Implications of f_1 mixing angle and mass spectrum of axial vector mesons

Yoshito Nakane, Daisuke Jido

Institute of Science Tokyo

International Workshop on Origin of Mass and Matter in the Universe
APCTP, 2026/07/23 (Thu.)

Table of Contents

Table of Contents

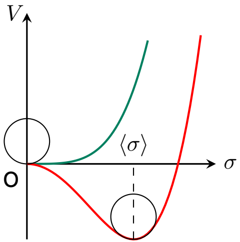
Background

Spontaneous breaking of chiral symmetry

- Perturbative expansion is difficult in low energy region because of infrared slavery in QCD

Partial restoration of chiral symmetry breaking

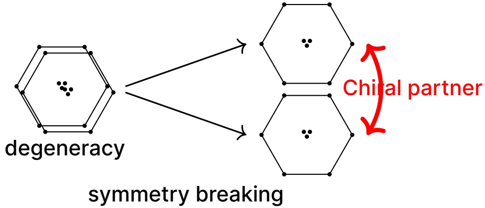
- Chiral symmetry breaking is restored in finite density and finite temperature

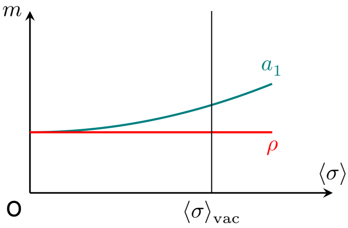


Chiral partner I

Chiral partner : A pair of fields that are transformed by chiral transformation each other

- Partner inside chiral representation
- **Degeneracy** between chiral partners if chiral symmetry is **completely restored**
- Residue of degeneracy might be seen even in vacuum
- Naturally thinking **similar properties are expected**





Chiral partner II

We focus on **vector meson – axial vector meson** chiral partner

Vector–Axial vector chiral partner

- Singlets are not chiral partner[Lee, 2024]
- but are parity partner

vector octet + axial vector octet

vector singlet

axial vector singlet

How does $\mathrm{SU}(3)$ breaking affect on chiral partner properties?

- $\mathrm{SU}(3)$ breaking effects are similar in **octet**

Isoscalar mixing I

We can not extract only singlet state

Isoscalar mixing

- $\overline{q}q$ nonet has two isoscalar states: singlet and octet isosinglet
- Same quantum number $I^G J^{PC}$
- Physical states have mixing between singlet and octet isosinglet parametrized by mixing angle θ

$$\begin{pmatrix} |\phi\rangle \\ |\omega\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |\omega^{\text{singlet}}\rangle \\ |\omega^{\text{octet}}\rangle \end{pmatrix}, \quad \omega^{\text{singlet}} = \frac{\overline{u}u + \overline{d}d + \overline{s}s}{\sqrt{3}}, \ \omega^{\text{octet}} = \frac{\overline{u}u + \overline{d}d - 2\overline{s}s}{\sqrt{6}} \tag{1}$$

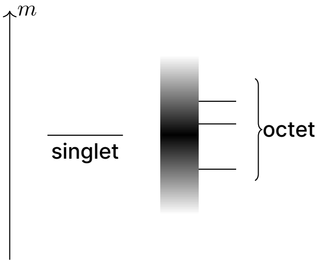
Ideal mixing

- Certain angle which separates contents into $(\overline{u}u + \overline{d}d)/\sqrt{2}, \overline{s}s$
- Assumed to be good in vector meson isoscalar ω – ϕ
- Natural consequence of $\mathrm{SU}(3)$ breaking

Isoscalar mixing II

Meaning of isoscalar mixing

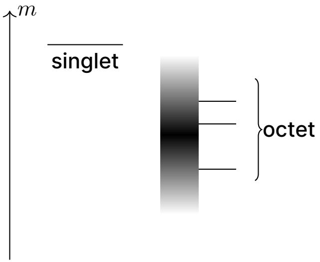
- Mixing angle is result of comparison between $\mathrm{SU}(3)$ breaking contribution and mass difference between singlet and octet
- How overlap singet to wide octet$\rightarrow$how natural mixing is
- We can expect ideal mixing when it mixes sufficiently



Isoscalar mixing II

Meaning of isoscalar mixing

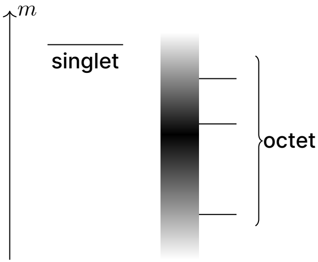
- Mixing angle is result of comparison between $\mathrm{SU}(3)$ breaking contribution and mass difference between singlet and octet
- How overlap singet to wide octet→how natural mixing is
- We can expect ideal mixing when it mixes sufficiently



Isoscalar mixing II

Meaning of isoscalar mixing

- Mixing angle is result of comparison between $\mathrm{SU}(3)$ breaking contribution and mass difference between singlet and octet
- How overlap singet to wide octet$\rightarrow$how natural mixing is
- We can expect ideal mixing when it mixes sufficiently



f_1 mesons

f_1 mesons in classification scheme

- Quantum numbers $I^G J^{PC} = 0^+ 1^{++}$ (Isoscalar in axial vector mesons)
- Two states in $\bar{q}q$ nonet: singlet and octet isosinglet

f_1 mesons in experiments

Name	Mass (MeV)	Width (MeV)	Decay destinations and branching ratio
$f_1(1285)$	1281.8±0.5	23.0±1.1	$4\pi : (32.7 \pm 1.8)\%$ $\eta\pi\pi : (52.2 \pm 1.9)\%$ $K\overline{K}\pi : (9.0 \pm 0.4)\%$
$f_1(1420)$	1427.7±0.5	56.7±3.3	$K\overline{K}\pi$?

