Lattice QED$_3$ with staggered and Wilson fermions: finite density and topology

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Why QED$_3$ with fermions at finite density? Two perspectives

Particle-physics perspective. Stepping stone toward Hamiltonian QCD: retains confinement and a nonperturbative gauge sector; no sign problem at *finite density* in the Hamiltonian formulation – but needs quantum hardware to scale.

Condensed-matter perspective. QED$_3$ with N_f flavors and chemical potentials is the effective theory of U(1) Dirac spin liquids and candidate cuprate phases, and a gauge-theoretic host of *topological phases* with nontrivial Chern numbers : Chern insulators and quantum spin Hall physics.

The bridge between the two is the choice of fermion discretization and it decides which physics the simulation can see at all.

Particle-physics perspective: finite density on quantum hardware

Model: $N_f = 2$ staggered QED$_3$ with chemical potential

Kogut–Susskind Hamiltonian with two staggered flavors and a *relative* chemical potential (total charge fixed at half filling):

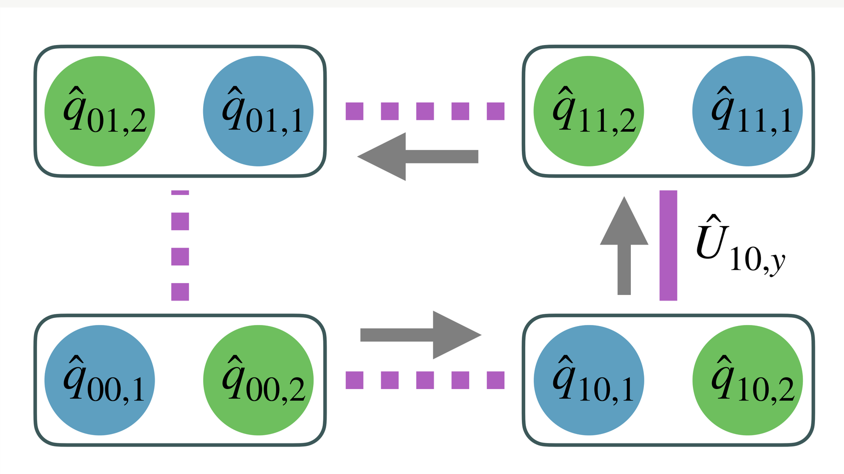
$$\hat{H} = \hat{H}_E + \hat{H}_B + \hat{H}_m + \hat{H}_{\mathrm{kin}} + \sum_{\vec{r},f} \mu_f \hat{\phi}^\dagger_{\vec{r},f}\hat{\phi}_{\vec{r},f}, \qquad \hat{G}_{\vec{r}}|\Psi_{\mathrm{phys}}\rangle = 0.$$

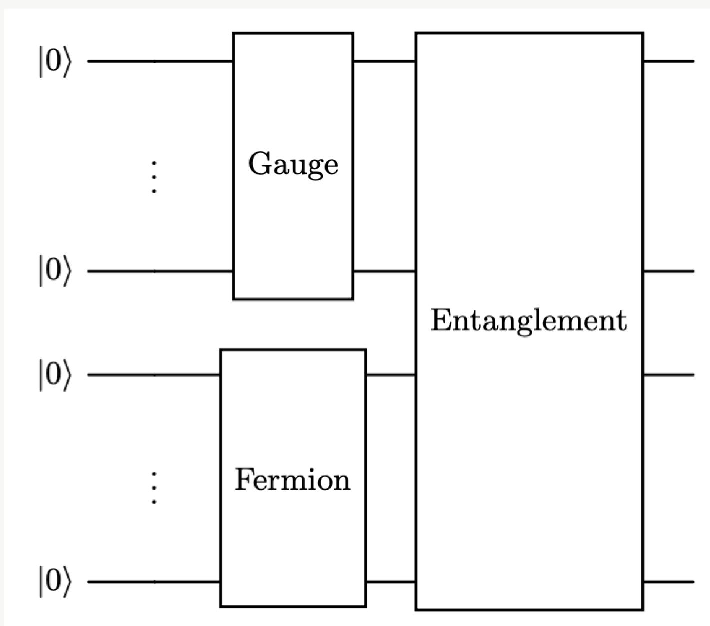
The phase diagram depends only on $\mu_1 - \mu_2$: dialing μ redistributes a *fixed* total charge between flavors, producing density-induced level crossings at

$$\mu_{\mathrm{LC}} = 2\,\frac{E_{(N_1,N_2)}(\mu^*) - E_{(\tilde{N}_1,\tilde{N}_2)}(\tilde{\mu}^*) - N_2\mu^* + \tilde{N}_2\tilde{\mu}^*}{\Delta N(\tilde{N}_1,\tilde{N}_2) - \Delta N(N_1,N_2)}.$$

