Theory and Phenomenology of Fundamental Interactions
UNIVERSITY AND INFN · BOLOGNA

INFN Istituto Nazionale di Fisica Nucleare
Sezione di Bologna

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Digital Rydberg simulation of dynamical quantum phase transitions in the Schwinger model

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THE PROBLEM: real time out-of-equilibrium dynamics on noisy quantum hardware

in collaboration with S. Pascazio, P. Facchi, F.V. Pepe, G. Magnifico, D. Pomarico, R. Cioli, F. Dell'Anna

Gauge theories are hard

Physical states must obey Gauss-law constraints; noise can leak states outside the gauge-invariant subspace.

DQPTs are delicate

They are detected through zeros of the Loschmidt echo; noise and finite-time errors can wash out sharp signatures.

Rydberg platforms are promising

Neutral atoms offer high gate fidelity, but pulses are slow enough that circuit duration matters.

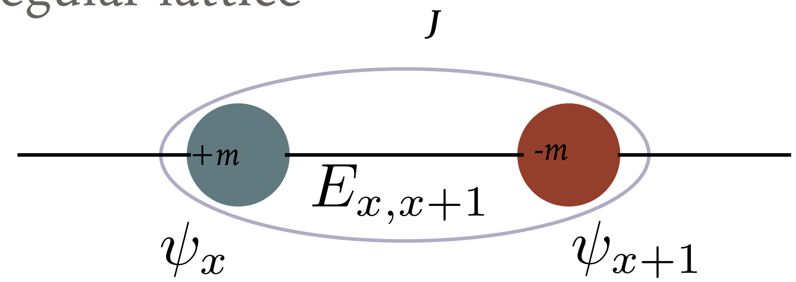
the model -
static phase diagram & real-time dynamics

PHYSICAL REVIEW D **98,** 074503 (2018)

1d QED: lattice $\mathbb{Z}_n$ Schwinger model

Lattice discretization: continuum space is replaced by a regular lattice

- fermions defined on lattice sites
- gauge fields on links



Schwinger's *staggered fermions*: index J for a "physical site" is split in two, +/- energy solutions

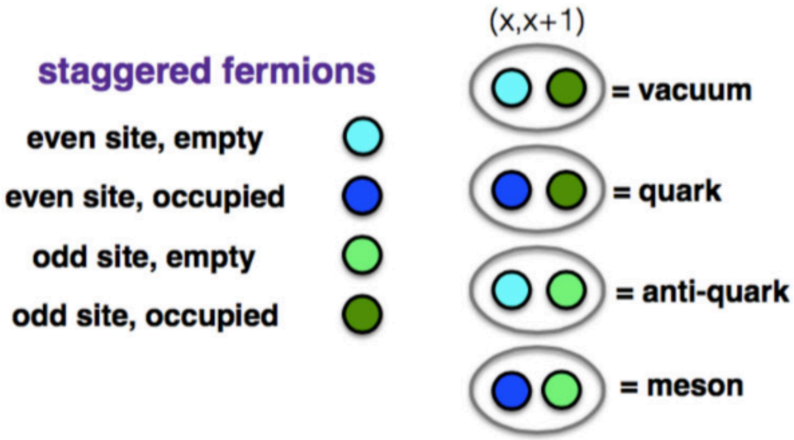
$$\Psi_J = \begin{pmatrix} \psi_x \\ \psi_{x+1} \end{pmatrix}$$

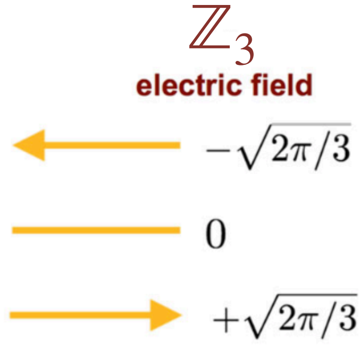
Z_n Quantum Model: Weyl quantisation scheme,unitary finite-dim representation

$$\mathcal{H}_{x,x+1} = \{|v_k\rangle\,,\, k = 1, 2, \cdots, n\}$$

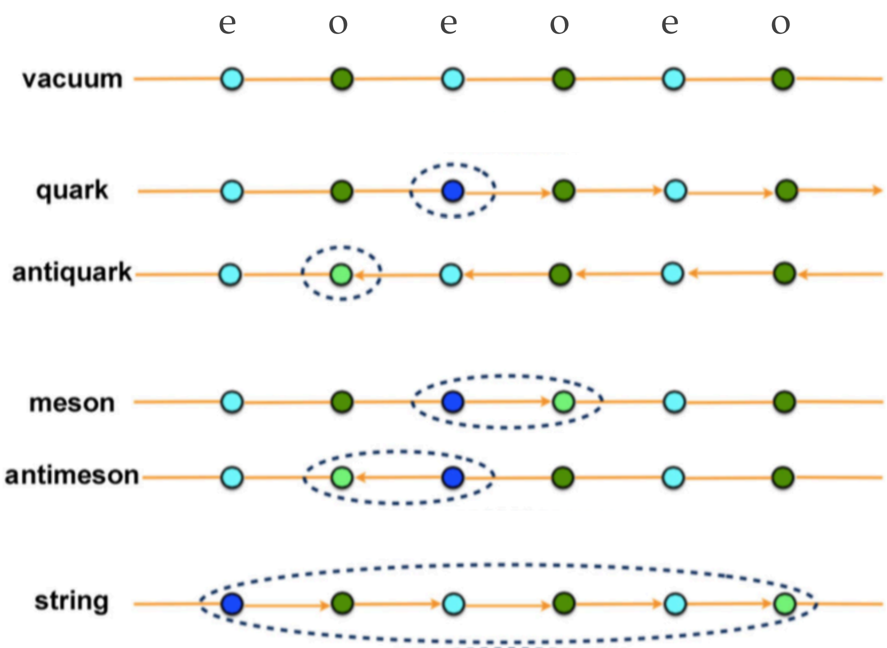
$$U_{x,x+1}(\eta) = e^{-i\eta A_{x,x+1}} \qquad V(x, x+1)|v_k\rangle = e^{\imath \frac{2\pi}{n}k}|v_k\rangle$$

