# No.28 A Quantum Algorithm for the Quantum Mpemba Effect and Its Application to the Schwinger Model

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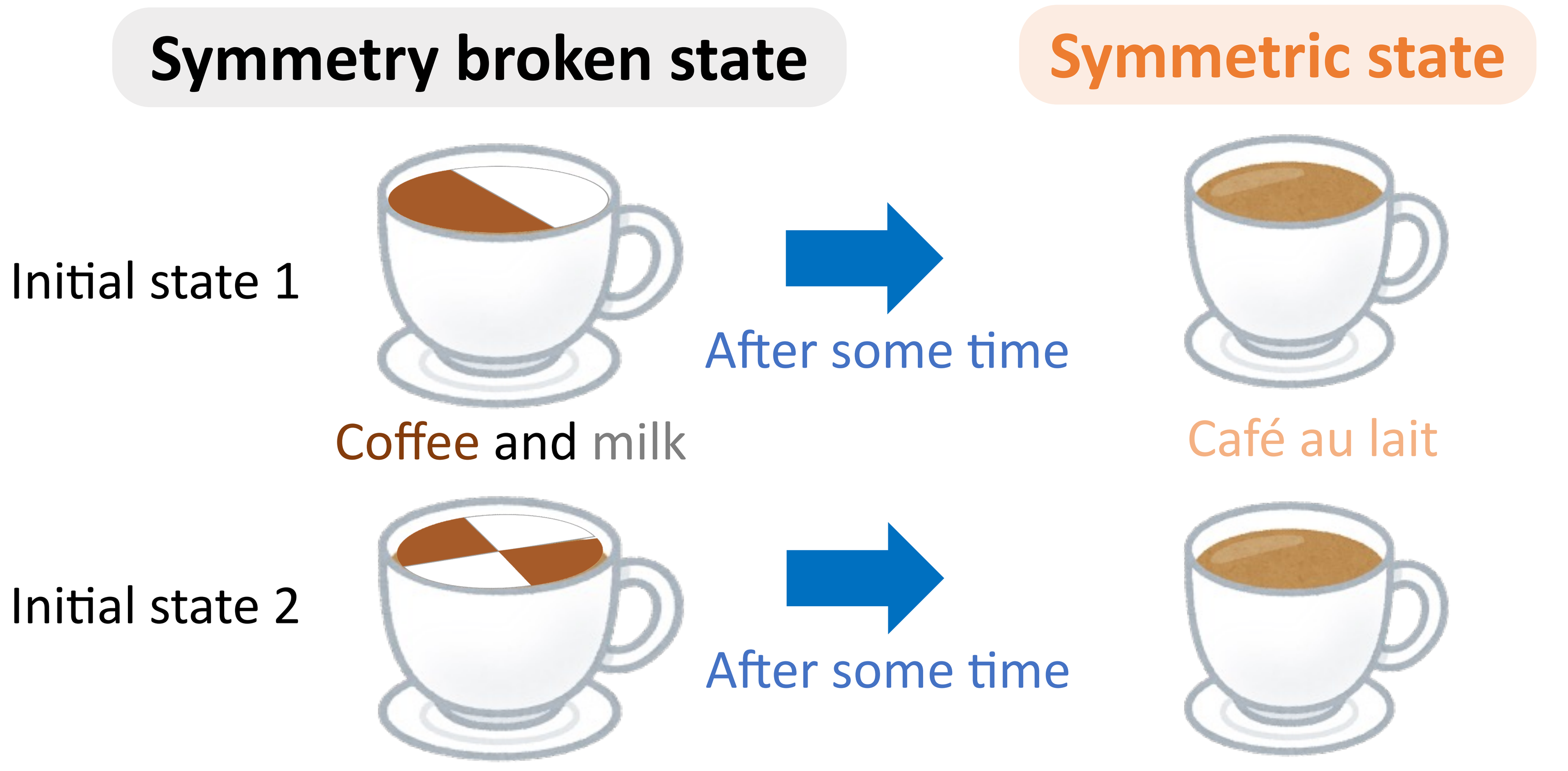
## 1. Introduction

### Quantum Mpemba Effect

[Mpemba-Osborn, 1969] [Ares et al, 2022]




Intuitively, one expects the initial state 2 relax faster than the initial state 1.
However, the opposite behavior is observed under certain conditions!

