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Scattering Amplitudes on a Quantum Annealer

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Exponential Feynman Combinatorics

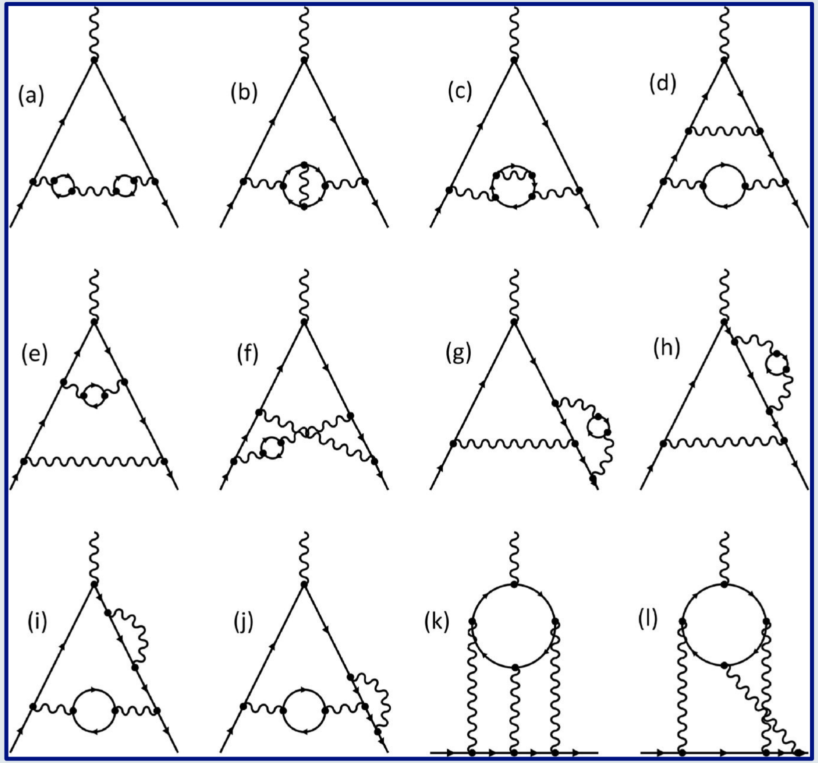
- Perturbation theory organises amplitudes as sums over allowed Feynman graph topologies.

- Tr[Φ^3] (cubic scalar) QFT:

$$\boxed{N_n^{\text{tree}} = C_{n-2} = \frac{1}{n-1}\binom{2n-4}{n-2} \sim \frac{4^n}{n^{3/2}}}$$

- Enhanced by loops, color d.o.f.s

$$\boxed{N_n^{\text{1-loop}} = \frac{1}{2}\binom{2n}{n} \sim \frac{4^n}{\sqrt{n}}}$$



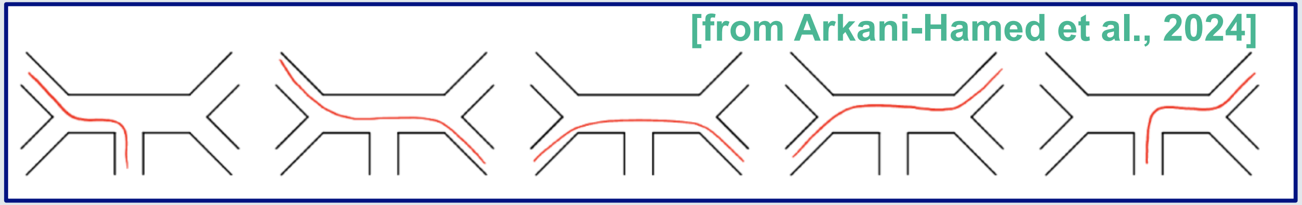
[Harlander 2024]

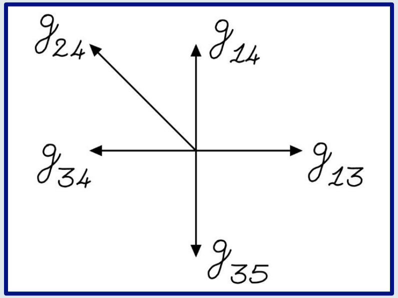
How compactly can the complete set of diagrams be encoded?

“Surfaceology” (a la Arkani-Hamed et al.)

<u>Defn:</u> Replacing a sum over diagrams with one global Schwinger integral

- Modern *amplitudes* methods seek complete descriptions w/o redundancies.





- Can select a fatgraph (at fixed multiplicity, loop, genus)
    - Induces a whole-amplitude integration domain.

$$\mathcal{A}_{n,L} = \int_{\mathbb{R}^E/\mathrm{MCG}} d^E t \left(\frac{\pi^L}{\mathcal{U}(t)}\right)^{D/2} \exp\left[\frac{\mathcal{F}_0(t)}{\mathcal{U}(t)} - \mathcal{Z}(t)\right]$$

All Loop Scattering as a Counting Problem 2023	$\longrightarrow$	*All Loop Scattering for All Multiplicity* 2023	$\longrightarrow$	*Tropical Amplitudes for Colored Lagrangians* 2024	$\longrightarrow$	*Surfaceology for Colored Yukawa Theory* 2024	$\longrightarrow$	*Surface Kinematics and “The” Yang–Mills Integrand* 2024