Gauge-invariant VQE ansatz (10 qubits)

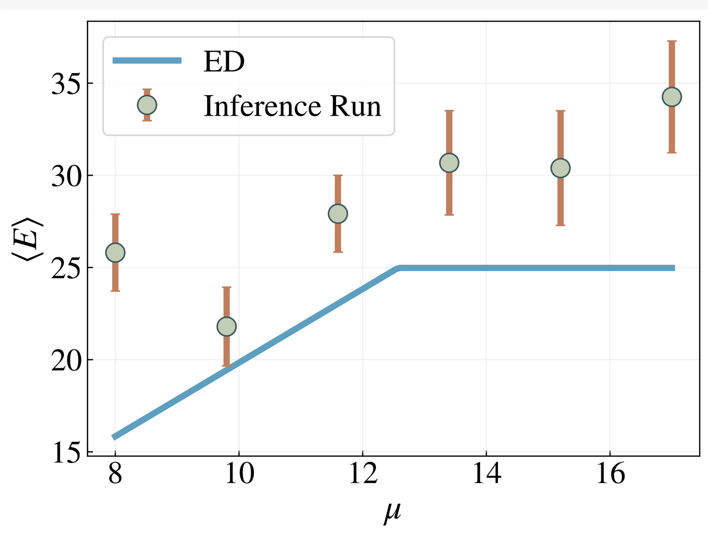
Gauss's law solved at the circuit level: on 2×2 OBC only *one* gauge link is dynamical. Truncation $U(1)\to\mathbb{Z}_{2l+1}$, l=1 using Gray-encoding on two qubits

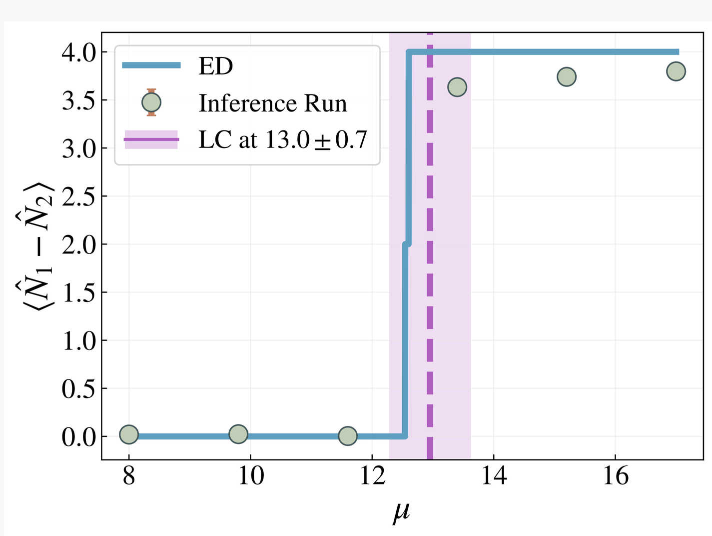




Results: density-induced level crossings on IBM hardware

Noiseless VQE reproduces the ED staircase in $\langle \hat{N}_1 - \hat{N}_2\rangle$; inference runs on `ibm_marrakesh` (8,192 shots, zero-noise extrapolation) locate the crossing at $\mu_{\mathrm{LC}} = 13.0 \pm 0.7$, consistent with ED (12.540, 12.599).





Noise lesson: Noise sensitivity of the Hamiltonian operator $\gg$ Number operator

The old connection from field theory

Callan–Harvey (1985): the (2+1)D bulk Chern–Simons current and the (1+1)D edge anomaly are two faces of one massive fermion, locked by *anomaly inflow*.

Kaplan (1992): sidesteps Nielsen–Ninomiya: doubler-free, chiral fermions on the lattice via domain walls.

Golterman-Jansen-Kaplan (1993): induced CS current of the (2+1)D Wilson fermion; level jumps at the doubler corners.

Sen (2020): CS-level transitions driven by lattice *anisotropy* at fixed M/R; maps DWF levels to Chern-insulator physics, difference traced to the *discrete time direction*.

