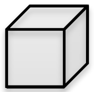
Quantum anomaly for benchmarking quantum computing

Arata Yamamoto (RIKEN)

Tomoya Hayata & AY, arXiv:2603.03697 [hep-lat]

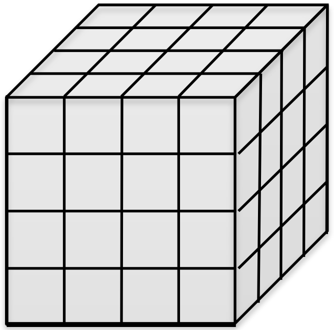
Introduction

how to verify quantum simulation?



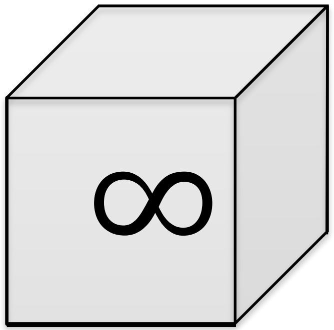
small lattice

exact diagonalization



large lattice

tensor networks etc.



quantum field theory

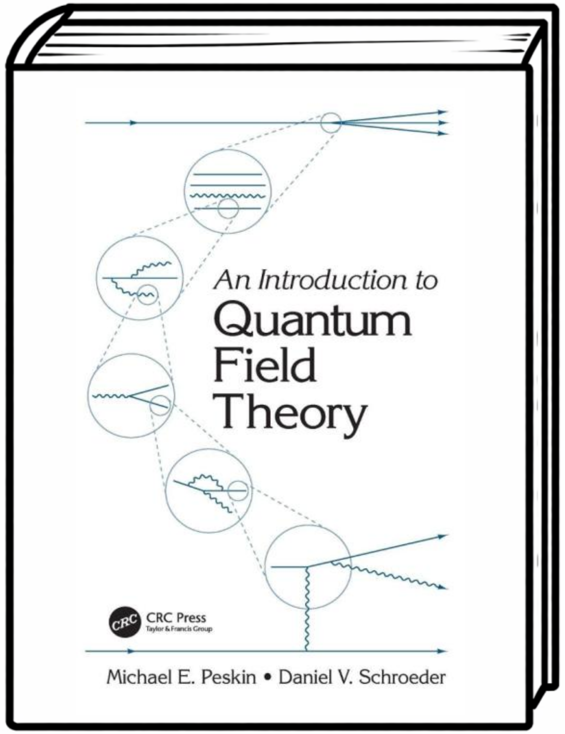
???

Introduction

how to verify quantum simulation?

- ✓ exact answer is known
- ✓ non-trivial (interacting) theory
- ✓ calculable on present small devices
- ✓ scalable to future large devices

Chiral anomaly



chiral anomaly in 1+1 dim.

Fourier transform of the field equation

$$\partial_\mu j^{\mu 5} = \frac{e}{2\pi}\epsilon^{\mu\nu}F_{\mu\nu}.$$

the axial vector current is not conserved in the pres
fields, as the result of an anomalous behavior of its
iagram.

chiral anomaly in 1+3 dim.

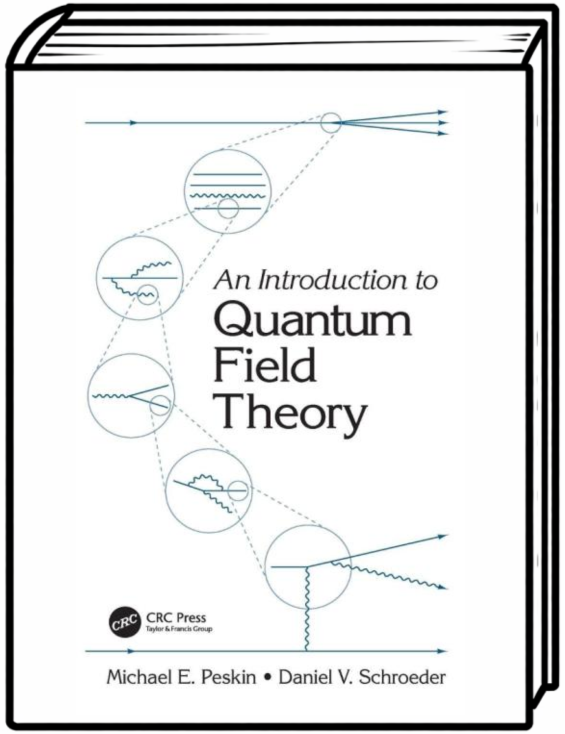
e symmetric limit $\epsilon \to 0$ in four dimensions. We find

