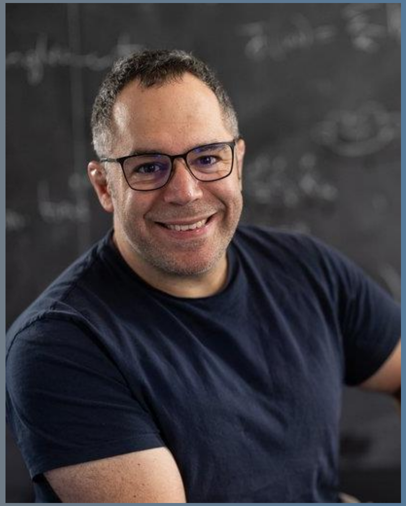
Hilbert space fragmentation and disorder-free localisation in the lattice Schwinger model



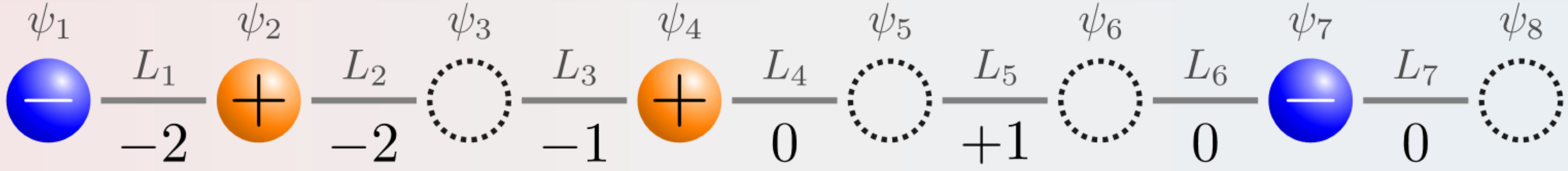

Jared Jeyaretnam *Nottingham*

Tanmay Bhore *Leeds*

Zlatko Papic *Leeds*

Jad C. Halimeh *Munich*

Jesse J. Osborne *Queensland*



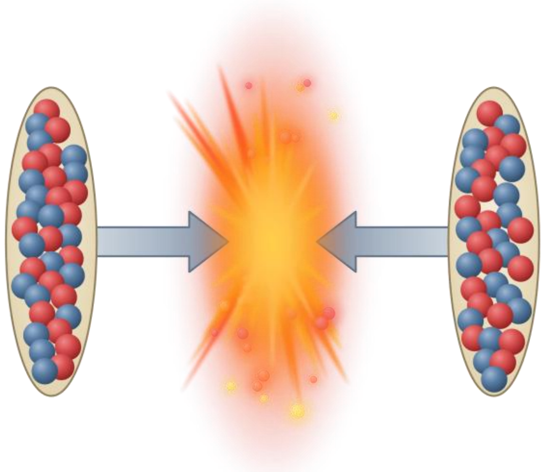
Jeyaretnam *et al.*, *Commun Phys* **8**, 172 (2025)

Outline

- Thermalisation and its breakdown in gauge theories
- Disorder-free localisation in the Schwinger model
- Conclusions & outlook

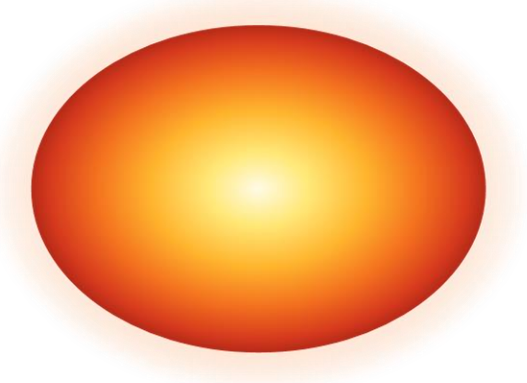
Thermalisation: classical and quantum

Room temperature



Unitary dynamics

$\sim 1\ \mathrm{fm/c}$



Memory of initial conditions is lost!

Arresting thermalisation in a gauge theory?

- Recent Google experiment simulated a $\mathbb{Z}_2$ LGT:

$$\hat{H}_{\mathrm{LGT}} = \sum_{\langle j,k\rangle}\left(\underbrace{\boxed{J\hat{\sigma}_j^Z \hat{Z}_{j,k}\hat{\sigma}_k^Z}}_{\text{coupling}} + \underbrace{\boxed{h\hat{X}_{j,k}}}_{\text{gauge field}}\right) + \underbrace{\boxed{\mu\sum_j \hat{\sigma}_j^X}}_{\text{matter}}$$

Gauge invariance: $\hat{G}_j$ =

Superselection sector

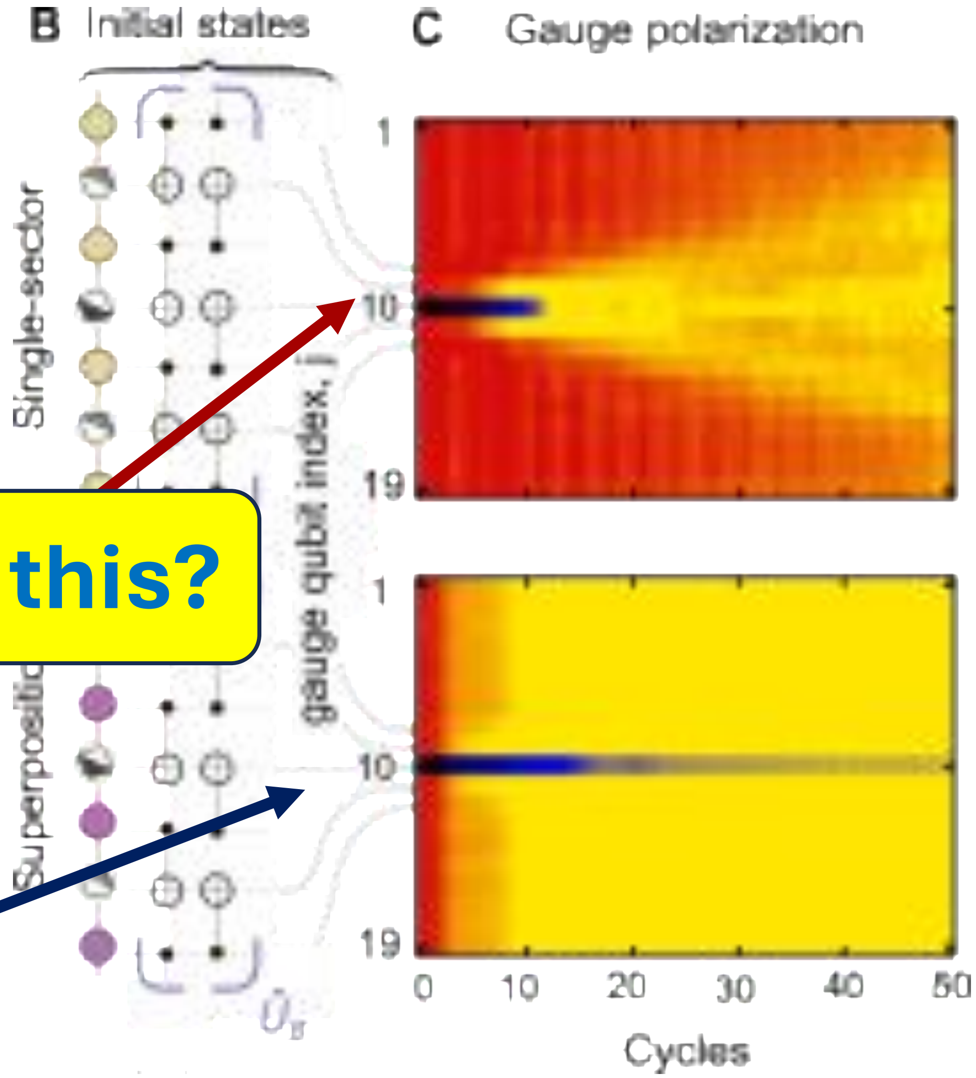
How do we explain this?

