Fermionic Non-Gaussianity, Complexity, and Quantum Simulation of High-Energy Physics Models

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Motivation

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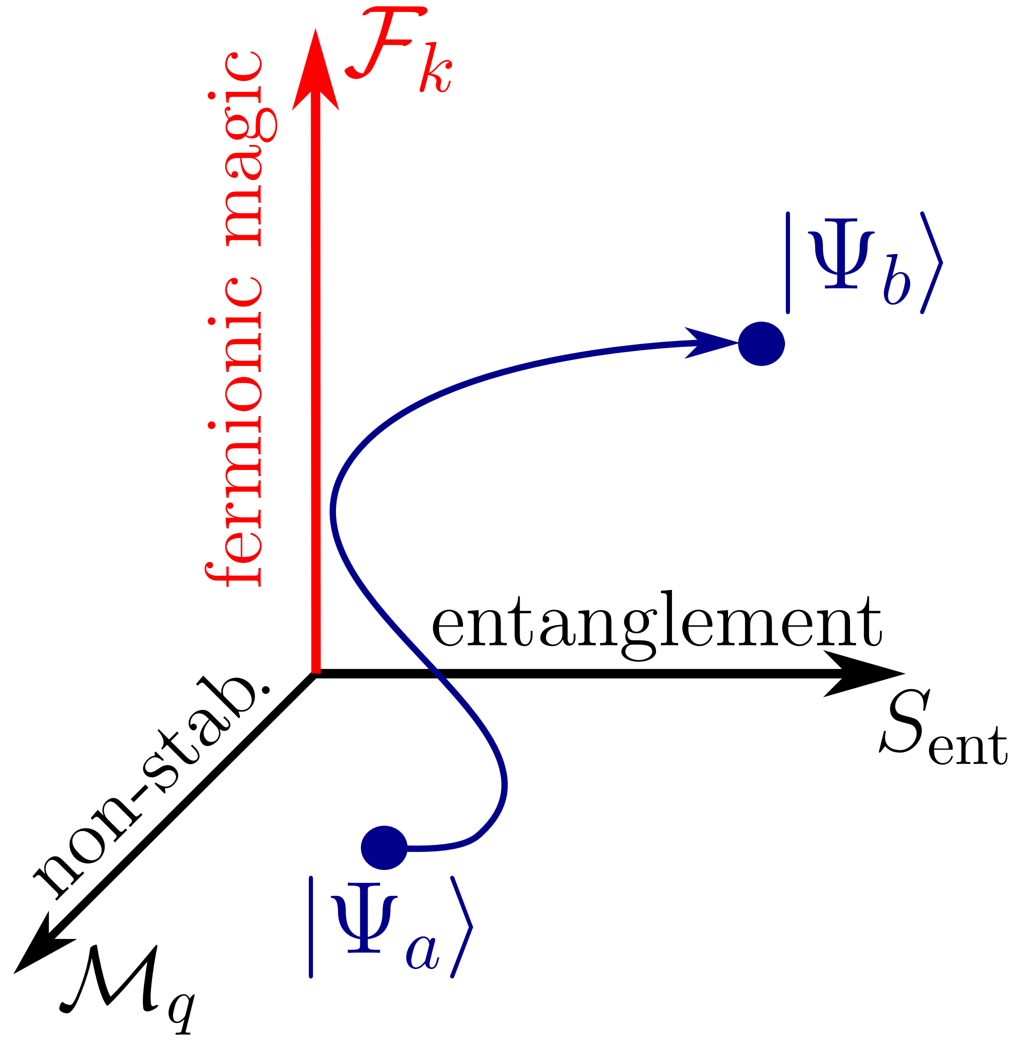
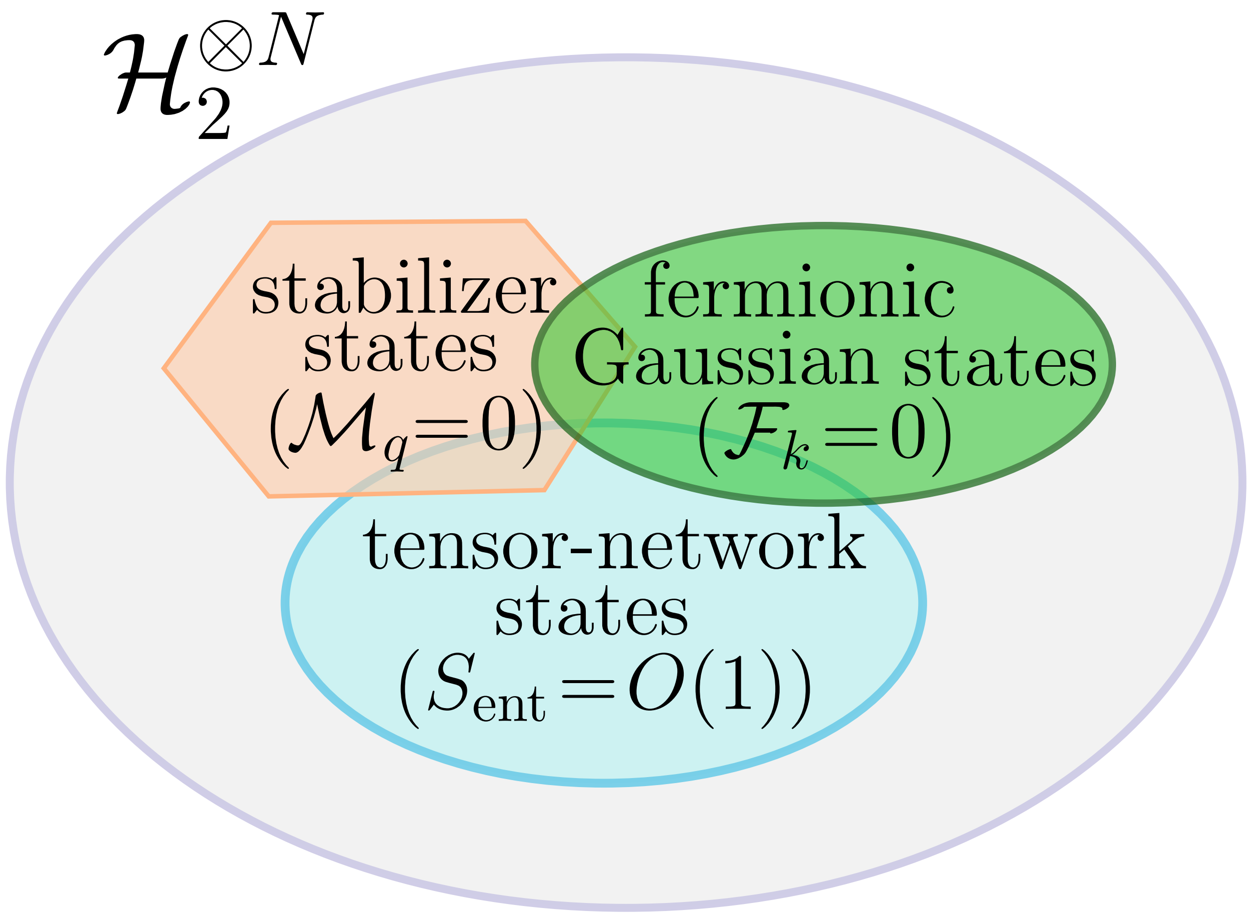
- Resource theories provide a framework to quantify and compare computational complexity of quantum states.

- Different classes of states (stabilizer, Gaussian, low-entanglement tensor networks) are classically tractable.

- Beyond these, magic resources (SRE, non-Gaussianity, entanglement) are needed to reach genuine quantum advantage.





E.Chitambar and G. Gour Quantum resource theories Rev. Mod. Phys. 91, 025001

Fermionic systems

- We consider N fermionic modes or via the Jordan-Wigner transformation a system of n qubits

- The $2n$ Majorana operators γ_k, can be represented in terms of the Pauli as:

$$\gamma_{2k-1} = \left(\prod_{m=1}^{k-1} Z_m\right) X_k \,, \quad \gamma_{2k} = \left(\prod_{m=1}^{k-1} Z_m\right) Y_k$$

- The covariance matrix $M = (M_{m,n})$ is defined for any state $|\Psi\rangle$ by its matrix elements:

$$M_{mn} = -\frac{I}{2}\langle\Psi|[\gamma_m, \gamma_n]|\Psi\rangle$$

Fermionic Gaussian states

- Gaussian states are defined as:

$$|\Psi> = e^{\frac{1}{4}\sum_{m,n=1}^{2N} H_{m,n}\gamma_m\gamma_n}|\mathbf{0}>$$

- The covariance matrix M fermionic Gaussian state encodes, through Wick's theorem, the complete information about all correlation functions

- Gaussian states are efficiently simulatale classically in polynomial time

S. Bravyi, QI&C, 5, 3, 216-238 (2005)   J. Surace, L. Tagliacozzo, SciPost Phys. Lect. Notes 54 (2022)

- Volume law entanglement and high SRE

M. Collura, J. De Nardis, V. Alba, G. Lami, Quantum 10, 2036 (2026)

Measure of non gaussianity

- Fermionic Gaussian rank: minimal k such that:

M. Hebenstreit, R. Jozsa, B. Kraus, S. Strelchuk, M. Yoganathan, PRL 123, 080503 (2019)

$$|\Psi\rangle = \sum_k b_\alpha |\psi_{G,\alpha}\rangle$$

- Gaussian fidelity:

J. Cudby, S. Strelchuk, arXiv:2307.12654

$$F_G(|\Psi\rangle) = \sup_{s \in G} \langle s | \psi \rangle$$

- Both hard to measure, they need an optimisation over infinite set of gaussian states

Fermionic Anti Flatness

- We define the Fermionic Anti Flatness **FAF** as:

$$\mathscr{F}_k(|\Psi\rangle) = N - \frac{1}{2}\mathrm{tr}\left[(M^T M)^k\right]$$

- A measure φ of fermionic magic resources is a function mapping an arbitrary state $|\Psi\rangle$ to a non-negative real number, satisfying the following properties:

1. Faithfulness — $\varphi\left(|\Psi\rangle\right) = 0$ iff $|\Psi\rangle$ is a fermionic Gaussian state
2. Gaussian invariance — $\varphi\left(U_G|\Psi\rangle\right) = \varphi\left(|\Psi\rangle\right)$ for all Gaussian unitaries U_G
3. (sub)additivity — $\varphi\left(|\Psi\rangle \otimes |\Phi\rangle\right) = \varphi\left(|\Psi\rangle\right) + \varphi\left(|\Phi\rangle\right)$ for states with fixed parity

P. Sierant, **PS**, X Turkeshi, Fermionic Magic Resources of Quantum Many-Body Systems, PRX Quantum **7**, 010302 (2026)

Intuition behind FAF

- The covariance matrix M is a $2N \times 2N$ antisymmetric matrix, and can be brought to the canonical form:

$$M = Q^T \bigoplus_{m=1}^{N} \begin{pmatrix} 0 & \lambda_m \\ -\lambda_m & 0 \end{pmatrix} Q$$

- where $\lambda_i > 0$ are the Williamson eigenvalues of M, and Q is an orthogonal matrix.

