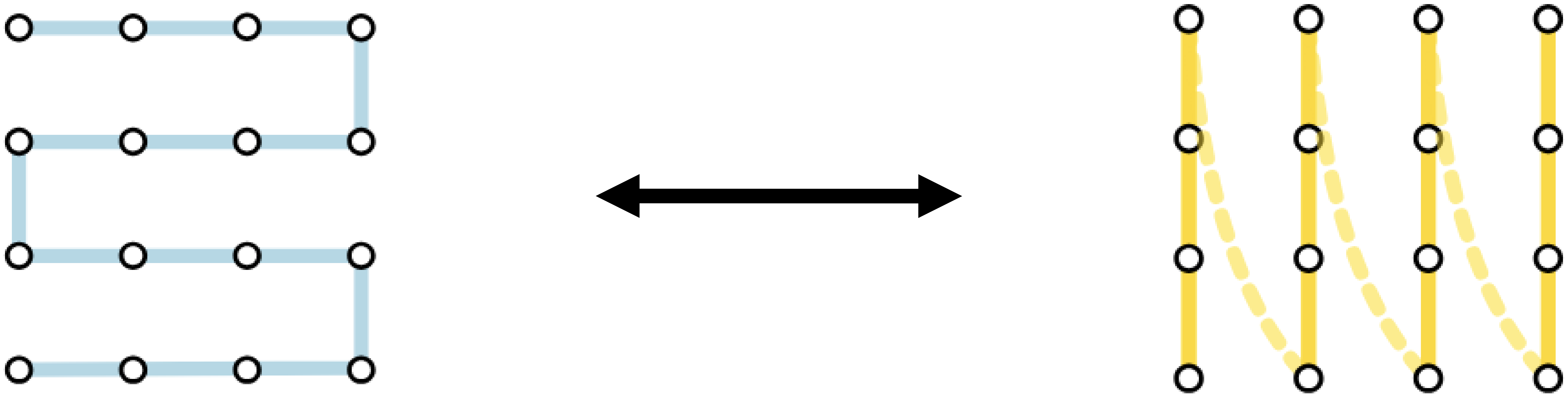
Dynamical F2Q encoding: fermionic modes that would be **distant** under a single JW encoding now **interact**.

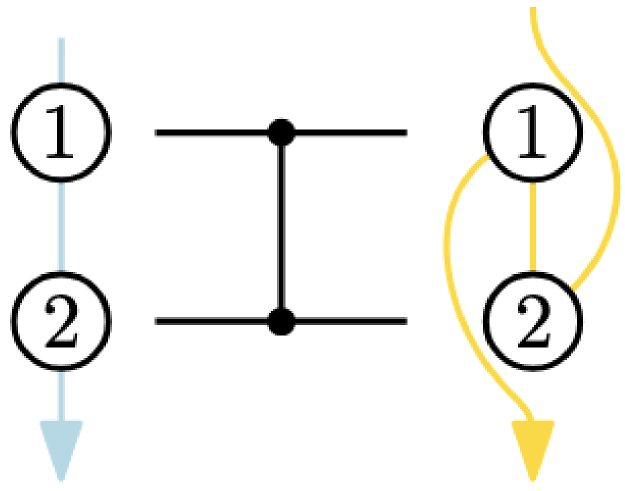


G. Aigner, B. Klaver, M. Lanthaler, and W. Lechner, (2026) arXiv: 2605.12600

The method

A global **S-pattern** is transformed into a **Z-pattern** and then transferred back into a NEW **S-pattern**.





CZ changes only ***JW ordering***, leaving ***physical*** positions of two fermionic modes unchanged.