**Quantum Mpemba Effect (QME)**

**The more symmetry is initially broken, the faster it is restored.**

An anomalous out-of-equilibrium relaxation.

### Background

The QME has attracted increasing attention across diverse fields, including condensed matter physics, high-energy physics, and quantum information science.

**Challenges**

- Analytical studies are limited to integrable models and conformal field theories.
- The origin of the QME remains elusive.
- Classical simulations suffer from exponential scaling with system size.
- Quantum field theories remain largely inaccessible.

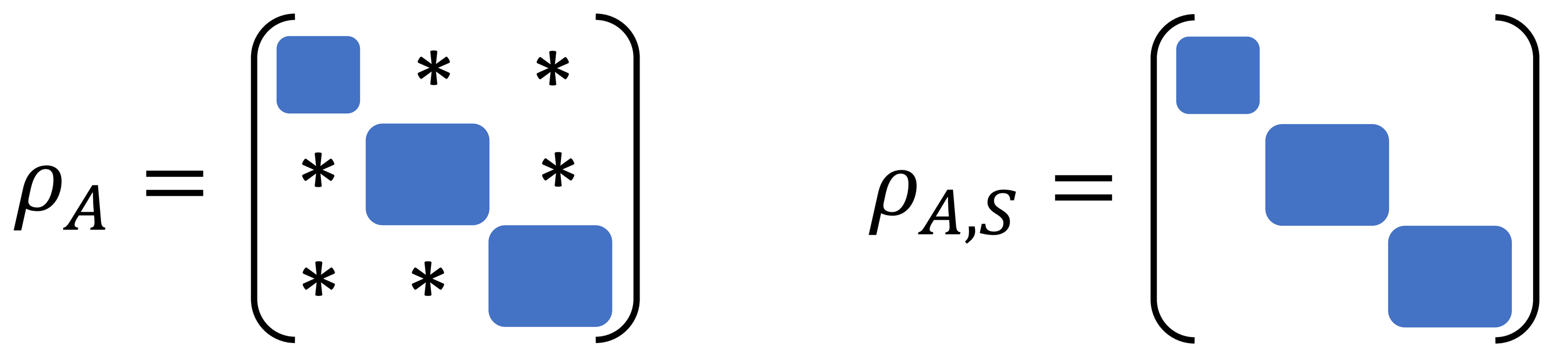
**A scalable approach is needed to investigate the QME in large quantum systems!**

### Our work

- We develop a quantum algorithm for investigating the QME.
- Our approach avoids exponential computational cost and is therefore scalable to large systems.
- As a first application, we study the Schwinger model (1+1D QED).
- Our results highlight the potential of quantum simulation for exploring the QME in quantum field theories.

## 2. Entanglement Asymmetry

Let $A$ be a subsystem of interest and consider the reduced density matrix in the eigenbasis of the symmetry charge.




Off-diagonal = symmetry breaking — Diagonal = symmetric state

### Rényi Entanglement Asymmetry (REA)

$$\Delta S_A^{(n)} \equiv \frac{1}{1-n}\left(\log \mathrm{Tr}[\rho_{A,S}^n] - \log \mathrm{Tr}[\rho_A^n]\right)$$

○ REA provides a quantitative measure of symmetry breaking and serves as a probe of the QME