"Surfaceology": Benefits and Limitations

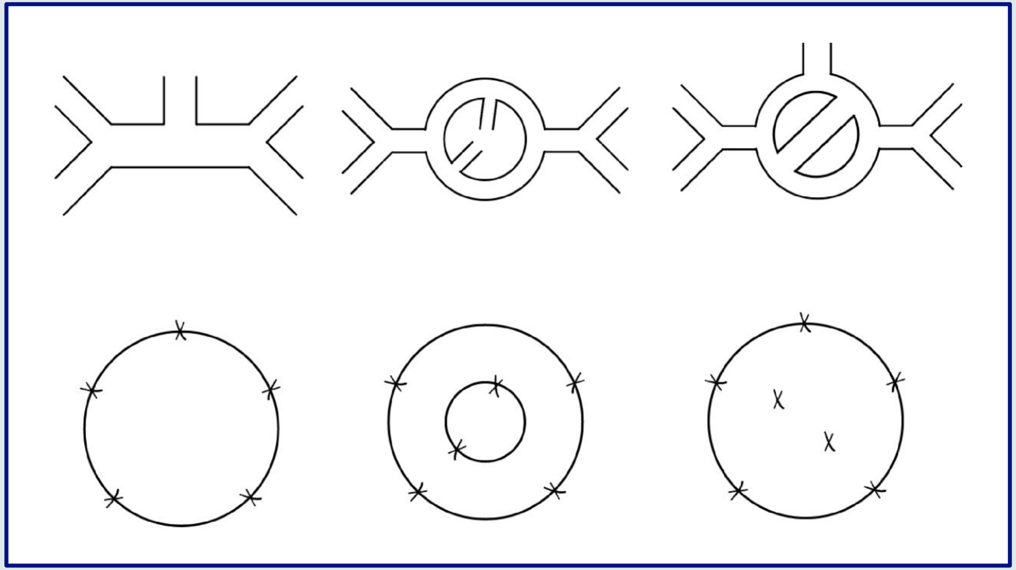
- Scalar, tree-level: $O(4^n)$ diagrams $\rightarrow$ $O(n^2)$ curve functions
- Given a **compact tropical numerator**, exponential evaluation averted
  - Shown even for colored $\mathrm{Tr}[\Phi^3]$, colored Yukawa (still 2^L species)

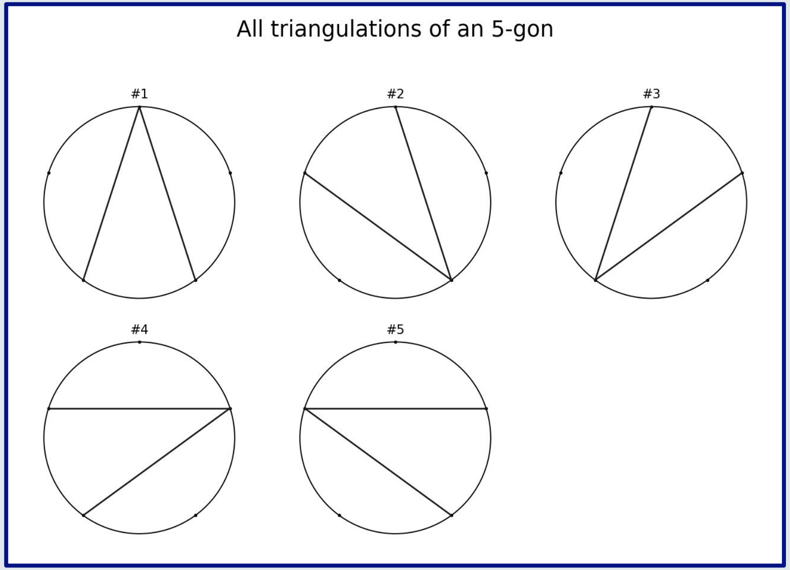
- For pure YM, compression at one-loop for all multiplicity, not far beyond…
- For gauge theories with matter, compact representation remains open

Select QFT, multiplicity, loop order, genus $\longrightarrow$ Enumerate triangulations $\longrightarrow$ Assemble integrand $\longrightarrow$ Evaluate amplitude

