Momentum Dependence of Charmonia and K1 in Medium

A QCD Sum Rules Approach

International workshop on origin of mass and matter in the universe

Seokwoo Yeo

Department of Physics, Yonsei University

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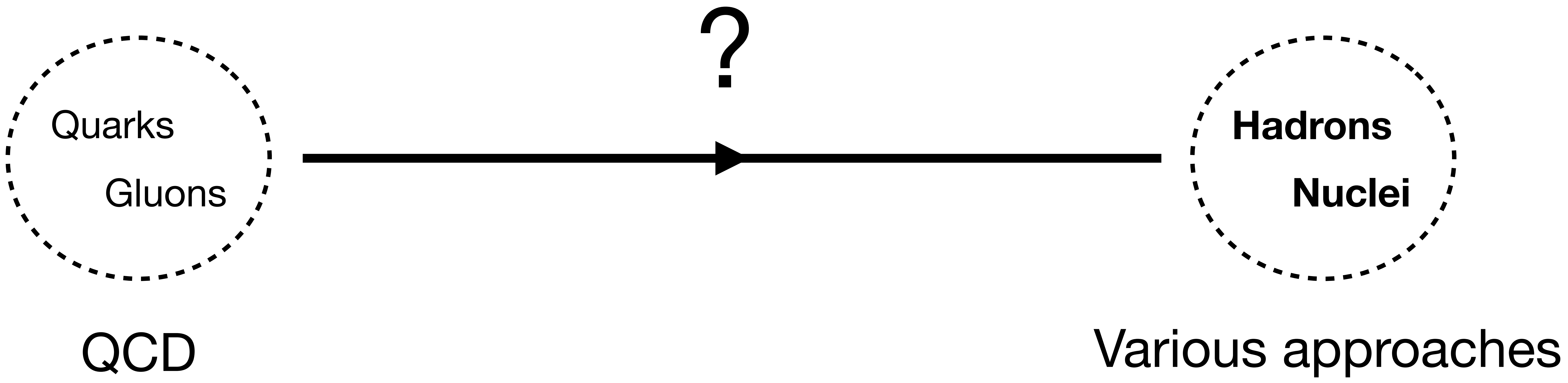

Hadrons are the most direct manifestations of QCD in the low energy regime.

We study hadrons using QCD sum rules.

What aspects of hadrons?

→ We investigate hadrons in medium.

- **In-medium modification**: Changes in mass and width broadening in hot or dense environments.
- Probes for **QCD behavior** under extreme conditions (Early Universe, Neutron Stars, Heavy-ion Collisions).
- The medium introduces an additional four-momentum, which increases the number of possible Lorentz scalars. As a result, the correlation function admits **additional tensor structures.**
- Quantities equivalent in vacuum split into distinct **longitudinal (L) and transverse (T)** modes in medium.

Momentum dependence

- ϕ
  - H. Kim and P. Gubler, Phys. Lett. B 805, 135412 (2020).
  - A. J. Arifi, P. Gubler, and K. Tsushima, arXiv:2603.15971 [hep-th] (2026).
- Charmonia $(\eta_c,\ \chi_{c0},\ J/\psi, \chi_{c1})$
  - H. Kim, S. Yeo, S. Cho, S. H. Lee, Phys. Rev. D 107, 114004 (2023).
- $K_1^{\pm}$
  - S. Yeo, H. Kim, S. H. Lee, Phys. Rev. D 110, 014013 (2024).

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Today's Topic

We investigate J/ψ in medium

A useful probe of in-medium QCD dynamics

- Accessible through **dilepton decays**
- Mainly sensitive **gluonic properties** of the medium
- Its compact size suppresses **long-range hadronic effects**
- Provides a relatively **simple picture** of in-medium modification

QCD Sum Rules

A **current** couples to some hadron

$\langle 0 | j | h \rangle \neq 0$

For the J/ψ

$$j_\mu(x) = \bar{c}(x)\gamma_\mu c(x) \longrightarrow \Pi_{\mu\nu}(q) = i\int d^4x e^{iq\cdot x}\langle T\{j_\mu(x) j_\nu^\dagger(0)\}\rangle$$