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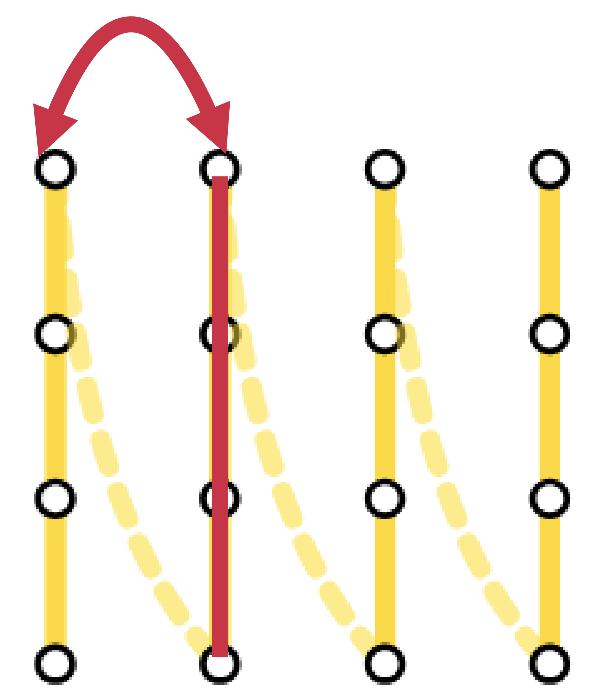