Single gauge sector, $g_j = +1$:

→ initial "bump" **spreads** throughout system

Superposition of sectors, $g_j \in \{+1, -1\}$:

→ **bump is preserved**, system does not equilibrate



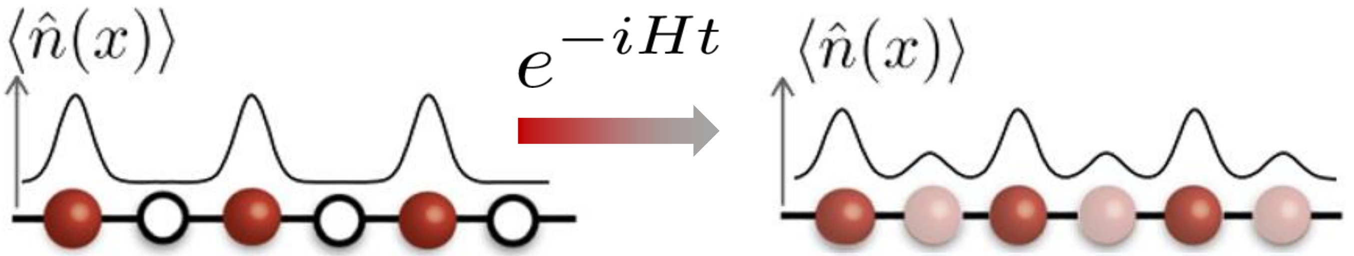
[1] Google Quantum AI *et al.*, Science **393**, 71-75 (2026)

Many-Body Localisation (MBL)

- **Interacting** system + strong **disorder**

⇒ Memory of initial conditions

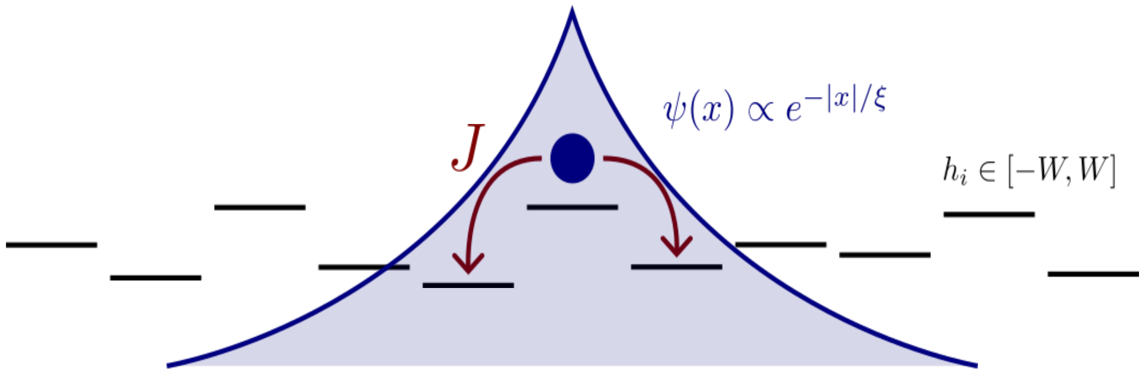
⇒ **Generic** violation of thermalisation



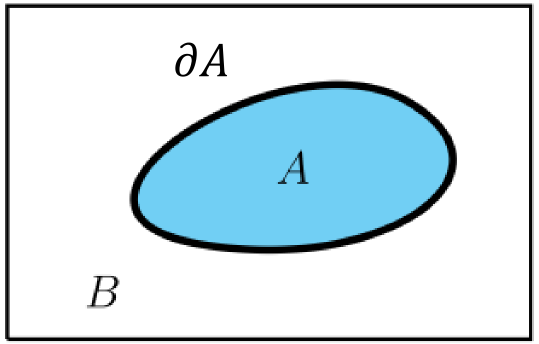
21001 (2019)

Can this physics arise in LGTs without disorder?

⇒ Interacting cousin of Anderson localisation [P.W. Anderson, 1958]

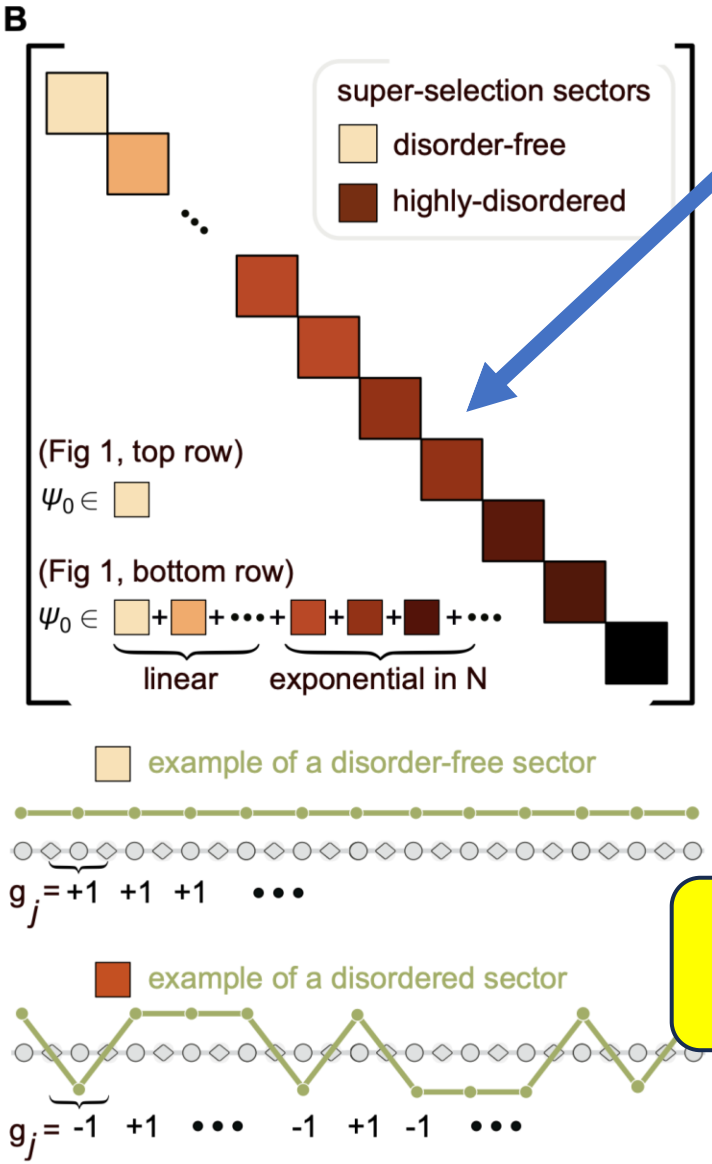


⇒ Eigenstates are short range correlated with **"area law"** entanglement entropy $S_A \propto |\partial A|$



