Scattering in Staggered Massive Schwinger Model on a Circle

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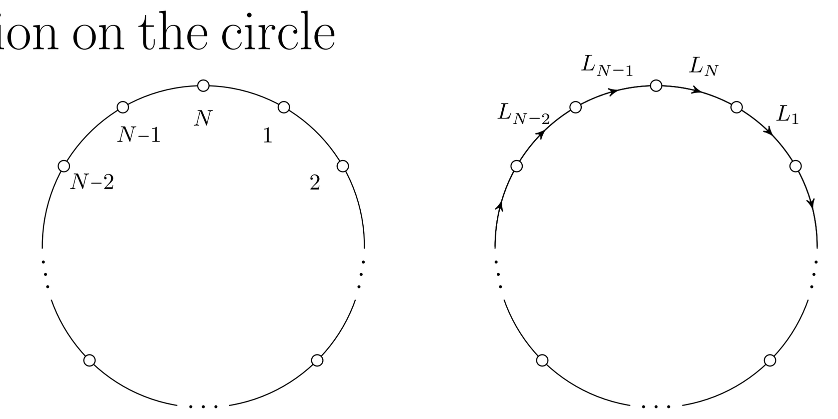
1. Introduction

Following Banks, Susskind and Kogut [1], we consider the Schwinger model via staggered fermion on the circle



with the rescaled Hamiltonian

$$\frac{2}{g^2 dx} H = N\bar{L}^2 + \mu \sum_{n=1}^{N} (-)^n Q_n + \sum_{n,m=1}^{N} G(n,m) Q_n Q_m + x \sum_{n=1}^{N} \left(\sigma_n^+ \bar{U} \sigma_{n+1}^- + h.c. \right)$$

and constraints —vanishing total fermion charge and large gauge transformation,

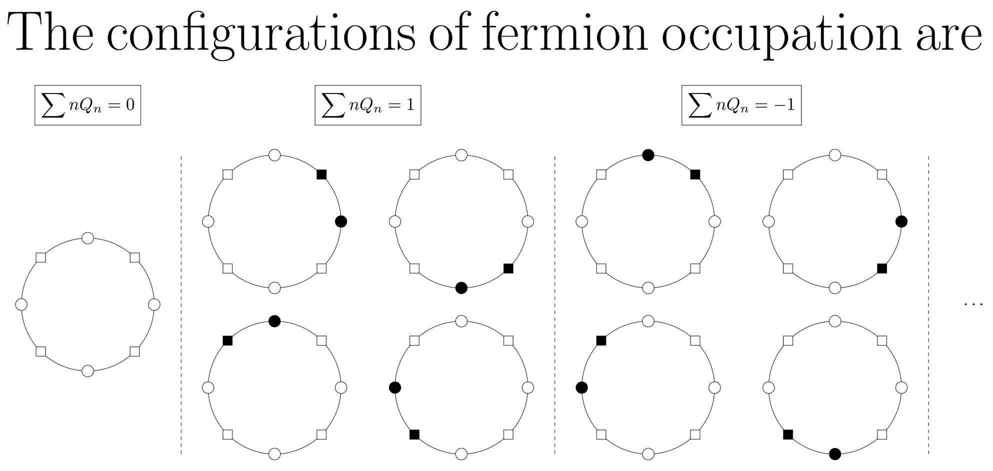
$$\sum_n Q_n |s\rangle = 0, \; \exp\left(-i\frac{2\pi}{N} \sum_{n=1}^{N} (N\bar{L} + nQ_n) \right) |s\rangle = |s\rangle$$

where Q_n is the fermion charge on n-th site and $\bar{L}$ is the averaged electric field. The Coulomb term is reminiscent of the Green function on the discrete circle

