Exact Stabilizer Scars in Two-Dimensional U(1) Lattice Gauge Theory

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Motivation & Background

Quantum many-body scars (QMBS)- Special eigenstates that **evade Eigenstate Thermalization Hypothesis (ETH).**
Stabilizer states- States with zero magic (Stabilizer Renyi Entropy = 0) - **efficiently classically simulable**, easily prepared on Quantum Computers
QMBS in Lattice Gauge Theories - QMBS occur in typical **constrained models**, like the U(1) LGT with kinetic constraints and Gauss law constraints.
Our study - We analyze the 2D Rokhsar–Kivelson model in the gauge-invariant subspace and identify **scarred stabilizer eigenstates** with **high entanglement but zero magic**, and give Clifford circuits to prepare them on quantum hardware.

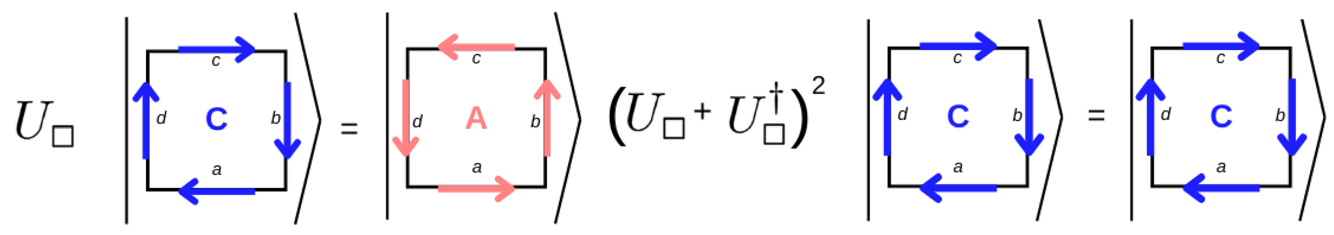
Rokhsar-Kivelson Model Hamiltonian

Simplest LGT with ring-exchange plaquette terms

Gauge fields live on the links, matter fields live on vertices

$$H_{\mathrm{RK}} = O_{\mathrm{kin}} + \lambda O_{\mathrm{pot}}$$
$$= -\sum_{\square} O_{\mathrm{kin},\square} + \lambda \sum_{\square} O_{\mathrm{pot},\square}$$
$$= -\sum_{\square} \left(U_\square + U_\square^\dagger\right) + \lambda \sum_{\square} \left(U_\square + U_\square^\dagger\right)^2$$

$U_\square = S_a^+ S_b^+ S_c^- S_d^-$ $\quad U_\square^\dagger = S_a^- S_b^- S_c^+ S_d^+$ $\quad S^+|\downarrow\rangle = |\uparrow\rangle$



$U_\square^\dagger |A\rangle = |C\rangle$ $\qquad (U_\square + U_\square^\dagger)^2 \, |A\rangle = |A\rangle$

Kinetic term flips cyclic plaquettes , annihilates non-cyclic ones

Potential term counts # of cyclic plaquettes in a configuration

Gauss law – electric flux at each vertex is conserved

$$G_x = \sum_{i=x,y} \left(S^z_{x,i} - S^z_{x-\hat{i},i}\right)$$

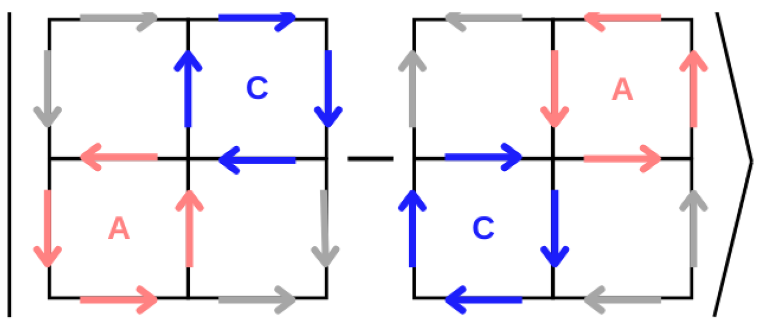
$$G_x = S^z_{\mathrm{right}} - S^z_{\mathrm{left}} + S^z_{\mathrm{up}} - S^z_{\mathrm{down}}$$

