Symmetry-protected topology and deconfined solitons in a multi-link $\mathbb{Z}_2$ gauge theory

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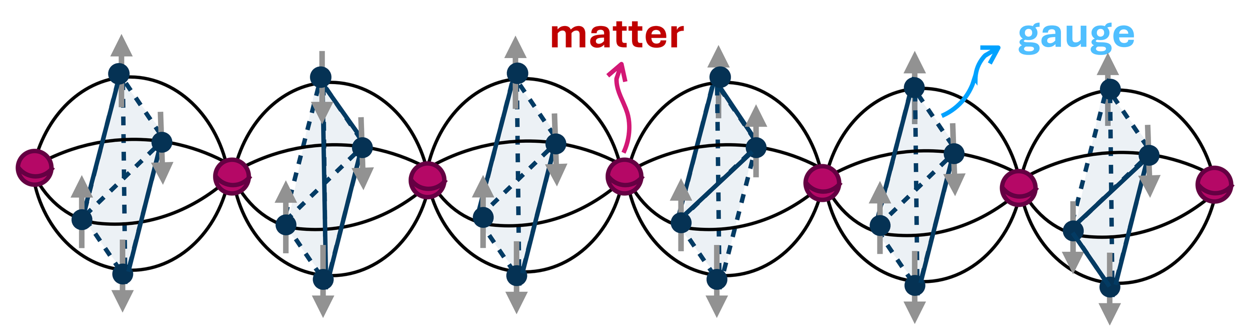
Summary

- Multi-link modification of the standard $\mathbb{Z}_2$ gauge theory
- Elementary plaquettes formed by pair of links enclose a dynamical gauge-magnetic flux
- Configurations of the gauge-invariant flux give rise to state-dependent tunneling amplitudes
- This drives a Peierls-like instability leading to the spontaneous breaking (SSB) of translational invariance
- Coexistence of SSB and symmetry protected topology
- Emergence of deconfined, fractionally-charged quasi-particles in the groundstate, upon doping above half-filling

1. The model



Electric term H_e

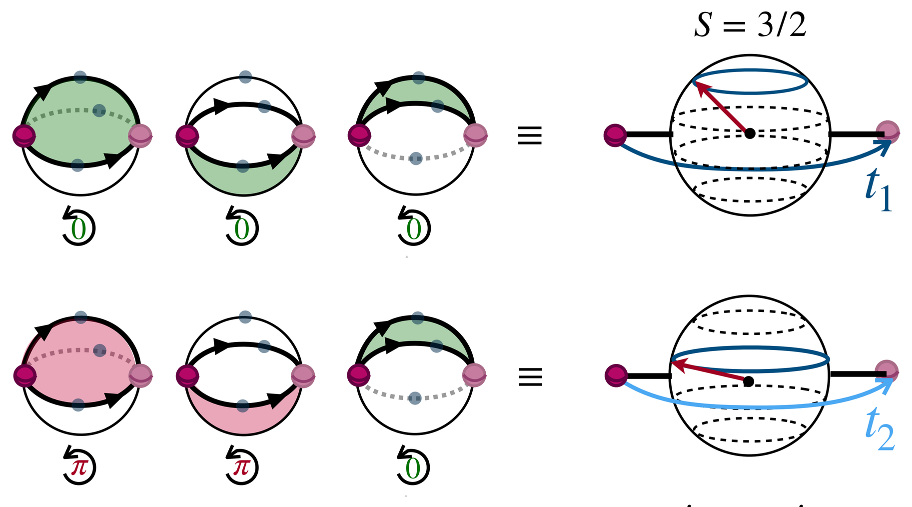
$$H = \underbrace{\frac{t}{2}\sum_{i,a}(\psi_i^\dagger\psi_{i+1} + \mathrm{H.c.})\,\sigma^z_{i_\ell,a}}_{\text{Gauge-matter interaction}} + \underbrace{\frac{J}{2}\sum_{i,a}\sum_{b<a}\sigma^z_{i_\ell,a}\sigma^z_{i_\ell,b}}_{\text{Magnetic term } H_m} + \frac{h}{2}\sum_{i,a}\sigma^x_{i_\ell,a}$$