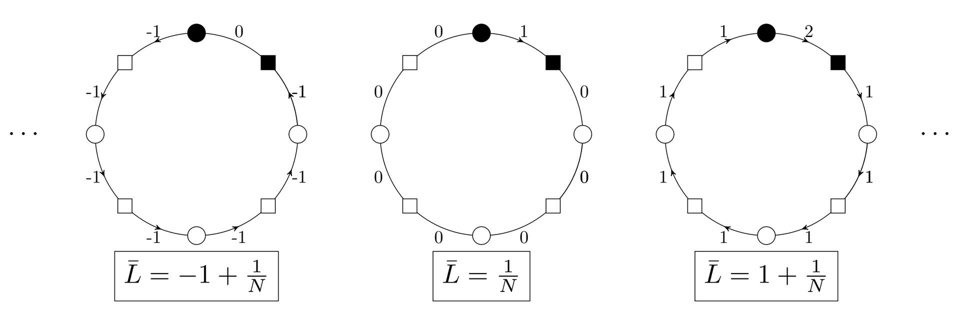
$$G(n-m) = \frac{(n-m)^2}{2N} - \frac{|n-m|}{2} + \frac{N^2-1}{12N}$$

2. Properties of model

◇ The configurations of fermion occupation are

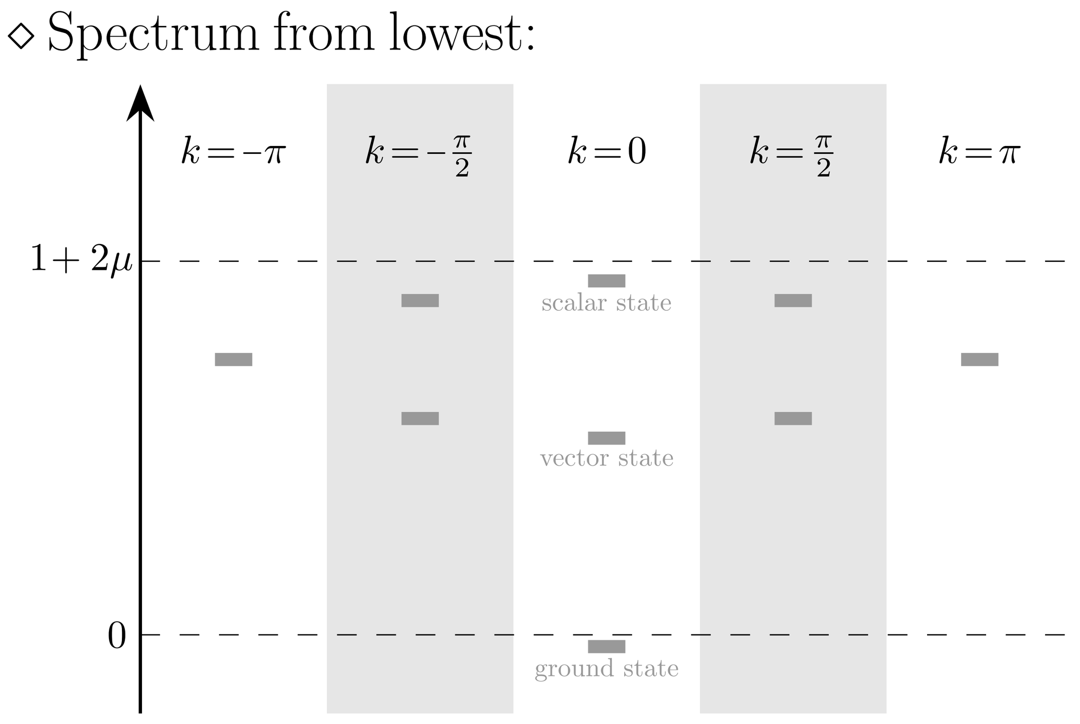


$N = 8$ for example. For each occupation, the large gauge transformation constraint gets the gauge field almost fixed, for instance,



To cover all fermion configurations, the range of the truncated gauge representation should not be smaller than N, i.e., with at least $\log_2(N)$ qubits.