Table.: The list of f_1 mesons in PDG live[Navas et al., 2024].

- Narrow width compared to $a_1(1260)$

Standard understanding for f_1 structure

$\overline{q}q$ and ideal mixing

- $f_1(1285)$ and $f_1(1420)$ are components of nonet
- $f_1(1285)$ is $(\overline{u}u + \overline{d}d)/\sqrt{2}$, $f_1(1420)$ is $\overline{s}s$

Reason for standard understanding

- Small $f_1(1285) \to K\overline{K}\pi$ width and it decays via $a_0\pi \to$ Based on OZI rule $\overline{s}s$ structure is small
- Small $f_1(1420) \to \eta\pi\pi$ width
- This understanding explains thier chiral partner vector mesons

Problems in standard understanding

- Lattice QCD data shows difference from ideal mixing with 27°[Dudek et al., 2013].
- $K^- p \to K\overline{K}\pi\Lambda$ experiment does not display $f_1(1420)$ peak[Aston et al., 1988].
- Axial vector spectrum does not looks like equal mass difference counting constituent quarks

We must investigate mixing angle θ_{f_1} to see whether standard understanding is approvable or not

f_1 mixing angle in previous Lattice QCD result

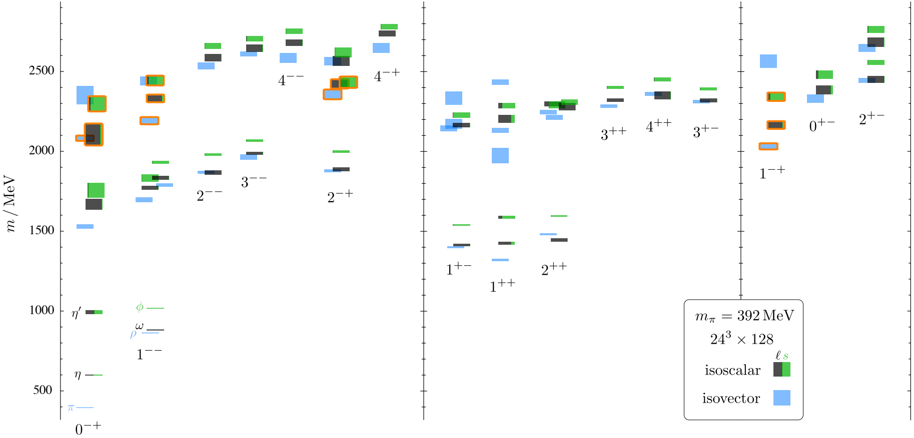


FIG. 11: Isoscalar (green/black) and isovector (blue) meson spectrum on the $m_\pi = 391\,\mathrm{MeV}$, $24^3 \times 128$ lattice. The vertical height of each box indicates the statistical uncertainty on the mass determination. States outlined in orange are the lowest-lying states having dominant overlap with operators featuring a chromomagnetic construction – their interpretation as the lightest hybrid meson supermultiplet will be discussed later.

f_1 mixing angle in previous Lattice QCD result

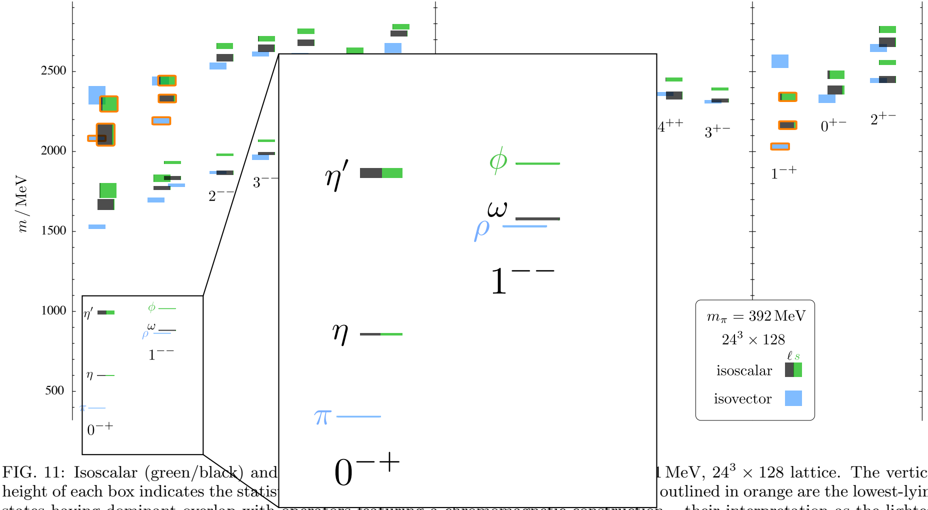
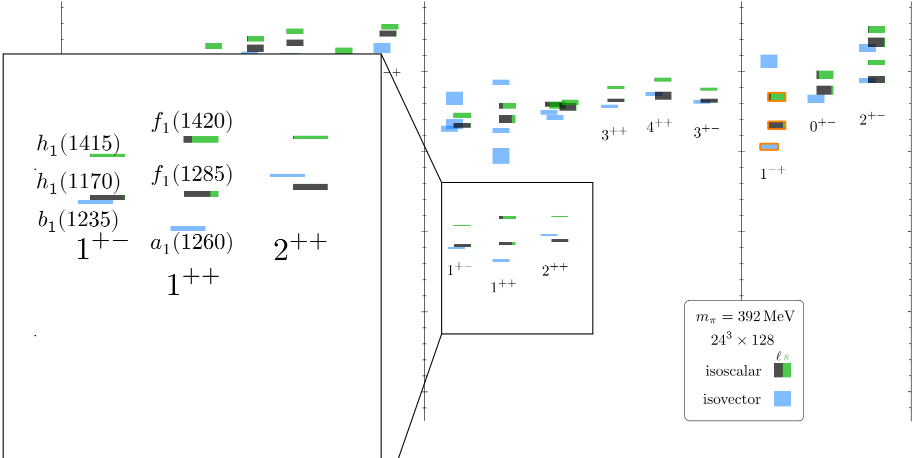