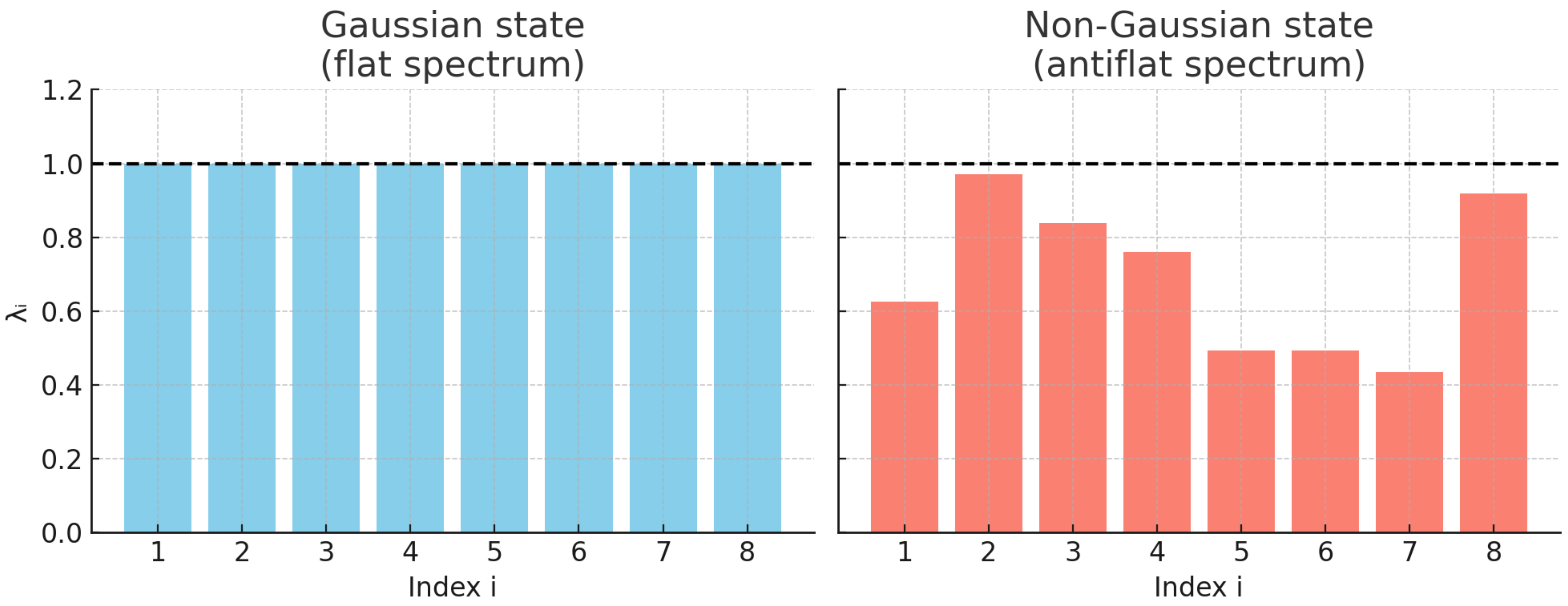
- In terms of Williamson eigenvalues, the FAF can be written as

$$\mathcal{F}_k(|\Psi\rangle) = N - \sum_{i=1}^{N} \lambda_i^{2k}$$

Intuition behind FAF

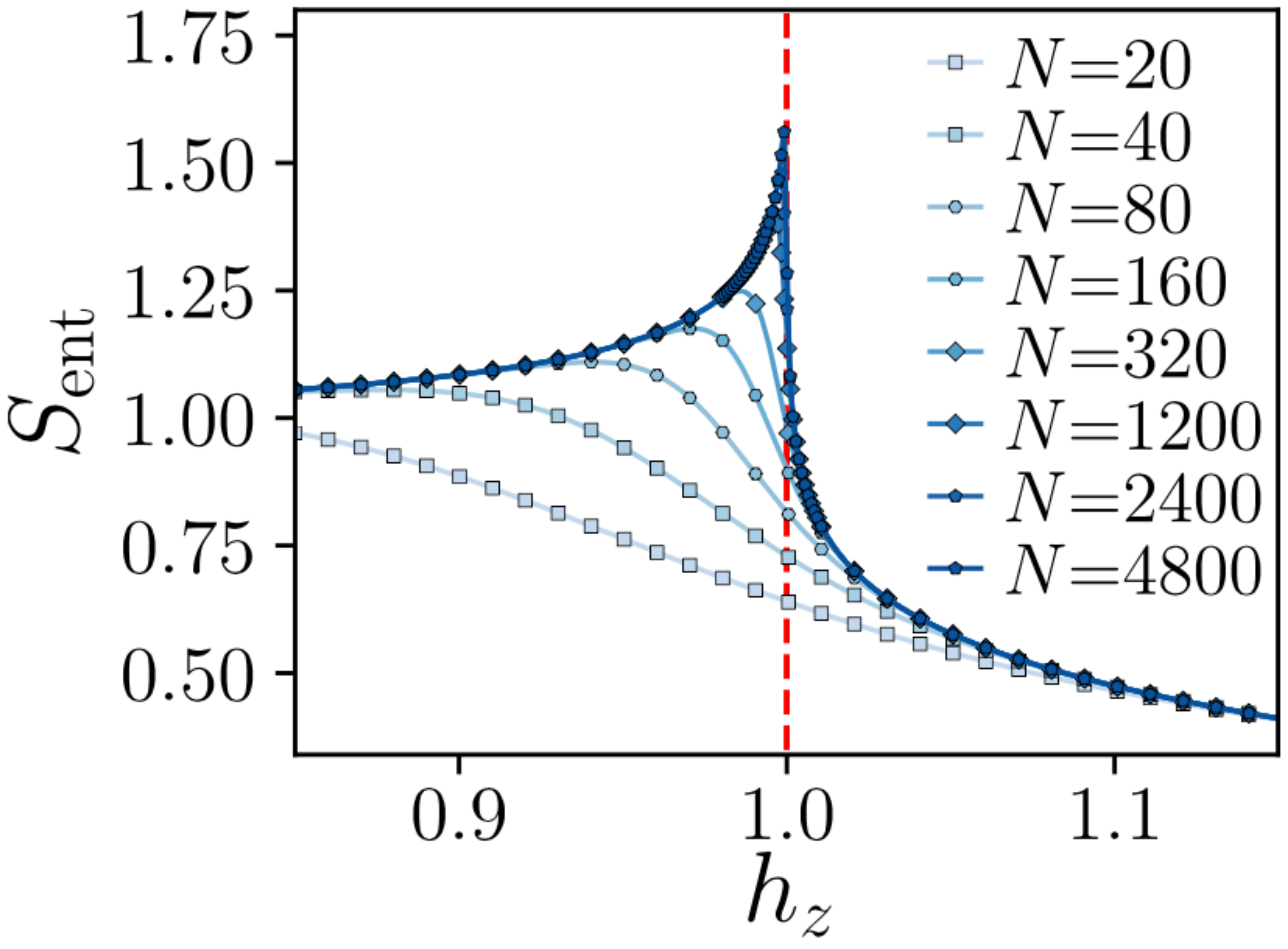
- When $|\Psi\rangle$ is a fermionic Gaussian state, the spectrum is completely flat, with $\lambda_i = 1$