We will cast the enumeration stage in a new light

Feynman Diagrams are Triangulations

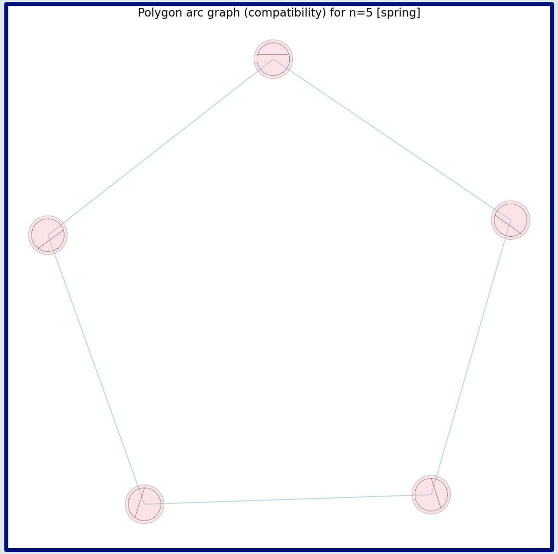




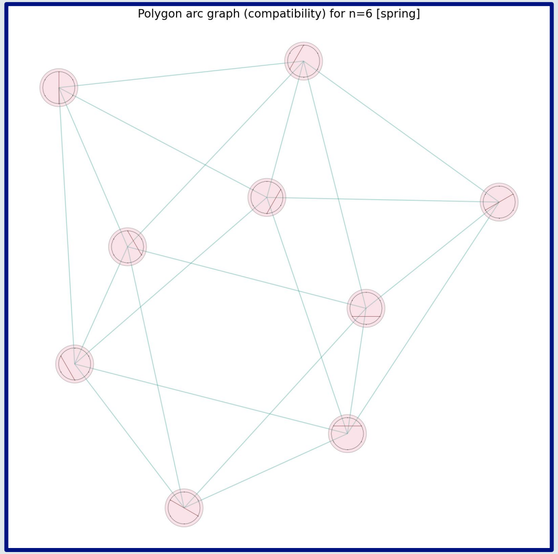
[Arkani-Hamed et al., 2024]

- Cubic interactions give triangulations; higher-valence, polyangulations
- Color data, matter, impose decorations on the surface and its arcs

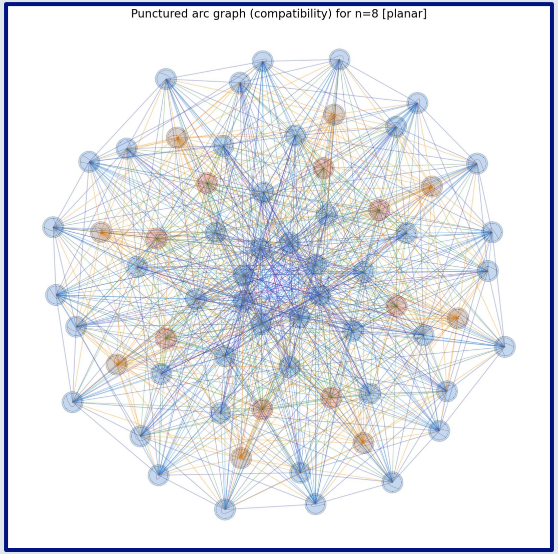
Triangulations are Max Cliques



n=5, L=0



n=6, L=0

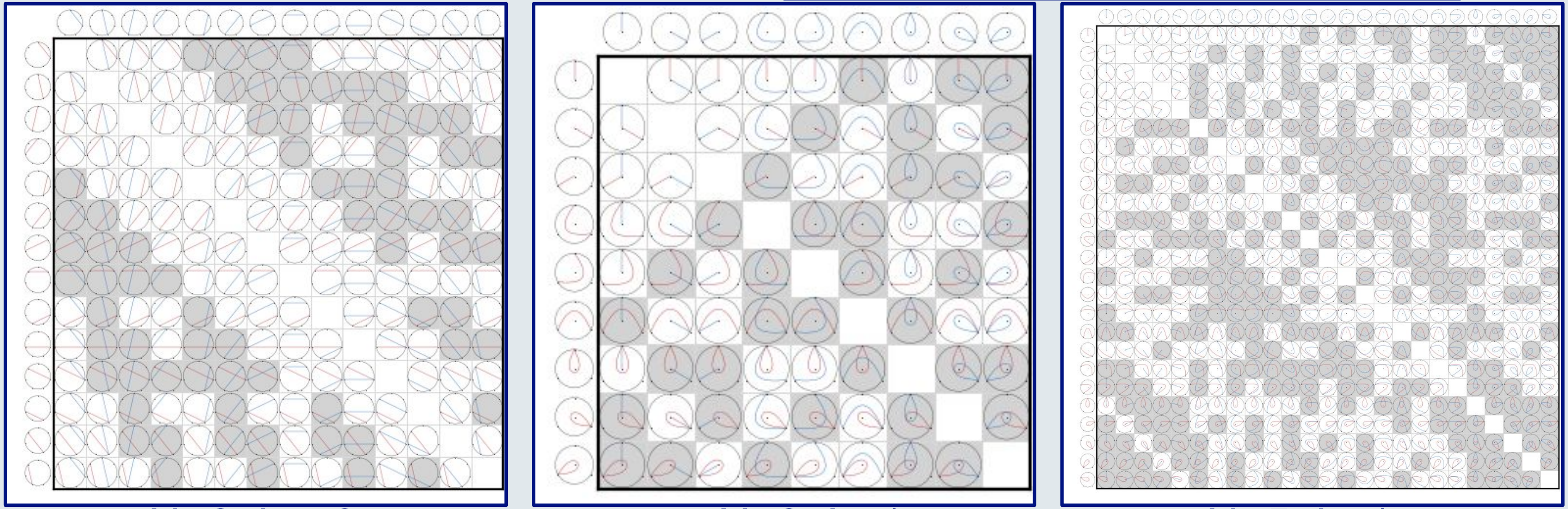


n=8, L=1

- “Arc complex” defines undirected compatibility graph between arcs
- For polynomially-many arcs, adjacency matrix is manageable and sparse
- **Max cliques:** maximal subsets of vertices which are fully-connected
  - (equivalently, maximum independent sets of the intersection graph)

Max Cliques are Ising Ground States (QUBO)