Gauge invariant subspace - physical states must fulfill

$G_r|\psi\rangle = 0 \quad \forall r$

Quantum Many Body Scars in U(1) LGT

Sublattice scars -Subclass of QMBS with total spin **singlet character** under bipartition of the lattice into sublattices A and B [1]

$\frac{1}{\sqrt{2}}$ 

Scarring characteristic signatures

Singlet states have bipartite entanglement entropy **ln(2) per dimer , not volume law**

According to **ETH**, all eigenstates at given energy, have local observables close to the **microcanonical average** $\langle O_{\mathrm{pot/kin}}\rangle_{\mathrm{thermal}} \approx$ fractional value

But **singlet scars** have **extreme expectation values** of both local observables

$\langle O_{\mathrm{pot/kin}}\rangle_{\mathrm{scar}} =$ exact integer

Tile the lattice with singlet states in larger lattice

Identifying Stabilizer States

Stabilizer states- simultaneous +1 eigenstates of an Abelian subgroup of the Paulis
Stabilizer Rényi entropy of order *n* [2] – **measures magic of a state**

$$M_n(|\psi\rangle) = \frac{1}{1-n} \log\left(\sum_{P \in \mathcal{P}_N} \frac{|\langle\psi|P|\psi\rangle|^{2n}}{2^N}\right)$$

$\mathcal{P}_N$ is N -qubit Pauli group
$M_n = 0$ **stabilizer**

For small systems, SRE can be computed exactly for all the eigenstates. Because of ***4^N*** Pauli strings it becomes infeasible to do this exactly

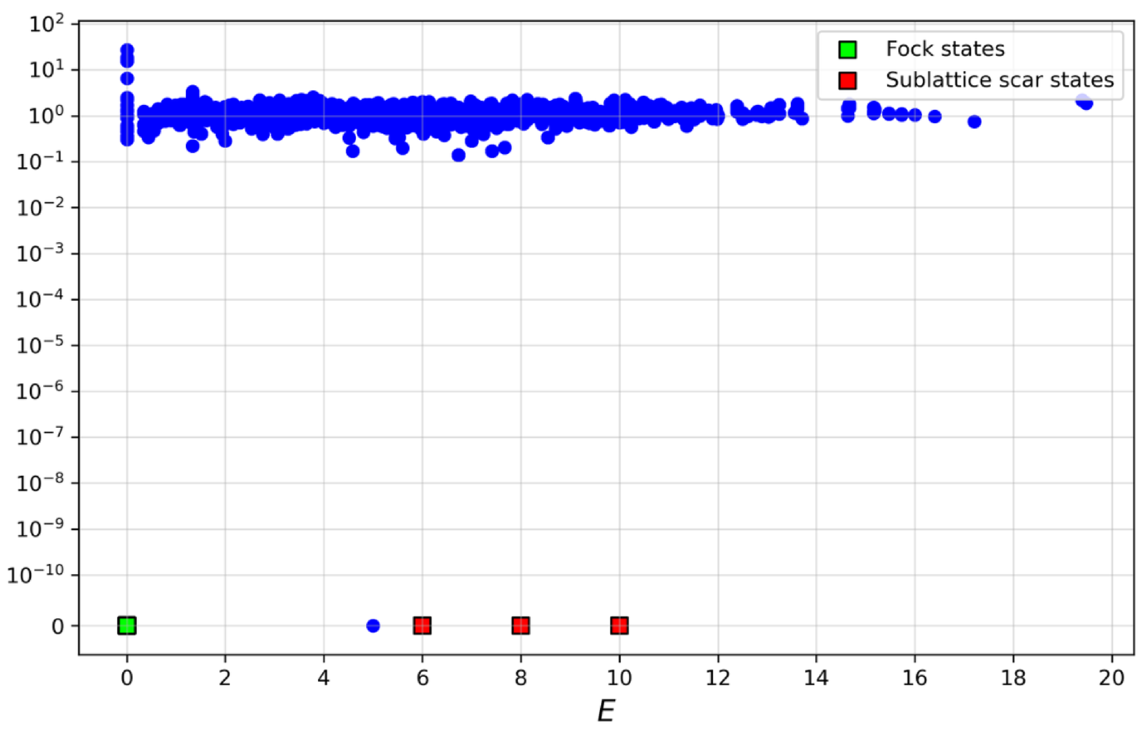
Multifractal flatness $F(|\psi\rangle)$ for states in computational basis [3], more feasible