$$V_{x,x+1}(\xi) = e^{+i\xi E_{x,x+1}} \qquad U(x, x+1)|v_k\rangle = |v_{k+1}\rangle$$





$$G_x \equiv \psi_x^\dagger \psi_x + \frac{1}{2}[(-1)^x - 1] - (E_{x,x+1} - E_{x-1,x}) \approx 0$$



$$H = -\overset{J}{\boxed{\frac{t}{2a}}} \sum_x \left(\psi_x^\dagger U_{x,x+1} \psi_{x+1} + \mathrm{H.c.}\right)$$

$$+ \boxed{m} \sum_x (-1)^x \psi_x^\dagger \psi_x + \frac{\boxed{g^2} a}{2} \sum_x E_{x,x+1}^2$$

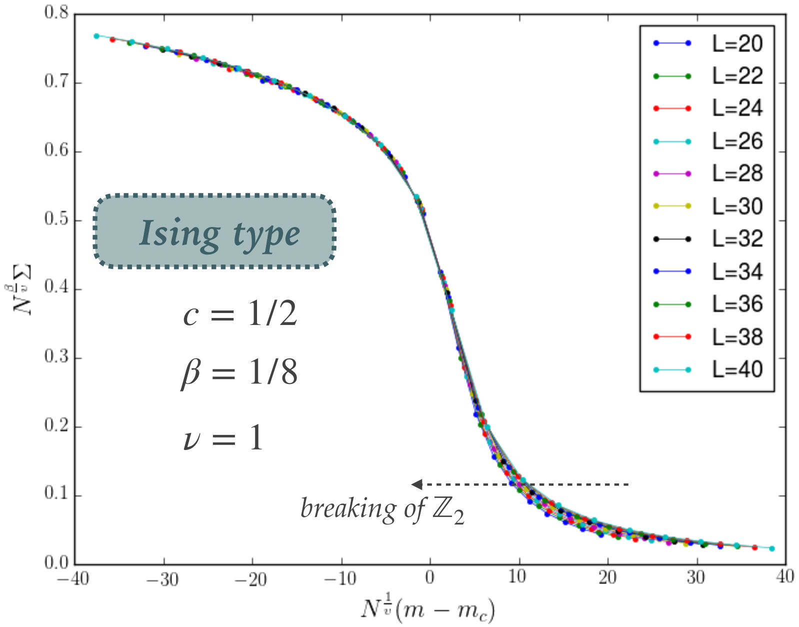
PHASE DIAGRAM

$\mathbb{Z}_n$ *fixed*

$(\mathbb{Z}_2 \cdots \mathbb{Z}_8)$

$$\Sigma = \frac{1}{N}\sum_x \langle E_{x,x+1}\rangle$$

order parameter



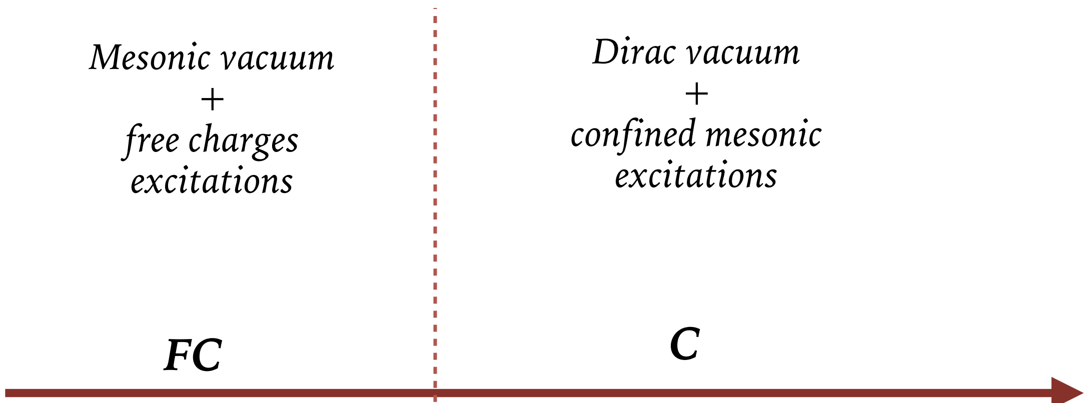
Density Matrix Renormalization Group Algorithm (**DMRG**)

The DMRG code strictly enforces Gauss law

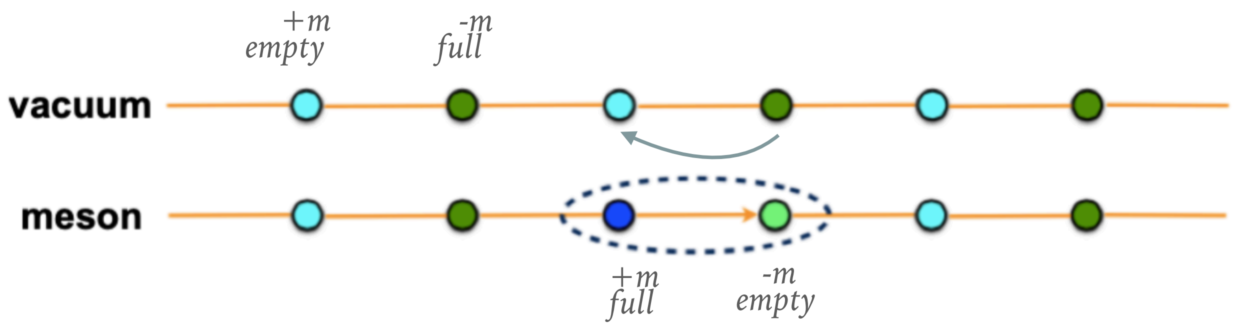
We work up to $L=N/2=40$ couples of sites & 1000 DMRG states