- **Three** links between pairs of neighboring sites
- One $\mathbb{Z}_2$ gauge field per link
- Gauss's law: $G_i|\Psi\rangle = |\Psi\rangle$

$$G_i = \prod_b \sigma^x_{i_\ell-1,b}\,(-1)^{\psi_i^\dagger\psi_i}\prod_b \sigma^x_{i_\ell,b}$$

2. Magnetic term: Aharonov-Bohm interference

State-dependent tunneling amplitudes



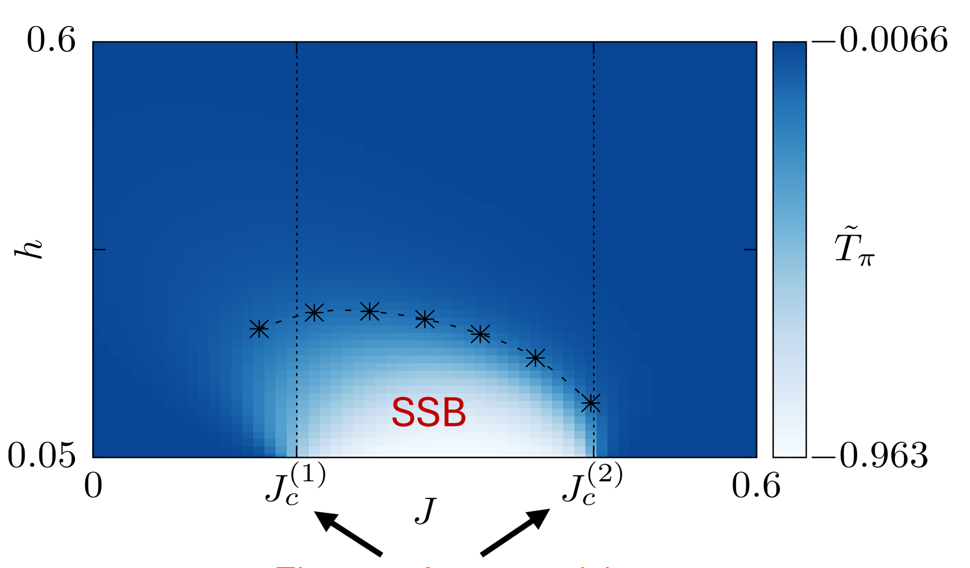
HALF-FILLING ANALYSIS

3. Spontaneous Symmetry Breaking (SSB) of translational invariance: inhomogeneous flux ordering

Order parameter: staggered flux

$$\tilde{T}_\pi = \frac{1}{L}\sum_j e^{i\pi j}\,T_{j_\ell} = \frac{1}{L}\sum_j (-1)^j\,T_{j_\ell}$$

Local flux operator



4. SSH model at $h = 0$

$$H_{SSH} = \sum_i (J_1\psi^\dagger_{i,A}\psi_{i,B} + J_2\psi^\dagger_{i,B}\psi_{i+1,A} + \mathrm{H.c.})$$

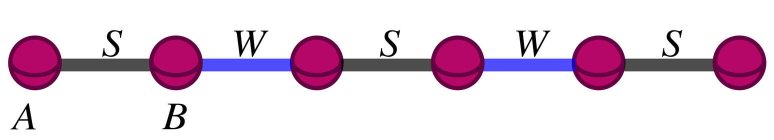
- BDI-class topological insulator: SPT protected by chiral symmetry
- Two distinct orderings distinguished by bulk topological invariant

$$\gamma = \int_{\mathrm{BZ}}\frac{\mathrm{d}k}{2\pi}\langle u_k|\partial_k|u_k\rangle$$

Zak phase

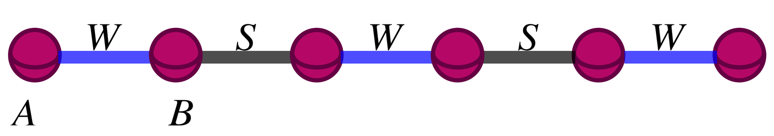
Trivial ordering: strong-weak (S-W) bond alternation