- Let's define a binary optimisation problem, with one variable per arc:
  - Compatibility rules, clique size: $H_{\mathrm{QUBO}}(\mathbf{x}) = A \sum_{(i,j)\in\mathcal{I}} x_i x_j + B\left(\sum_i x_i - k\right)^2$

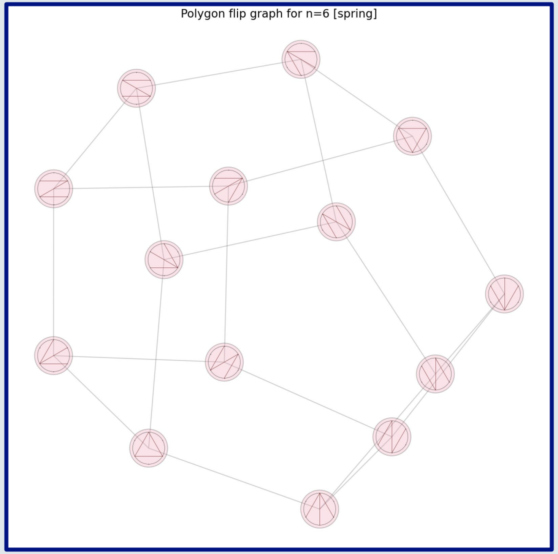


N=6, L = 0 N=3, L=1 N=5, L=1

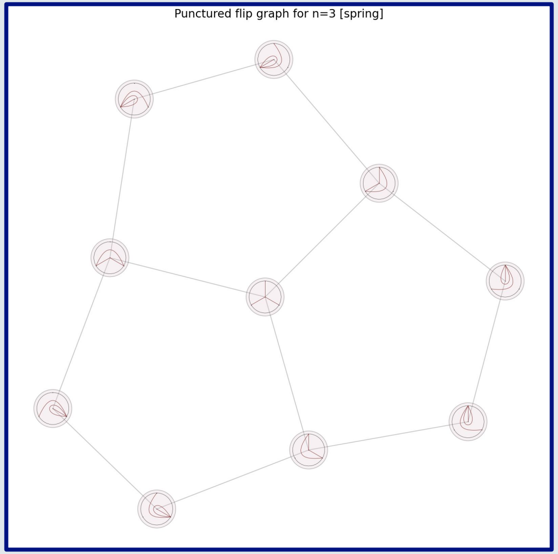
- Map to spin variables: $x_i = \frac{1-Z_i}{2} \implies H_{\mathrm{Ising}} = \sum_i h_i Z_i + \sum_{i<j} J_{ij} Z_i Z_j$

Flip Graphs and Implicit Complexity

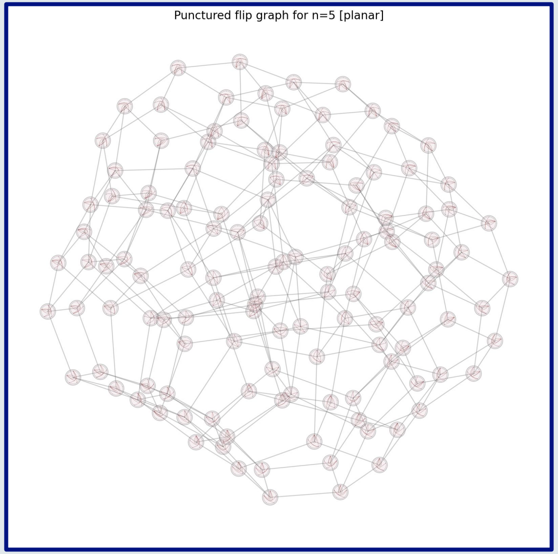
Define flip operation as unique replacement of arc within a triangulation