$$\partial_\mu j^{\mu 5} = -\frac{e^2}{16\pi^2}\epsilon^{\alpha\beta\mu\nu}F_{\alpha\beta}F_{\mu\nu}.$$

on, which expresses the anomalous nonconservation of th
axial current, is known as the Adler-Bell-Jackiw anomaly

Chiral anomaly

© CRC Press



“exact” in all orders

chiral anomaly in 1+1 dim.

Fourier transform of the field equation

$$\partial_\mu j^{\mu 5} = \frac{e}{2\pi}\epsilon^{\mu\nu}F_{\mu\nu}.$$

the axial vector current is not conserved in the pres
fields, as the result of an anomalous behavior of its
iagram.

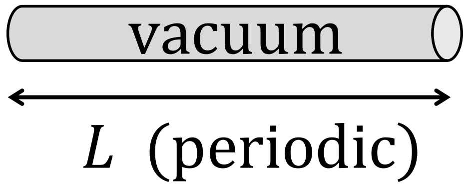
chiral anomaly in 1+3 dim.

e symmetric limit $\epsilon \to 0$ in four dimensions. We find

$$\partial_\mu j^{\mu 5} = -\frac{e^2}{16\pi^2}\epsilon^{\alpha\beta\mu\nu}F_{\alpha\beta}F_{\mu\nu}.$$

on, which expresses the anomalous nonconservation of th
axial current, is known as the Adler-Bell-Jackiw anomaly

Chiral anomaly

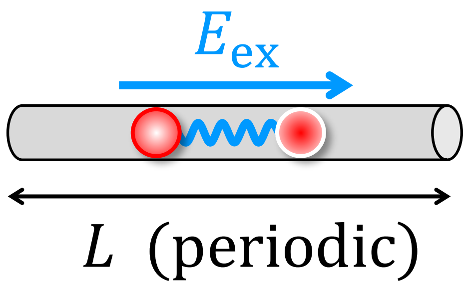


Chiral anomaly

external electric field

$$\frac{d}{dt}\langle Q\rangle = \frac{1}{\pi} L e E_{\mathrm{ex}}$$

exact number



$\langle Q\rangle$ = # of right-handed electrons − # of left-handed electrons

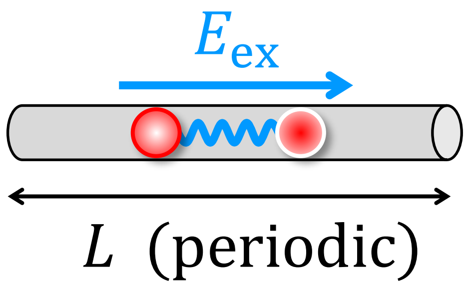
Chiral anomaly

input parameter

$$\frac{d}{dt}\langle Q\rangle = CLeE_{\mathrm{ex}}$$

output data

fitting result



if $C = \frac{1}{\pi} \pm$ (statistical error) $\Longrightarrow$ simulation is correct

Nielsen-Ninomiya no go theorem   Nielsen, Ninomiya (1981)

"chiral symmetry (or one of basic properties) is violated on lattices"

lattice perturbation   Ambjorn, Greensite, Peterson (1983)

$$\frac{d}{dt}\langle Q\rangle = \frac{d}{dt}\int dx\,\langle\psi^{+}\gamma^{5}\psi\rangle = O(a) \to 0$$

no anomaly???