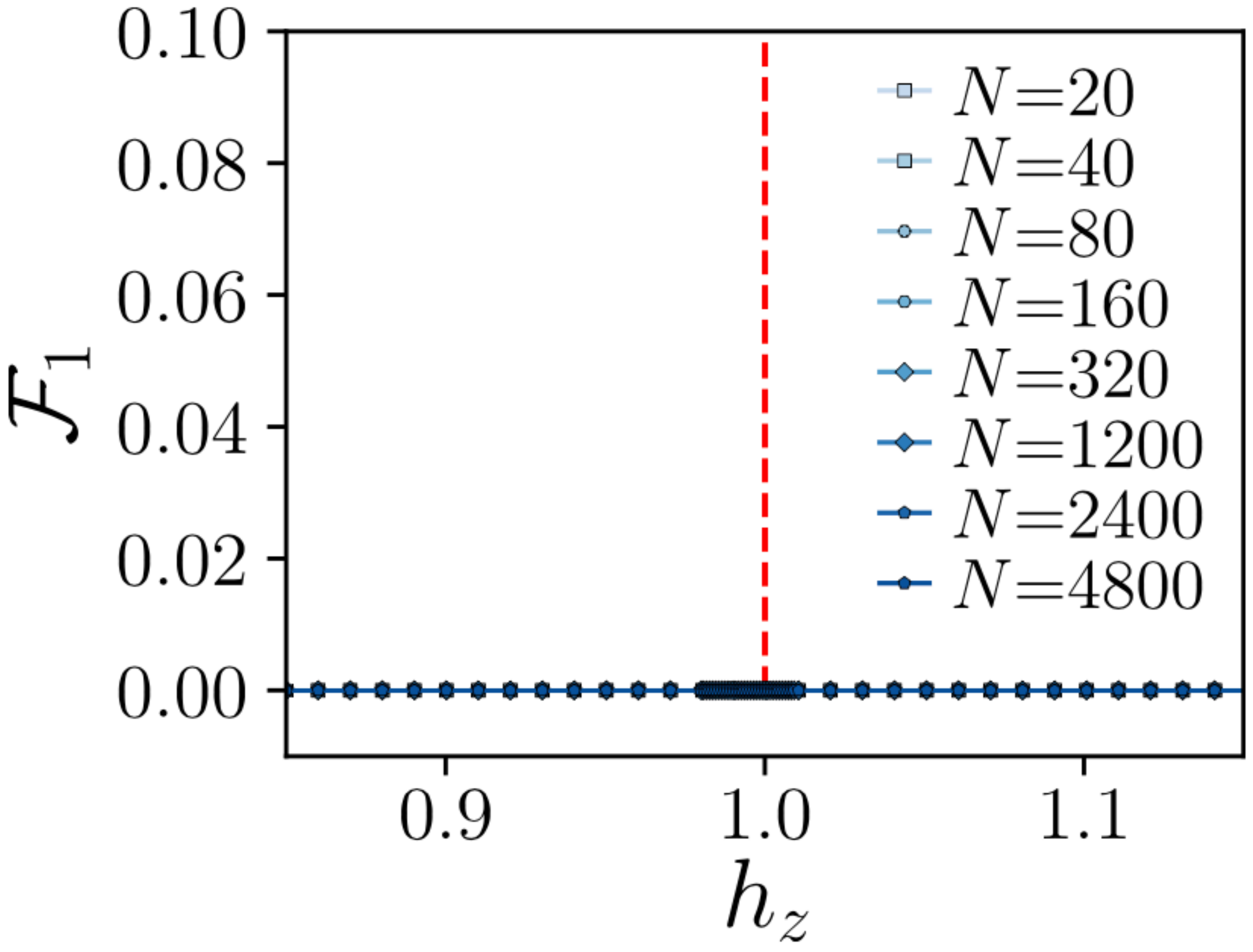
- When $|\Psi\rangle$ is non Gaussian, the Williamson eigenvalues decrease, with $\lambda_i \in [0,1]$, and are no longer equal, which Is indirectly measured by $\mathscr{F}_k$



FAF in GS: Ising

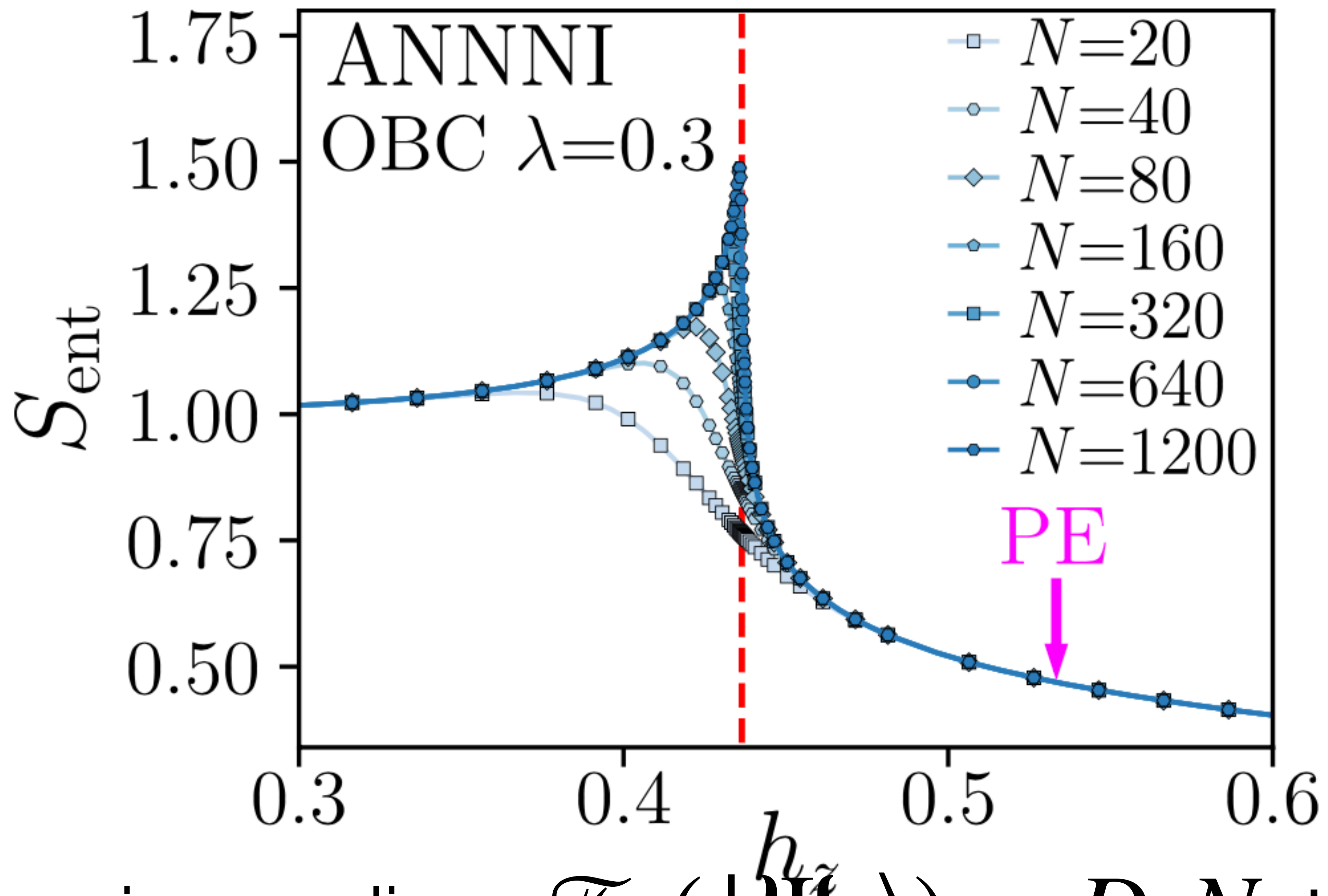
The Ising model: $$H_{ising} = -h_z \sum_m Z_m - \sum_m X_m X_{m+1}$$