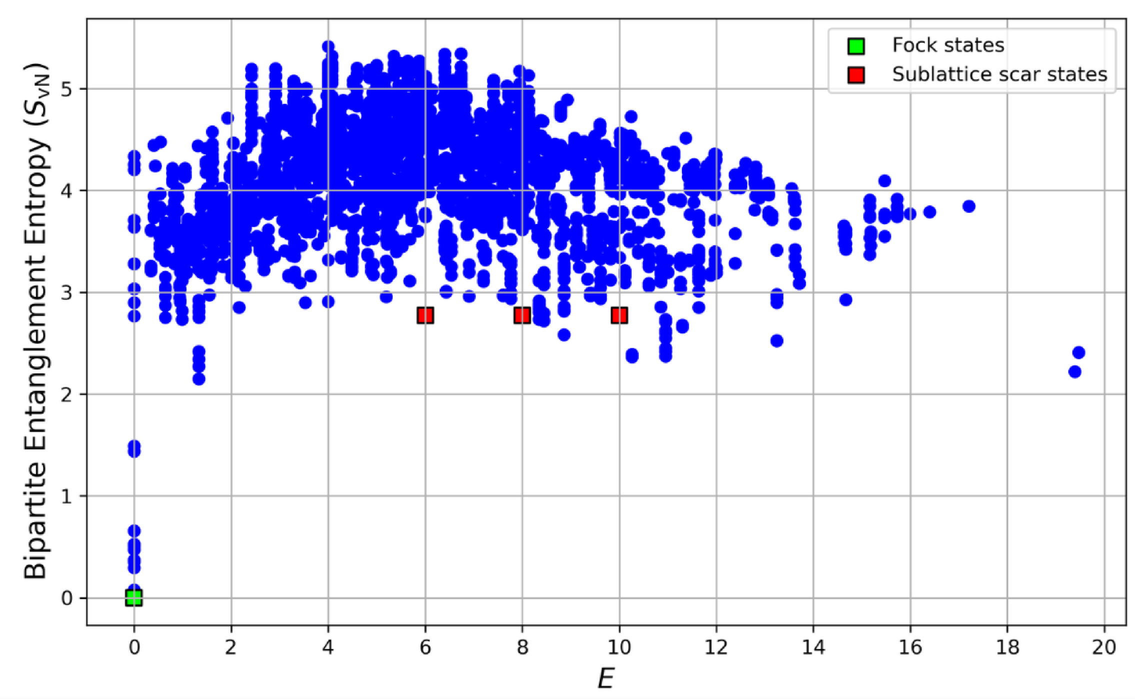
$F(|\psi\rangle) = \sum_\sigma p_\sigma^3 - \left(\sum_\sigma p_\sigma^2\right)^2$, $p_\sigma = |\langle\sigma|\psi\rangle|^2$ **Flatness *F=0*** states are good stabilizer candidates

+ if states is in **canonical form** [4] $\quad |\psi\rangle = \frac{1}{\sqrt{|A|}} \sum_{x \in A} i^{\ell(x)} (-1)^{q(x)} |x\rangle \quad A \subset F_2^n$

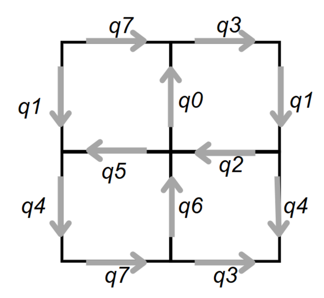
State is Stabilizer ! $\qquad \ell : \{0,1\}^n \to \{0,1,2,3\} \qquad q : \{0,1\}^n \to \{0,1\}$