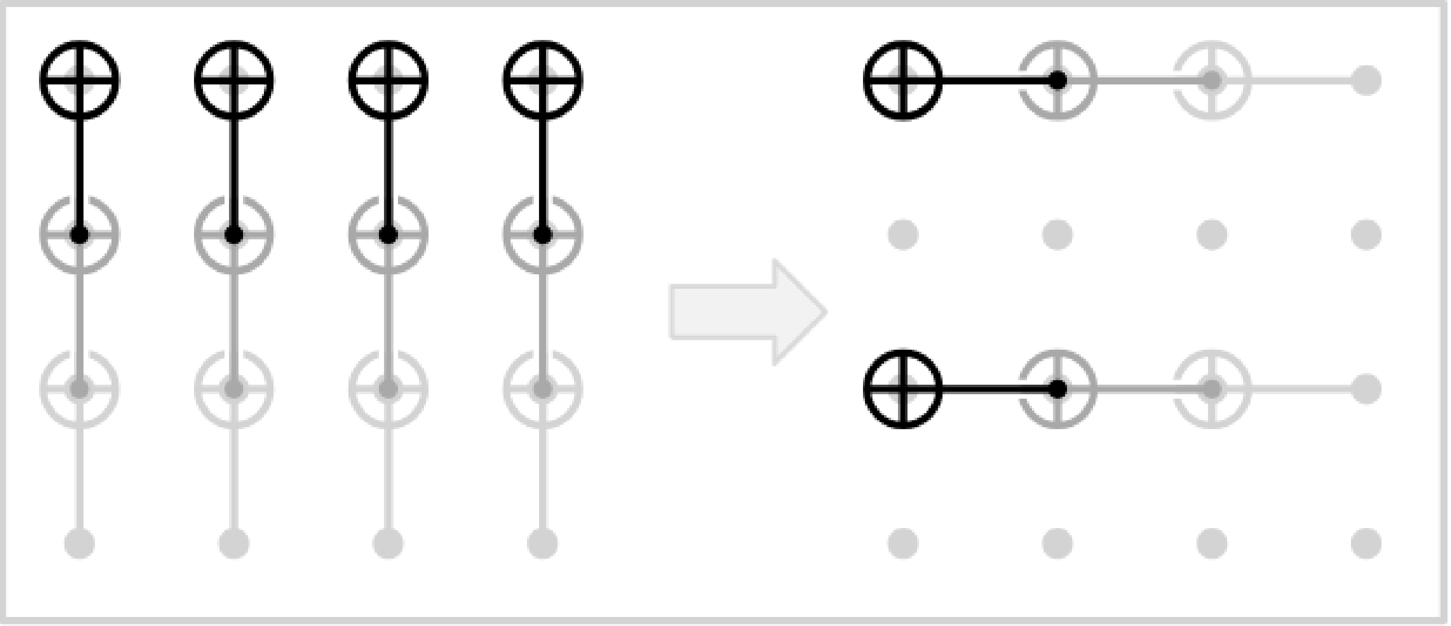
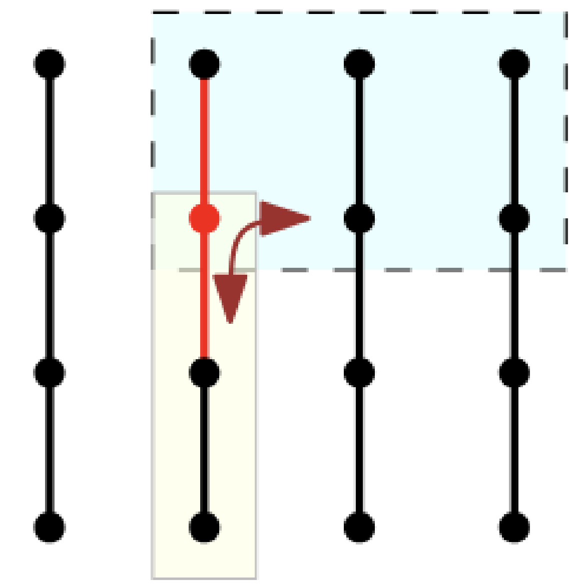
The method: local interactions