⇒ **Slow growth** of entanglement after a quench $S_A \sim \log(t)$

Gauge constraint induces effective disorder



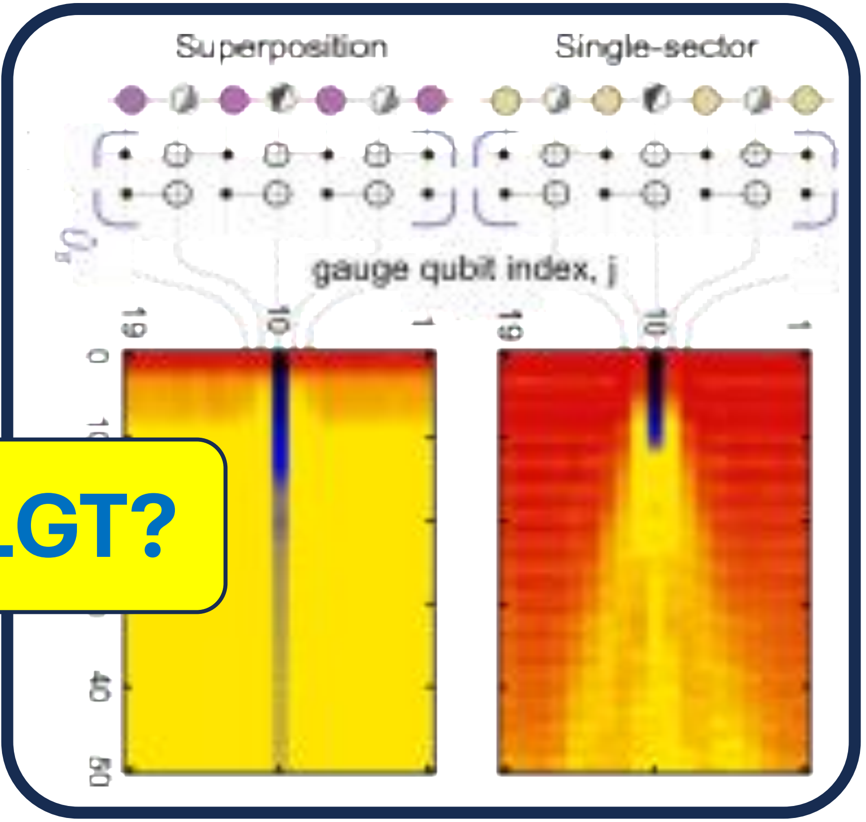
Hamiltonian is **block diagonal** in superselection sectors $\{g_j\}$

Superposition of local d.o.f. $\leftrightarrow$ superposition over sectors

Each sector has a **different effective Hamiltonian** $H_{\text{eff}}(\{g_j\}) \longrightarrow$ effective **disorder ensemble**

Expectation values $\sim$ disorder average

Quantum parallelism can efficiently perform disorder average!



Paredes, Verstraete, Cirac, PRL **95**, 140501 (2005)
A. Smith *et al.*, PRL **118**, 266601 (2017)
Google Quantum AI *et al.*, Science **393**, 71-75 (2026)

What about in a U(1) LGT?

Figure: Google Quantum AI *et al.*, Science **393**, 71-75 (2026)

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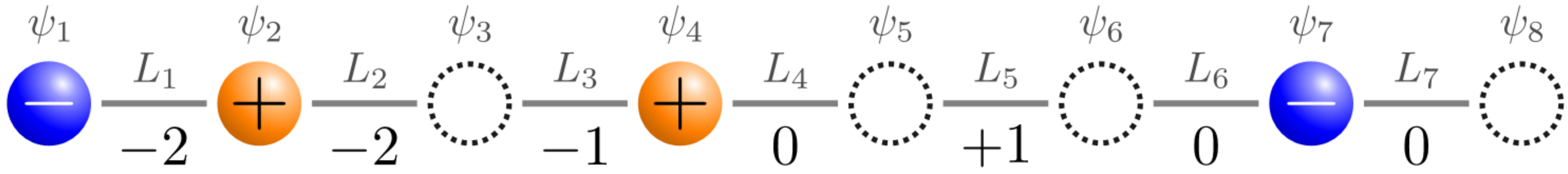
The Schwinger model: a U(1) LGT in 1+1D



$$\hat{H}_{\mathrm{sch}} = -iw \sum_{n=1}^{N-1} \left(\hat{\psi}_n^\dagger e^{i\hat{\theta}_n} \hat{\psi}_n - \text{ h.c. } \right) + m \sum_{n=1}^{N} (-1)^n \hat{\psi}_n^\dagger \hat{\psi}_n + J \sum_{n=1}^{N-1} \hat{L}_n^2$$