◇ Spectrum from lowest:



In the free theory ($x = 0$),

$$\text{vector state}: \quad |k^V\rangle \equiv |k^F\rangle - e^{-i\frac{k}{2}} |k^B\rangle$$

$$\text{scalar state}: \quad |k^S\rangle \equiv |k^F\rangle + e^{-i\frac{k}{2}} |k^B\rangle$$

where $|k^{F/B}\rangle$ are the forward/backward plane waves whose existence is due to the periodic boundary condition.

3. Quantum Circuit

◇ Initialization: Translation invariant W-state [2]

$$|W\rangle = \sum_k c_k |k^V\rangle = \sum_{n^F} c_n^F |n^F\rangle + \sum_{n^B} c_n^B |n^B\rangle$$

with $c_k = \exp(-ikx_0 - (k-k_0)^2/4\sigma_k^2)$ in practice.

◇ Time evolution:

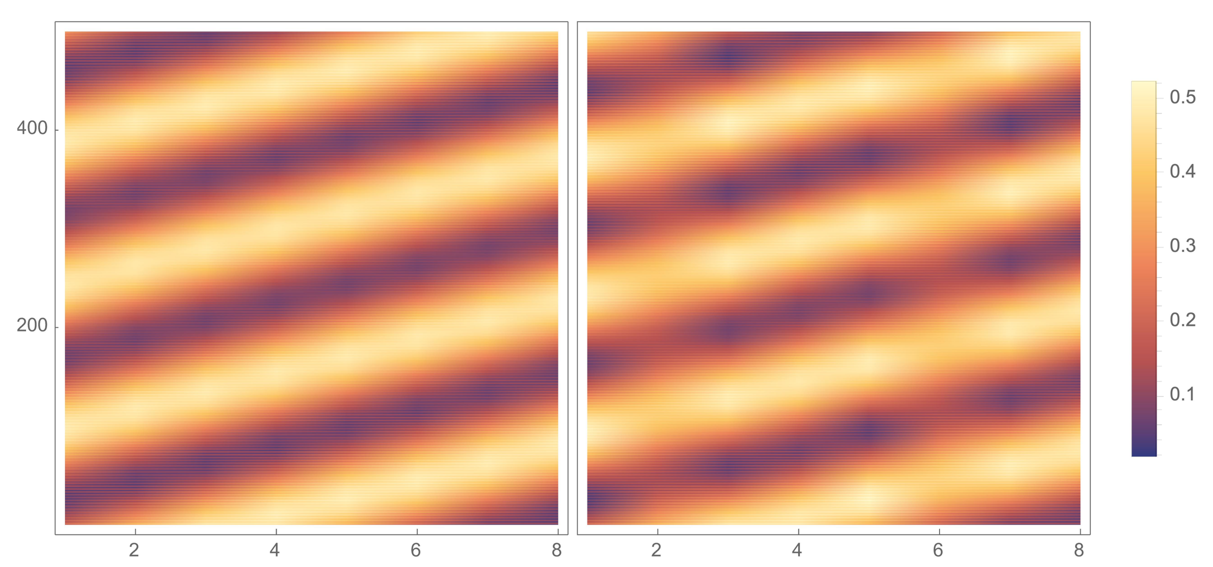
$$e^{-idtH} \approx \exp\left(-i\frac{\theta_L}{2}\bar{L}^2\right) \prod_{m,n} \exp\left(-i\frac{\theta_C}{2}\left[\frac{(n-m)^2}{N} - |n-m|\right] Q_m Q_n\right) \prod_n \exp\left(-i\frac{\theta_Q}{2}(-)^n Q_n\right) \prod_{n=even} \exp\left(-i\frac{\theta_I}{2}\left[\sigma_n^+ \bar{U} \sigma_{n+1}^- + h.c.\right]\right) \prod_{n=odd} \exp\left(-i\frac{\theta_I}{2}\left[\sigma_n^+ \bar{U} \sigma_{n+1}^- + h.c.\right]\right)$$

All terms on the right hand side can be implemented precisely, and so the gauge invariance will be conserved [3].