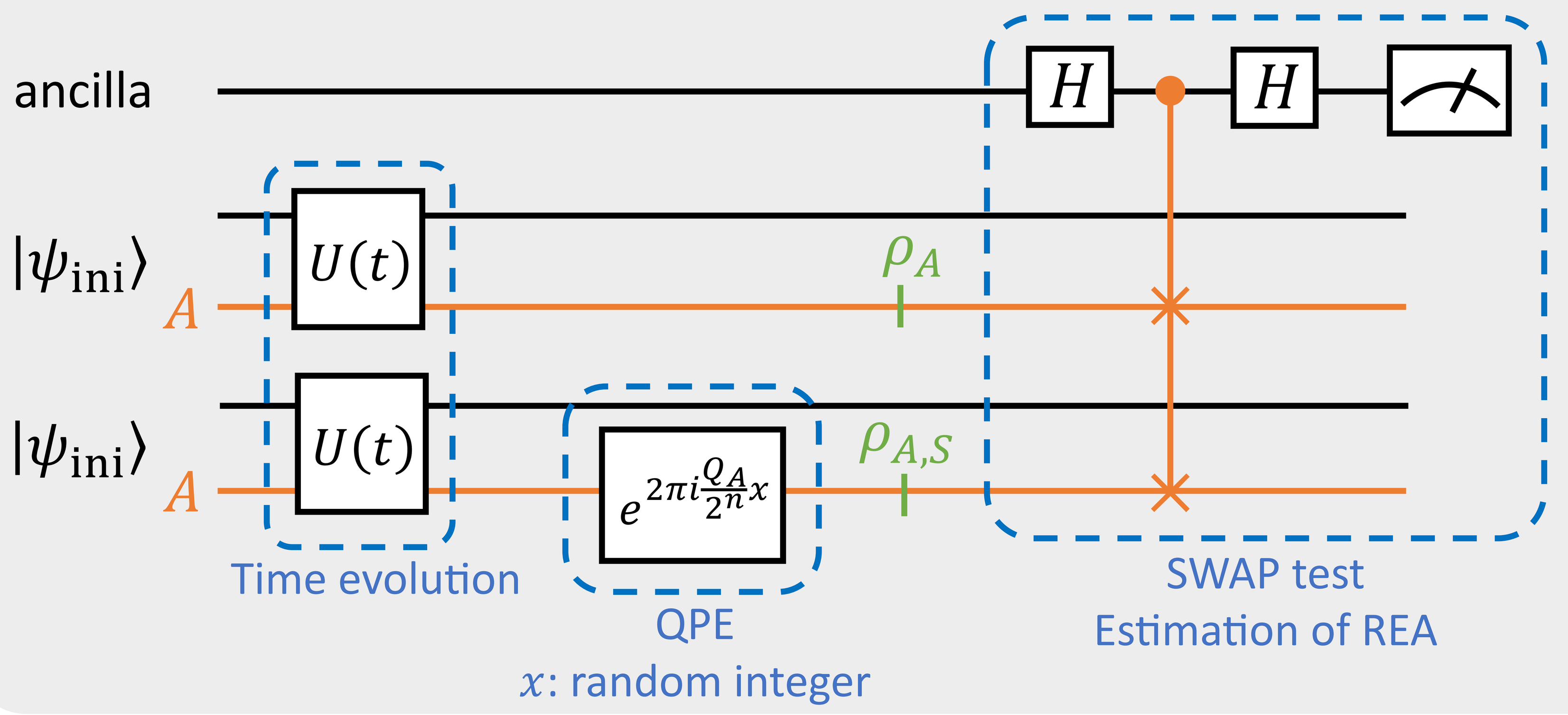
▲ Naïve classical simulation is exponentially expensive: $\mathcal{O}(2^{3N_A})$, where $N_A$ is the system size

## 3. Quantum Algorithm

### Our idea

- **Quantum phase estimation (QPE)** → Symmetry projection
- **SWAP test** → REA estimation
- Combining QPE and SWAP test, we can efficiently investigate the QME.

### Our quantum circuit $(n = 2)$




### Key Features

- Measurement cost is independent of the system size.

  $N_{\rm shot} = \mathcal{O}(\delta^{-2})$, where $\delta$ is statistical error of SWAP test.
- Gate complexity scales polynomially with the system size.

  (Gate complexity) $\leq \mathcal{O}(N^{\#})$, where $N$ is the system size

No exponential computational cost!