staggered fermions

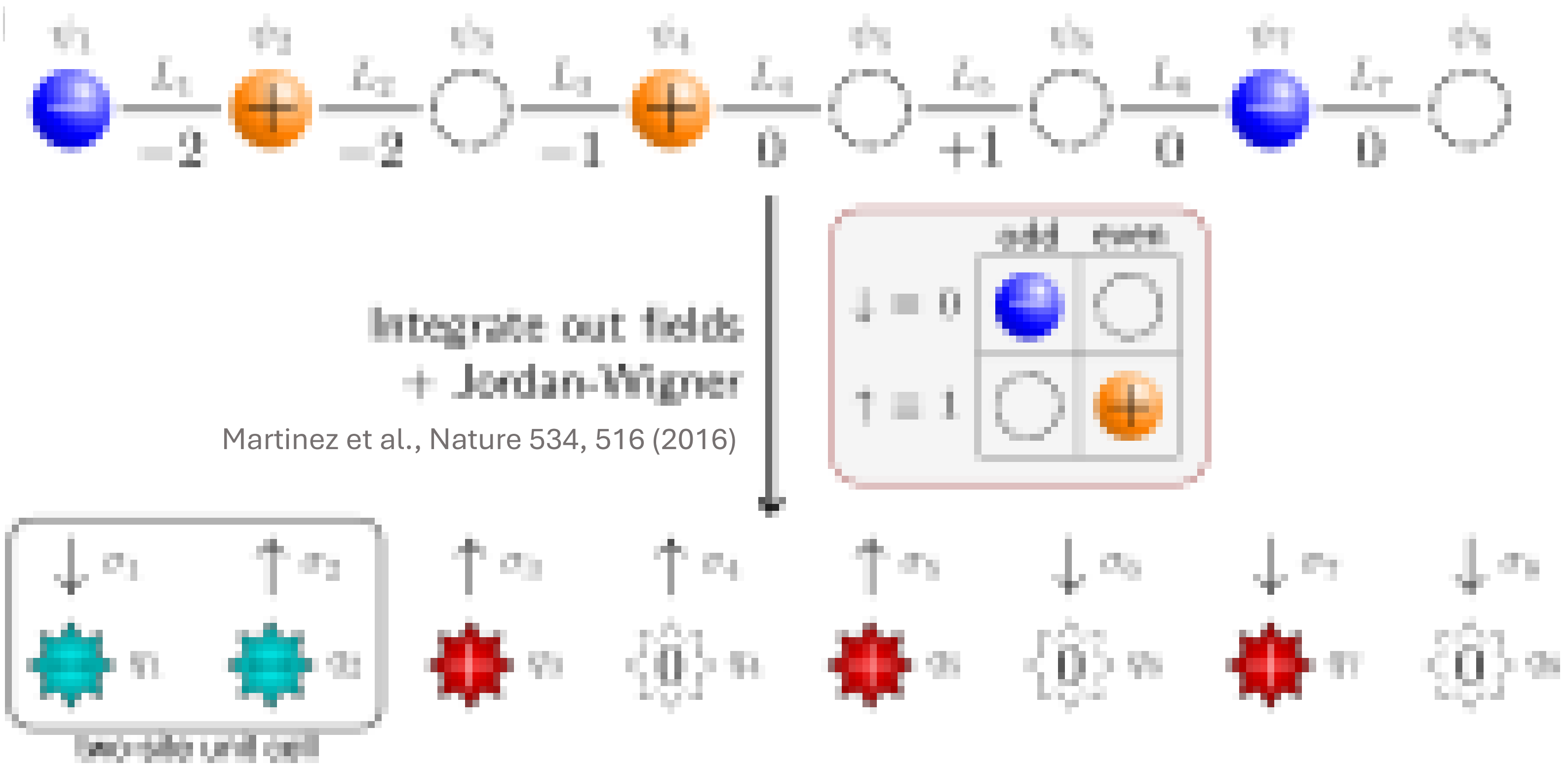
electric field

J. Schwinger, Phys. Rev. 82, 914 (1951)
J. Kogut and L. Susskind, Phys. Rev. D 11, 395 (1975)
T. Banks, L. Susskind, and J. Kogut, Phys. Rev. D 13, 1043 (1976)

- Gauss's law: $\hat{G}_n = \hat{L}_n - \hat{L}_{n-1} - \psi_n^\dagger \psi_n + (n \bmod 2), \ [\hat{G}_n, \hat{H}] = 0$
- Physical states $\hat{G}_n|\psi\rangle = q_n|\psi\rangle$ defined by a set of **conserved charges** $q_n \in \mathbb{Z}$

Effective model in superselection sectors?

Mapping the Schwinger model to a spin chain

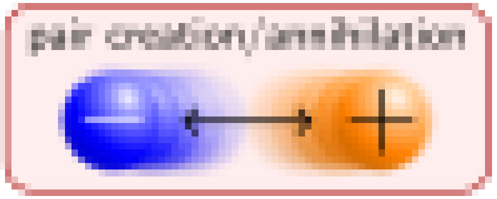


Martinez et al., Nature 534, 516 (2016)

$$L_j - L_{j-1} = q_j + \begin{cases} -n_j & j \text{ odd} \\ 1 - n_j & j \text{ even} \end{cases}$$

Background charges

$$\hat{H}_\pm = w \sum_{j=1}^{N-1} \left[\hat{\sigma}_j^+ \hat{\sigma}_{j+1}^- + \text{h.c.} \right]$$



Coulomb force between fermions and background

$$\hat{H}_q = \sum_{k=1}^{N} \left(h_k + \frac{m}{2}(-1)^k \right) \hat{\sigma}_k^z \ ,$$

$$h_k = \frac{J}{2} \left((N-k)\frac{\theta}{\pi} - \left\lceil \frac{N-k}{2} \right\rceil + 2 \sum_{j=k}^{N-1} \sum_{i=1}^{j} q_i \right)$$

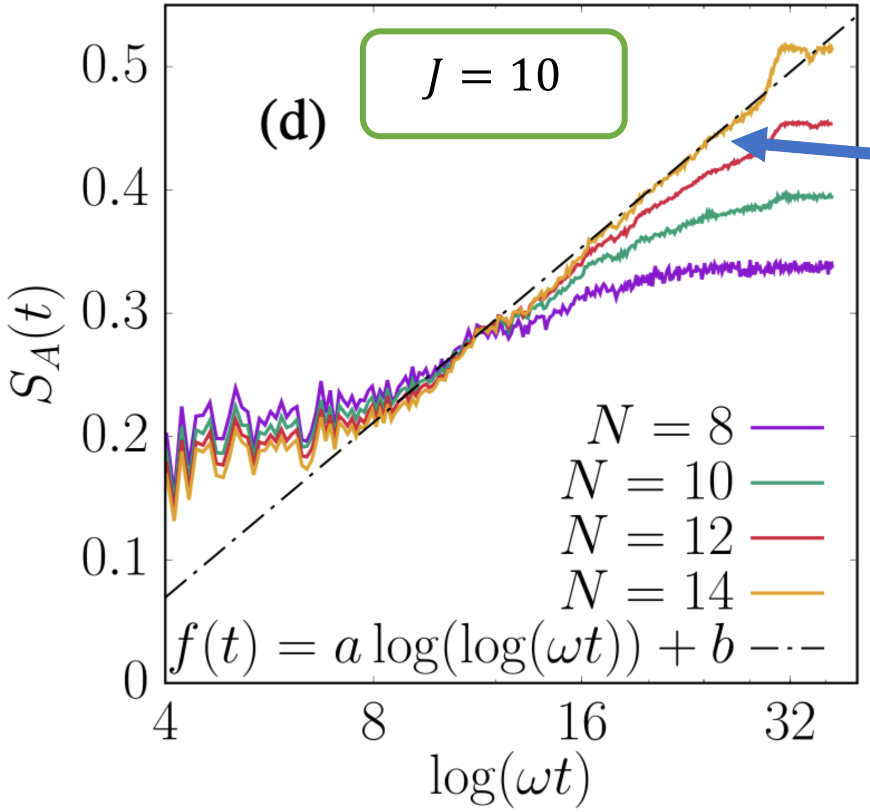
Depends explicitly on $\{q_i\}$!

$$\hat{H}_{ZZ} = \frac{J}{2} \sum_{j=1}^{N-2} \sum_{k=j+1}^{N-1} (N-k) \hat{\sigma}_j^z \hat{\sigma}_k^z$$

Encodes Coulomb force in 1D

Previous studies claimed ultraslow dynamics

Initial state: $|\mathrm{vac}\rangle = |101010\ldots\rangle$, $q_n \in \{-1, 0, +1\}$



M. Brenes, M. Dalmonte, M. Heyl, and A. Scardicchio
Phys. Rev. Lett. **120**, 030601 (2018)

See also:
G. Giudici *et al.*, PRR **2**, 032034(R) (2020)
C. Muschik *et al*, *New J. Phys.* **19** 103020 (2017)

Entanglement growth appears even slower than MBL:

$S \sim \log\log t?$

But its origin has not been identified and J is very large.