N=6, L = 0



N=3, L=1

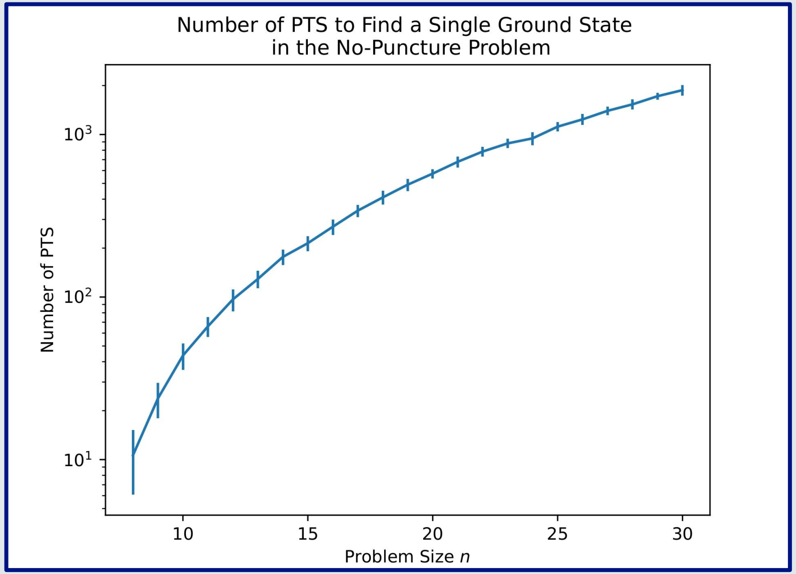


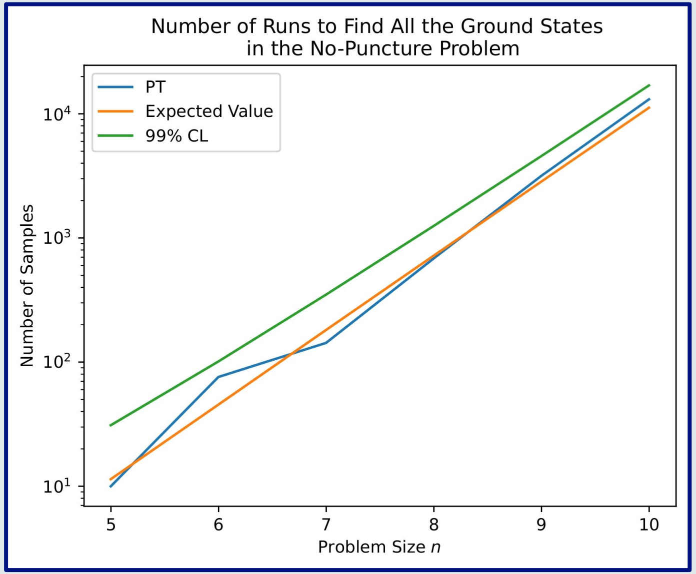
N=5, L=1

- Tree level: flip graph is regular – indeed, it is the *associahedron*.
- Loops grow search space rapidly, and introduce graph irregularities

Classical Sampling Methods

- **Simulated Annealing:** random spin flips accepted with cooling schedule
- **Parallel Tempering:** annealer replicas also exchange configurations





Massive thanks to **Will Gilmore** (Soton ‘26) for getting these methods running!

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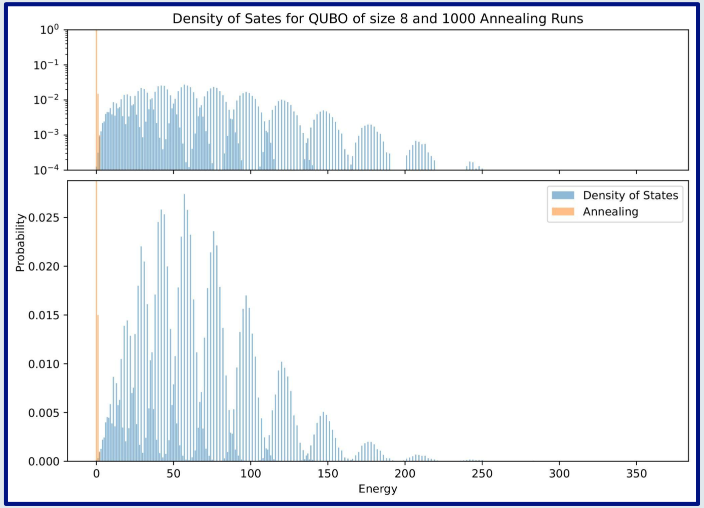

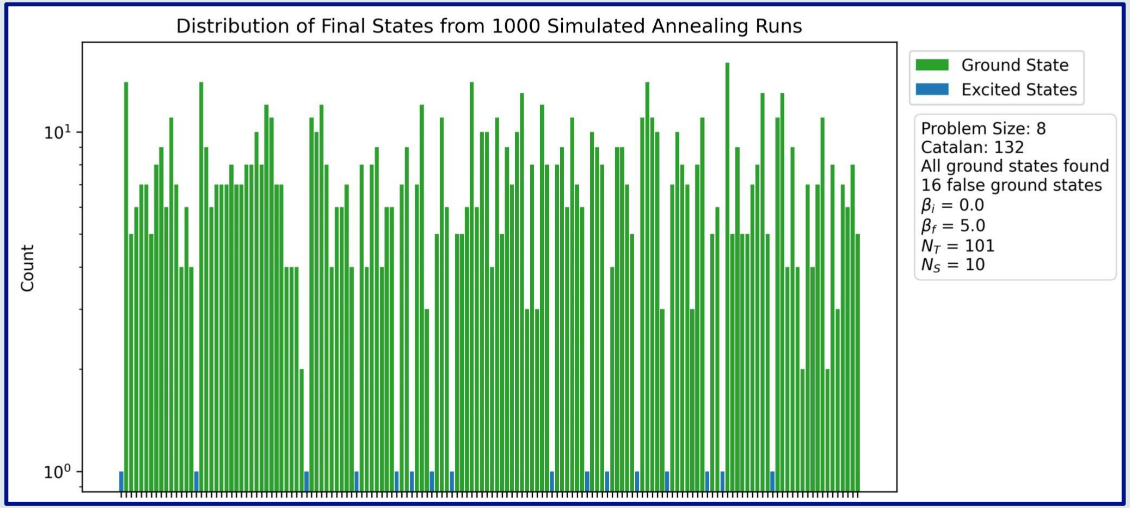

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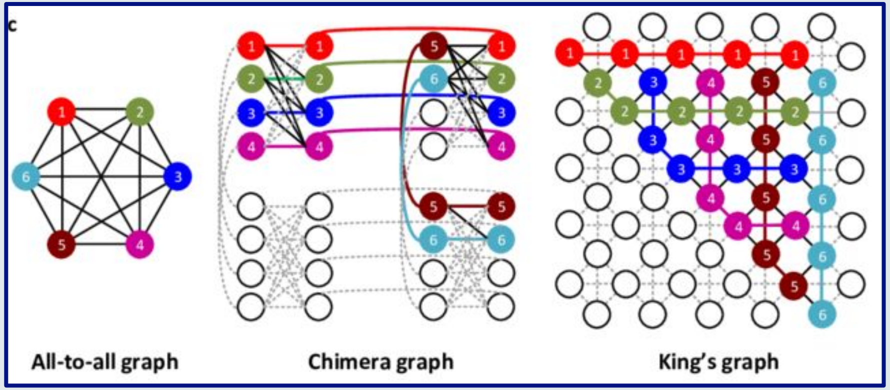
Quantum Annealing with D-WAVE