**Toward scalable investigations of the QME in quantum field theories!**

## 4. Application

### Schwinger model (1+1D quantum electrodynamics)

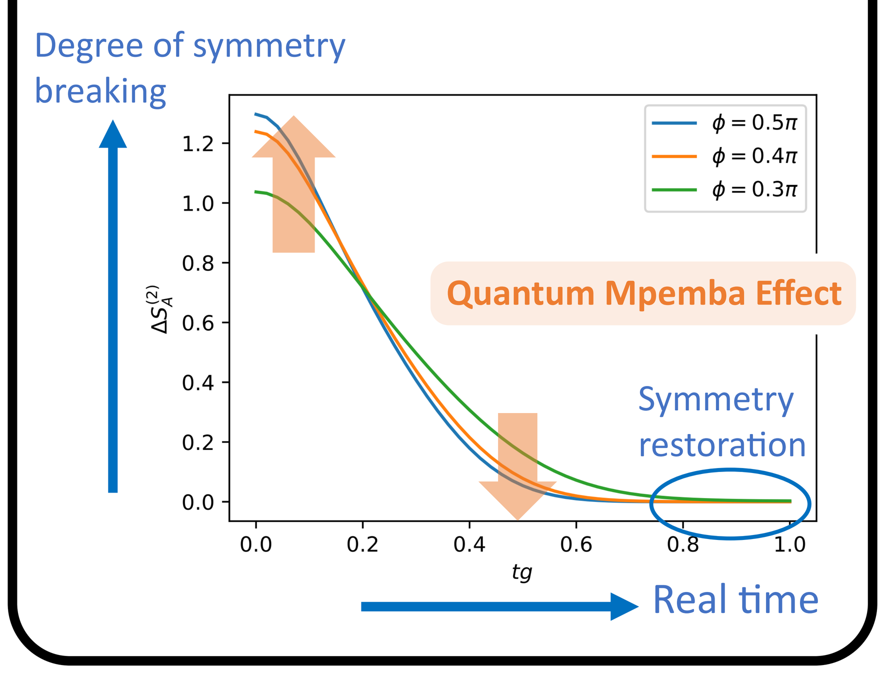
After lattice discretization, this model is described by the following Hamiltonian.

$$H = H_{ZZ} + H_{\pm} + H_Z \qquad X_n, Y_n, Z_n\text{: Pauli operators.}$$

$$H_{ZZ} \sim \sum_n \sum_{1\leq k<\ell\leq n} Z_k Z_\ell\ , \quad H_{\pm} \sim \sum_n (X_n X_{n+1} + Y_n Y_{n+1})\ , \quad H_Z \sim \sum_n Z_n$$

**Spin-product state (Benchmark)**

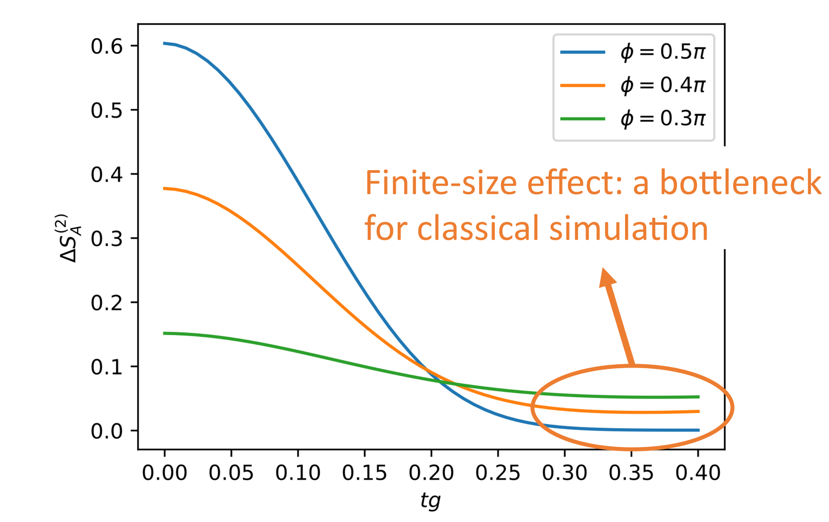
Initial state: $|\psi_{\rm ini}\rangle = e^{i\frac{\phi}{2}\sum_n Y_n}|01\cdots01\rangle$




**Fermion-parity-preserving state (Physical state)**

Initial state: $|\psi_{\rm ini}\rangle = e^{i\frac{\phi}{2}\sum_n Y_n Y_{n+1}}|01\cdots01\rangle$

➡Well defined in fermionic theories. [Nicetu Tibau Vidal et al, 2021]




Parameters: $N = 18, N_A = 4, ag = 0.25, m/g = 4, \theta = \pi/3, \delta t = 0.02/g$

**We demonstrate the utility of our algorithm for investigating the QME and find a regime where quantum computers can make a difference.**

## 5. Summary

- We propose a quantum algorithm for the QME with measurement cost independent of the system size and gate complexity scaling polynomially.
- Using the Schwinger model as an example, we investigate the QME and highlight its potential as a promising target for quantum simulation.

**Future directions**

- Detailed resource estimation for quantum simulation (ongoing).
- Implementation on NISQ devices (ongoing).

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