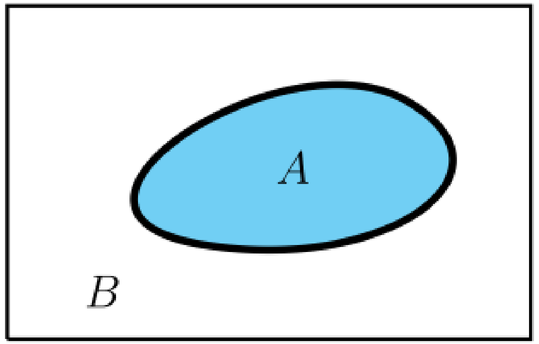
Two questions:

1. Could **fragmentation** play a role in the strong-coupling regime?
2. Is there (conventional) **MBL at intermediate J**?

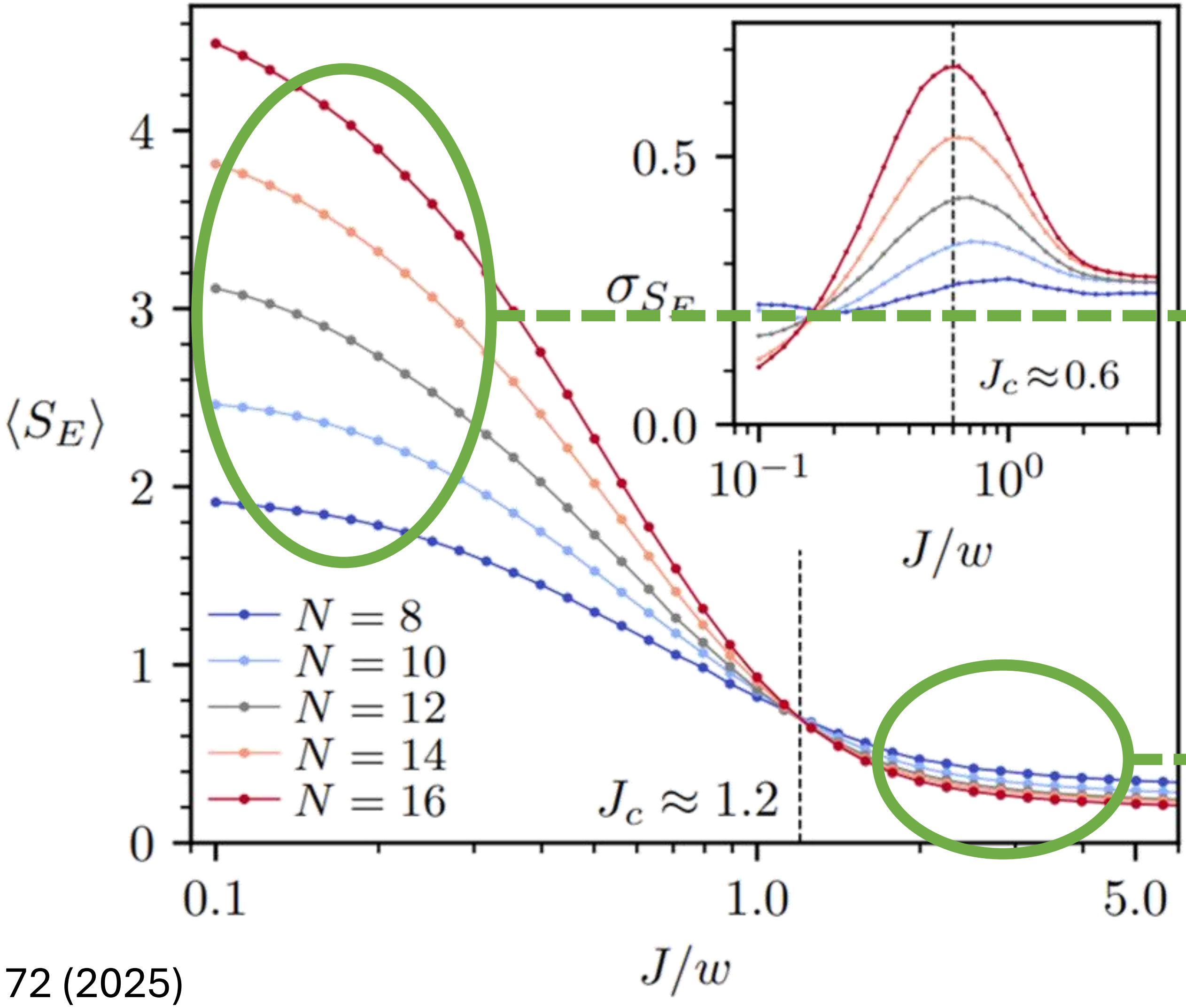
$J \sim w$ regime: MBL eigenstates

$$H = wH_{\pm} + J[H_{ZZ} + H_q(\{q_n\})]$$

Bipartite entanglement entropy of eigenstates:



$$\rho_A = \mathrm{Tr}_B(|\psi\rangle\langle\psi|)$$
$$S_A = \mathrm{Tr}(\rho_A \log \rho_A)$$



$S \sim O(N)$

Localisation!

$S \sim O(1)$

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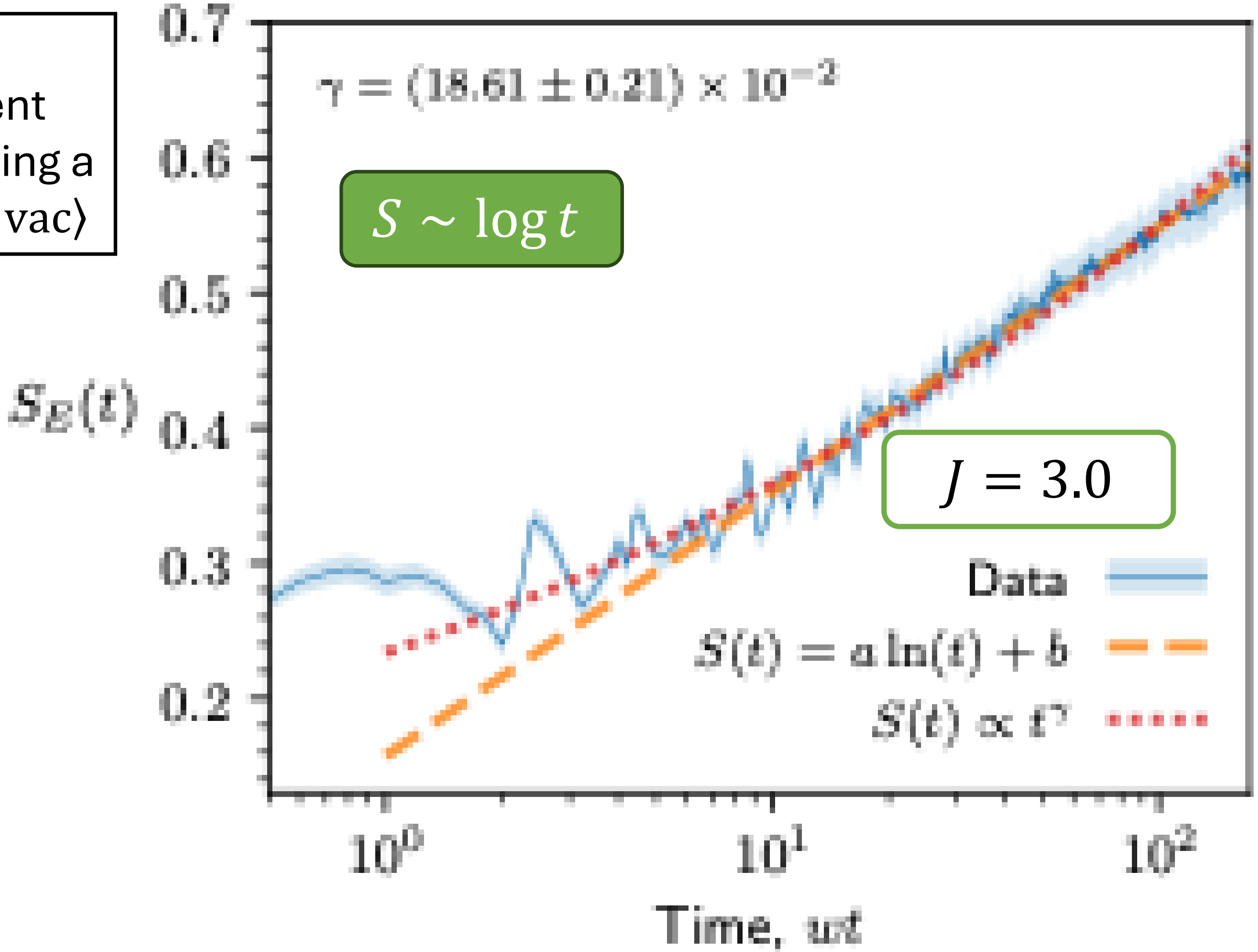
$J \sim w$ regime: MBL dynamics