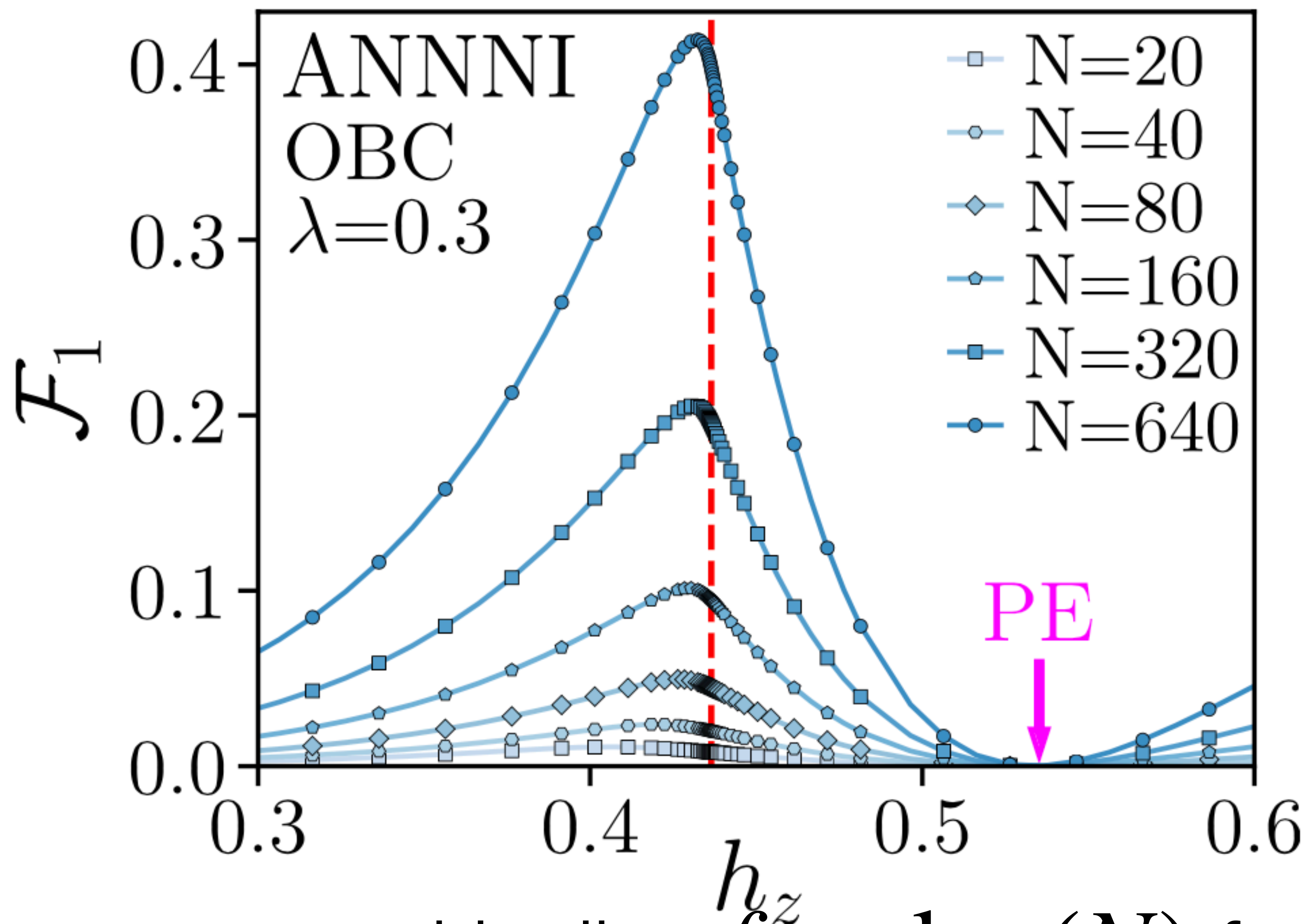




FAF in GS: ANNNI

The ANNNI model: $$H = H_{Ising} - \lambda \sum_m X_m X_{m+2}$$



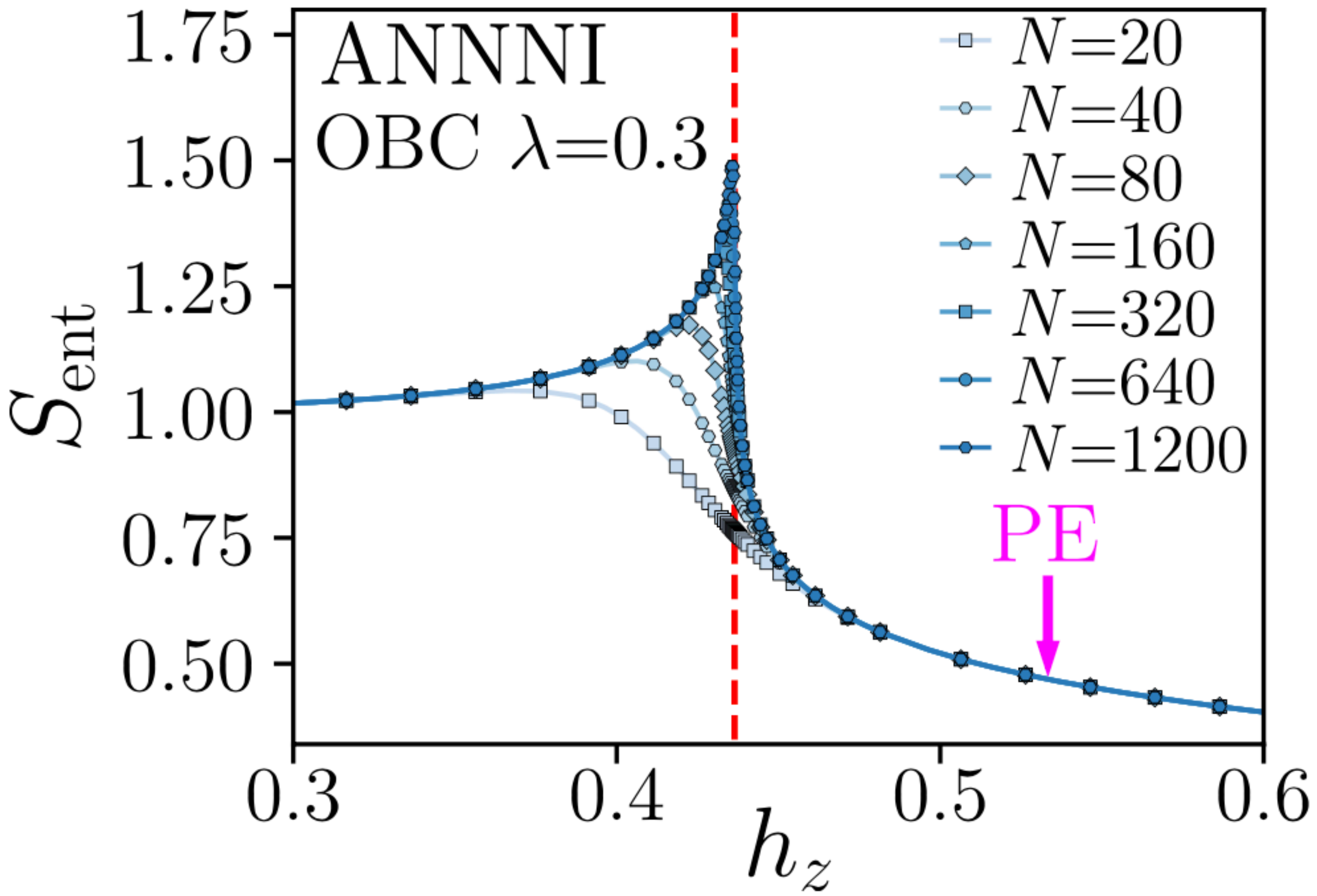


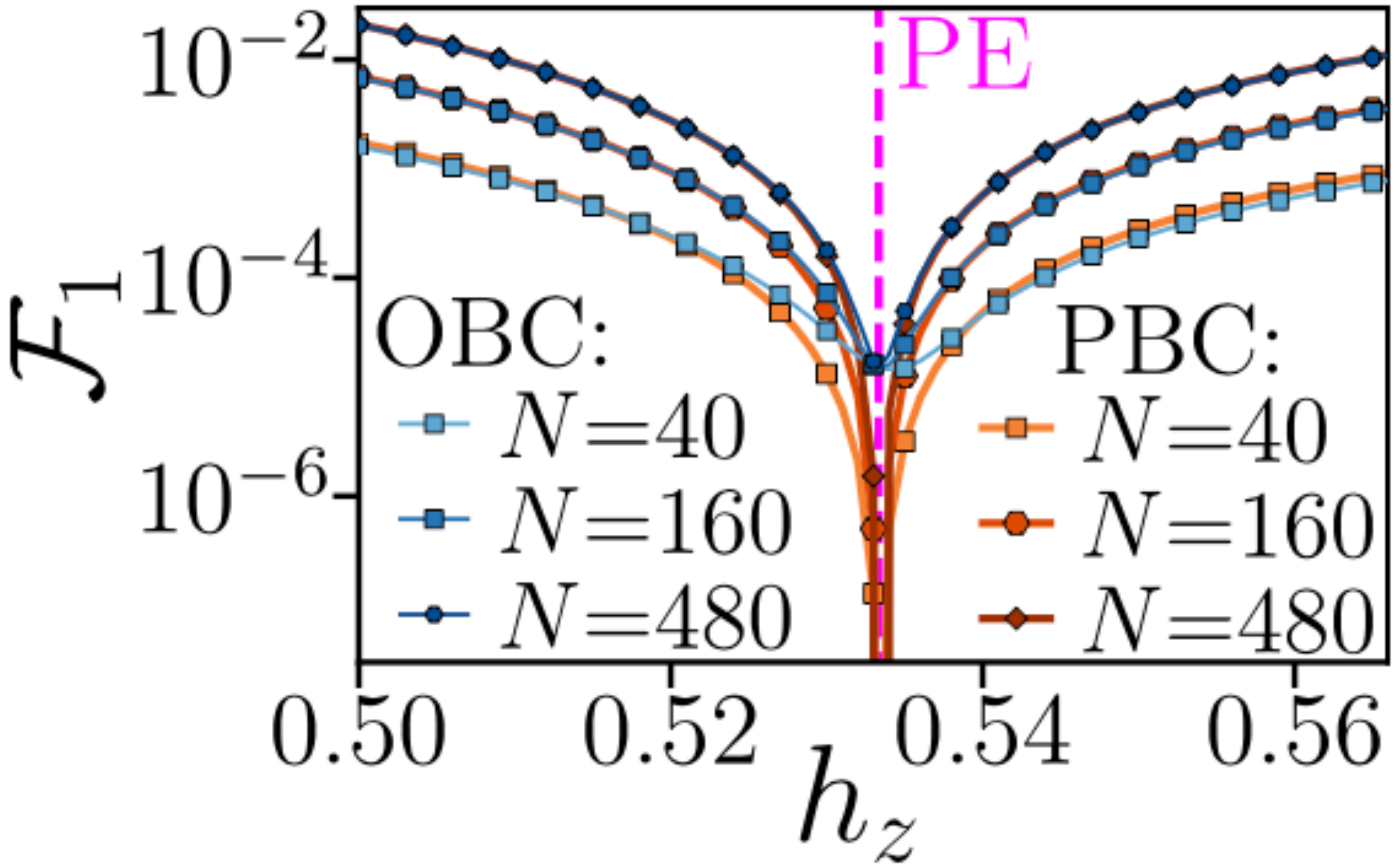
Extensive scaling: $\mathscr{F}_1(|\Psi_0\rangle) = D_d N + f_k$; at criticality $f_k \sim \log(N)$ for OBC

Similar to SRE and Participation entropies

FAF in GS: PE point

The ANNI model: $$H = -h_z \sum_m Z_m - \sum_m X_m X_{m+1} - \lambda \sum_m X_m X_{m+2}$$