Changing into intersection basis



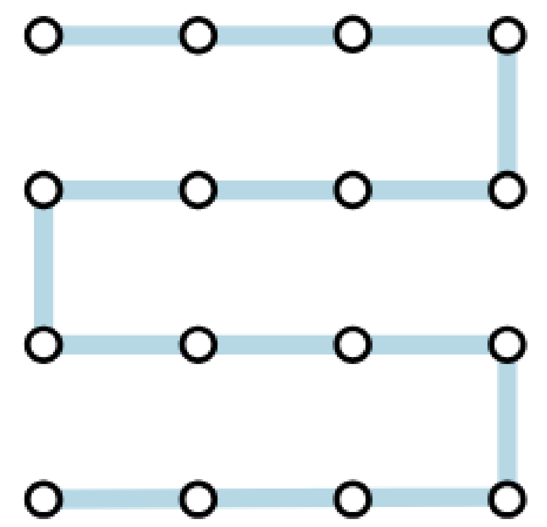
$C_{\uparrow}$ $C_{\leftarrow}$



C_Z

Undoing the initial basis transformation.

$C_{\uparrow}^{\dagger} C_{\leftarrow}^{\dagger}$



To exchange information **many CZ** needed.

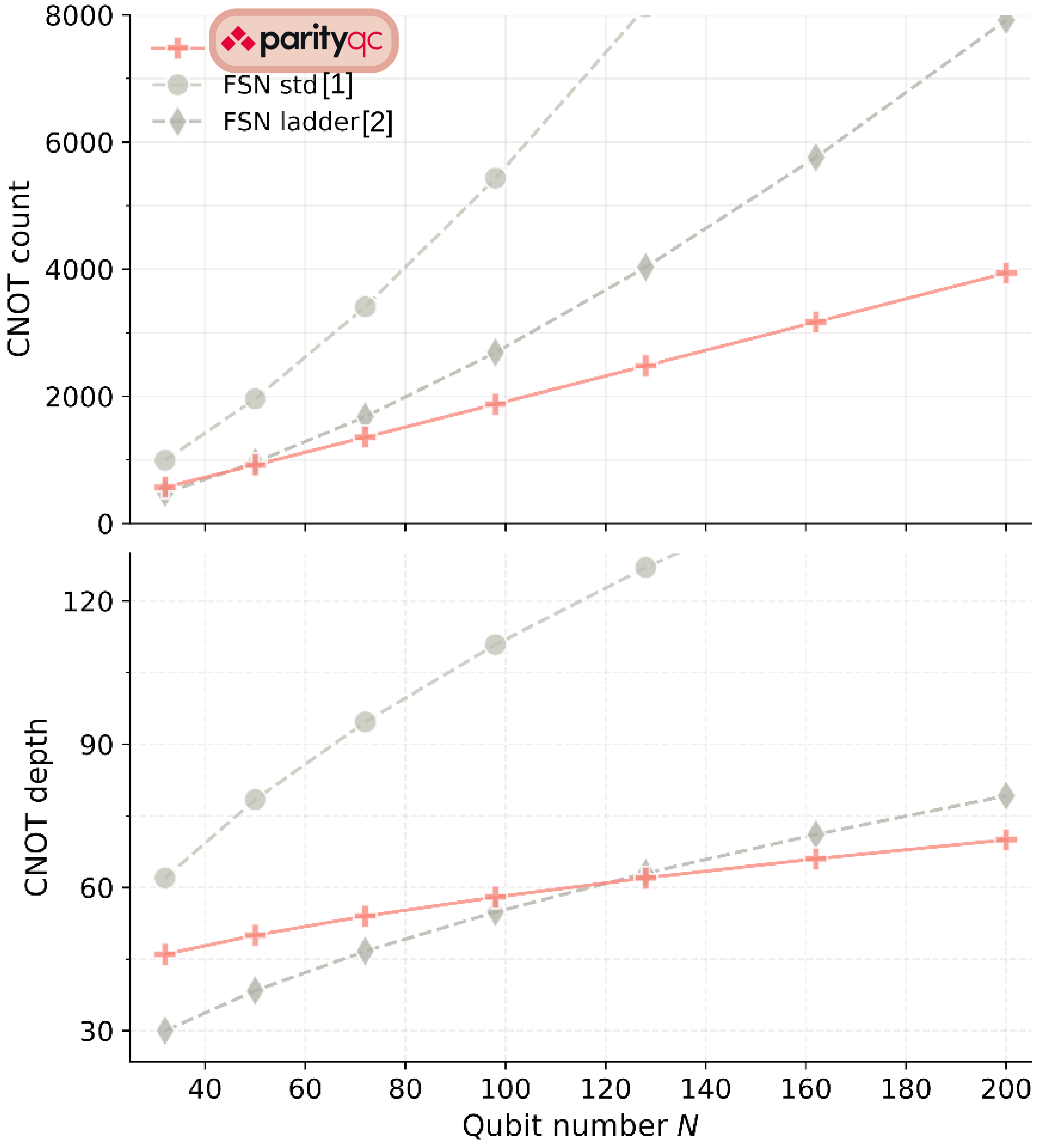
Basis change + parity flow allows us to understand and simplify interactions between row and column parities.

A **single** CZ is needed to allow **column** information to be exchanged with **row** information.