QCD Sum Rules

$$\Pi_{\mu\nu}(q) = i\int d^4x e^{iq\cdot x}\langle T\{j_\mu(x) j_\nu^\dagger(0)\}\rangle \text{ with } j_\mu(x) = \bar{c}(x)\gamma_\mu c(x)$$

$$\downarrow$$

$$\rho(s) = \frac{1}{\pi}\,\mathrm{Im}\,\Pi(s) = \sum_n |\langle 0|J|n\rangle|^2\,\delta(s - m_n^2)\,.$$

$$\longrightarrow f\,\delta(s - m_{J/\psi}^2) + \theta(s - s_0)\,\frac{1}{\pi}\mathrm{Im}\,\Pi^{\mathrm{pert}}(s)$$

pole + continuum ansatz

Borel transform suppresses the continuum

QCD Sum Rules

$$\Pi_{\mu\nu}(q) = i\int d^4x e^{iq\cdot x}\langle T\{j_\mu(x)j_\nu^\dagger(0)\}\rangle$$ with $j_\mu(x) = \bar{c}(x)\gamma_\mu c(x)$

$$\Pi(Q^2) = \sum_n C_n(Q^2)\,\langle \mathcal{O}_n\rangle$$ at $Q^2 = -q^2 \gg 0$

(to avoid physical spectrum)

- Wilson coefficients
- short-distance
- perturbative

- local operators / condensates
- long-distance
- nonperturbative

$$
\begin{aligned}
\Pi(q) &= i\int d^4x\, e^{iqx}\, \langle T\{\bar{c}(x)\,\Gamma_1 c(x)\bar{c}(0)\,\Gamma_2 c(0)\}\rangle \\
&= i\int d^4x\, e^{iqx}\Big\langle : \mathrm{Tr}\big[\Gamma_1 S(0,x)\,\Gamma_2 S(x,0)\big] : +i : \bar{c}(x)\,\Gamma_1 S(x,0)\,\Gamma_2 c(0) : \\
&\qquad\qquad + i : \bar{c}(0)\,\Gamma_2 S(0,x)\,\Gamma_1 c(x) : + : \bar{c}(x)\,\Gamma_1 c(x)\bar{c}(0)\,\Gamma_2 c(0) : \Big\rangle
\end{aligned}
$$

$$\Pi(q) = i\int d^4x\, e^{iqx}\,\langle T\{\bar{c}(x)\Gamma_1 c(x)\bar{c}(0)\Gamma_2 c(0)\}\rangle$$

$$= i\int d^4x\, e^{iqx}\Big\langle : \mathrm{Tr}[\Gamma_1 S(0,x)\Gamma_2 S(x,0)] : + i : \bar{c}(x)\Gamma_1 S(x,0)\Gamma_2 c(0) :$$

$$+ i : \bar{c}(0)\Gamma_2 S(0,x)\Gamma_1 c(x) : + : \bar{c}(x)\Gamma_1 c(x)\bar{c}(0)\Gamma_2 c(0) : \Big\rangle$$

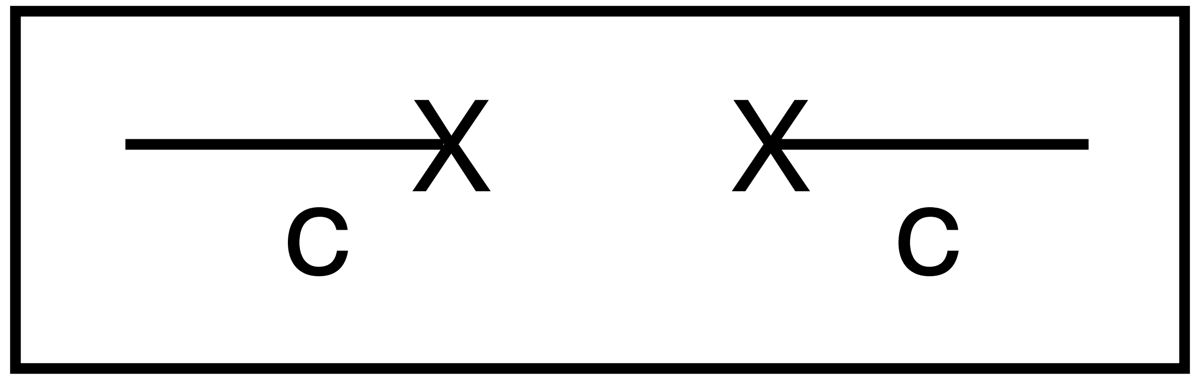
In trivial vacuum

c $|0\rangle = 0$

In **nontrivial** QCD vacuum

c $|0\rangle \neq 0$

$\langle \bar{c}c\rangle \neq 0$ $\longrightarrow$



$$\Pi(q) = i\int d^4x\, e^{iqx}\, \langle T\{\bar{c}(x)\Gamma_1 c(x)\bar{c}(0)\Gamma_2 c(0)\}\rangle$$