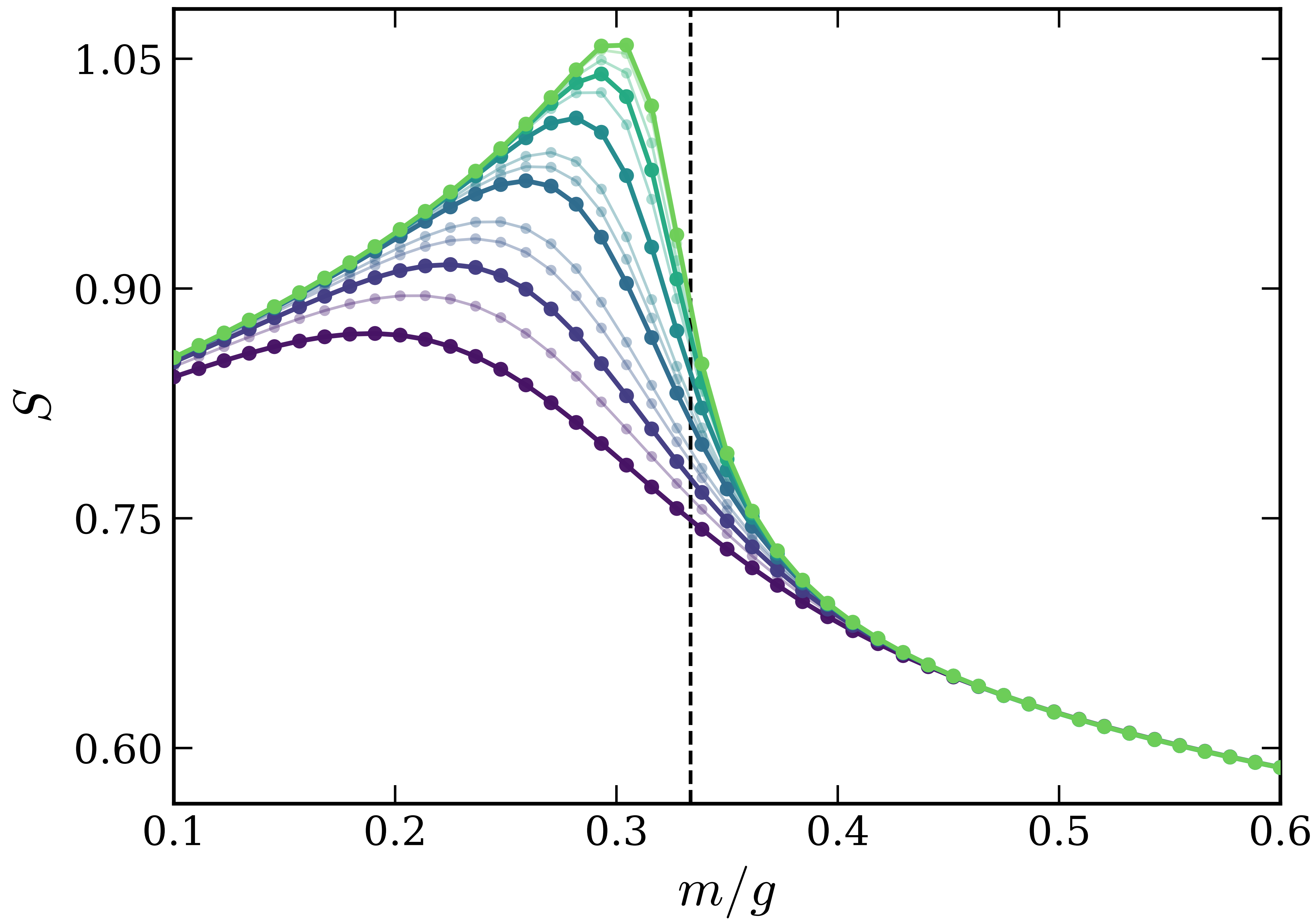


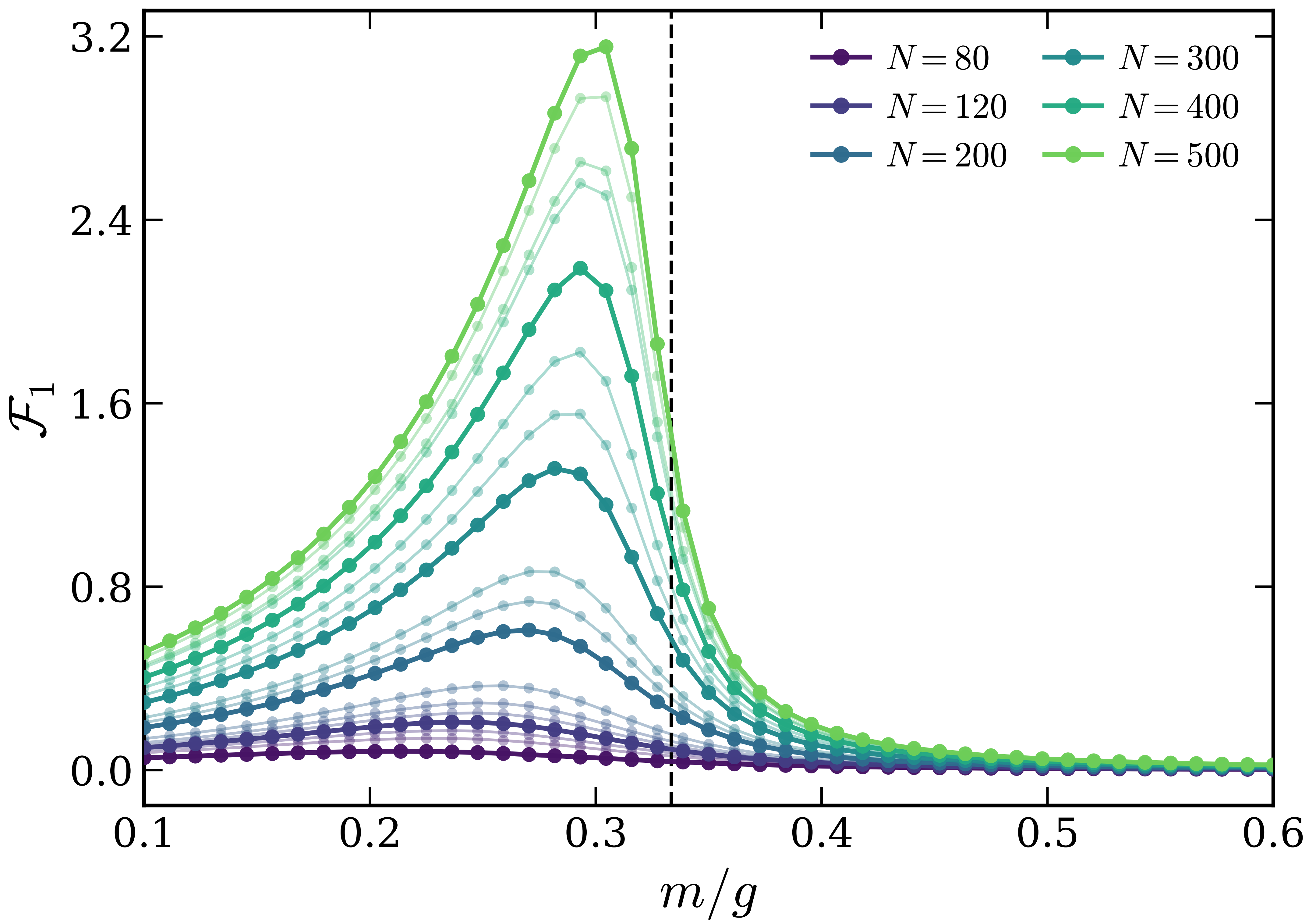


PE= Peschel- Emery point

I. Peschel, V. Emery, Z. Phys. B **43**, 241 (1981)

FAF in Schwinger model $\theta = \pi$

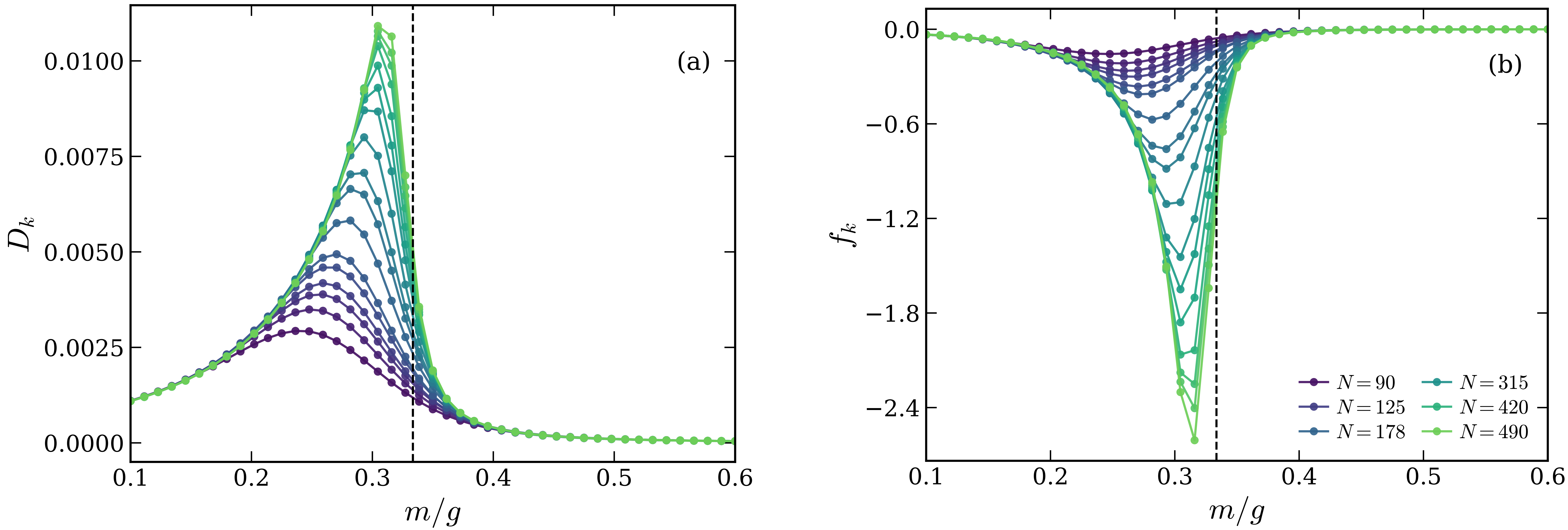




$$H_{\rm S} = \frac{g^2 a}{2} \sum_{n=0}^{N-2} \left(L_n + \frac{\theta}{2\pi} \right)^2 + m \sum_{n=0}^{N-1} (-1)^n c_n^\dagger c_n - \frac{i}{2a} \sum_{n=0}^{N-2} \left(c_n^\dagger e^{i\phi_n} c_{n+1} - c_{n+1}^\dagger e^{-i\phi_n} c_n \right) .$$

R. Picañol, **PS**, FAF in 1+1d QED, In preparation

FAF in Schwinger model $\theta = \pi$



Extensive scaling: $\mathscr{F}_1(|\Psi_0\rangle) = D_d N + f_k$; at criticality $f_k \sim \log(N)$ for OBC

R. Picañol, **PS**, FAF in 1+1d QED, In preparation

Summary

- Presented a framework for investigating fermionic magic resources (FAF) in quantum of many-body systems
- It be experimentally computed in polynomial time
- Static properties of GS
- Critical scaling at the phase transition points