Open boundary conditions

PHASE DIAGRAM

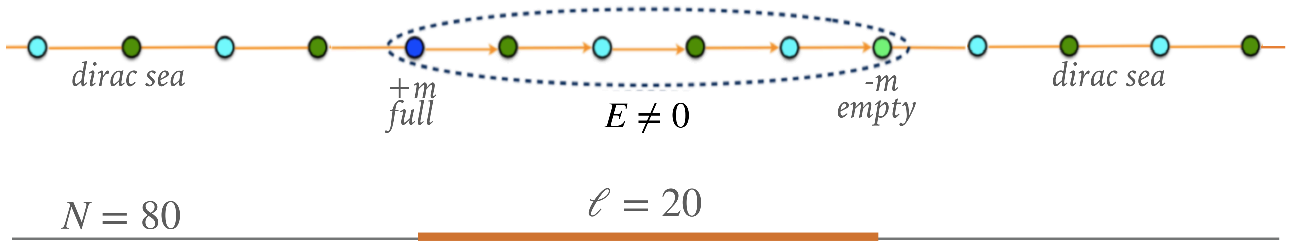


$m_c = -0.33$ (large n, i.e U(1) limit)

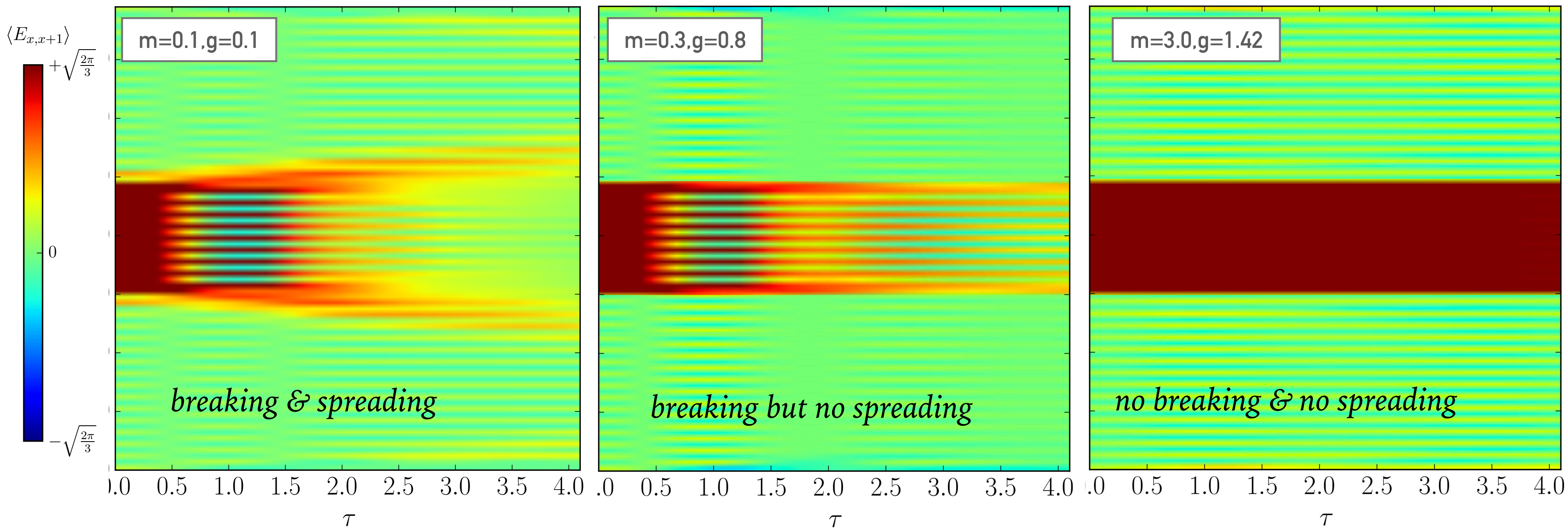


STRING BREAKING

Real time evolution of the string excitation: does it break?