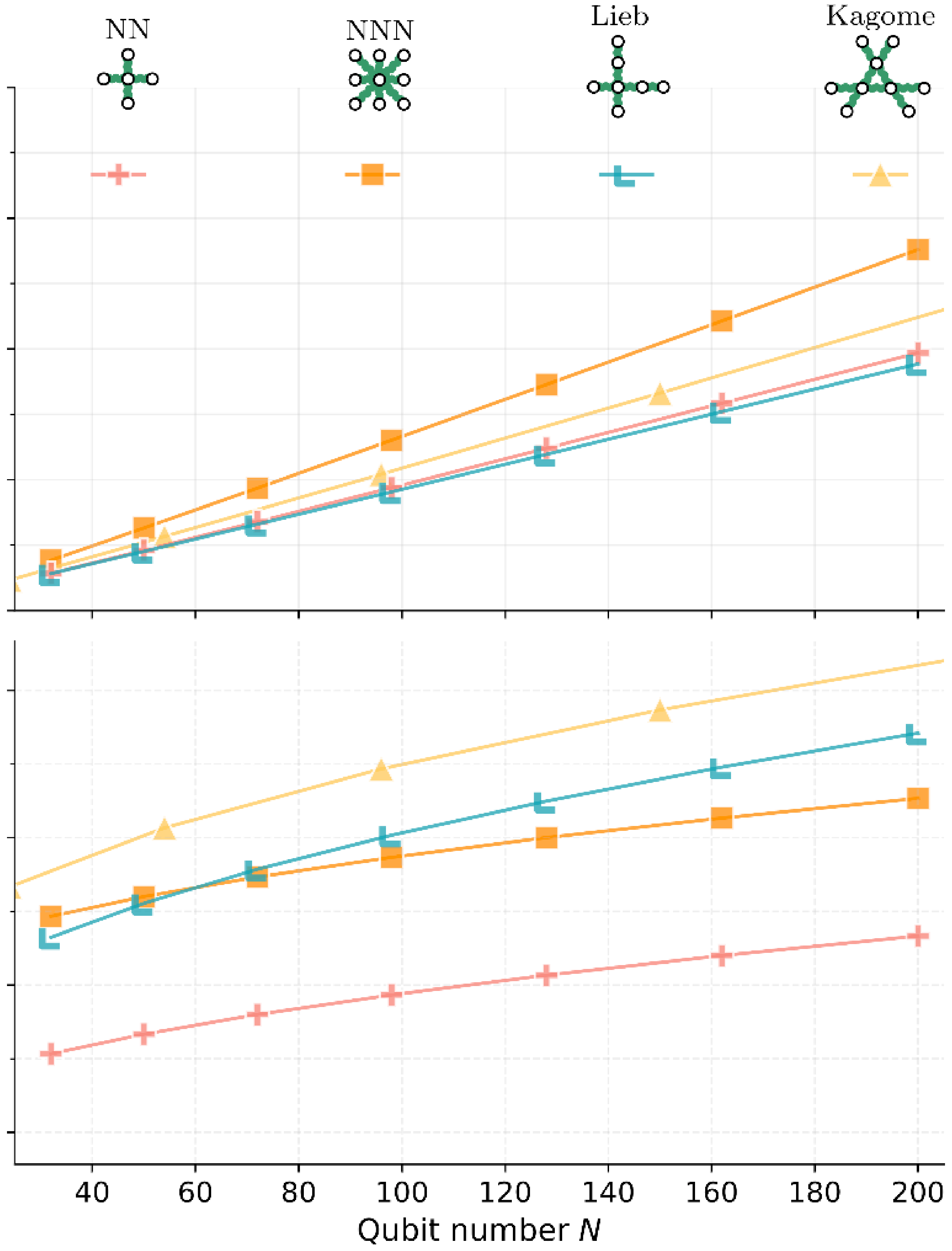
Application to the Fermi-Hubbard model

Single 2nd-order Trotter step for the Fermi-Hubbard model on **square qubit lattice**:

ParityQC method vs two fermionic-SWAP networks (FSN) approaches.



More complex interactions using ParityQC dynamical encoding.



[1] I. D. Kivlichan, et al. Physical Review Letters 120, 110501 (2018).
[2] K. Hemery et al. PRX Quantum 5, 030323 (2024).