FIG. 11: Isoscalar (green/black) and ... MeV, $24^3 \times 128$ lattice. The vertical height of each box indicates the statis... outlined in orange are the lowest-lying states having dominant overlap with operators featuring a chromomagnetic construction – their interpretation as the lightest hybrid meson supermultiplet will be discussed later.

f_1 mixing angle in previous Lattice QCD result



F…) meson spectrum on the $m_\pi = 391\,\mathrm{MeV}$, $24^3 \times 128$ lattice. The vertical h… on the mass determination. States outlined in orange are the lowest-lying states having dominant overlap with operators featuring a chromomagnetic construction – their interpretation as the lightest hybrid meson supermultiplet will be discussed later.

Determination of axial vector nonet spectrum

$a_1(1260)$ width

- Wide width of $\Gamma_{a_1(1260)} \sim 350\mathrm{MeV}$ leads to difficulty in determination its mass precisely.
- Errorbar of $40~\mathrm{MeV}$

K_1 mixing

- K sector does not have C parity because charge conjugate does not itself
- Existence of pseudovector (1^{+-}) near axial vector (1^{++}) which mixes only K_1 part

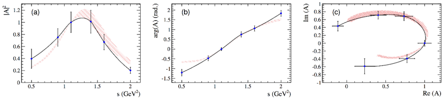


Figure 9. Magnitude-squared (a), phase (b) and Argand diagram (c) of the quasi-model-independent $a_1(1260)$ lineshape. The fitted knots are displayed as points with error bars and the black line shows the interpolated spline. The Breit-Wigner lineshape with the mass and width from the nominal fit is superimposed (red area). The latter is chosen to agree with the interpolated spline at the point $\Re(A) = 1$, $\Im(A) = 0$.

$$\begin{pmatrix} |K_1(1400)\rangle \\ |K_1(1270)\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta_{K_1} & \sin\theta_{K_1} \\ -\sin\theta_{K_1} & \cos\theta_{K_1} \end{pmatrix} \begin{pmatrix} |K_{1A}\rangle \\ |K_{1B}\rangle \end{pmatrix} \tag{2}$$

K_1 mixing

K_1 mixing

- An explanation for axial vector spectrum
- Another axial vector (pseudovector) meson nonet ($h_1(1170)$, b_1, K_{1B}, $h_1(1415)$)
- Mixing between K_{1A} and K_{1B} produce $K_1(1270)$ and $K_1(1400)$
- K_1 mixing angle is not determined yet[Shi et al., 2023, Cheng, 2012]

Relationship between f_1 mixing and K_1 mixing

- Ladder shape spectrum if $\theta_{K_1} \approx 45^\circ$
- If θ_{f_1} is ideal **and** $\theta_{K_1} \approx 45^\circ$, it is consistent with constituent quark counting

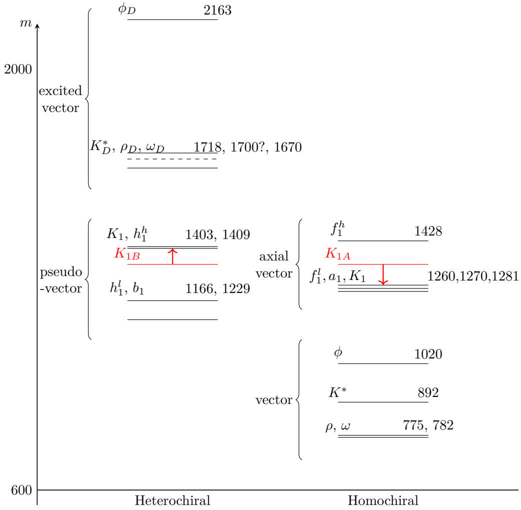


Table of Contents

Gell-Mann-Okubo mass formula

Gell-Mann-Okubo (GMO) mass formula

$$\frac{1}{3}m_{a_1}^2 - \frac{4}{3}m_{K_{1A}}^2 + m_{f_1^{octet}}^2 = 0 \tag{3}$$

- **The relation inside octet**
- In case masses are propotional to those $\mathrm{SU}(3)$ Clebsch-Gordan coefficients f and d
- Assumption: proportional to chiral condensate $\langle\sigma\rangle$
- No restrictions for singlet

Utilizing Gell-Mann-Okubo mass formula

- We can obtain f_1 octet mass from other octet

Chiral properties

Gell-Mann-Okubo type Lagrangian

Minimal Lagrangian produces Gell-Mann-Okubo mass relation

$$\mathcal{L}_{\mathrm{GMO}} = m_V^2 \mathrm{Tr}(V_\mu V^\mu) + a\mathrm{Tr}(V_\mu[\langle\Sigma\rangle, V^\mu]) + b\mathrm{Tr}(V_\mu\{\langle\Sigma\rangle, V^\mu\}) \tag{4}$$

where $\langle\Sigma\rangle = \mathrm{diag}(\langle\bar{q}q\rangle, \langle\bar{q}q\rangle, \langle\bar{s}s\rangle)$
This is not a chiral symmetric one

Extended linear sigma model

By including chiral symmetry breaking mechanism, [Parganlija et al., 2013]

$$\mathcal{L}_{eLSM} = \mathcal{L}_{sca+kin} + \frac{m_0^2}{2}\mathrm{Tr}\left[(L_\mu L^\mu + R_\mu R^\mu)\right] + h_2\mathrm{Tr}(L_\mu \Phi\Phi^\dagger L^\mu + R_\mu \Phi^\dagger M R^\mu) + 2h_3\mathrm{Tr}(L_\mu \Phi R^\mu \Phi^\dagger) \tag{5}$$