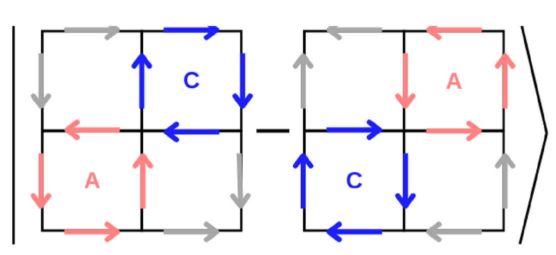
Results

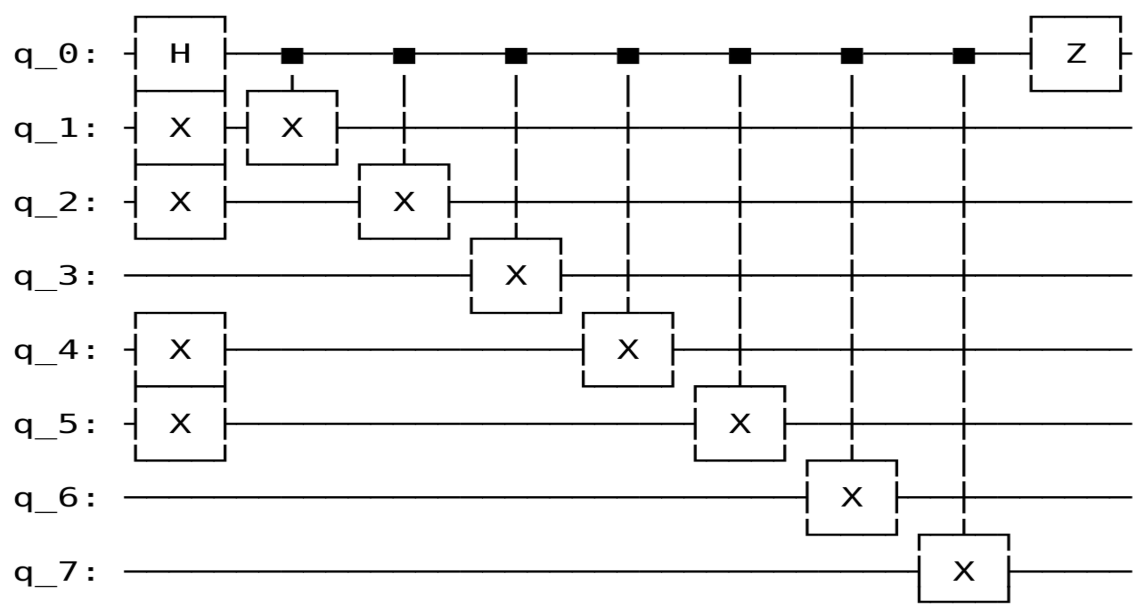




System	No. of spins	No. of physical states	No. of stabilizer scars
2×2	8	18	2
4×2	16	114	2
6×2	24	858	2
4×4	32	2,970	8



$\frac{1}{\sqrt{2}}$ 



Superpose more qubits, repeat the 2x2 circuit, for bigger lattice

Stabilizer algebra

Each flippable plaquette is a **logical qubit** with basis states |C⟩ and |A⟩ flux loops.

With projectors P_C = |C⟩⟨C| , P_A = |A⟩⟨A| , the logical operators are X_L = |C⟩⟨A| + |A⟩⟨C| and Z_L = P_C − P_A .

Across the sublattice bipartition, active plaquettes pair into **Bell-like logical singlets**

|Ψ⁻⟩ = (|C⟩$_m$|A⟩$_n$ − |A⟩$_m$|C⟩$_n$) / √2

stabilized by the emergent group

Z_L ⊗ Z_L = X_L ⊗ X_L = −1 .

Each scar is a product of these singlets over the active plaquettes,

|ψ_SS⟩ = ⊗ |Ψ⁻⟩ ⊗ |inactive⟩

scars form an **intrinsic stabilizer manifold** in the gauge-invariant Hilbert space

Conclusion and further developments

Certain **high-energy eigenstates** of the 2D Rokhsar–Kivelson model are **exact stabilizer scar states** with vanishing stabilizer Rényi entropy but finite entanglement entropy, representing a nonthermal, classically simulable sector within the gauge-invariant Hilbert space.

This construction provides an experimentally accessible routine to initialize the **nonthermal stabilizer eigenstates**, offering insight into their potential for **controllable state preparation** in constrained quantum systems. Stabilizer structure as probe for QMBS.

References

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[4] A. Montanaro. Learning stabilizer states by Bell sampling. arXiv:1707.04012