nonconserved chiral charge

$$Q = \int dx\, \psi^{+} \gamma^{5} \psi$$

conserved chiral charge

$$Q = \int dx\, \psi^{+} \gamma^{5} \psi + O(a)$$

$$\mathrm{s.t.} \quad [Q, H_{\mathrm{fermion}}] = 0$$

Hidaka, Yamamoto (2025)

nonconserved chiral charge

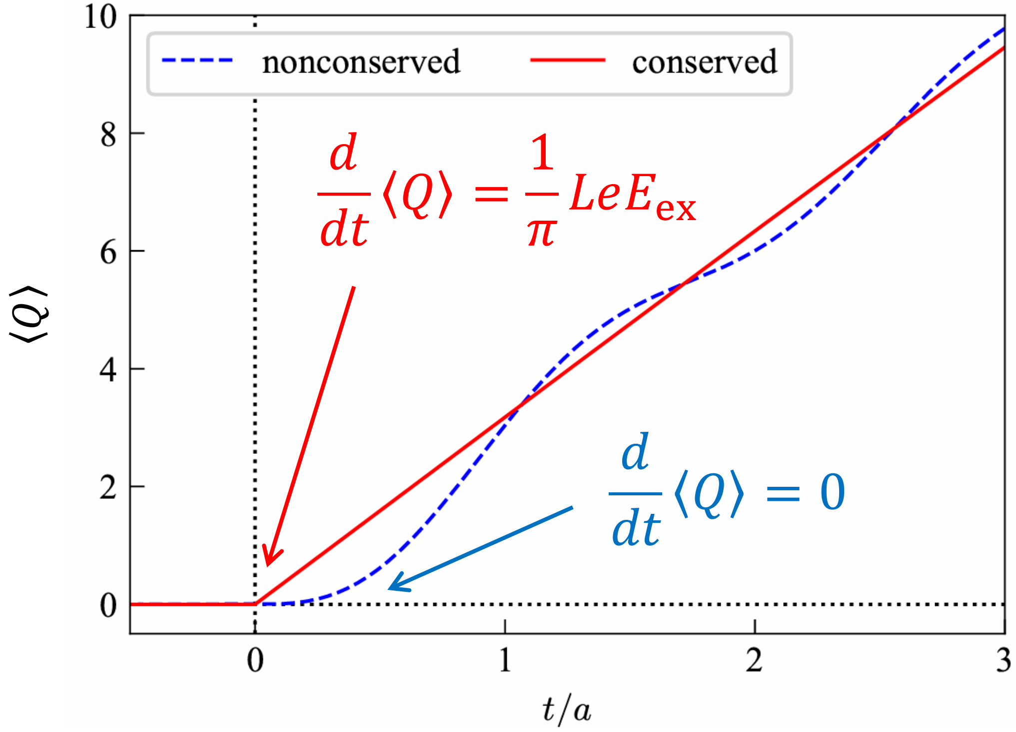
$$Q = \int dx\, \psi^{+}\gamma^{5}\psi$$

conserved chiral charge

$$Q = \int dx\, \psi^{+}\gamma^{5}\psi + O(a)$$

$$\text{s.t.} \quad [Q, H_{\text{fermion}}] = 0$$

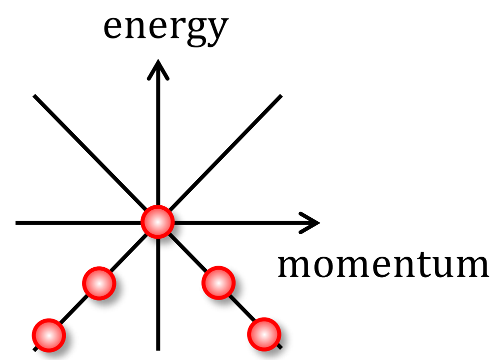
classical simulation of free fermion



anomaly is reproduced by **infinitesimal time evolution = a single Trotter step**