Even field power is required
Naturally masses are proportional to $\langle\sigma\rangle^2$

Model construction

Extended linear sigma model

- Extended linear sigma model forces into ideal mixing
- Single $\mathrm{SU}(3)$ breaking source

Our model

- We include another term represent singlet mass difference
- To control effectiveness of $\mathrm{SU}(3)$ breaking effect, we include another $\mathrm{SU}(3)$ source by indipendent way; explicit $\mathrm{SU}(3)$ breaking term

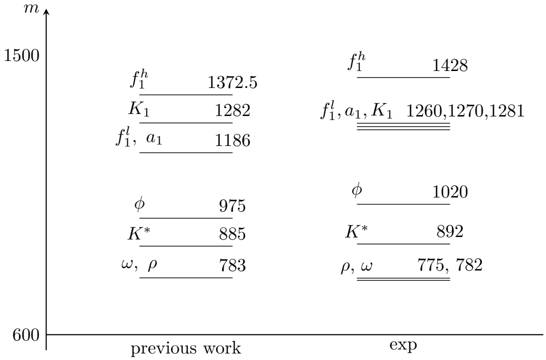


Fig.: Previous study which reflects equal space spectrum[Parganlija et al., 2013] and experimental result[Navas et al., 2024]. We apply values of $m_{K_{1A}} = 1270$ and $m_{a_1} = 1260$ at first.

Model Lagrangian

Model Lagrangian

$$\begin{aligned}\mathcal{L} &= \mathrm{Tr}[(D_\mu\Phi)^\dagger(D^\mu\Phi)] - \mu^2\mathrm{Tr}[\Phi^\dagger\Phi] - \lambda_1\left(\mathrm{Tr}[\Phi^\dagger\Phi]\right)^2 - \lambda_2\mathrm{Tr}[(\Phi^\dagger\Phi)^2] \\ &+ c_1\left(\det\Phi + \det\Phi^\dagger\right) + \epsilon\mathrm{Tr}[\mathcal{M}\Phi^\dagger + \Phi\mathcal{M}] - \frac{1}{4}\mathrm{Tr}[L_{\mu\nu}L^{\mu\nu} + R_{\mu\nu}R^{\mu\nu}] \\ &+ \frac{m_0^2}{2}\mathrm{Tr}\left[(L_\mu L^\mu + R_\mu R^\mu)\right] + h_2\mathrm{Tr}(L_\mu\Phi\Phi^\dagger L^\mu + R_\mu\Phi^\dagger M R^\mu) + 2h_3\mathrm{Tr}(L_\mu\Phi R^\mu\Phi^\dagger) \\ &+ h_2'\mathrm{Tr}(L_\mu\mathcal{M}\Phi^\dagger L^\mu + L_\mu\Phi\mathcal{M}L^\mu + R_\mu\mathcal{M}\Phi R^\mu + R_\mu\Phi^\dagger\mathcal{M}R^\mu) + 2h_3'\mathrm{Tr}(L_\mu\mathcal{M}R^\mu M^\dagger + L_\mu\Phi R^\mu\mathcal{M})\end{aligned} \tag{6}$$

$$D_\mu\Phi = \partial_\mu\Phi + igL_\mu\Phi - ig\Phi R_\mu^\dagger + ig'L_\mu^0\Phi - ig'\Phi R_\mu^{0\dagger} \tag{7}$$

$$L_{\mu\nu} = \partial_\mu L_\nu - \partial_\nu L_\mu + ig[L_\mu, L_\nu], \quad R_{\mu\nu} = \partial_\mu R_\nu - \partial_\nu R_\mu + ig[R_\mu, R_\nu] \tag{8}$$

- Red parameters affect the vector and axial vector meson masses

Table of Contents

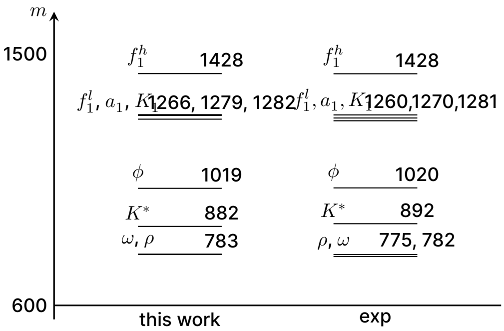
Result1: Fitting with our model

Fitting condition

- χ^2 fitting calculation
- Under already broken phase $\rightarrow$ only involved parameters
- 8 masses for 5 parameters
- $m_{K_{1A}} = 1270$ and $m_{a_1} = 1260$ at first

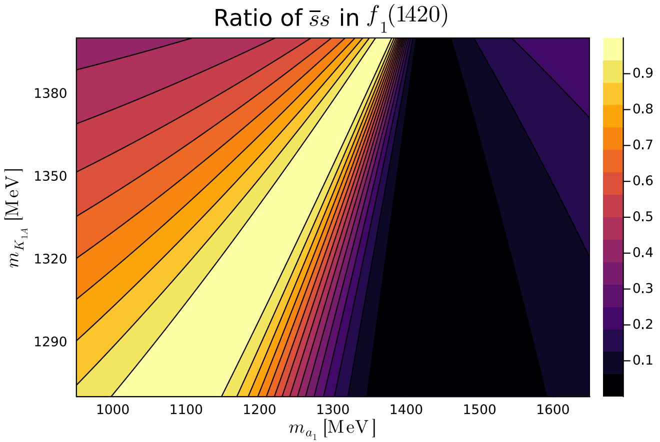
Fitting result

- Good agreement for the shape of spectrum
- Ideal result for θ_ω
- **Algebraic result** for θ_{f_1} $\rightarrow$ singlet-octet splitting