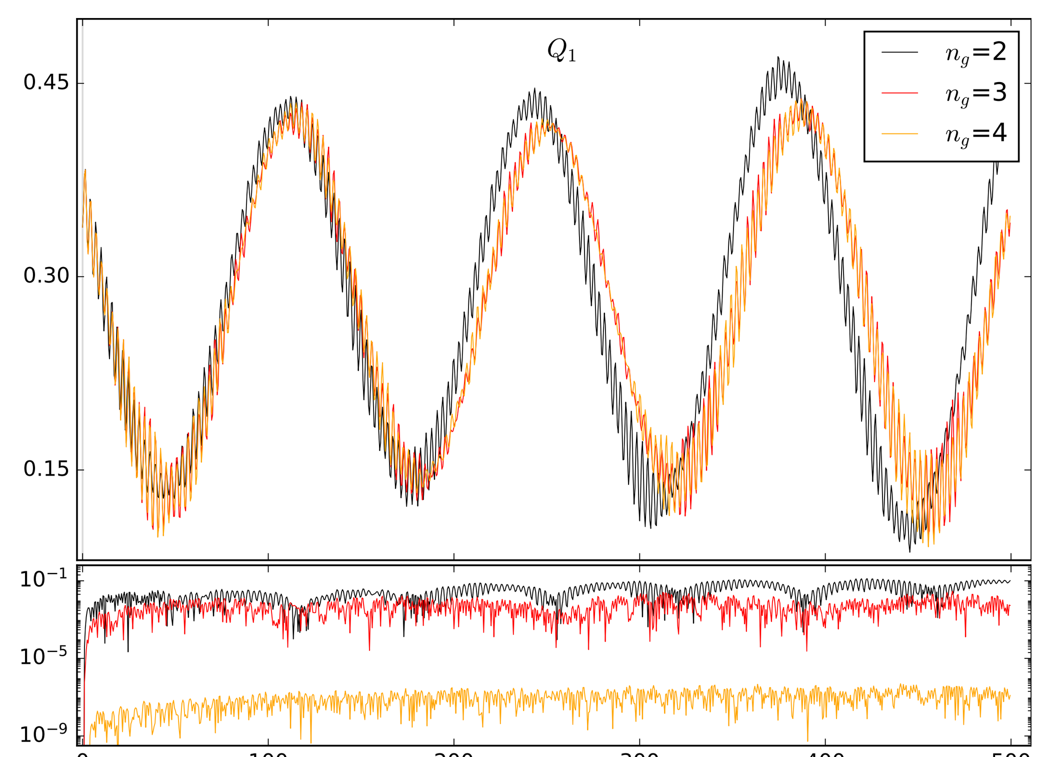
◇ Quantum Error Mitigation: With Dynamical Decoupling and Pauli Twirling in practice, the Operator Decoherence Renormalization [4] can be used to calibrate the measurement.

4. Simulation results

◇ Abs[Q]: Exact Diagonalization vs $dt = 0.5$:

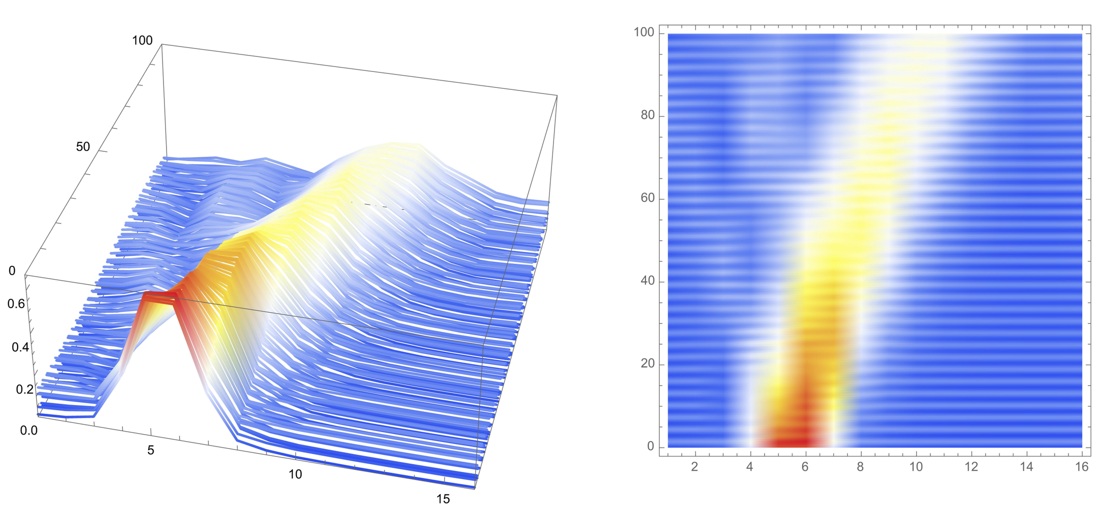


◇ As n_g, the qubit number of the gauge field, increases, a convergence can be spotted.

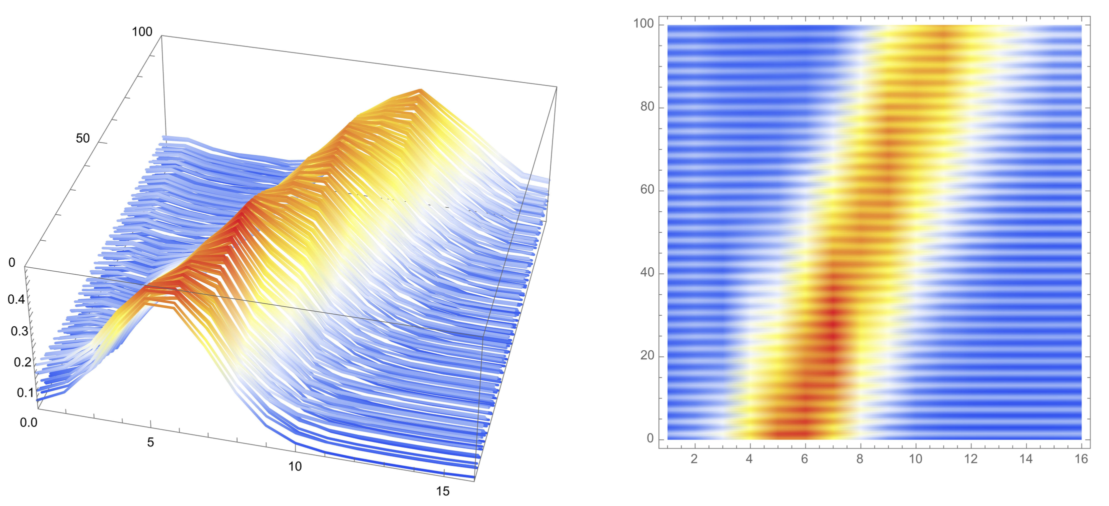


where the lower panel shows the values subtracted by Q of $n_g = 5$.