Circuit

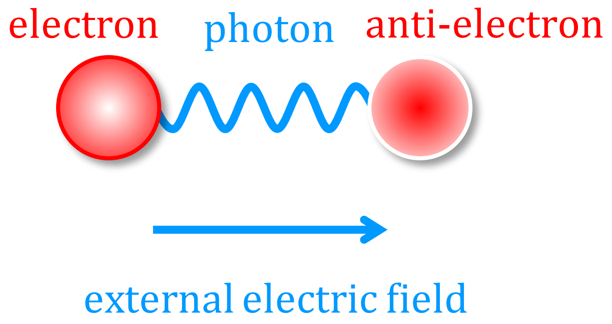
1. initial state preparation



free electron vacuum

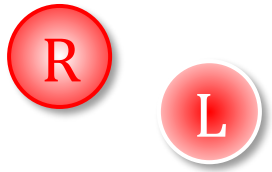
$$\langle Q \rangle_{t=0} = 0$$

2. time evolution



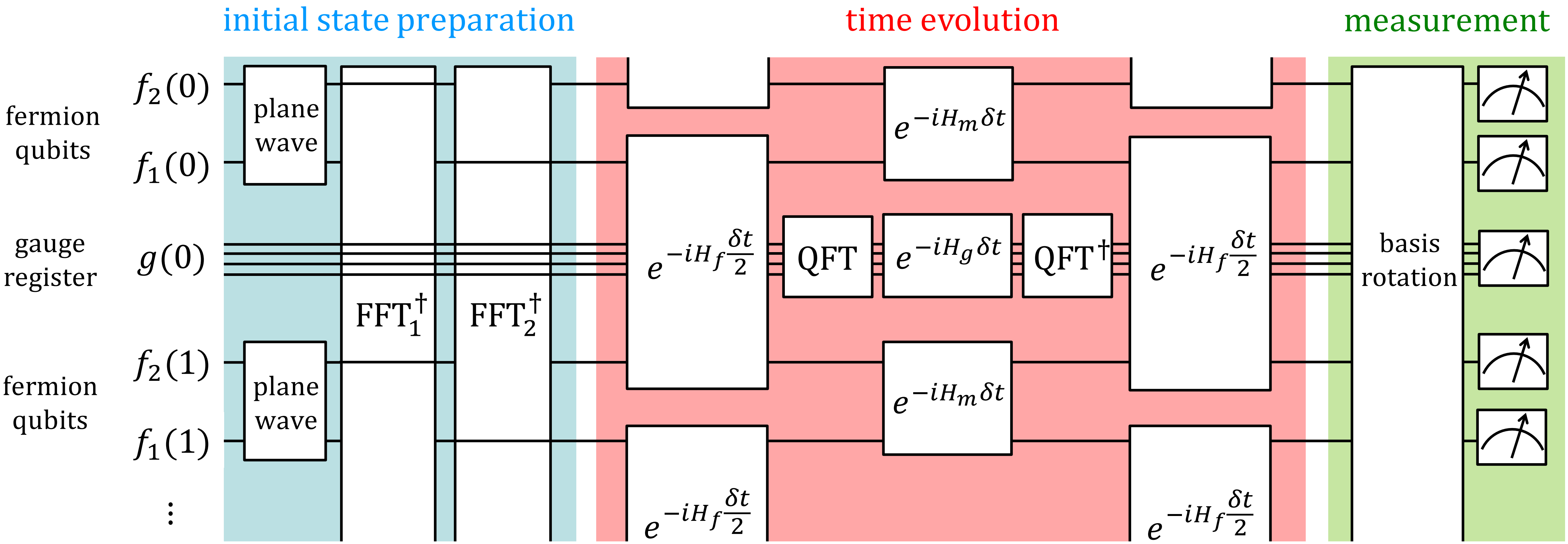
Trotter step δt

3. measurement

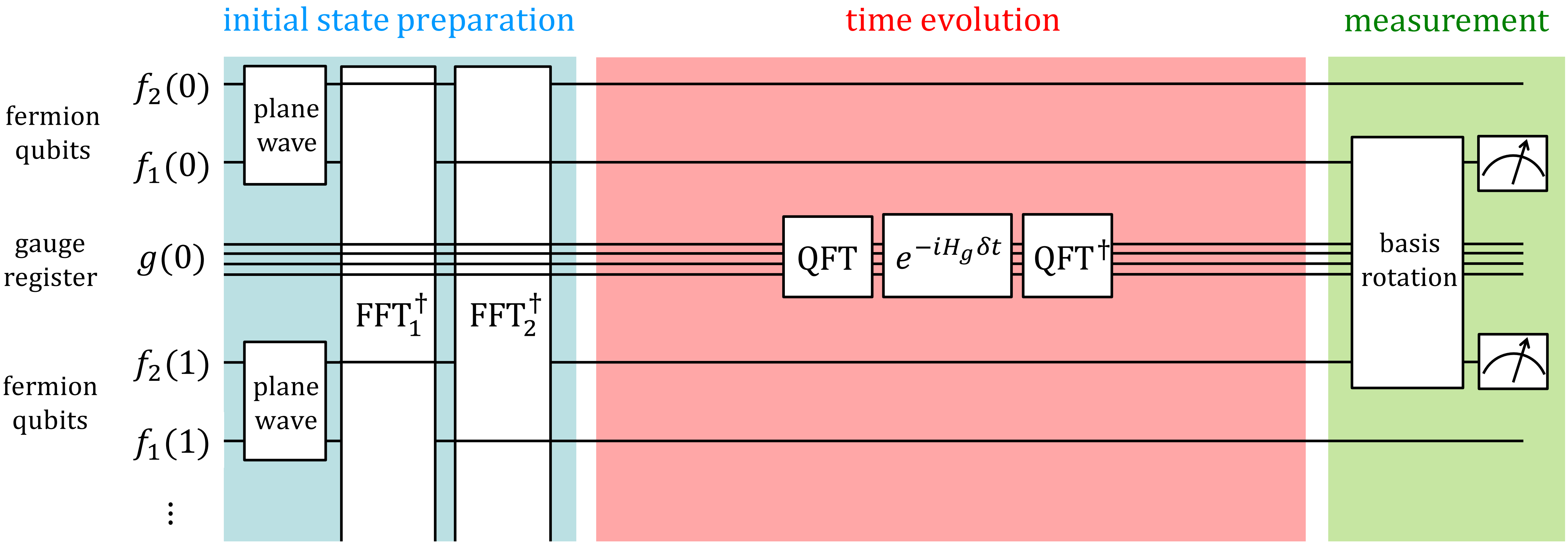


$$\langle Q \rangle_{t=\delta t} > 0$$

Circuit


initial state preparation
time evolution
measurement
fermion qubits
gauge register
fermion qubits
$f_2(0)$
$f_1(0)$
$g(0)$
$f_2(1)$
$f_1(1)$
⋮
plane wave
plane wave
$\mathrm{FFT}_1^\dagger$
$\mathrm{FFT}_2^\dagger$
$e^{-iH_f\frac{\delta t}{2}}$
$e^{-iH_m\delta t}$
QFT
$e^{-iH_g\delta t}$
$\mathrm{QFT}^\dagger$
$e^{-iH_m\delta t}$
$e^{-iH_f\frac{\delta t}{2}}$
basis rotation

Circuit



drastic circuit simplification !!

“Reimei”

- ✓ deployed at RIKEN in Japan
- ✓ Quantinuum System Model H1
- ✓ all-to-all connectivity
- ✓ 20 qubits

Z_N gauge theory on a finite lattice

1. U(1) gauge $Z_4, Z_8, Z_{16} \to Z_\infty$
2. infinitesimal time $\delta t \to 0$
3. infinite volume $L = 3, 4, 5 \to \infty$
4. continuum limit $e \to 0$