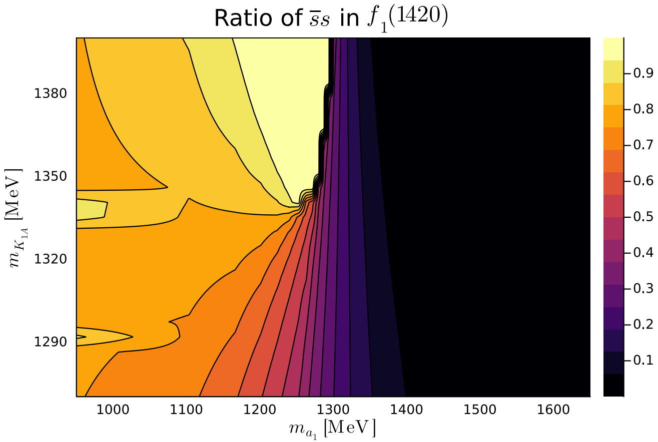
χ^2	θ_ω [deg]	θ_{f_1} [deg]	Z_π	Z_K	H_R
0.000910	-35.26	-86.10	0.8639	0.8303	0.3801

a_1 and K_1 dependence of f_1 mixing angle with Gell-Mann-Okubo mass formula



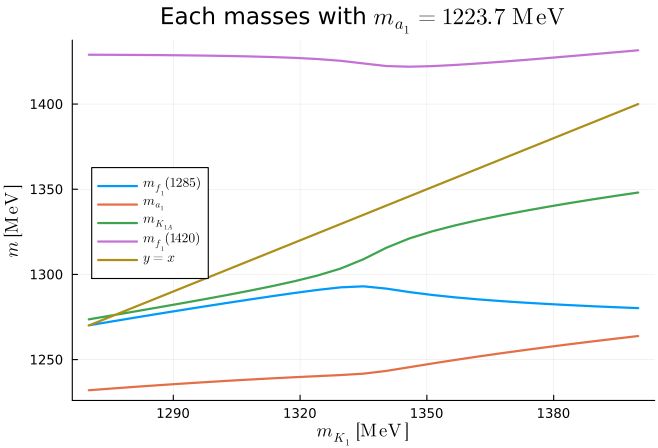
- If yellow ideal mixing is favored
- GMO mass formula is analytical solution for θ_{f_1}

Result2: a_1 and K_1 dependence of f_1 mixing angle with our model



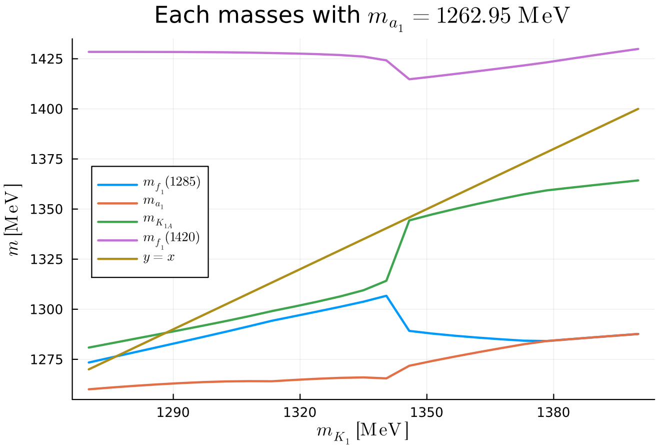
- Ideal region is located in $1335 \gtrsim m_{K_{1A}}$
- Difference from GMO is cliff and lower K_{1A}
- Cliff endpoint is located in possible region

Result2: a_1 and K_1 dependence of each masses with our model



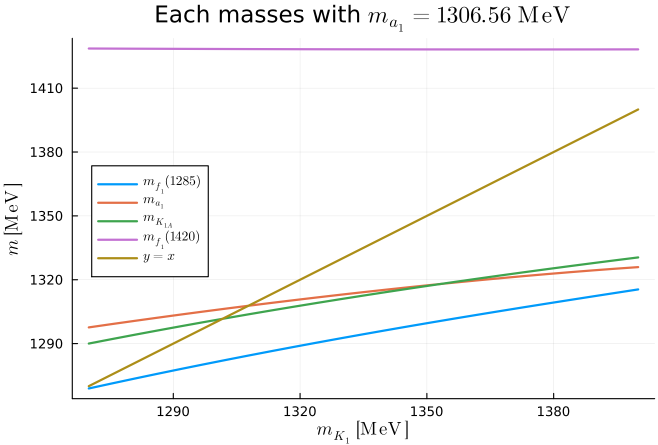
- Lower m_{a_1}: $\mathrm{SU}(3)$ breaking is large and it mixes as desired
- Intermediate m_{a_1}: $\mathrm{SU}(3)$ breaking is small and sudden mixing occurs
- Higher m_{a_1}: inverse mass: very high octet and $\mathrm{SU}(3)$ breaking effect is large