Outlook

- Fermionic Augmented Matrix Product States — S Masot-Llima, A Garcia-Saez PRL 133, 230601 (2024)

- Understanding the relationship between the resource theories of fermionic non-Gaussianity: FAF, fermionic rank, fermionic gaussian extent

- Develop a description of FAF at quantum phase transitions, with CFT

  M. Hoshino, M. Oshikawa, and Y. Ashida, Stabilizer renyi entropy and conformal field theory Phys. Rev. X **16**, 011037 (2026)

- Study FAF in BKT transitions or topological scenarios

- FAF in HEP models: Gauge Theories ,SYK models …

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BSC

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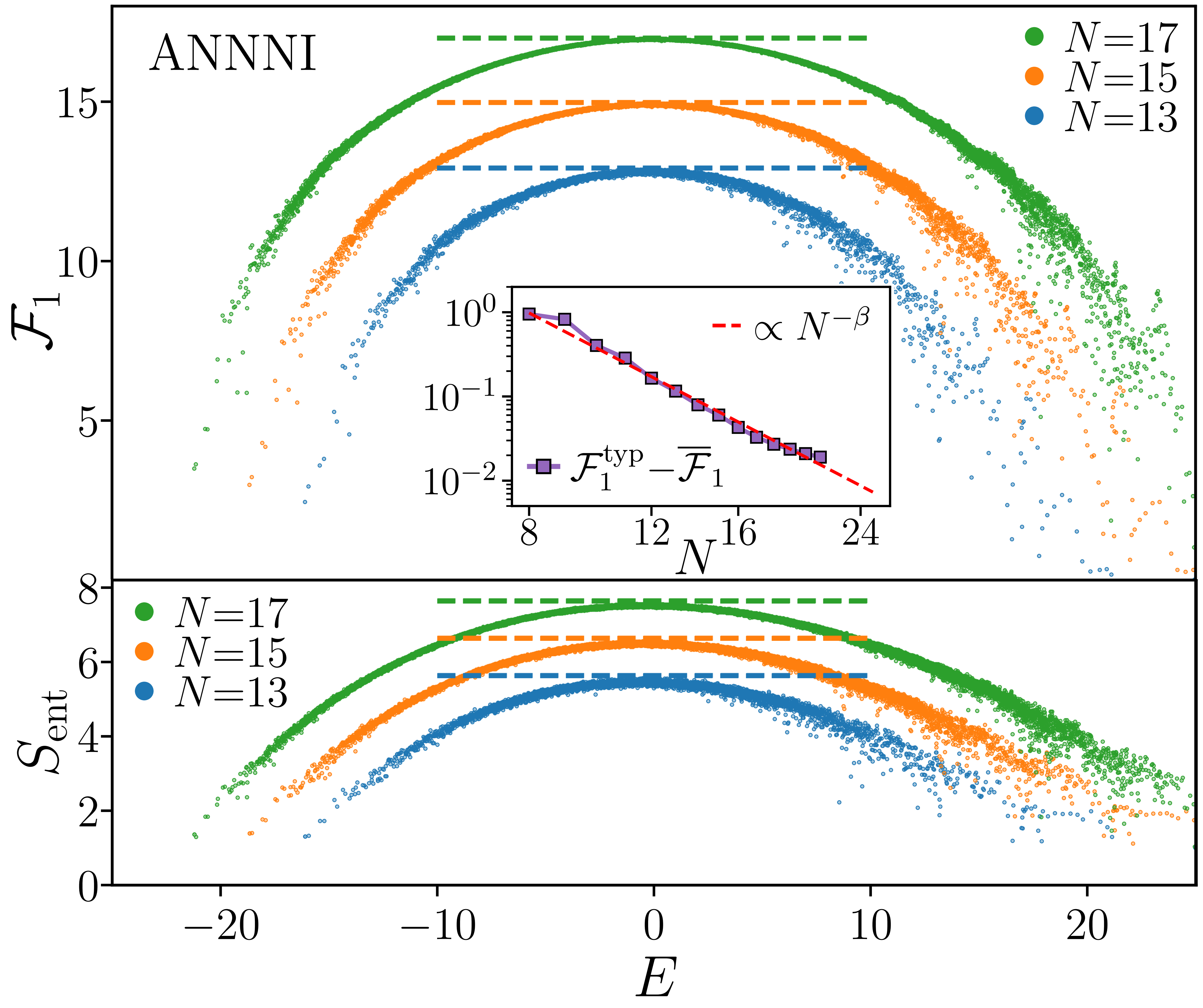
University of Cologne

FAF across the many-body spectrum

- ETH: excited states of ergodic systems share universal properties
- Middle-spectrum FAF ≈ typical FAF → signals ergodicity
- Mirrors behavior of entanglement entropy



Stabilizer Reny entropy

- A measure φ of magic resources is a function mapping an arbitrary state $|\Psi\rangle$ to a non-negative real number, satisfying the following properties:

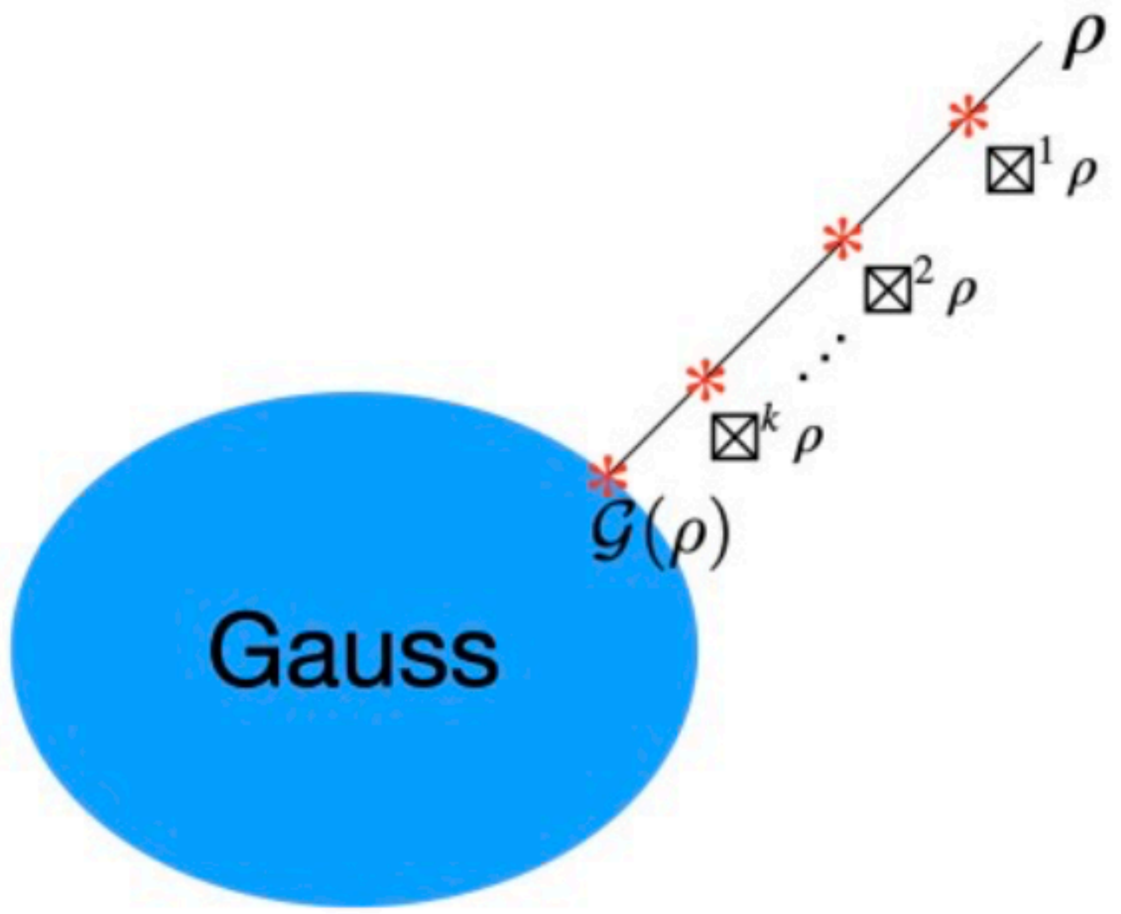
1. Faithfulness — $\varphi\left(|\Psi\rangle\right) = 0$ iff $|\Psi\rangle$ is a stabilizer state
2. Clifford invariance — $\varphi\left(U_C|\Psi\rangle\right) = \varphi\left(|\Psi\rangle\right)$ for all Clifford unitaries U_C
3. Additivity under $\otimes$ — $\varphi\left(|\Psi\rangle \otimes |\Phi\rangle\right) = \varphi\left(|\Psi\rangle\right) + \varphi\left(|\Phi\rangle\right)$

L Leone, SFE Oliviero, A Hamma Physical Review Letters 128 (5), 050402

Measure of non gaussianity

- Non gaussian entropy: Under convolution trace out part of the system

$$\boxtimes^1 \rho = \mathrm{tr}_2\left[W(\rho \otimes \rho)W^\dagger\right], \qquad W = \exp\left(\frac{\pi}{8}\sum_{j=1}^{2N}\gamma_j\gamma_{2N+j}\right)$$



$$NGE_\infty(\Psi) = S(|\Psi_F\rangle) = -\sum_i \log_2\left(\left(\frac{1+\lambda_i}{2}\right)^2 + \left(\frac{1-\lambda_i}{2}\right)^2\right)$$