Ising Graph → Minor Embedding → Physical Anneal → Decode Chain

- Physical qubits grouped into chains, which encode one logical variable.



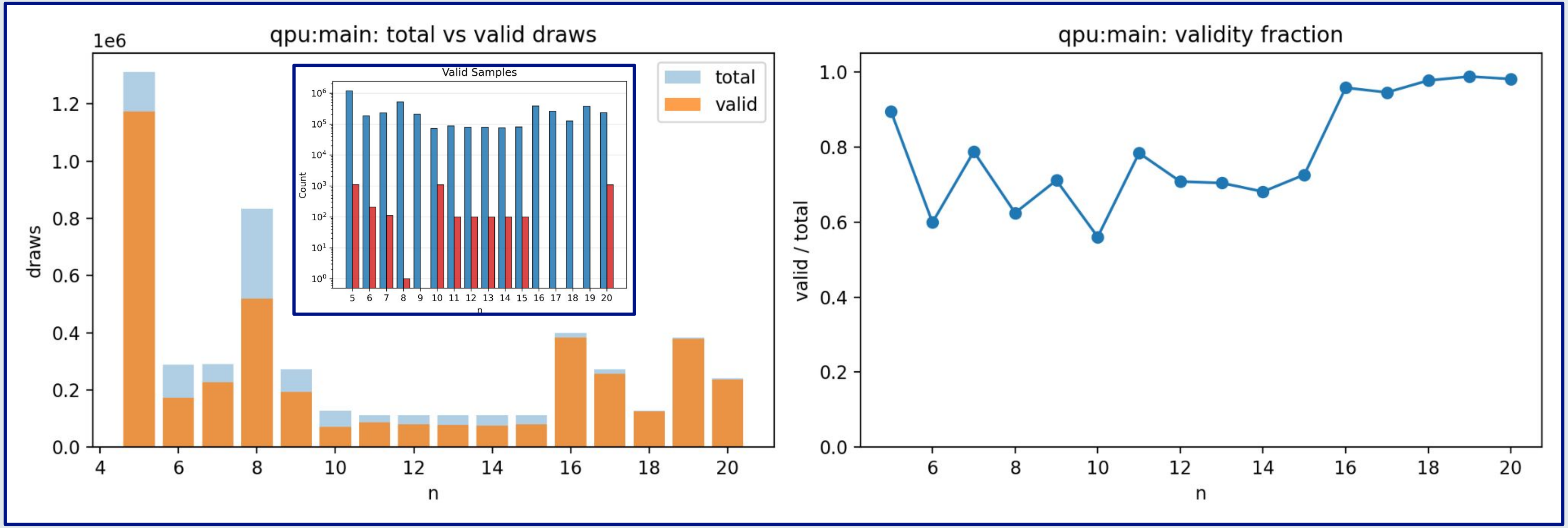
[from Lo et al., 2022]

- Chain strength J is ferromagnetic coefficient on chain agreement.

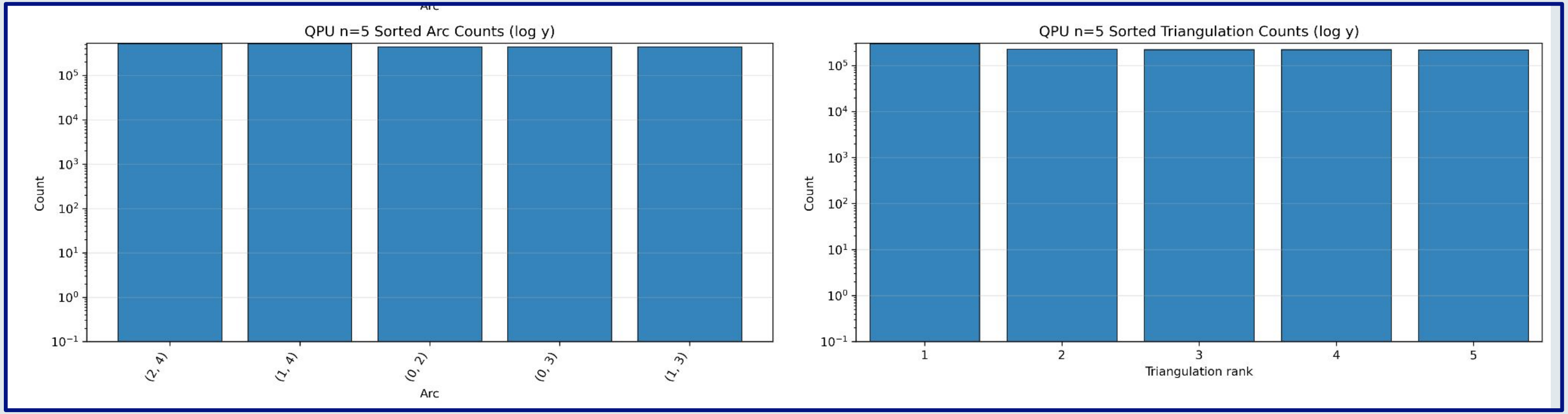
- Hamiltonian is driven gradually
  - Wide literature on fair sampling of degenerate ground states