$J_1 > J_2,\ \gamma = 0$



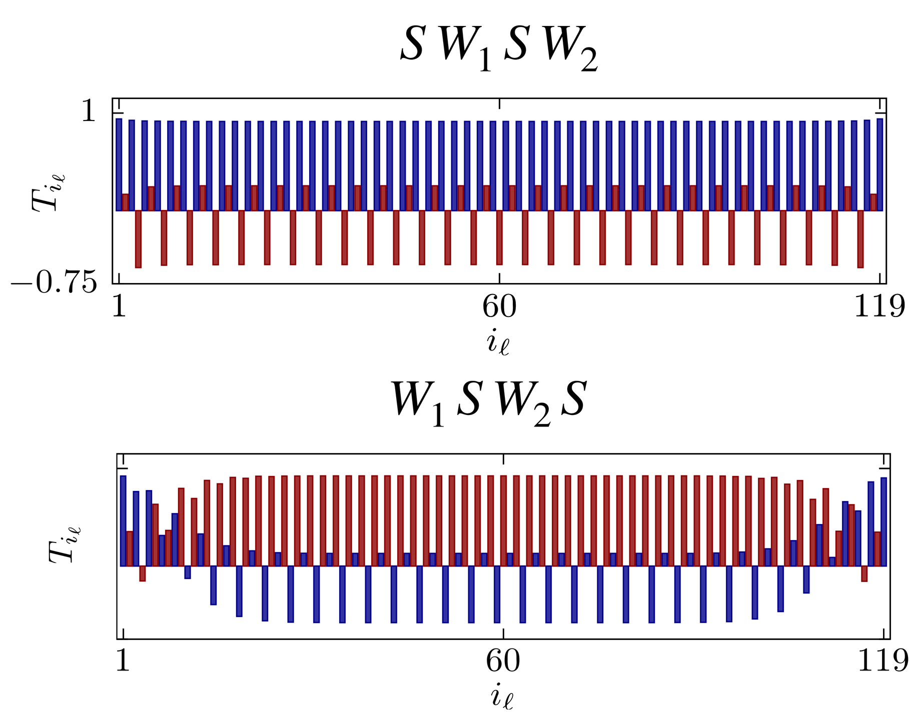
Topological ordering: weak-strong (W-S) bond alternation

$J_1 < J_2,\ \gamma = \pi$



5. SPT at $h \neq 0$

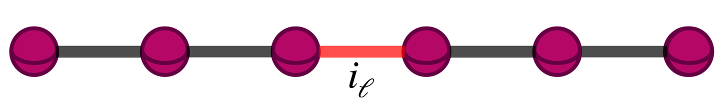
- Flux configuration in two degenerate ground states
- Odd links in blue, even in red
- Different bulk orderings



6. Local Berry Phase

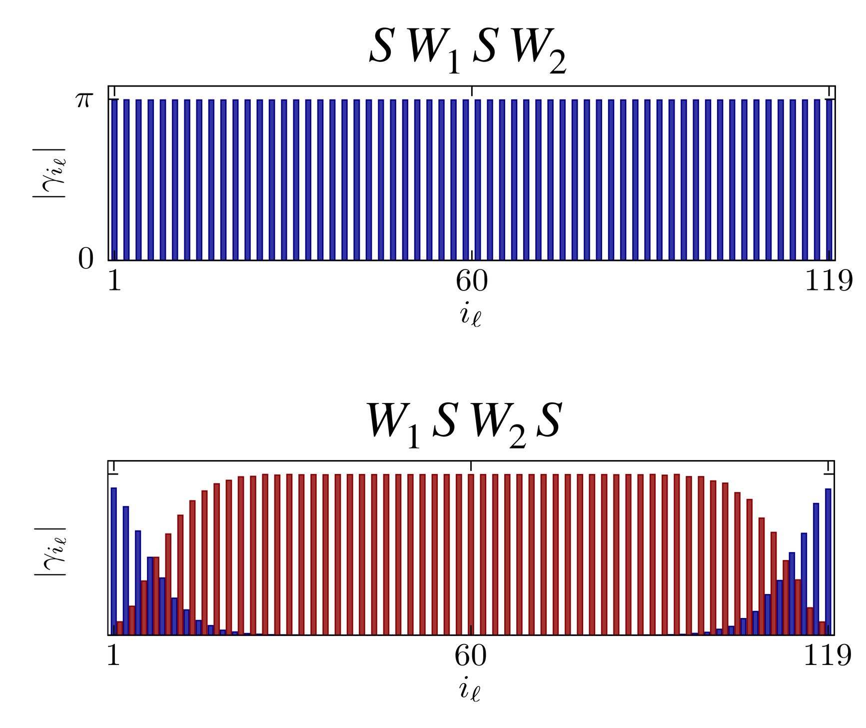
Local twist of the tunneling amplitude on a single bond