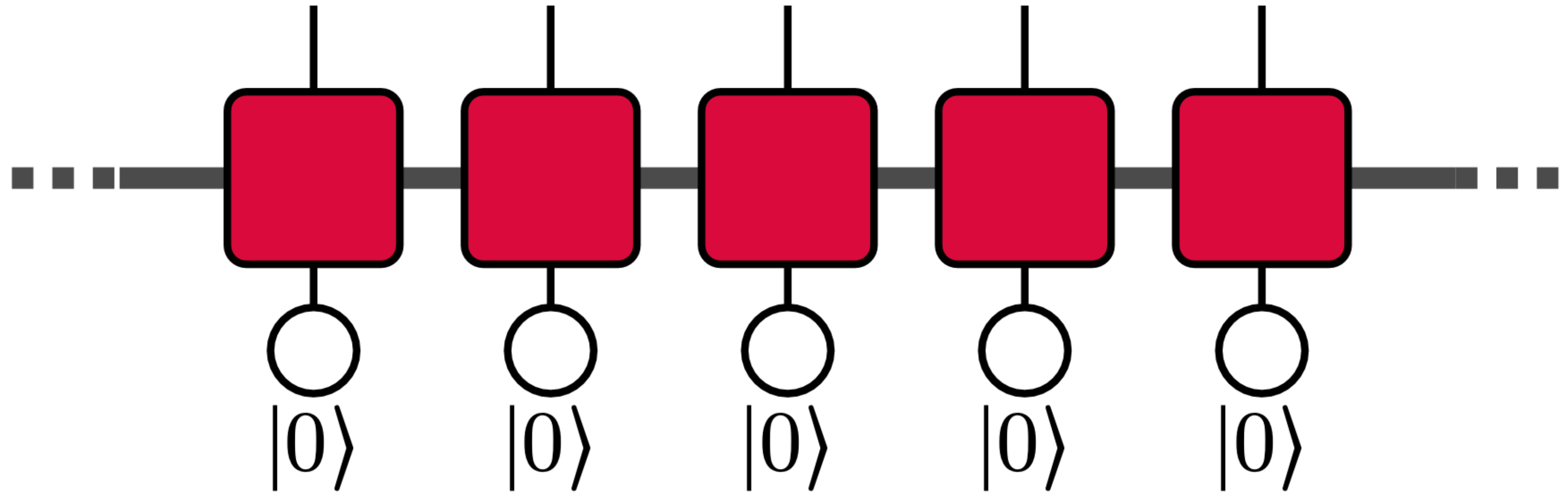
Where λ_i are the Williamson eigenvalues

L. Coffman, G. Smith, X. Gaok, arXiv:2307.12654

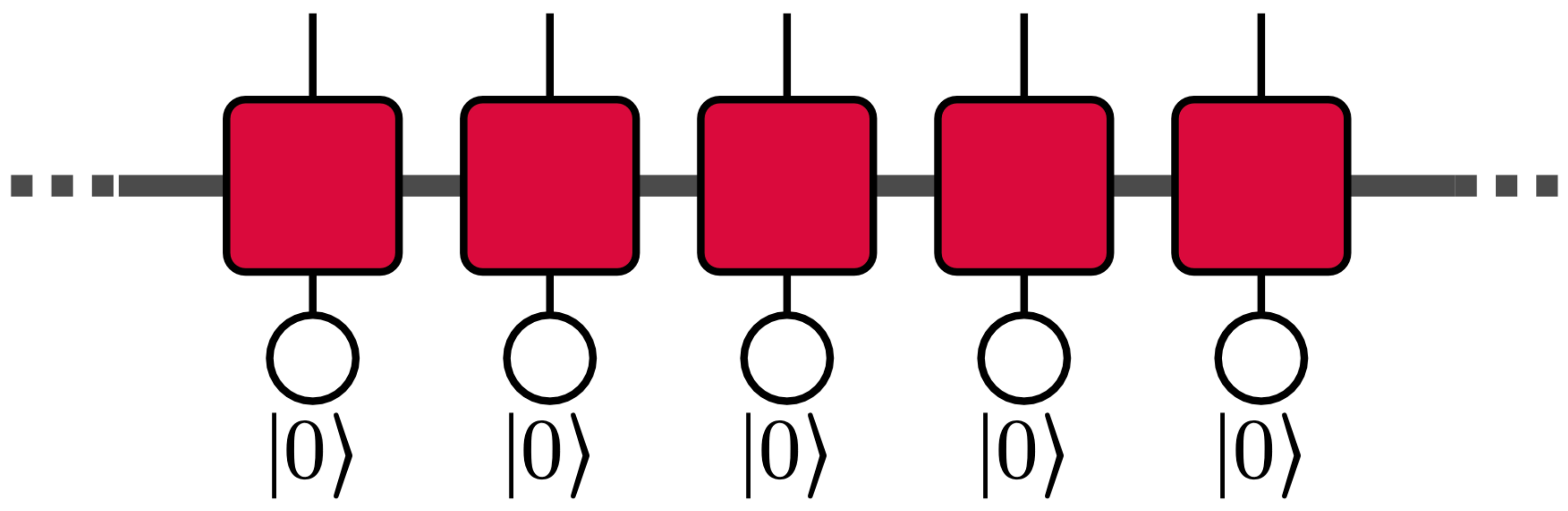
N. Lyu , K. Bu arXiv:2409.08180

Harr random (MPS)

- Sampled uniformly from the full Hilbert space (Haar measure).
- Typical states are **highly entangled** (volume-law).
- Almost maximally “magical” (non-stabilizer, non-Gaussian).
- Serve as a **reference point**: generic quantum states are complex.
- Efficient classical simulation is impossible → benchmark for resource measures (SRE, FAF)
- Harr random MPS are equivalent but at finite χ



FAF in RMPS



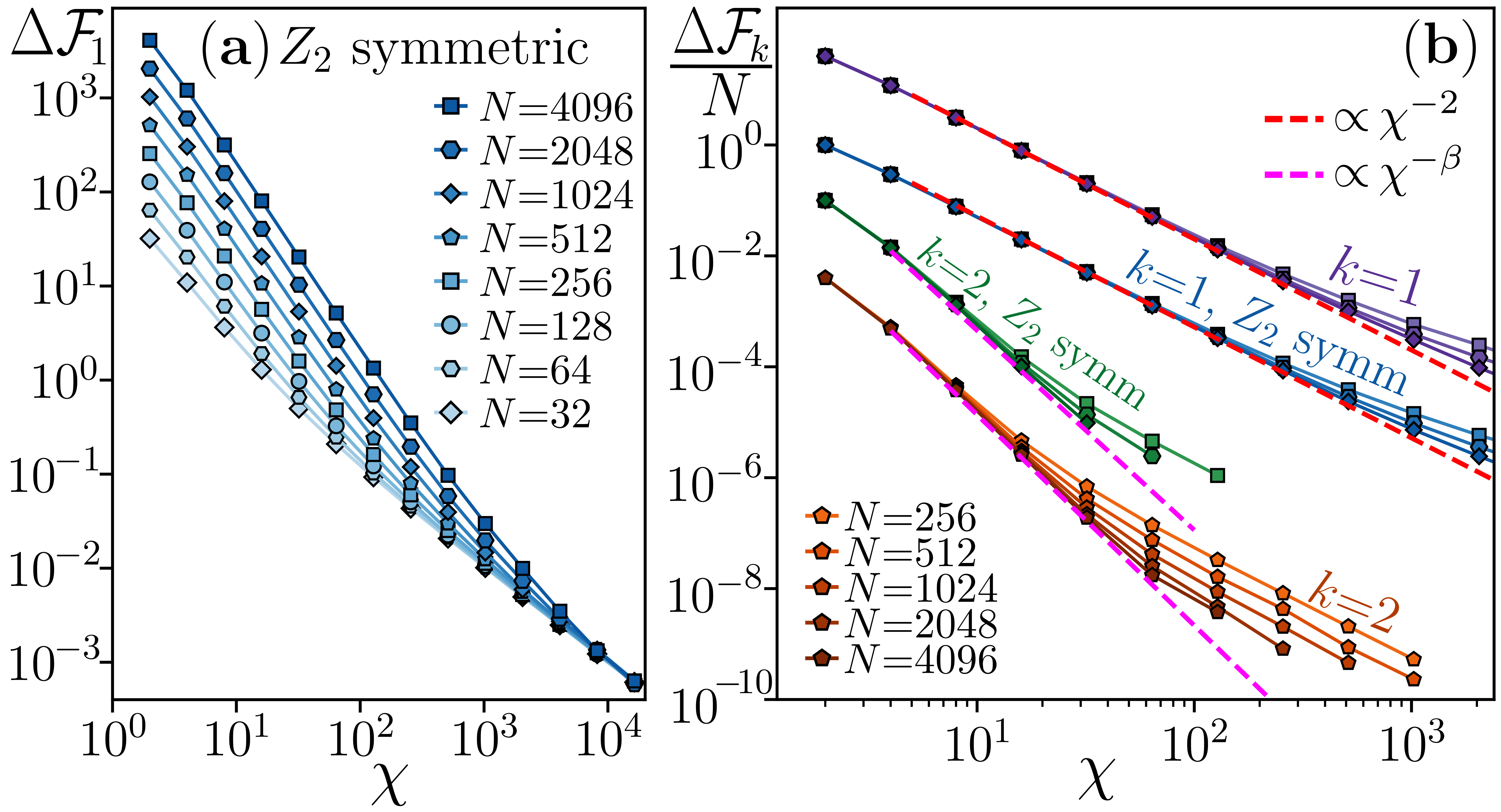
$$\mathcal{F}_k^{\mathrm{typ}} = N - C_k \frac{2^k N^{k+1}}{d^k} + O\left(\frac{N^k}{d^k}\right) \qquad \Delta\mathcal{F}_k = \mathcal{F}_k^{\mathrm{typ}} - \mathcal{F}_k,$$

- The Typical FAF for Harr random states can be computed using Wingarden calculus