- Direct QPU (more control) vs Hybrid (better scaling)

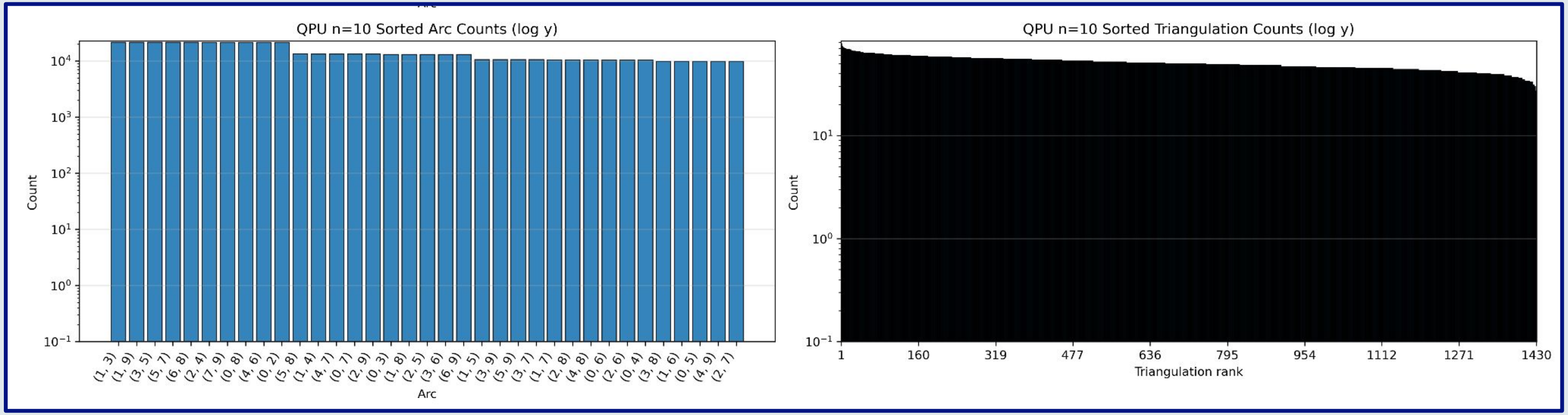
Results: Validity, Coverage and Uniformity