$t \to t\,e^{i\theta}$



$$\gamma_{i_\ell} = -\,i\oint_{\mathscr{C}\equiv[0:2\pi]}\langle\psi(\theta)|\partial_\theta|\psi(\theta)\rangle$$

- Quantized in the presence of symmetries of the system
- Topological invariant
- Different patterns → topologically distinct ground states



7. Doping above half-filling

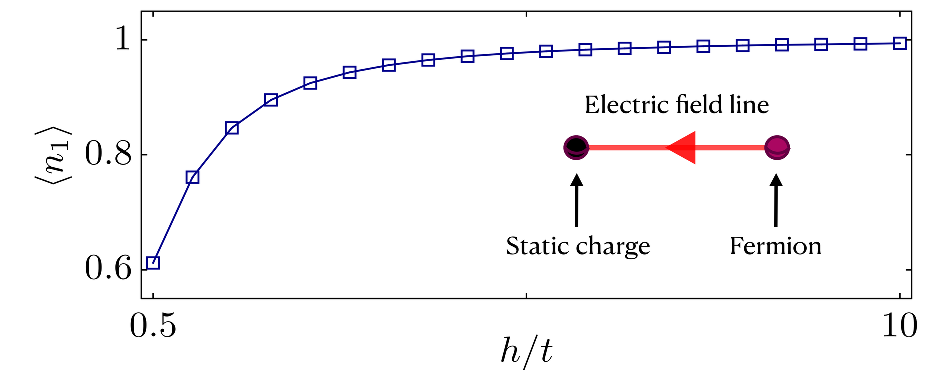
- Parity constraint in the neutral gauge sector