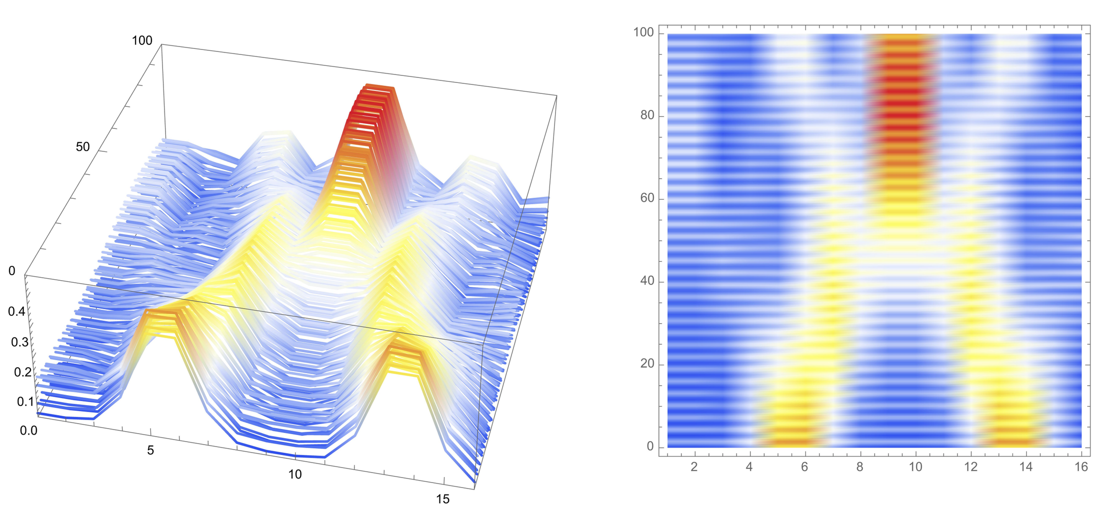
◇ The velocity of the wavepacket is proportional to x^2, and the spread of the wavepacket is determined by σ_k. The propagation of a narrow wavepacket in Abs[Q]:



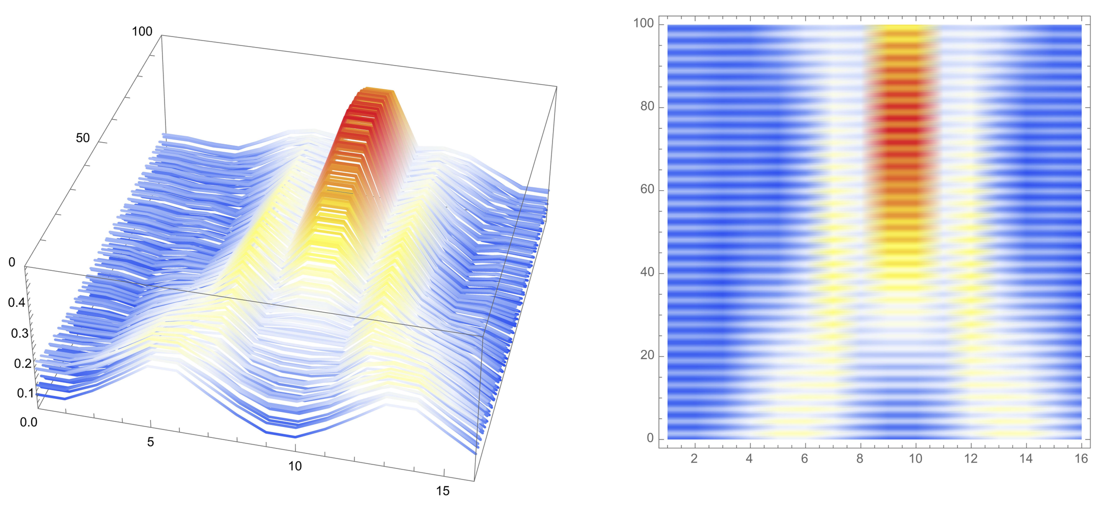
The propagation of a wide wavepacket:



◇ A scattering can be arranged when two separated wavepackets travel towards each other. Scattering of two narrow wavepackets:

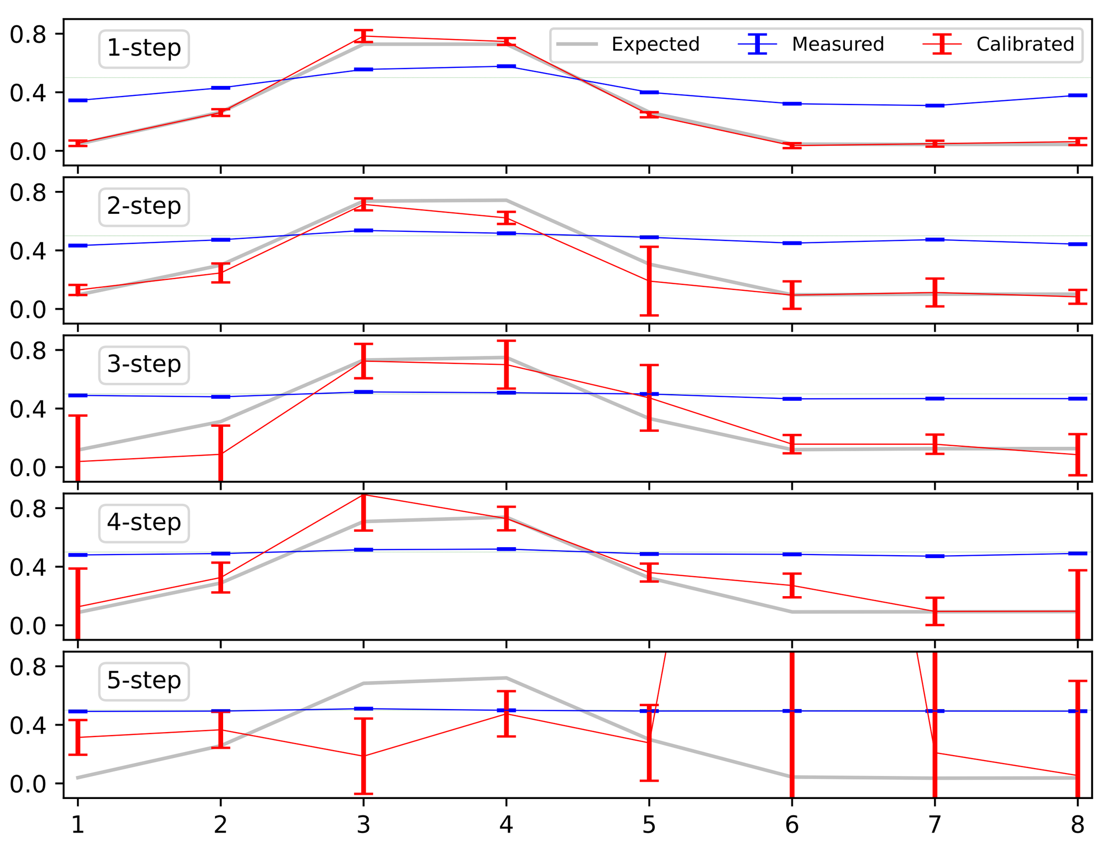


Scattering of two wide wavepackets:



5. Quantum Hardware

◇ For $N = 8$ up to 5 Trotter steps on *ibm_boston*



With Quantum Error Mitigation technique, the calibrated results can match the exact results up to 4 steps.

[1] T. Banks, L. Susskind, and J. B. Kogut, "Strong Coupling Calculations of Lattice Gauge Theories:(1+1)-Dimensional Exercises," Phys. Rev. D 13 (1976) 1043.

[2] R. C. Farrell, N. A. Zemlevskiy, M. Illa, and J. Preskill, "Digital quantum simulations of scattering in quantum field theories using W states," arXiv:2505.03111 [quant-ph].

[3] Z.-G. Mou and B. Chakraborty, "Scalable quantum computation of Quantum Electrodynamics beyond one spatial dimension," arXiv:2510.27668 [hep-lat].

[4] R. C. Farrell, M. Illa, A. N. Ciavarella, and M. J. Savage, "Scalable Circuits for Preparing Ground States on Digital Quantum Computers: The Schwinger Model Vacuum on 100 Qubits," PRX Quantum 5 no. 2, (2024) 020315, arXiv:2308.04481 [quant-ph].

Paper will be on arXiv soon!