Result2: a_1 and K_1 dependence of each masses with our model



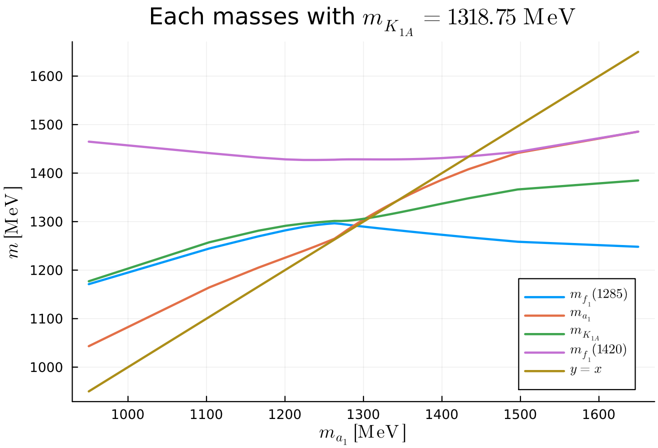
- Lower m_{a_1}: $\mathrm{SU}(3)$ breaking is large and it mixes as desired
- Intermediate m_{a_1}: $\mathrm{SU}(3)$ breaking is small and sudden mixing occurs
- Higher m_{a_1}: inverse mass: very high octet and $\mathrm{SU}(3)$ breaking effect is large

Result2: a_1 and K_1 dependence of each masses with our model



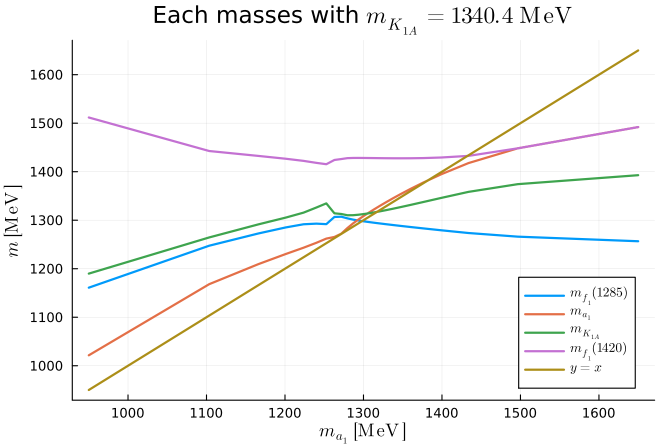
- Lower m_{a_1}: $\mathrm{SU}(3)$ breaking is large and it mixes as desired
- Intermediate m_{a_1}: $\mathrm{SU}(3)$ breaking is small and sudden mixing occurs
- Higher m_{a_1}: inverse mass: very high octet and $\mathrm{SU}(3)$ breaking effect is large

Result2: a_1 and K_1 dependence of each masses with our model



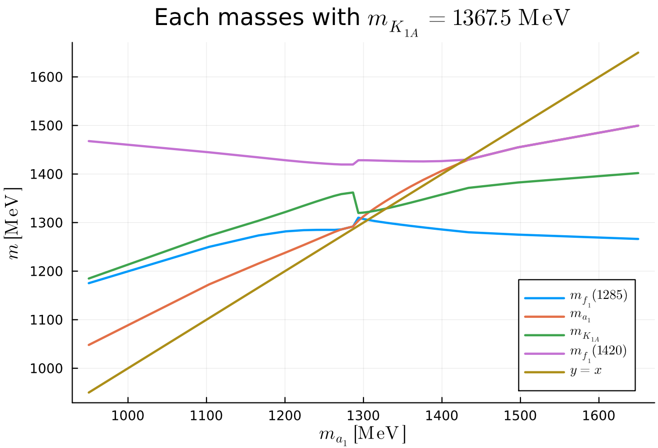
- There is mass inversion point
- Depending on $m_{K_{1A}}$, inversion occurs before or after mixing

Result2: a_1 and K_1 dependence of each masses with our model



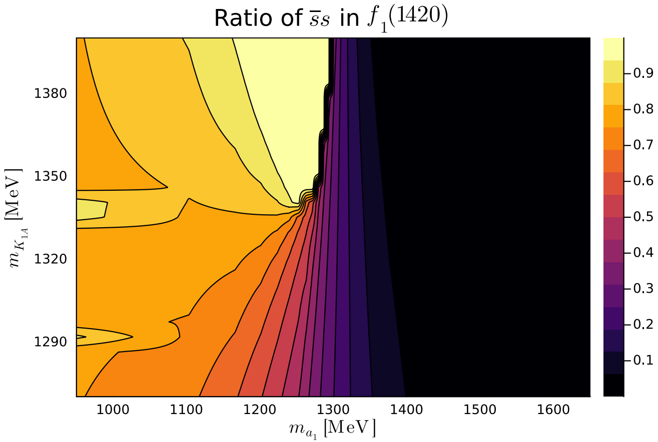
- There is mass inversion point
- Depending on $m_{K_{1A}}$, inversion occurs before or after mixing

Result2: a_1 and K_1 dependence of each masses with our model



- There is mass inversion point
- Depending on $m_{K_{1A}}$, inversion occurs before or after mixing

Result2: a_1 and K_1 dependence of f_1 mixing angle with our model



- Cliff is mass inversion line
- Difference in lower K_{1A} is smallness of $\mathrm{SU}(3)$ breaking effect

Summary

Summary

- Try to investigate chiral partner property by investigating $\mathrm{SU}(3)$ breaking effect
- Confirm that we are eager to get mixing agnle to obtain occupation ratio for singlet
- Check f_1 meson is understood in standard (ideal) way but it has some problems
- Classical Gell-Mann-Okubo mass relation can give us some implications but it also have problems of K_1 mixing and a_1 width
- As another model chiral partner included our model is described by extended linear sigma model plus g' and h' terms
- Investigate where ideal mixing is approvable in $m_{a_1} - m_{K_{1A}}$ plot in both Gell-Mann-Okubo mass relation and our model
- With our model, $m_{K_{1A}} \gtrsim 1330$ is desired to achieve ideal mixing
- To introduce picture of comparison between $\mathrm{SU}(3)$ breaking effect and mass difference between singlet and octet