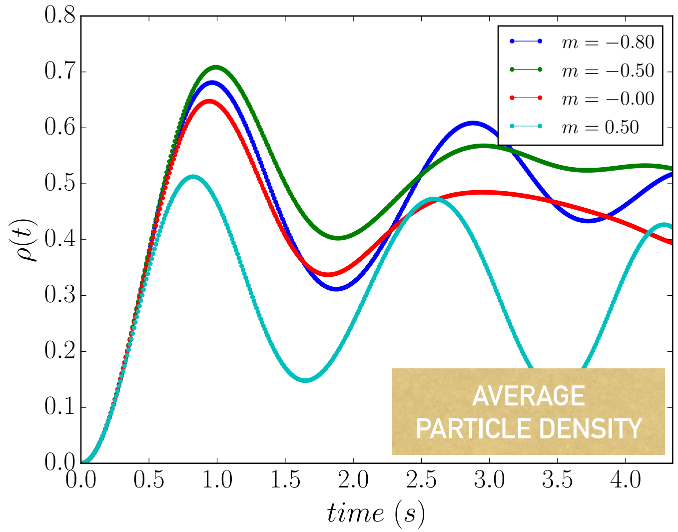
$\mathbb{Z}_3$

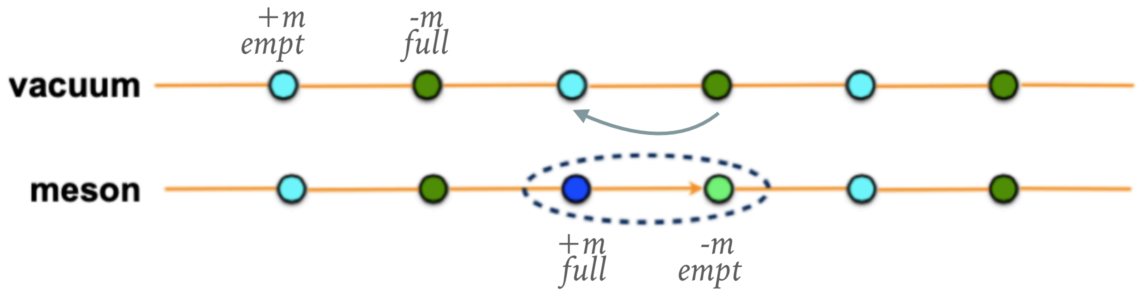


stronger confinement

DYNAMICS OF PAIR PRODUCTION

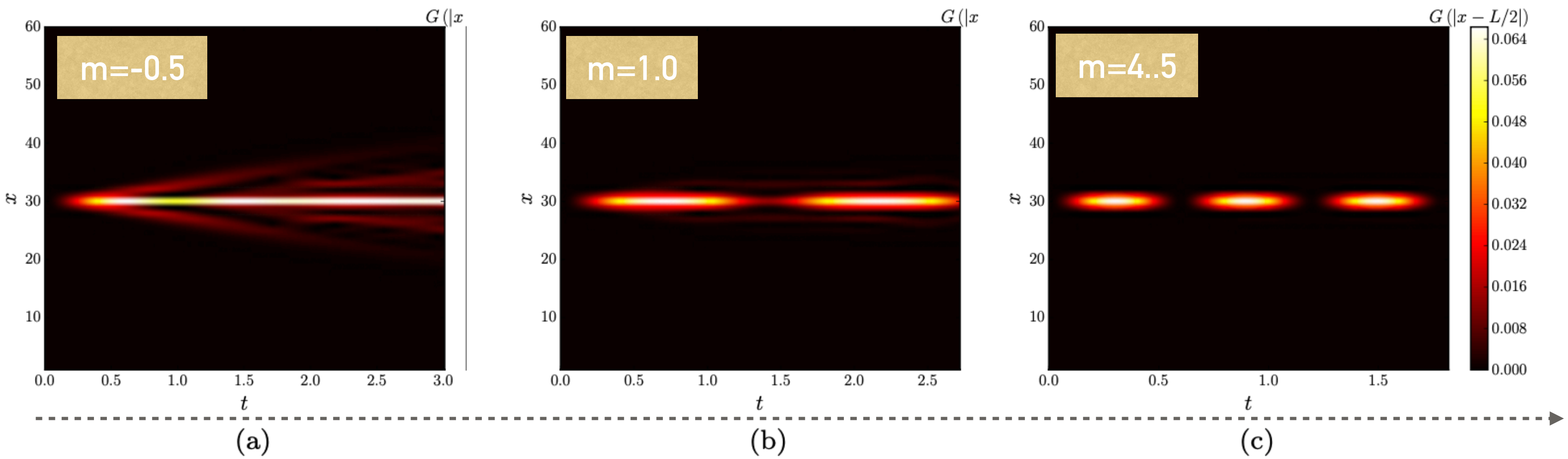
Quench protocol: system prepared in Dirac vacuum (m large) evolving with H with a chosen value of m





DENSITY-DENSITY CORRELATIONS

$\mathbb{Z}_3$

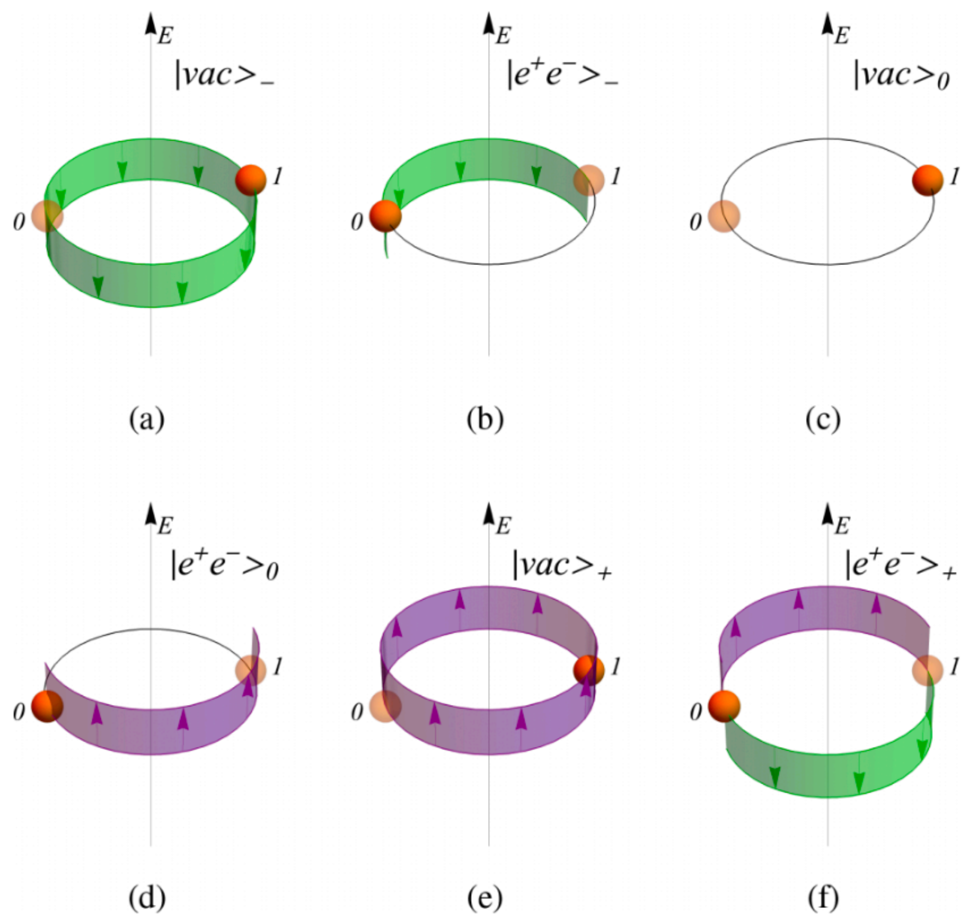


stronger coupling ~ stronger confinement

non-equilibrium dynamics &
dynamical phase transitions
om noisy Rydberg platform

$\mathbb{Z}_3$ Schwinger model on 2 sites

gauge-invariant subspace



$\mathscr{H}$ commutes with parity $\qquad [\mathscr{H}, P] = 0$

We consider only the even sector,
to which Dirac vacuum belongs

$$|\psi_1^+\rangle = |\text{vac}\rangle_0, \quad |\psi_2^+\rangle = \frac{|e^+e^-\rangle_0 + |e^+e^-\rangle_-}{\sqrt{2}},$$