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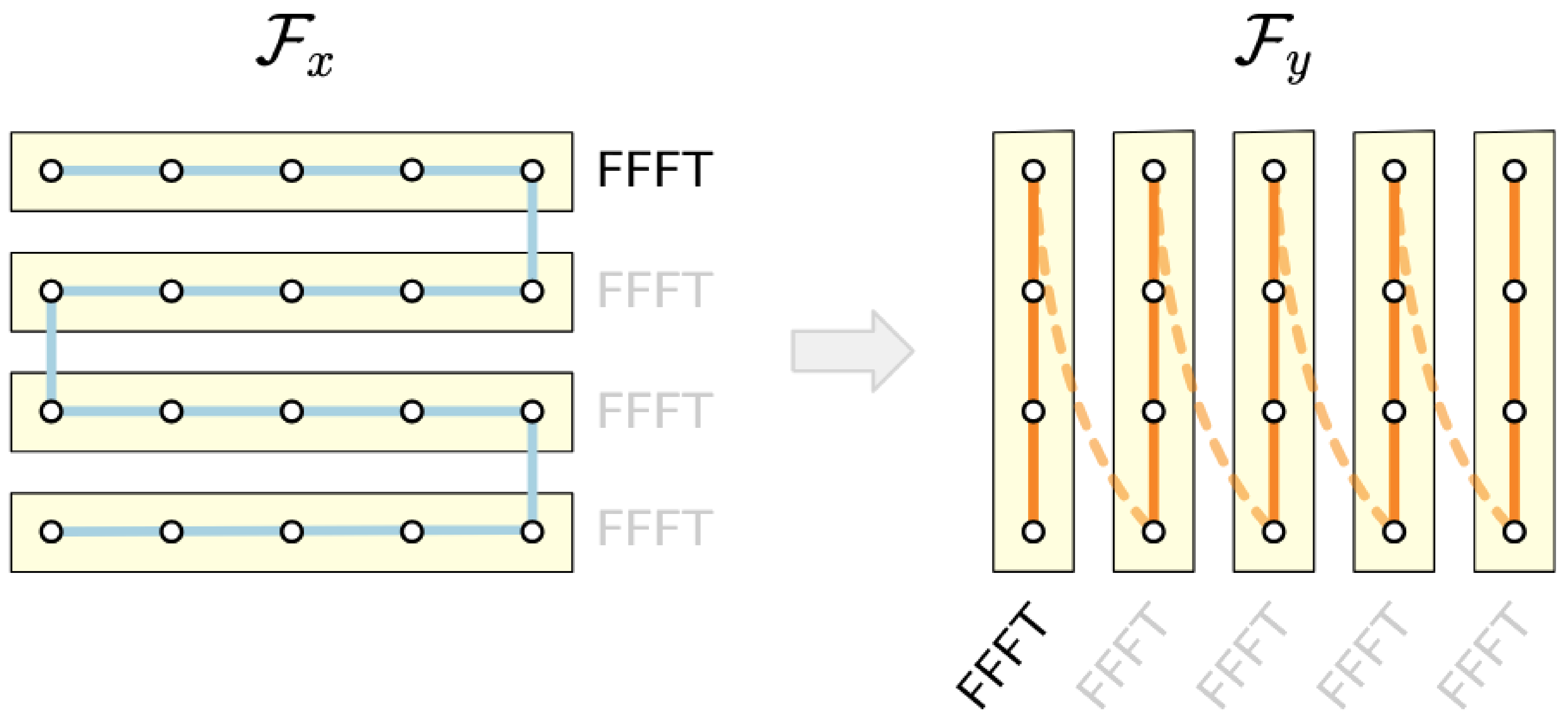
Application to the 2D Fermionic Fast Fourier Transform

Fermionic Fast Fourier transform (**FFFT**) switches fermionic operators between position and momentum bases.

- 2D FFFT can be used for **state preparation** (e.g. superconducting states).
- **Hubbard model**: the kinetic term is diagonal in momentum space, so FFFT diagonalizes it.
- **2D lattices**: the 2D FFFT decomposes into successive 1D transforms