$$H = wH_{\pm} + J[H_{ZZ} + H_q(\{q_n\})]$$

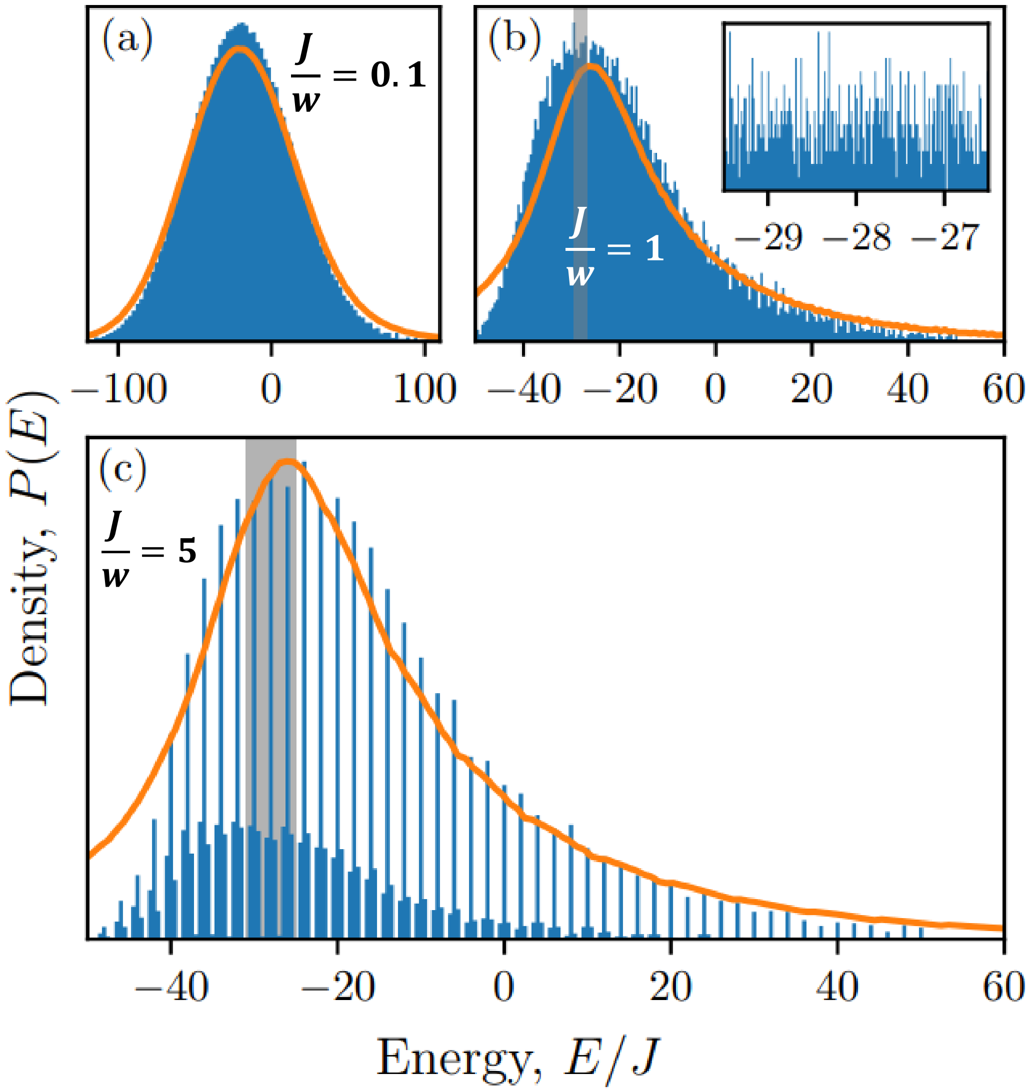
Bipartite entanglement entropy following a quench from $|\mathrm{vac}\rangle$