$$|\psi_3^+\rangle = \frac{|\text{vac}\rangle_+ + |\text{vac}\rangle_-}{\sqrt{2}}, \quad |\psi_4^+\rangle = |e^+e^-\rangle_+,$$

$$\tilde{\mathcal{H}}^+ = \begin{pmatrix} -\mu & \frac{\xi}{\sqrt{2}} & & \\ \frac{\xi}{\sqrt{2}} & \mu + \frac{\pi}{3} & \frac{\xi}{2} & \\ & \frac{\xi}{2} & -\mu + \frac{2\pi}{3} & \frac{\xi}{\sqrt{2}} \\ & & \frac{\xi}{\sqrt{2}} & \mu + \frac{2\pi}{3} \end{pmatrix}$$

LOSCHDIMT ECHO & DYNAMICAL PHASE TRANSITIONS

Quench protocol: $\mathcal{G}(t) = \langle \psi_0 | e^{i\mathcal{H}_1 t} \rangle | \psi_0 \rangle$

L.E = probability to return to initial state, in terms of dynamical free energy density $\lambda(t)$

$$\mathcal{L}(t) = |\mathcal{G}(t)|^2 = e^{-N\lambda(t)}$$

DPTs = points of non-analyticity of $\lambda(t)$ -> isolated zeros of $\mathcal{L}(t)$

Signatures od QDPTs, such as Zeros of the L.E.,
can be identified already for *very small lattice sizes*,
whose dynamics can be simulated with high fidelity
on a realistic noisy quantum computer

NON-INTERACTING CASE $(g = 0)$

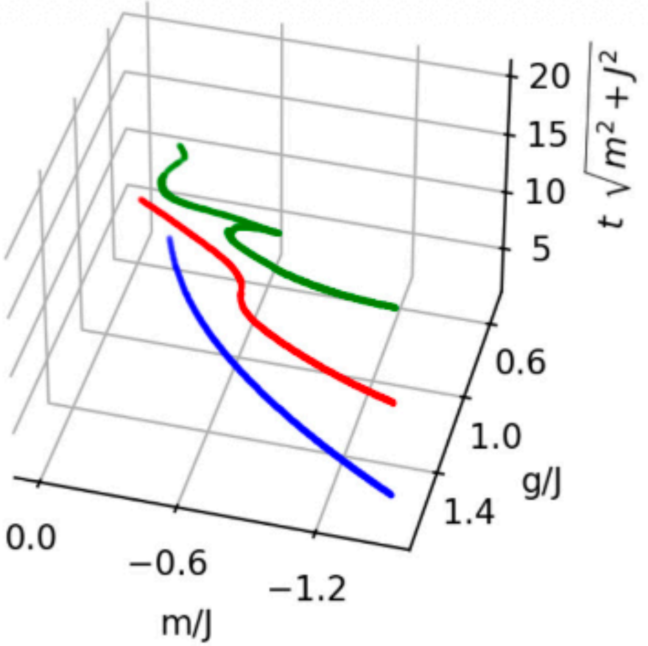
quench: $\bar{m} \to m$ - it can be solved exactly

$$\mathcal{G}(t) = \cos(\omega t) + \mathrm{i}\frac{\bar{m}m + J^2}{\bar{\omega}\omega}\sin(\omega t),$$

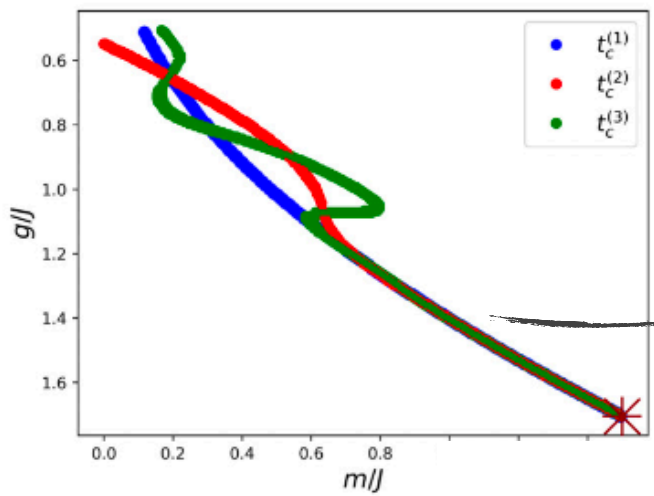
$$\omega = \sqrt{m^2 + J^2}$$

$$\mathscr{L}(t_c) \sim 0 \Leftrightarrow \bar{m}m + J^2 = 0\ , t_c\omega \sim 1$$

INTERACTING – CASE 1



long-time dynamics



periodic Rabi oscillations:
the initial Dirac vacuum evolving only in the subspace spanned by itself and the mesonic ground state