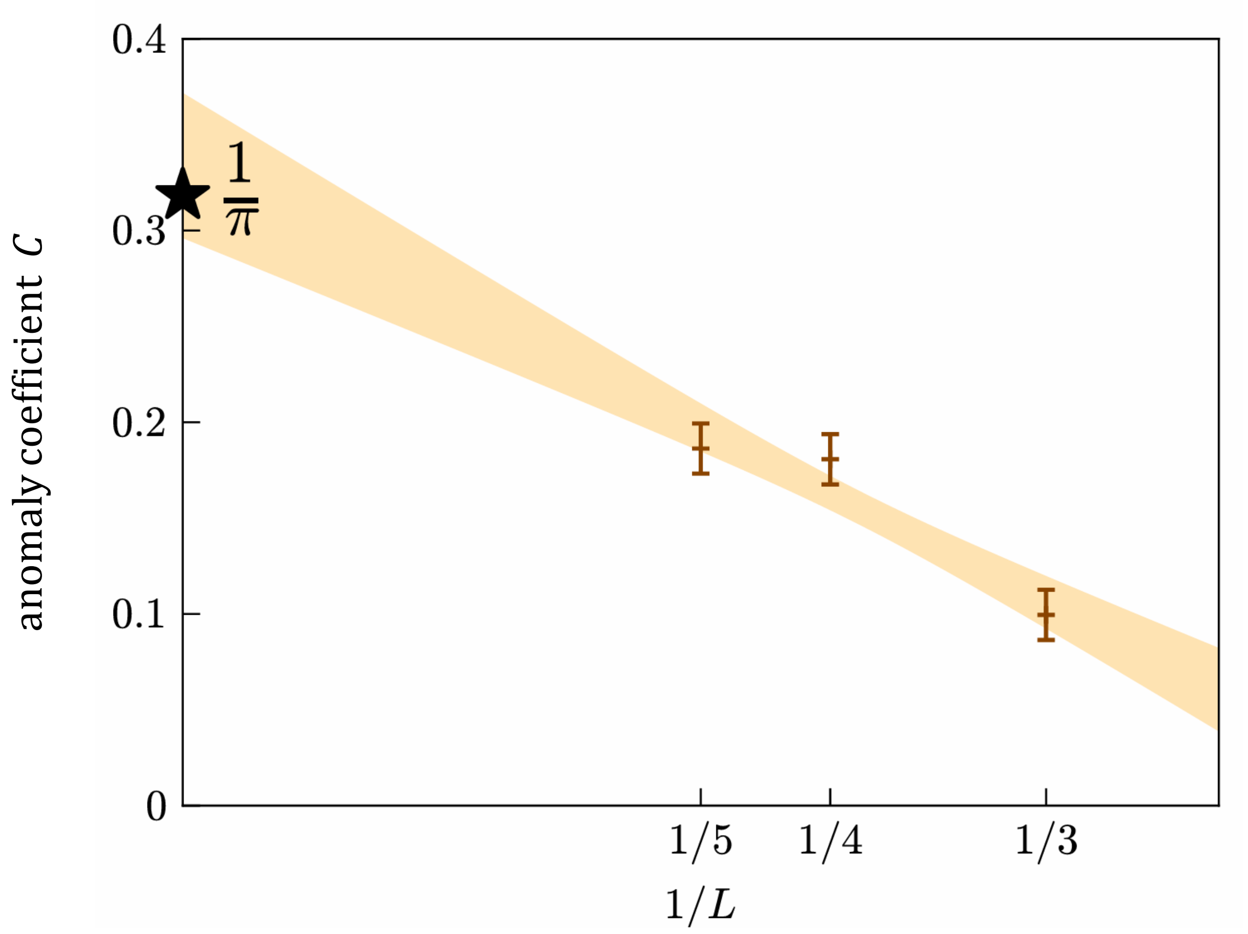
U(1) gauge theory in infinite continuous space

Quantum simulation

Z_N gauge theory on a finite lattice

1. U(1) gauge $Z_4, Z_8, Z_{16} \to Z_\infty$
2. infinitesimal time $e^2 \delta t = 0.3, 0.2, 0.1 \to 0$
3. infinite volume $L = 3, 4, 5 \to \infty$
4. ~~continuum limit~~ $\cancel{e \to 0}$ for a single Trotter step

U(1) gauge theory in infinite continuous space simulation on Reimei (**no error mitigation**)



good agreement !!

Summary

chiral anomaly has been reproduced on a quantum computer