Condensed-matter perspective: topological phases from Wilson fermions

The trap: a secretly symmetric lattice

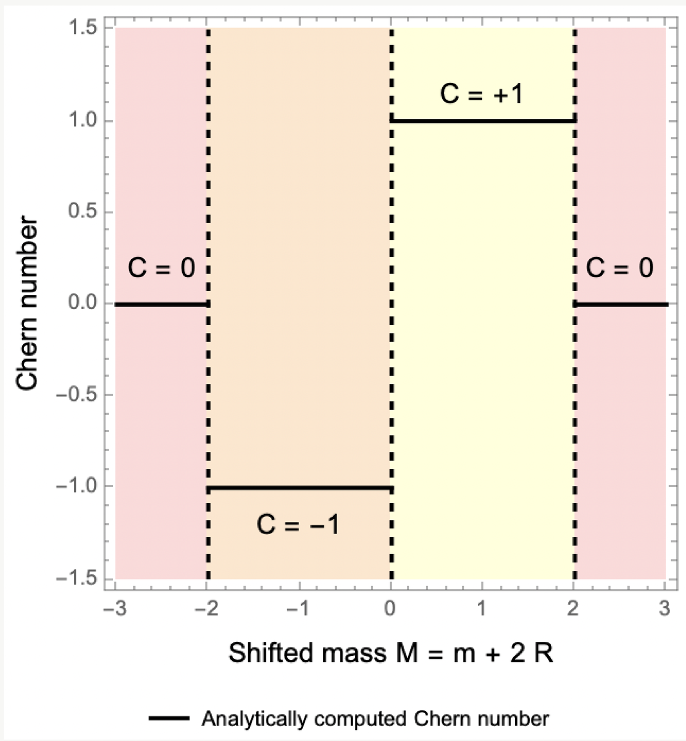
In the continuum, one Dirac fermion in (2+1)D exhibits the parity "anomaly": no gauge-invariant regularization preserves time reversal. On the lattice, **staggered fermions keep two doublers with equal and opposite masses** $\Rightarrow$ an *exact* antiunitary $\mathcal{T}$-symmetry $\Rightarrow$ Chern number $C = 0$ for any mass, any coupling. *The staggered theory is not an approximation to the anomalous theory – it is a different, topologically trivial one.*

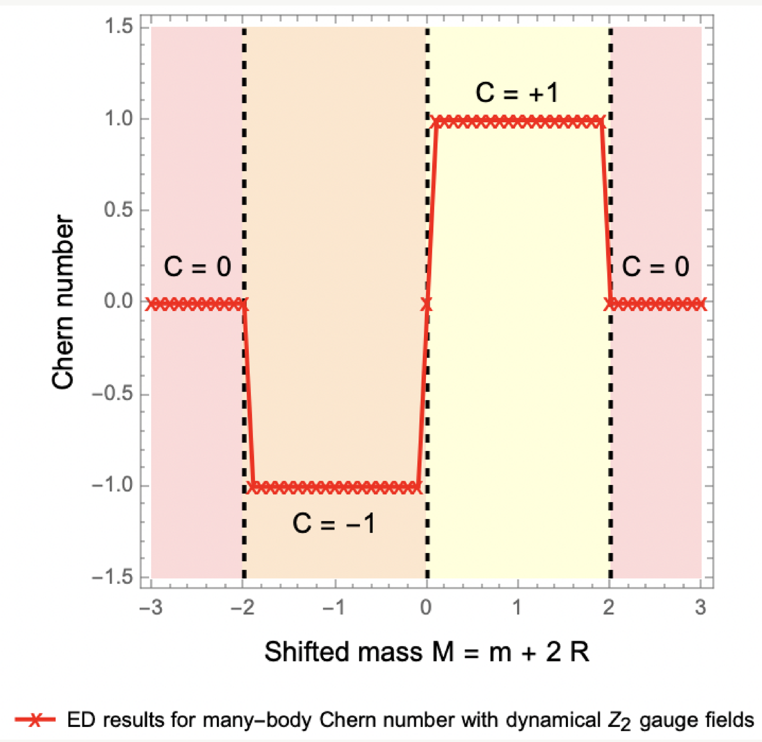
Fermion discretization	$\mathcal{T}$ broken?
Continuum N_f=1 Dirac	✓ any m
Continuum N_f=2 Dirac	× iff $m_1 = -m_2$
Lattice $\tilde{N}_f$=1 staggered	× any m
Lattice N_f=1 Wilson	✓ any (m, R)
Lattice N_f=2 Wilson	× iff $(m_1, R_1) \neq -(m_2, R_2)$

Wilson fermions: topology from doubler bookkeeping

Trivial-flux sector, $U_k = 1$: the Bloch Hamiltonian is the QWZ model, $\boldsymbol{h}(\boldsymbol{k}) = (\sin k_x, \sin k_y,\ M + \cos k_x + \cos k_y)$. Each doubler corner contributes $\pm\frac{1}{2}$:

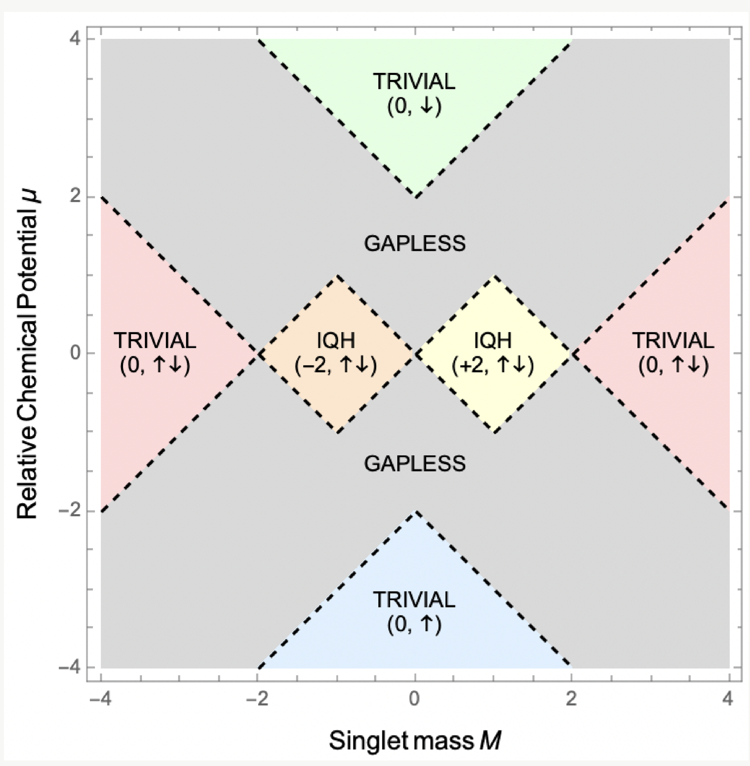
$$C = \tfrac{1}{2}[\mathrm{sgn}(M{+}2) - 2\,\mathrm{sgn}(M) + \mathrm{sgn}(M{-}2)] = \begin{cases} -1, & M \in (-2, 0) \\ +1, & M \in (0, 2) \\ 0, & \text{else} \end{cases}$$

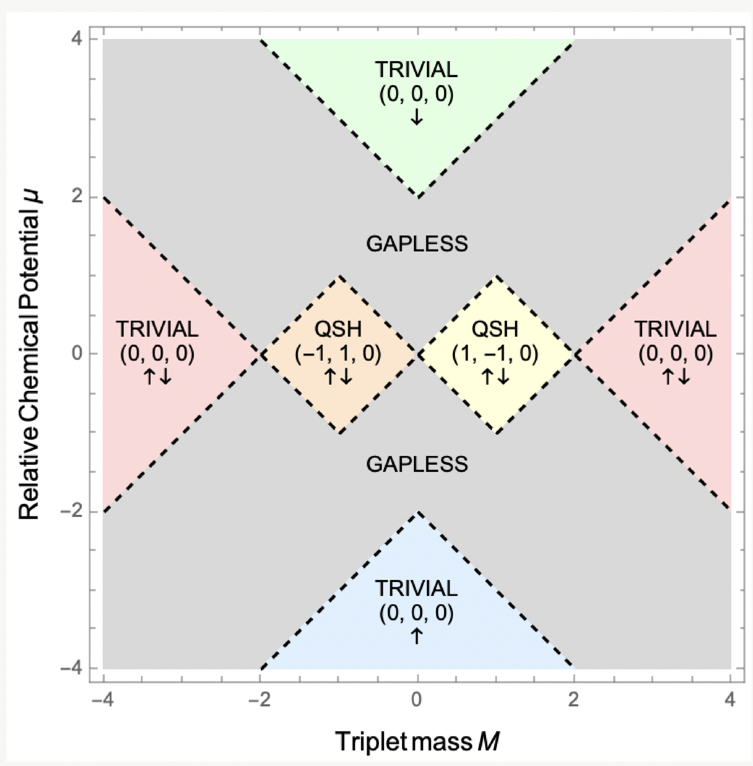




$N_f = 2$ at finite density: IQH and QSH phases

Half filling; relative μ redistributes fixed charge between flavors. **Singlet mass** (M_1=M_2): flavors add, $C_{\mathrm{tot}} = \pm 2$, net Hall current, $\mathcal{T}$ broken $\Rightarrow$ *IQH*. **Triplet mass** ($M_1 = -M_2$): $c_\uparrow = -c_\downarrow$, $C_{\mathrm{tot}} = 0$, spin current only, $\mathcal{T}$ restored $\Rightarrow$ *QSH* (BHZ at μ=0).





Take-home: the discretization is part of the definition of the theory

Choose the discretization after the target observable. Symmetry-protected *counting* observables (particle-number redistribution, density-induced level crossings) tolerate the cheaper staggered choice – demonstrated here on IBM hardware. *Anomalies, Chern–Simons response, and topological phases do not*: they require a discretization that breaks the symmetries the continuum theory breaks.

Outlook. Wilson fermions on hardware: the $M = 0, \pm 2$ corner-flip transitions as measurable level crossings; strong coupling and Aoki-like phases; ADAPT-VQE scaling. [2mm] **References:**

arXiv:2504.21828 • arXiv:2603.05616 • arXiv:2509.20558