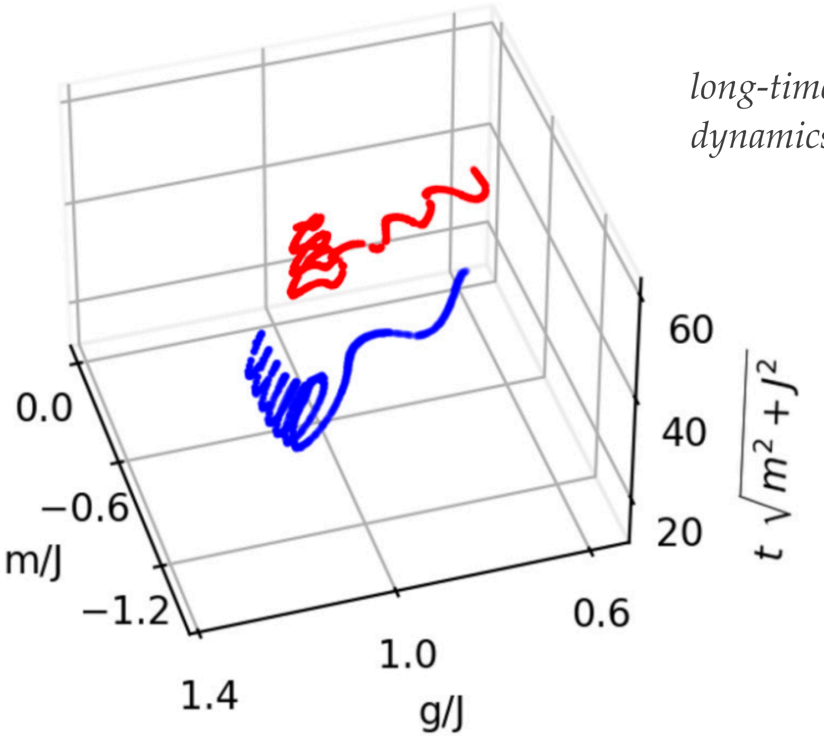
$$loci\ for\ \mathscr{L} = 0$$

$$|\psi_0\rangle = |vac\rangle_0$$

$$\mathcal{H}^+ \equiv \frac{g^2}{J}\tilde{\mathcal{H}}^+ = \begin{pmatrix} 0 & \frac{J}{\sqrt{2}} & & \\ \frac{J}{\sqrt{2}} & 0 & \frac{J}{2} & \\ & \frac{J}{2} & 2\Delta & \frac{J}{\sqrt{2}} \\ & & \frac{J}{\sqrt{2}} & \Delta \end{pmatrix} - \boldsymbol{m}\mathbf{1},$$

$$\Delta = -2m$$

INTERACTING – CASE 2



long-time dynamics

periodic Rabi oscillations: additional oscillation of the initial Dirac vacuum, evolvingin the subspace spanned by itself and the fourth state

$$\mathcal{H}^{+} \equiv \frac{g^2}{J}\tilde{\mathcal{H}}^{+} = \begin{pmatrix} -m & \frac{J}{\sqrt{2}} & & \\ \frac{J}{\sqrt{2}} & 0 & \frac{J}{2} & \\ & \frac{J}{2} & -3m & \frac{J}{\sqrt{2}} \\ & & \frac{J}{\sqrt{2}} & -m \end{pmatrix} - \boldsymbol{m}\mathbf{1},$$