$$G_i = 1\,\forall i \implies P_f = 1$$

- We add a static charge at the leftmost side

$$G_1 = -1$$

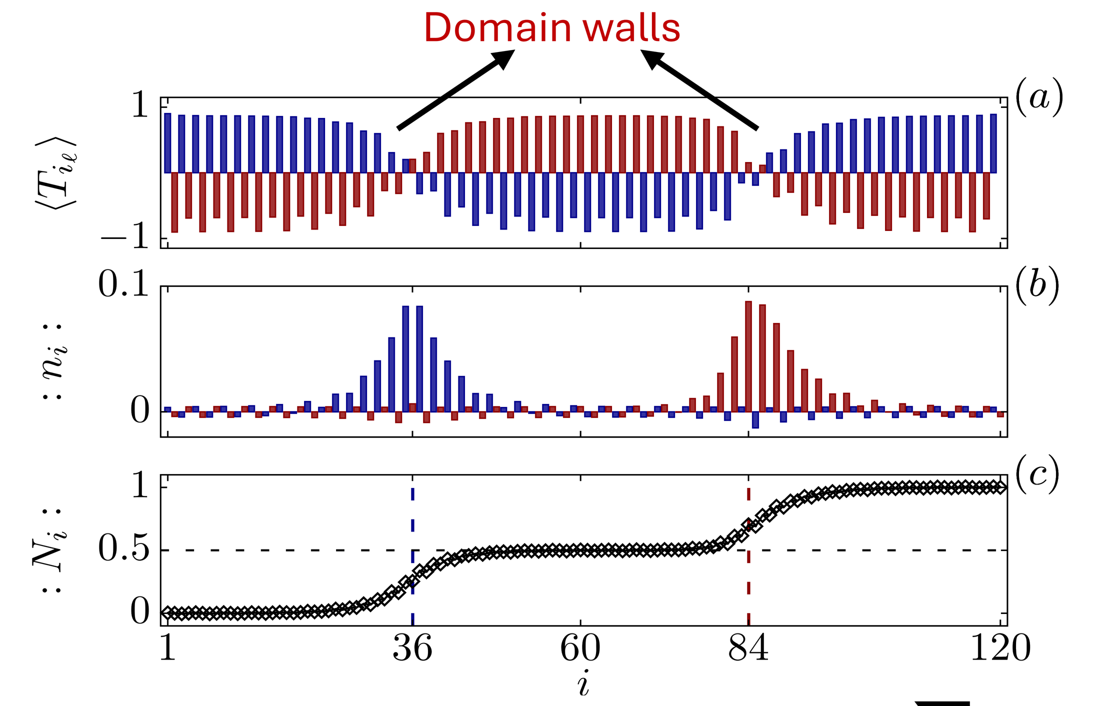
Outside of the SSB region



Confinement: one particle is bound to the static charge by an electric field line.

In the SSB region

Spontaneous generation of soliton/anti-soliton pairs interpolating between different bulk-orderings of the gauge-magnetic flux.



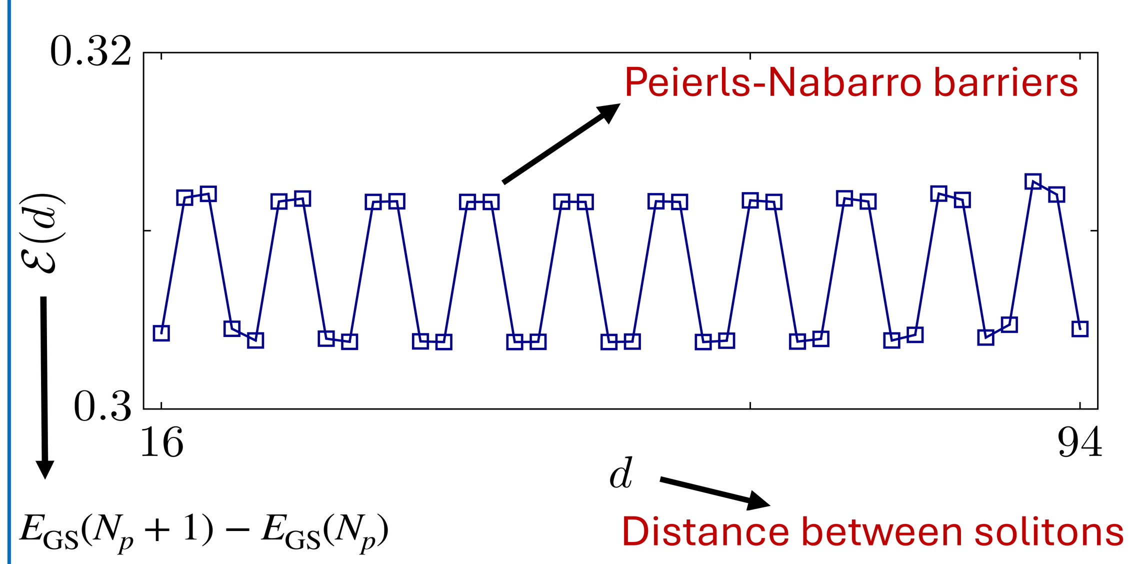
$$:n_i: = \langle\psi_i^\dagger\psi_i\rangle - \nu \qquad :N_i: = \sum_{j\leq i} :n_i:$$

- Solitons have support on different sublattices
- Charge-fractionalization: fractional charges localized at the soliton centers

8. Soliton deconfinement

Solitons are pinned at a distance d

$$H \to H + \epsilon V_{\mathrm{pin}}\,,\quad \epsilon \ll 1$$



- Periodic energy barrier: due to the SSB of translational invariance
- No linearly growing attractive potential between the pinned soliton centers: fractional quasi-particles are **deconfined**

References

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