$$\mathcal{F} = \mathcal{F}_x \mathcal{F}_y .$$

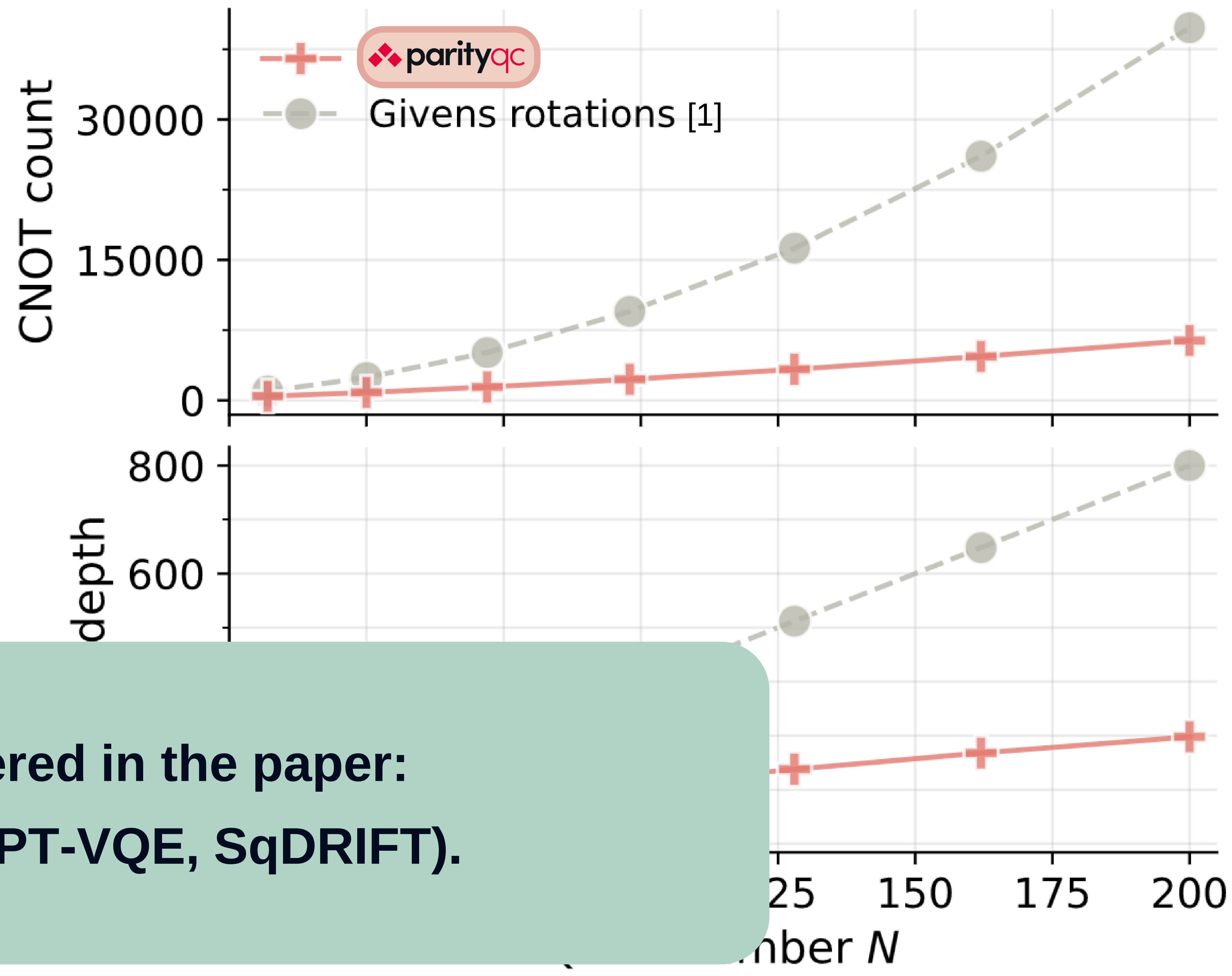


With ParityQC's dynamical encoding, we can efficiently implement the intermediate reorientations required to apply successive 1D FFFTs.

Application to the 2D Fermionic Fast Fourier Transform

Asymptotic resource scaling for the 2D FFFT.
Comparison with method that use (auxiliary-free) 2D **square qubit lattice**.

Method	Depth
Givens rotations [1]	$O(N)$
parityqc	$O(\sqrt{N})$



Other application considered in the paper:
molecular simulations (ADAPT-VQE, SqDRIFT).

[1] I. D. Kivlichan, et al. Physical Review Letters 120, 110501 (2018)