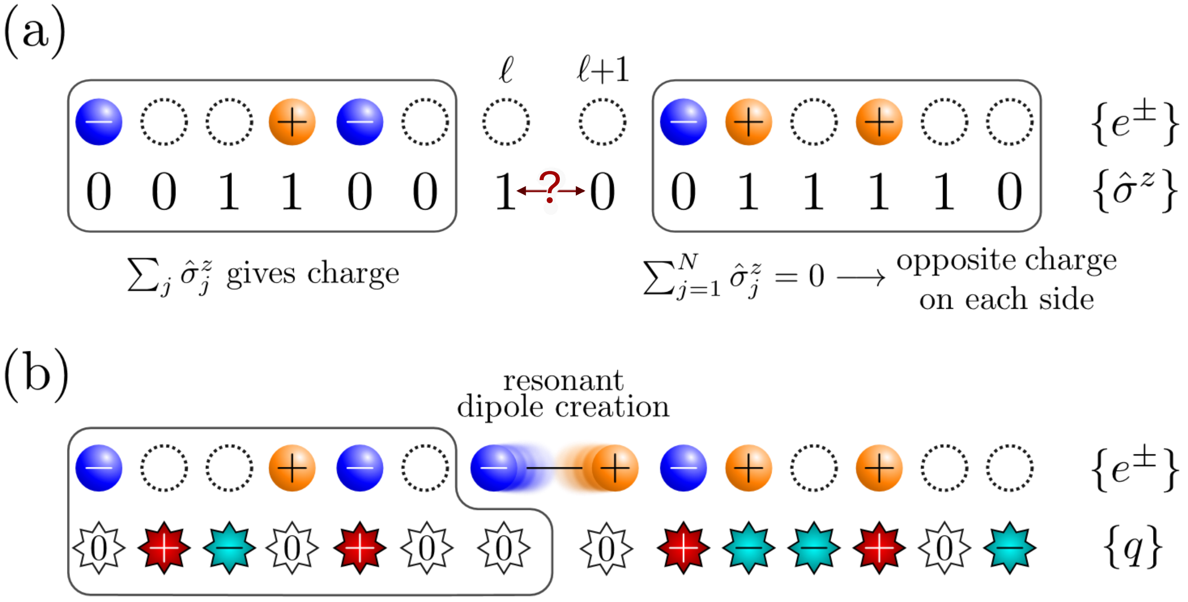


MPS simulation, $N = 50$

Expect:
- Power law for thermalisation
- $\log(t)$ for localisation

$J \gg w$: a fragmented limit





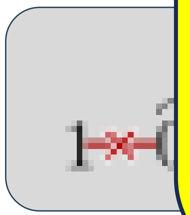
$$\boxed{\sum_{j=1}^{\ell-1} \hat{\sigma}^z_j + 2\sum_{j=1}^{\ell} q_j = (\ell \mod 2) - \frac{\theta}{\pi}}$$

Constraint: **dynamical charges** screen **background charges**

- Hopping only **resonant** if constraint met
- "J-level" splits into many weakly-mixed subspaces
- Nearly-degenerate eigenstates & atypical dynamics!

Deg (DPT)

- In J
chai



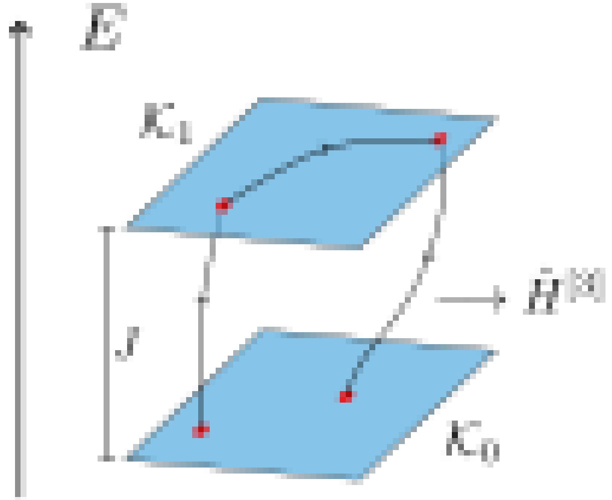
Accuracy of DPT:

- confirms **fragmentation picture**
- suggests $\log(t)$ **growth** at intermediate times
- ultraslow **power-law** approach to steady state

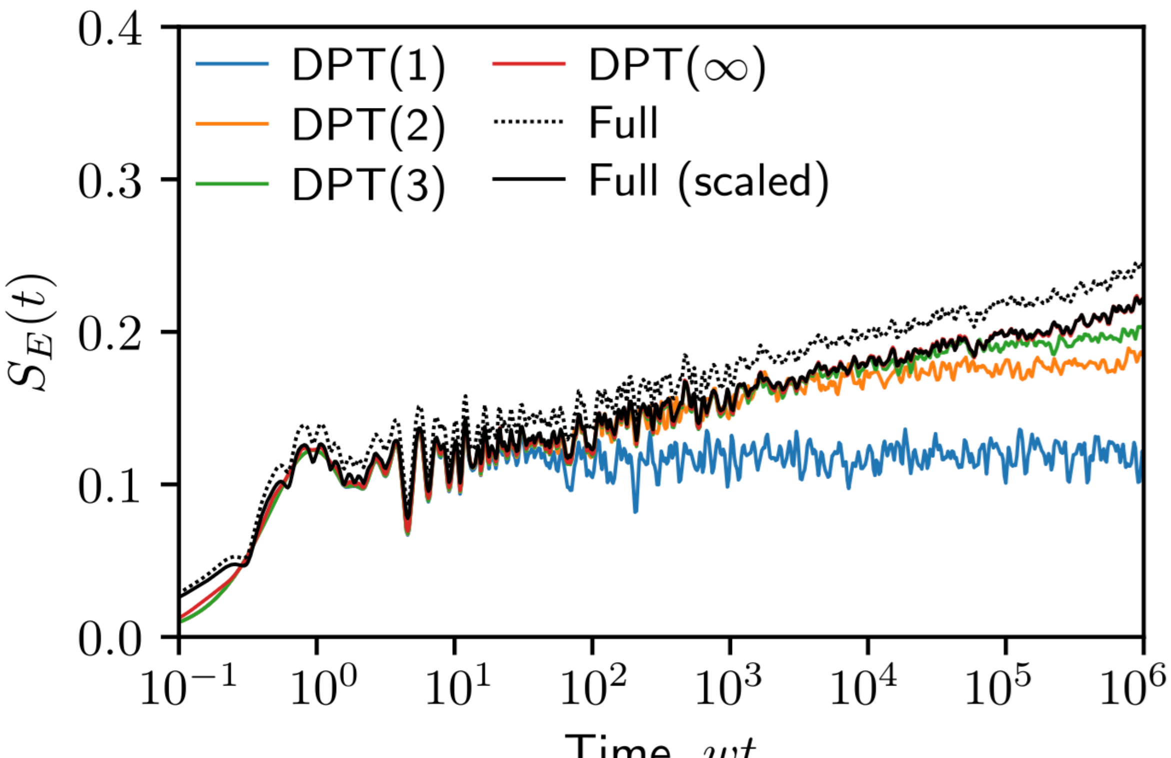
cs

- Finite coupling: expand perturbatively in J/w

$$\hat{H}_{\text{eff}} \propto \hat{H}^{[0]} + (w/J)\hat{H}^{[1]} + (w/J)^2\hat{H}^{[2]} + \ldots$$



virtual hops via
other energy levels



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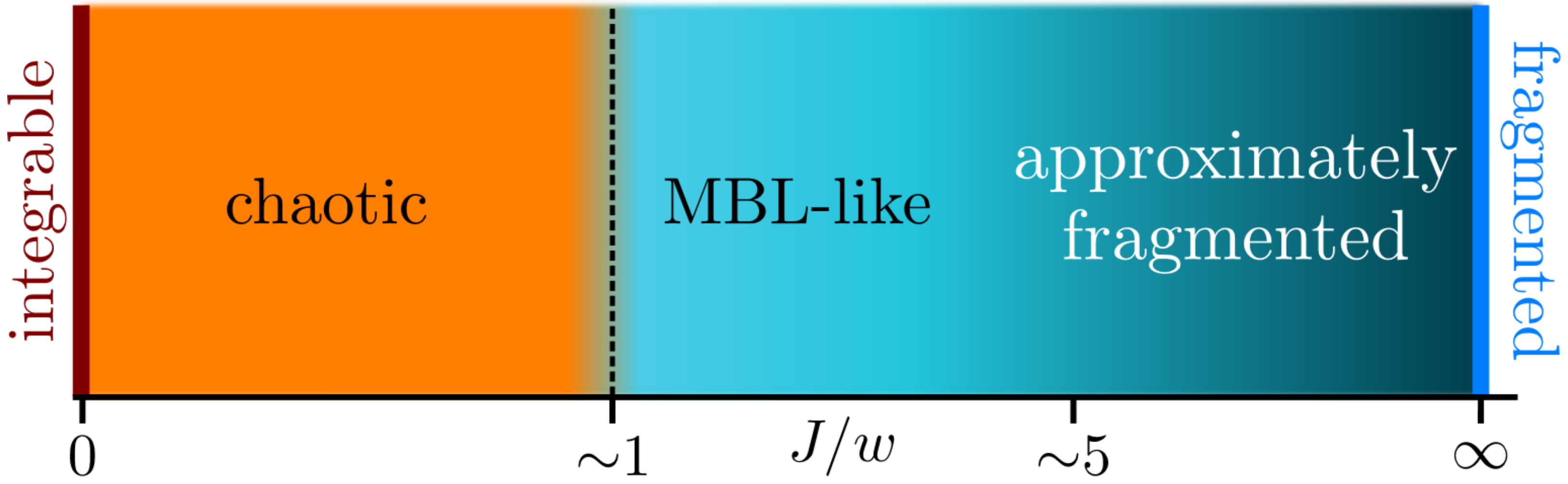
Conclusions