Future works

Future works

- Apply this approach to Birkel-Fritzsch discussion[Birkel and Fritzsch, 1996] to describe $f_1(1510)$
- Discussion of pseudoscalar – axial vector diagonalization and anomalous decays[Osipov et al., 2020]
- Try to explain mass difference between singlet and octet using $\mathrm{U}(1)_A$ anomaly [Kochelev et al., 2000]
- There is a discussion of possibility that in-medium f_1 peak is seen in J-PARC E88 experiment[Ejima et al., 2024]

Meson matrices

$$\Phi = \Sigma + i\Pi,\ V_\mu = L_\mu + R_\mu,\ A_\mu = L_\mu - R_\mu, \tag{9}$$

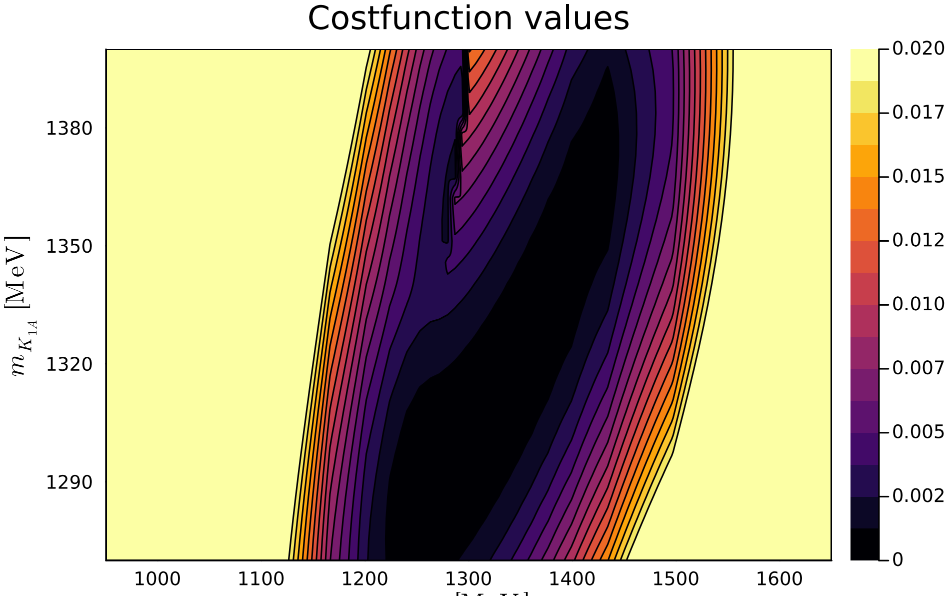
$$\Sigma = \begin{pmatrix} \frac{1}{\sqrt{3}}\sigma + \frac{1}{\sqrt{2}}a_0^0 + \frac{1}{\sqrt{6}}f_0 & a_0^+ & K_0^+ \\ a_0^- & \frac{1}{\sqrt{3}}\sigma - \frac{1}{\sqrt{2}}a_0^0 + \frac{1}{\sqrt{6}}f_0 & K_0^0 \\ K_0^- & \overline{K_0^0} & \frac{1}{\sqrt{3}}\sigma - \frac{2}{\sqrt{6}}f_0 \end{pmatrix}$$

$$\Pi = \begin{pmatrix} \frac{1}{\sqrt{3}}\eta_0 + \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & \frac{1}{\sqrt{3}}\eta_0 - \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \overline{K^0} & \frac{1}{\sqrt{3}}\eta_0 - \frac{2}{\sqrt{6}}\eta_8 \end{pmatrix}$$

$$V_\mu = \begin{pmatrix} \frac{1}{\sqrt{3}}\omega_{0\mu} + \frac{1}{\sqrt{2}}\rho_\mu^0 + \frac{1}{\sqrt{6}}\omega_{8\mu} & \rho_\mu^+ & K_{0\mu}^{*+} \\ \rho_\mu^- & \frac{1}{\sqrt{3}}\omega_{0\mu} - \frac{1}{\sqrt{2}}\rho_\mu^0 + \frac{1}{\sqrt{6}}\omega_{8\mu} & K_{0\mu}^{*0} \\ K_{0\mu}^{*-} & \overline{K_{0\mu}^{*0}} & \frac{1}{\sqrt{3}}\omega_{0\mu} - \frac{2}{\sqrt{6}}\omega_{8\mu} \end{pmatrix} \tag{10}$$

$$A_\mu = \begin{pmatrix} \frac{1}{\sqrt{3}}f_{10\mu} + \frac{1}{\sqrt{2}}a_{1\mu}^0 + \frac{1}{\sqrt{6}}f_{18\mu} & a_{1\mu}^+ & K_{1\mu}^+ \\ a_{1\mu}^- & \frac{1}{\sqrt{3}}f_{10\mu} - \frac{1}{\sqrt{2}}a_{1\mu}^0 + \frac{1}{\sqrt{6}}f_{18\mu} & K_{1\mu}^0 \\ K_{1\mu}^- & \overline{K_{1\mu}^0} & \frac{1}{\sqrt{3}}f_{10\mu} - \frac{2}{\sqrt{6}}f_{18\mu} \end{pmatrix}$$

Result


Costfunction values
1380
1350
1320
1290
$m_{K_{1A}}$ [MeV]
1000
1100
1200
1300
1400
1500
1600
0.020
0.017
0.015
0.012
0.010
0.007
0.005
0.002
0

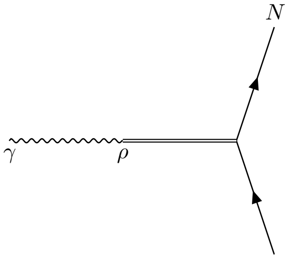
Vector meson dominance

Vector meson dominance

- Hadron electromagnetic interactions should have vector intermediate states
- This hypothesis supports the gauge-like couplings