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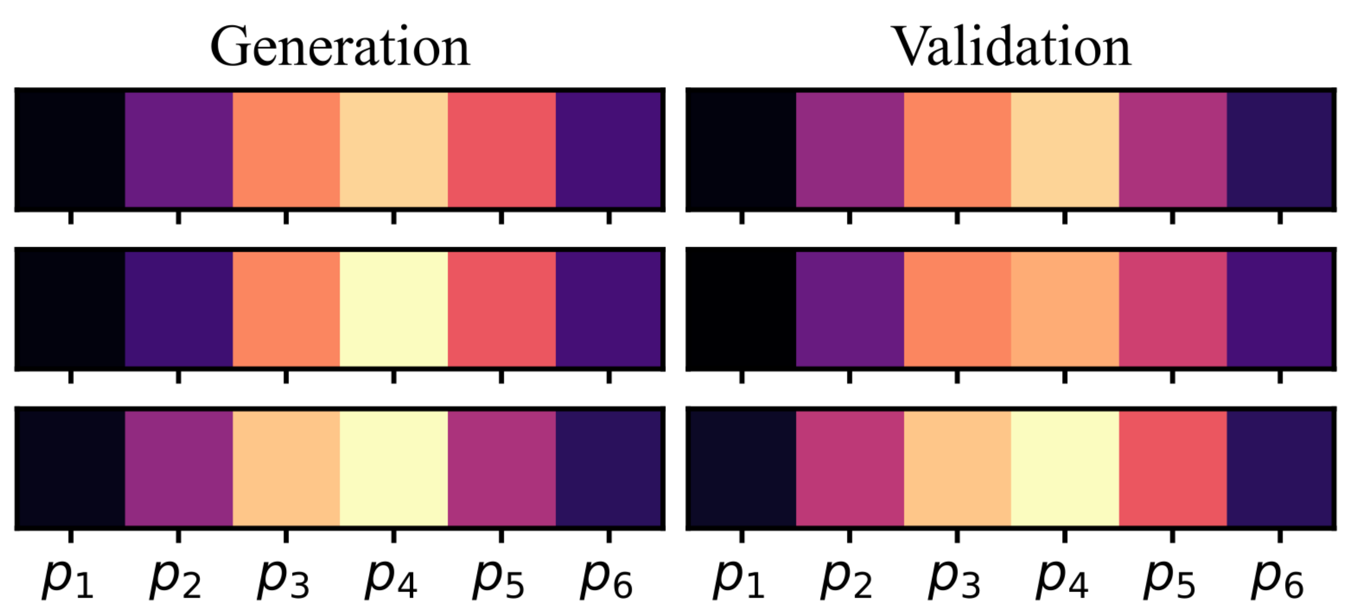
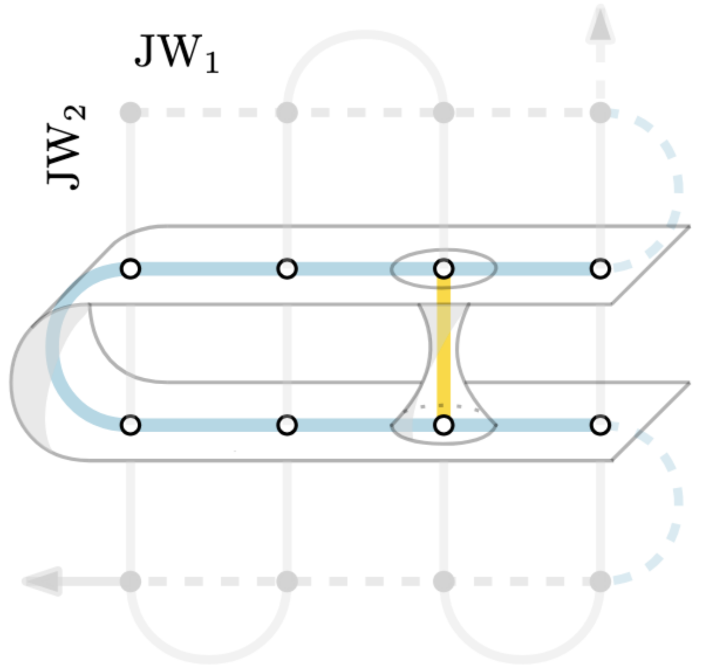
Qudit-based IQP circuits



Paper	arXiv: 2606.28236
Development	Extension of parameterized IQP circuits to integer data
Application	HEP datasets
Outlook	Hardware runs strategies *(Parity Twine)*

<u>See poster “Qudit extension of parameterized IQP circuits” by Robert J. Banks.</u>

Dynamical encodings



Paper	arXiv: 2605.12600
Development	Efficient dynamical encoding for fermionic d.o.f.
Application	Local and non-local fermionic interactions
Outlook	Hamiltonian compiler (*in progress*)