Results: Validity, Coverage and Uniformity

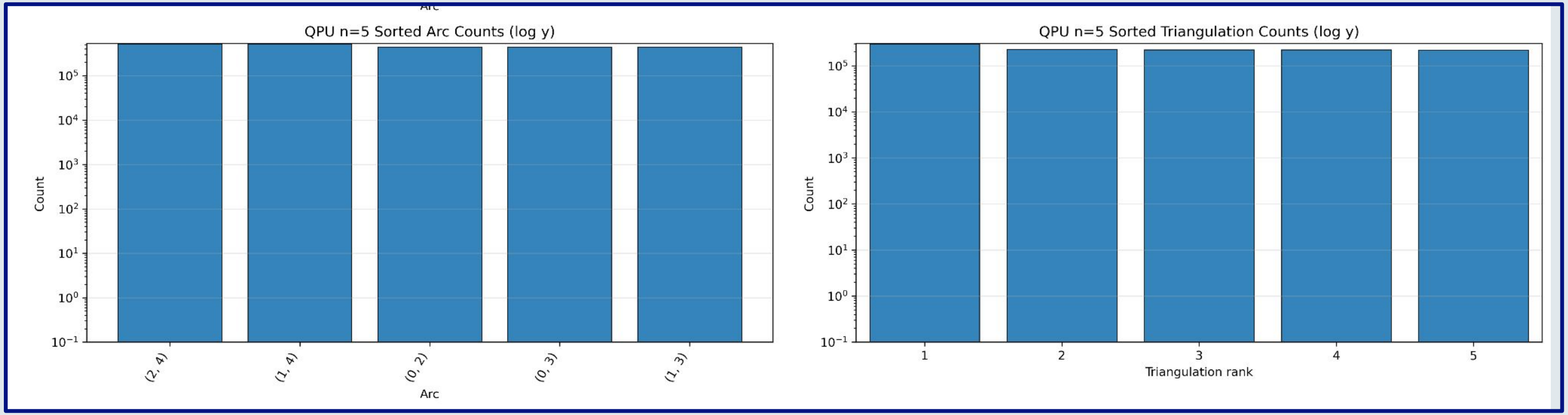


N=5
L=0
100% cov



N=10
L=0
100% cov

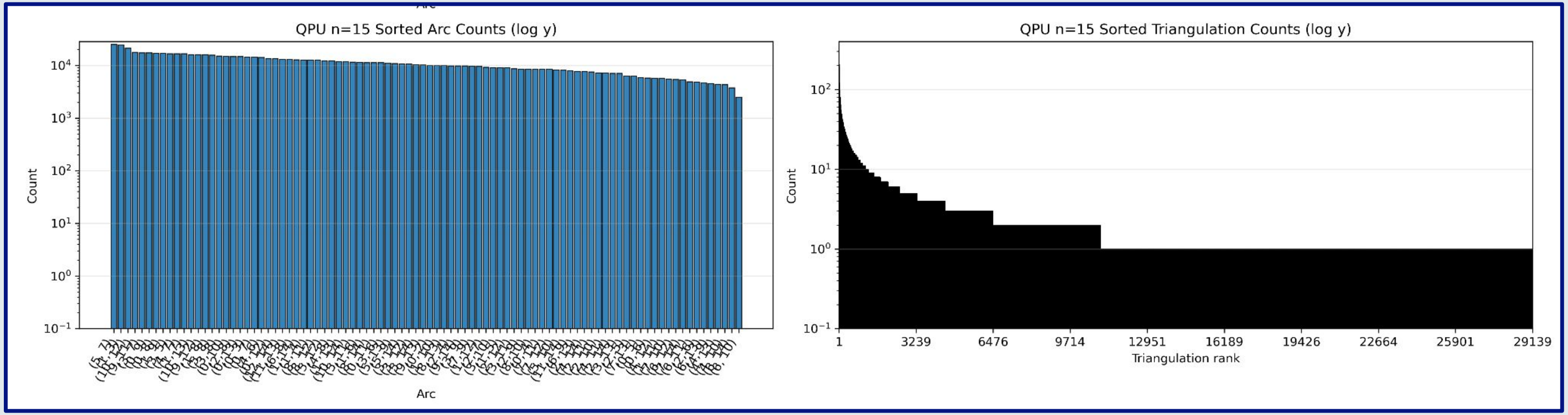
Results: Validity, Coverage and Uniformity



N=5

L=0

100% cov



N=15

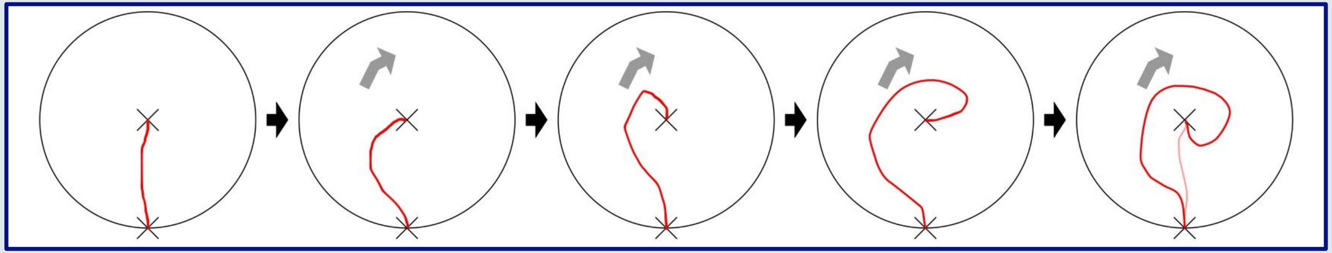
L=0

~50% cov

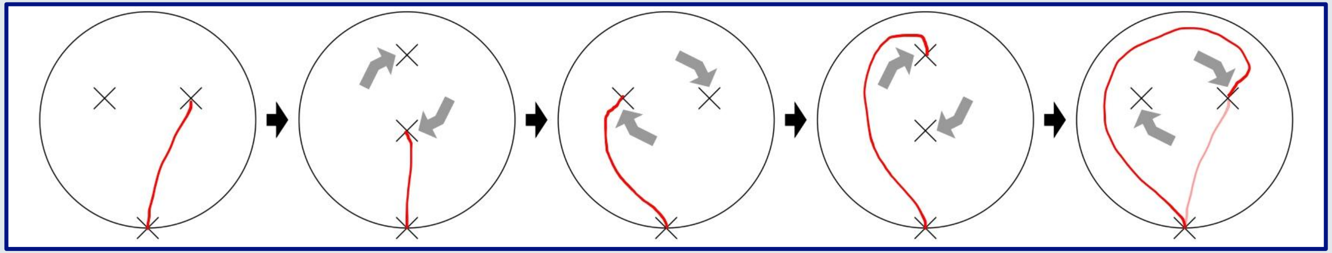
Coming Soon: 2-Loop and Beyond

- Two punctures introduce infinitely many winding representatives.
  - But the combinatorial problem is made finite by quotienting by MCG
  - Isotopy classes of arcs must be decorated by a truncated winding

L=1



L=2



What's Next?

Key Achievements:

- Feynman diagram combinatorics formulated as max-clique problem
- QUBO embedding made explicit for 0,1,2-loops, all multiplicity
- SA and PT classical benchmark results with exp. degeneracy
- **First quantum sampling of 0,1-loop Feynman diagrams**

<u>Coming Soon</u>

- Specify embedding for N-loops with all multiplicity (genus too?)
- Push SA/PT to the point of failure, study scaling
- Analysis of nonuniformity propagation to amplitudes

Thank you for listening!

Works Cited

Surfaceology and amplitudes

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Compute Resources

[Izubuchi, 2019]