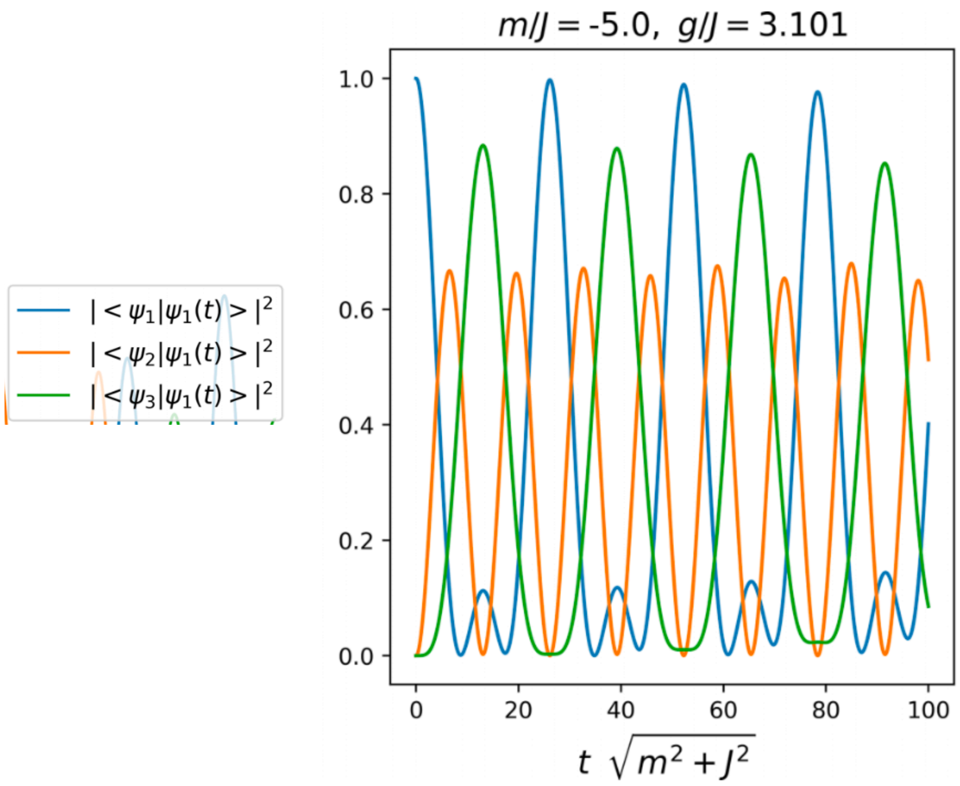
loci for $\mathscr{L} = 0$

$|\psi_0\rangle = |vac\rangle_0$

INCREASING THE SIZE

N=4 $dim\mathscr{H}^+ = 7$

CASE 1 3-states Rabi oscillations



2-states Rabi oscillations

CASE 2 between 1st and 4th

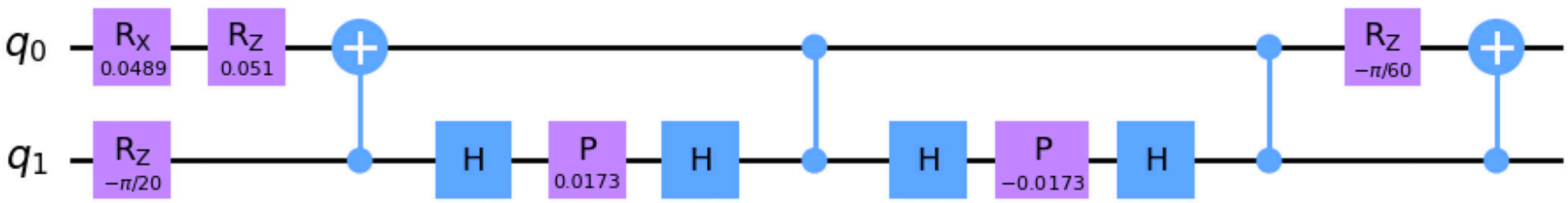
CASE 2′ between 1st and 6th

CASE 2″ between 1st and 7th

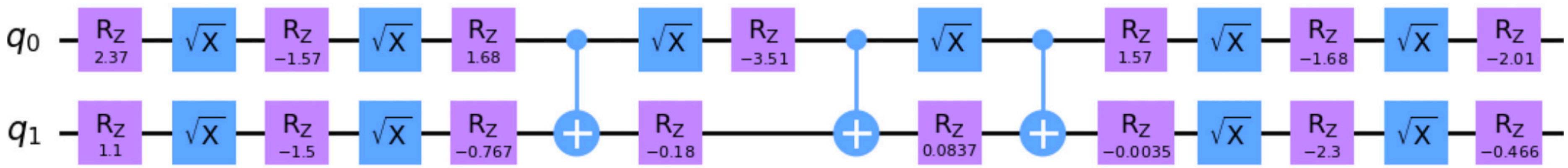
simulations on a noisy Rydberg atom device

CIRCUIT DESIGN

Trotter evolution on positive-parity sector, by writing $\mathscr{H}^+$ as composition of Pauli strings
Circuit compression with Qiskit



(a) ONE TROTTER STEP

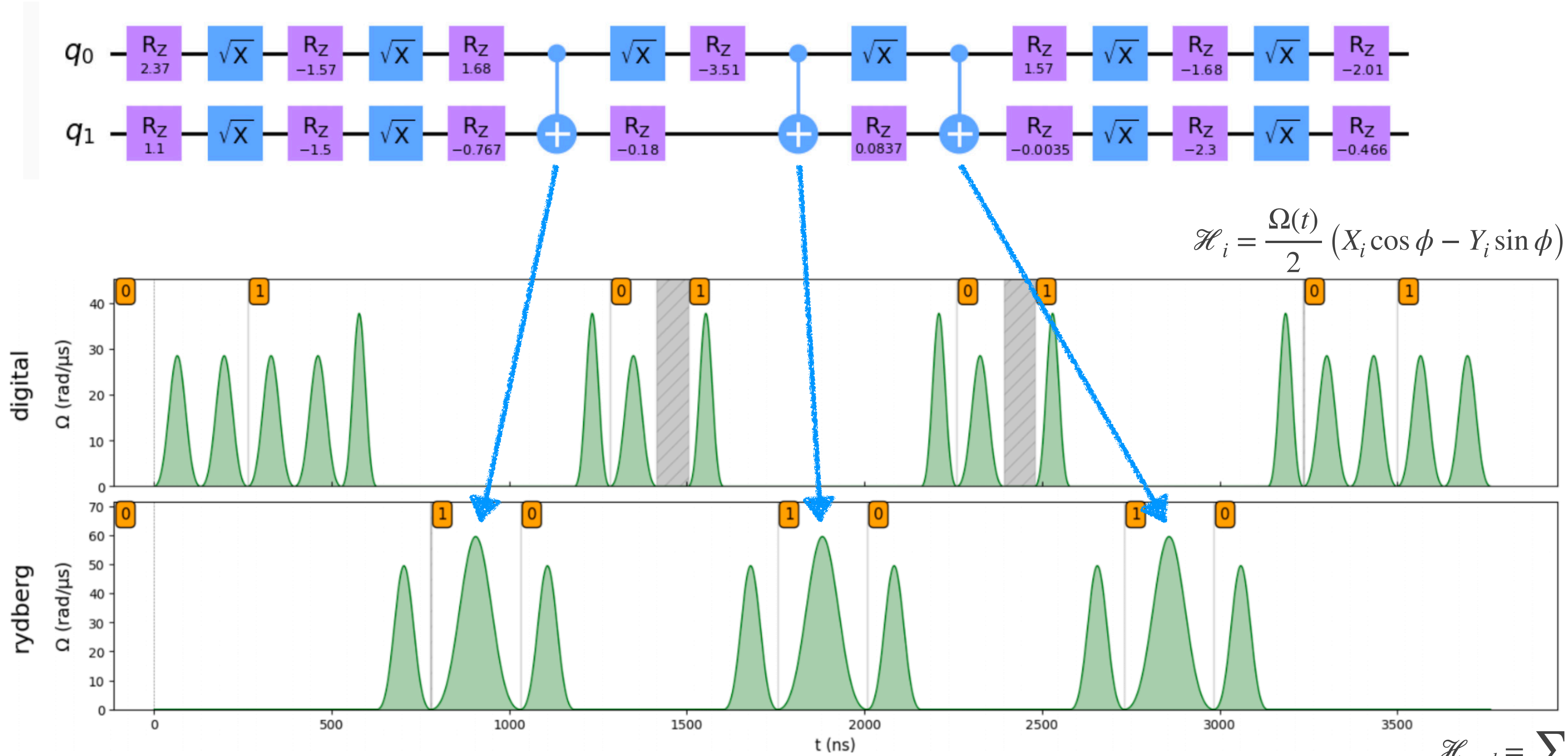


(b) ARBITRARY TROTTER STEPS (stable structure, different parameters)