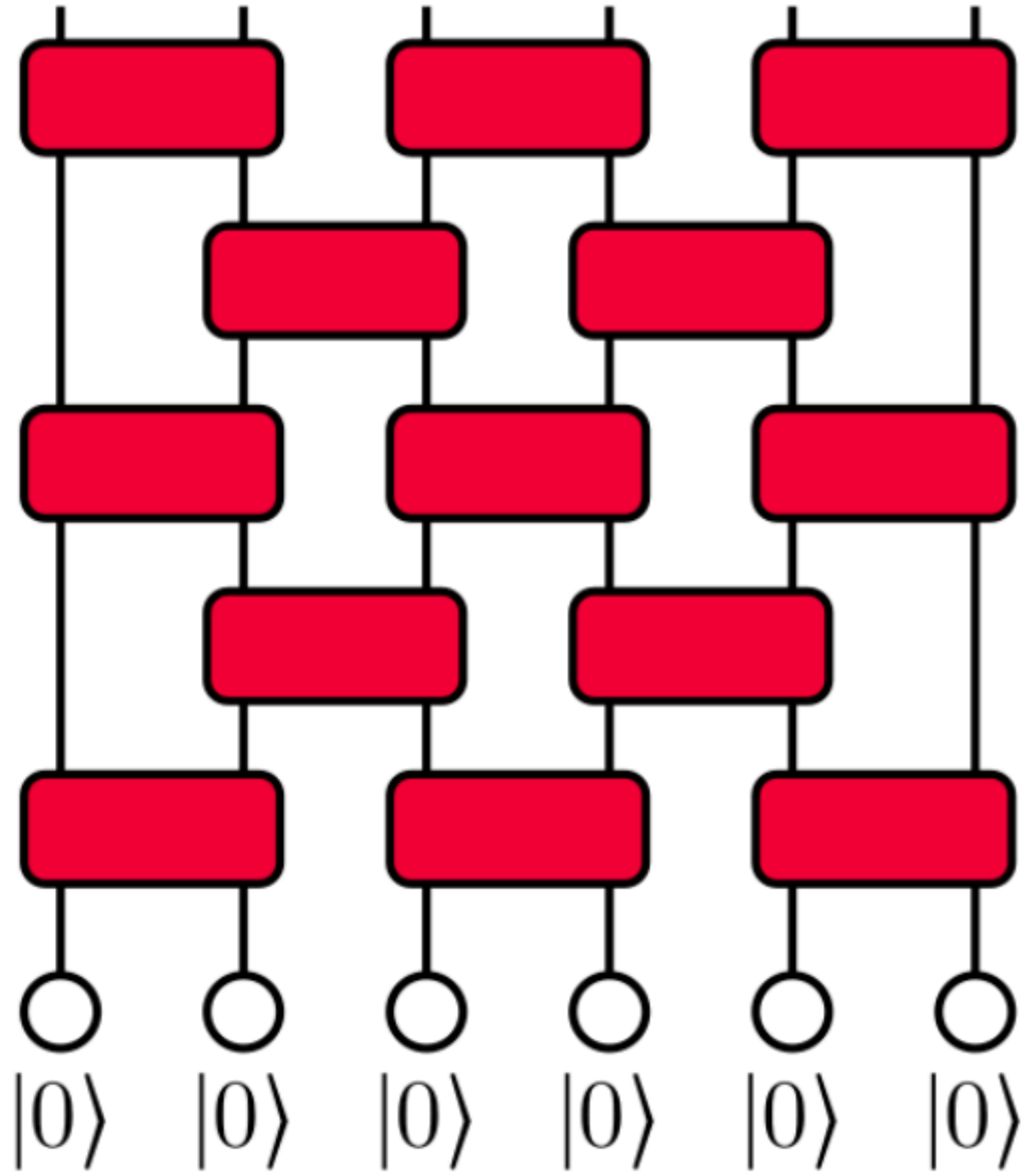
- Universal behaviour:

$$\mathcal{F}_k = \left(1 - \frac{a_k}{\chi^\beta}\right) N + b_k,$$

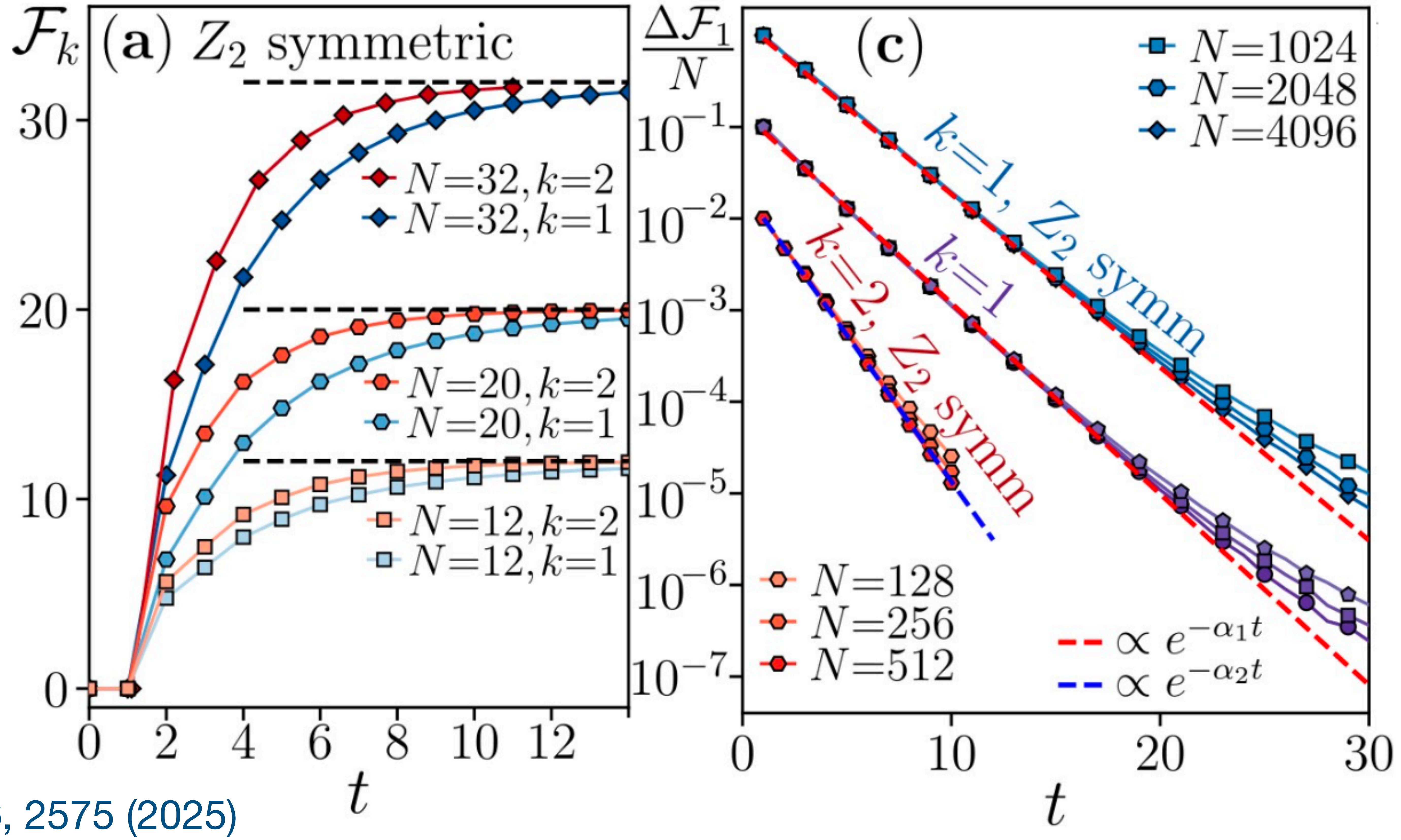
- This behavior mirrors that of the stabilizer Rényi entropy (SRE)



FAF Random quantum circuits



$$\mathcal{F}_k^{\mathrm{typ}} = N - C_k \frac{2^k N^{k+1}}{d^k} + O\left(\frac{N^k}{d^k}\right) \qquad \Delta\mathcal{F}_k = \mathcal{F}_k^{\mathrm{typ}} - \mathcal{F}_k,$$



- Saturation time at $t_{sat} \propto \log(N)$
- Similar to SRE

X. Turkeshi, E. Tirrito, P. Sierant, Nat. Comm. **16**, 2575 (2025)

Clifford Circuits and Stabilizer States

- Clifford group = gates that map Pauli operators → Pauli operators under conjugation.
- Generated by: H (Hadamard), S (phase), CNOT.
- Stabilizer states evolve into stabilizer states under Clifford circuits.
- Efficient classical simulation via *tableau representation*
- *Gottesman–Knill theorem: simulation cost is poly(n).*

$$|000> \to CNOT \otimes H|000>$$

	X	Z	Phase
Qubit 1	0	1	+
Qubit 2	0	1	+
Qubit 3	0	1	+

➡

	X	Z	Phase
Qubit 1	1	1	+
Qubit 2	1	1	+
Qubit 3	0	1	+

Scott Aaronson Daniel Gottesman Phys. Rev. A 70, 052328

Beyond Clifford and Stabilizer

- No quantum advantage in Clifford Circuits
- They can represent High Entanglement states: GHZ states are stabiliser
- We need a new gate:

$$T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$$

- The gate set $\{\text{Clifford} \cup T\}$ is dense in $SU(2^n)$
- T gate is called the magic gate from '*Magic gate insertion*'
- T gate count → complexity or 'Magic' in the system

Scott Aaronson Daniel Gottesman Phys. Rev. A 70, 052328

Stabilizer reny entropy

- The stabiliser Reny entropy **SRE** quantify the 'distance' of $|\psi\rangle$ from the set of Clifford states: L Leone, SFE Oliviero, A Hamma Physical Review Letters 128 (5), 050402

$$M_\alpha(\psi) = \frac{1}{1-\alpha}\log\left(\sum_{P\in\mathscr{P}_n}\left(\frac{|\langle\psi|P|\psi\rangle|^2}{2^n}\right)^\alpha\right)$$

- Where $P \in \mathscr{P}_n$ runs over all possible 4^n Pauli operator
- Exponentially costly in the volume to compute
- There are sampling technique for MPS PS Tarabunga, E Tirrito, T Chanda, M Dalmonte PRX Quantum 4 (4), 040317