Current-field identity

- The photon and vector mesons are almost identity
- External current is identical to vector mesons $J_{EM} \propto \rho^0$
- The problem of gauge variance



Chiral transformations

Chiral transformations

- Vector and axial vector singlet are not a chiral partner

$$v_\mu^a \xrightarrow{\text{axial}} v_\mu^a + f^{abc}\theta_A^b a_\mu^c, \qquad a_\mu^a \xrightarrow{\text{axial}} a_\mu^a + f^{abc}\theta_A^b v_\mu^c \tag{11}$$

- Antisymmetric structure constant for SU(3) f does not couple to singlet
- They are just a parity partner

S. H. Lee., Subatomic Part. Cosmol., 1-2:100008, 2024.

Chiral representation

Chiral hadrons

- Chiral hadrons composed of the valence quarks

$$s^a = \overline{q}t^a q, \quad p^a = -\overline{q}i\gamma_5 t^a q, \quad v^a_\mu = \overline{q}\gamma_\mu t^a q, \quad a^a_\mu = \overline{q}\gamma_\mu\gamma_5 t^a q \tag{12}$$

- The chirality of $s \sim \overline{q}_L q_R + \overline{q}_R q_L$ and $v \sim \overline{q}_L q_L + \overline{q}_R q_R$
- $\mathrm{SU}(3) + \mathrm{U}(1)$ generator t^a for $a = 0, 1, \ldots, 8$
- $J^{PC} = 0^{++}, 0^{-+}, 1^{--}, 1^{++}$ respectively

Chiral representation

- Chiral representation with notation (n_L, n_R) for n_L, n_R dimensional representations for each chirality
- Same representation for scalars $(\overline{\mathbf{3}}, \mathbf{1}) \times (\mathbf{1}, \mathbf{3}) + (\mathbf{1}, \overline{\mathbf{3}}) \times (\mathbf{3}, \mathbf{1}) = (\overline{\mathbf{3}}, \mathbf{3}) + (\mathbf{3}, \overline{\mathbf{3}})$
- **Different representation for vectors** $(\overline{\mathbf{3}}, \mathbf{1}) \times (\mathbf{3}, \mathbf{1}) + (\mathbf{1}, \overline{\mathbf{3}}) \times (\mathbf{1}, \mathbf{3}) = (\mathbf{8}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{8}) + (\mathbf{1}, \mathbf{1})$

References I

D. Aston, N. Awaji, T. Bienz, F. Bird, J. D'Amore, W. Dunwoodie, R. Endorf, K. Fujii, H. Hayashi, S. Iwata, W. Johnson, R. Kajikawa, P. Kunz, D. Leith, L. Levinson, T. Matsui, B. Meadows, A. Miyamoto, M. Nussbaum, H. Ozaki, C. Pak, B. Ratcliff, D. Schultz, S. Shapiro, T. Shimomura, P. Sinervo, A. Sugiyama, S. Suzuki, G. Tarnopolsky, T. Tauchi, N. Toge, K. Ukai, A. Waite, and S. Williams. Evidence for two strangeonium resonances with JPC=1++ and 1+- in K-p interactions at 11 GeV/c. *Physics Letters B*, 201(4):573–578, Feb. 1988. ISSN 03702693. doi: 10.1016/0370-2693(88)90620-X.

M. Birkel and H. Fritzsch. Nucleon spin and the mixing of axial vector mesons. *Physical Review D*, 53(11):6195–6202, June 1996. ISSN 0556-2821, 1089-4918. doi: 10.1103/PhysRevD.53.6195.

H.-Y. Cheng. Revisiting Axial-Vector Meson Mixing. *Physics Letters B*, 707(1):116–120, Jan. 2012. ISSN 0370-2693. doi: 10.1016/j.physletb.2011.12.013.

J. J. Dudek, R. G. Edwards, P. Guo, and C. E. Thomas. Toward the excited isoscalar meson spectrum from lattice QCD. *Physical Review D*, 88(9), Nov. 2013. ISSN 1550-7998, 1550-2368. doi: 10.1103/PhysRevD.88.094505.

R. Ejima, P. Gubler, C. Sasaki, and K. Shigaki. Toward a Direct Measurement of Partial Restoration of Chiral Symmetry at J-PARC E16 via Density-induced Chiral Mixing, Dec. 2024.

N. I. Kochelev, D.-P. Min, Y. Oh, V. Vento, and A. V. Vinnikov. New anomalous trajectory in Regge theory. *Physical Review D*, 61(9):094008, Apr. 2000. ISSN 0556-2821, 1089-4918. doi: 10.1103/PhysRevD.61.094008.

S. H. Lee. Chiral symmetry breaking, chiral partners, and the K_1 and K^* in medium, Oct. 2024.

S. Navas et al. Review of particle physics. *Phys. Rev. D*, 110(3):030001, 2024. doi: 10.1103/PhysRevD.110.030001.

A. A. Osipov, A. A. Pivovarov, M. K. Volkov, and M. M. Khalifa. Account for axial vector mesons in the $\eta \to \pi + \pi - \gamma$ and $\eta' \to \pi + \pi - \gamma$ decays. *Physical Review D*, 101 (9):094031, May 2020. ISSN 2470-0010, 2470-0029. doi: 10.1103/PhysRevD.101.094031.

D. Parganlija, P. Kovacs, G. Wolf, F. Giacosa, and D. H. Rischke. Meson vacuum phenomenology in a three-flavor linear sigma model with (axial-)vector mesons. *Physical Review D*, 87(1), Jan. 2013. ISSN 1550-7998, 1550-2368. doi: 10.1103/PhysRevD.87.014011.

Y.-J. Shi, J. Zeng, and Z.-F. Deng. Revisiting $K_1(1270)- K_1(1400)$ mixing in QCD sum rules, Oct. 2023.