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- U(1) LGTs can also exhibit disorder-free localisation
- Conventional MBL at intermediate coupling
- Strong coupling is controlled by proximity to a fragmented limit

Quantum simulation?

[1] Google Quantum AI *et al.*, Science **393**, 71-75 (2026)
[2] M. Meth, L. Dellantonio *et al.*, Nat. Phys. **21**, 570–576 (2025).
[3] R. Joshi *et al.*, arXiv:2507.12614 **[quant-ph]**
[4] A. Mil, P. Hauke *et al.*, Science, **367**, 1128-1130 (2020)
[5] V. Ale, T. Rainaldi, E. Rico *et al.*, JHEP **2026**, 122
[6] J. M. Alcaine-Cuervo, E. Rico *et al.*, arXiv:2601.23150 **[quant-ph]**
[7] See Simon William's talk, Monday

- Google experiment: (2+1)D $\mathbb{Z}_2$ gauge theory [1], but *two-level links only*
- How do we faithfully simulate **full** $U(1)$ LGT with matter and **infinite dimensional** links?
- Related: poster on how many levels we need to see MBL!

Digital

- e.g. trapped ion qudits for links [2-3]
- Truncation is a tunable dial
- Gate-based ⇒ high level of control

Analogue

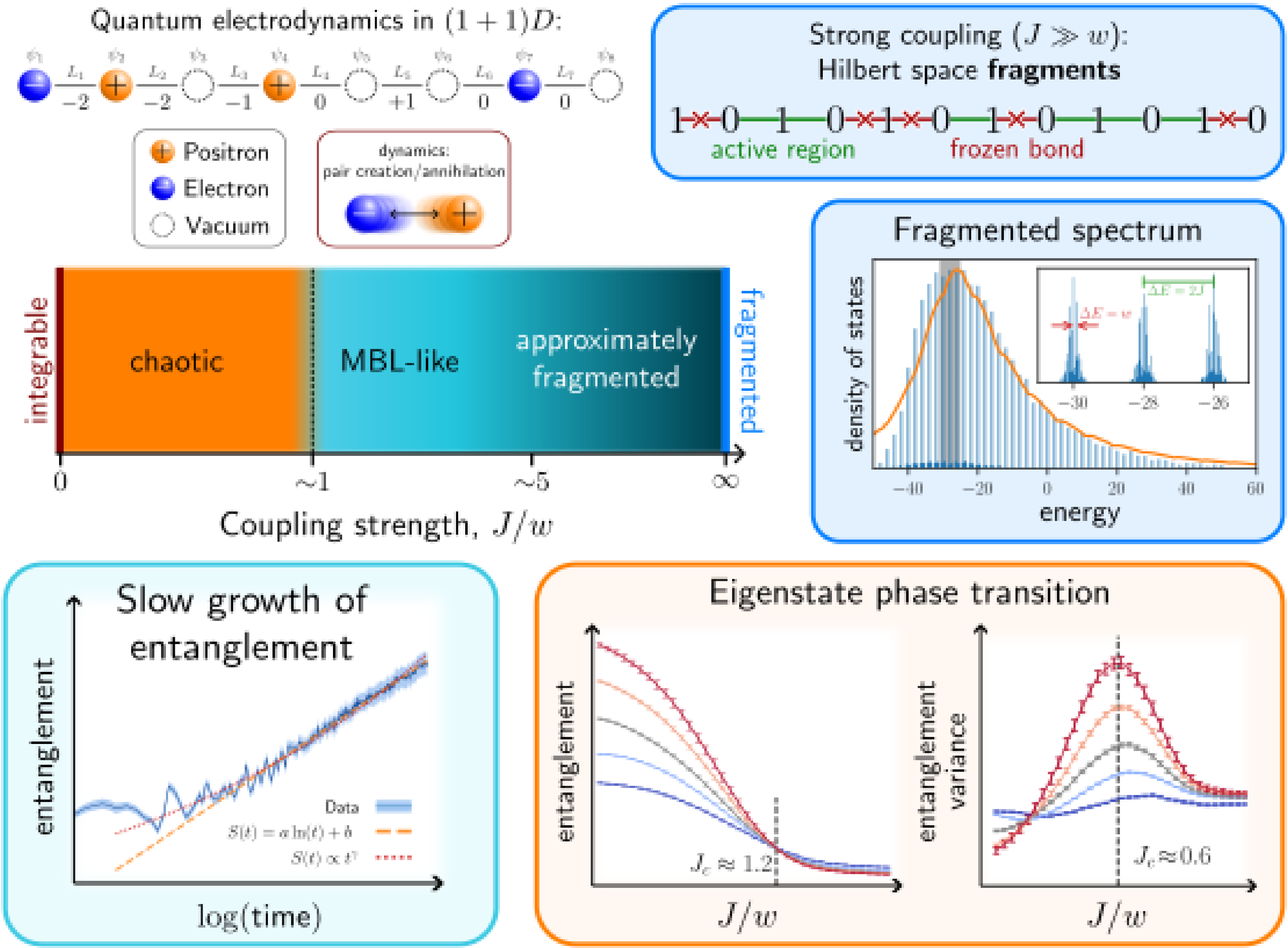
- e.g. cold atoms, using BECs for links [4]
- Close to continuum $U(1)$
- Scalable to very large systems!

Local control and readout

Faithful links
Large systems

Hybrid approaches with continuous variables? e.g. [5-7]

Thank you!



Quantum electrodynamics in $(1+1)D$:
Positron
Electron
Vacuum
dynamics:
pair creation/annihilation
integrable
chaotic
MBL-like
approximately fragmented
fragmented
0
~1
~5
∞
Coupling strength, J/w
Strong coupling ($J \gg w$):
Hilbert space **fragments**
active region
frozen bond
Fragmented spectrum
density of states
energy
Slow growth of entanglement
entanglement
Data
log(time)
Eigenstate phase transition
entanglement
$J_c \approx 1.2$
J/w
entanglement variance
$J_c \approx 0.6$
J/w