G. Vidal and C. M. Dawson, Universal quantum circuit for two-qubit transformations with three controlled-not gates, Phys. Rev. A **69**, 010301(R) (2004)

PULSE SEQUENCE DESIGN

PASQAL's platform, PULSER emulator



$$\mathcal{H}_i = \frac{\Omega(t)}{2}\left(X_i \cos\phi - Y_i \sin\phi\right) - \frac{\delta(t)}{2} Z_i$$

$$\mathcal{H}_{ryd} = \sum_{i \neq j} \frac{C_6}{r^6} n_i n_j$$

EMULATION

PASQAL's platform, PULSER emulator

NOISE MODEL (100 realizations)

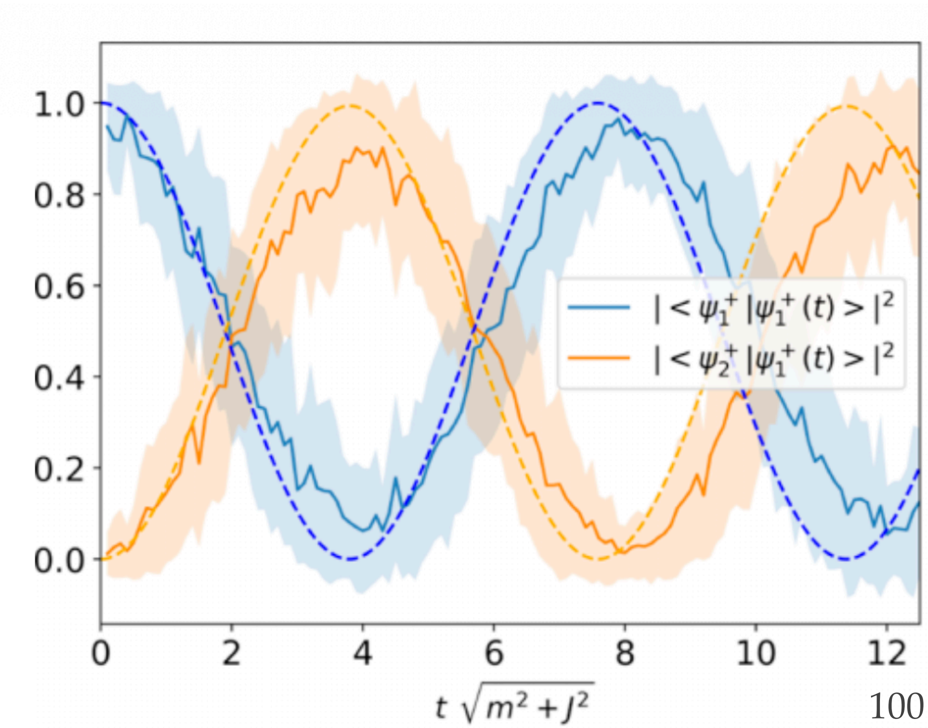
state preparation errors (with probability $\eta = 5 \times 10^{-3}$)

measurement errors (with false negative rates $\epsilon = 10^{-2}$ and false positive rates $\epsilon' = 5 \times 10^{-2}$)

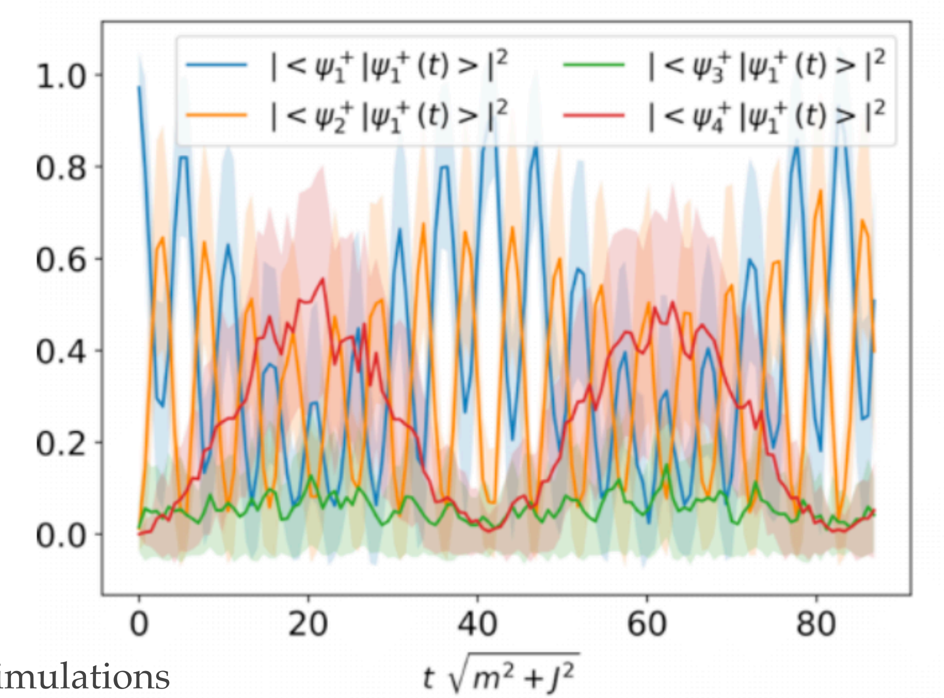
Doppler damping ($T \sim 50\ \mu K$ that induces a Doppler shift due to thermal motion)

Amplitude fluctuations (standard deviation of 0.05 μm)

CASE 1 $m/J \approx -1.49,\ g/J = 1.7$



CASE 2 $m/J \approx -0.87,\ g/J = 1.05$



100 noisy simulations

FINAL REMARKS

- Proof of concepts validation
- Encouraging preliminary results

- Extending to larger sizes
- Experiment on a real device

== THANK YOU ==