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background field method

$A^a_\mu = \bar{A}^a_\mu + a^a_\mu$, $\bar{A}^a_\mu$: long-distance, nonperturbative background,

a^a_μ : short-distance fluctuation

Fock-Schwinger gauge

$x^\mu A_\mu(x) = 0$

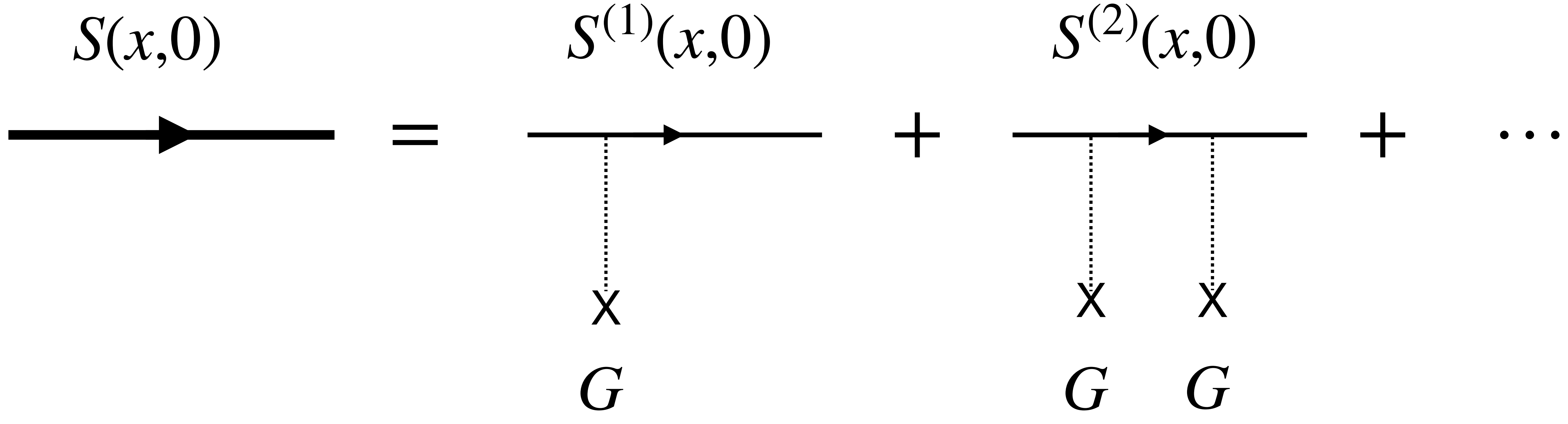
$\longrightarrow$ $S(x,0) = S^{(0)}(x,0) + S^{(1)}(x,0) + S^{(2)}(x,0) + \cdots,$

$$\Pi(q) = i\int d^4x\, e^{iqx}\, \langle T\{\bar{c}(x)\Gamma_1 c(x)\bar{c}(0)\Gamma_2 c(0)\}\rangle$$

$$= i\int d^4x\, e^{iqx}\Big\langle : \mathrm{Tr}[\Gamma_1 S(0,x)\Gamma_2 S(x,0)] : + i : \bar{c}(x)\Gamma_1 S(x,0)\Gamma_2 c(0) :$$

$$+\, i : \bar{c}(0)\Gamma_2 S(0,x)\Gamma_1 c(x) : + : \bar{c}(x)\Gamma_1 c(x)\bar{c}(0)\Gamma_2 c(0) : \Big\rangle$$

$$\xrightarrow[\text{background field method}]{} \quad S(x,0) = S^{(0)}(x,0) + S^{(1)}(x,0) + S^{(2)}(x,0) + \cdots,$$



$$\Pi(q) = i\int d^4x\, e^{iqx}\,\langle T\{\bar{c}(x)\Gamma_1 c(x)\bar{c}(0)\Gamma_2 c(0)\}\rangle$$

$$= i\int d^4x\, e^{iqx}\Big\langle \underline{:\mathrm{Tr}[\Gamma_1 S(0,x)\Gamma_2 S(x,0)]} : +i : \bar{c}(x)\Gamma_1 S(x,0)\Gamma_2 c(0) :$$

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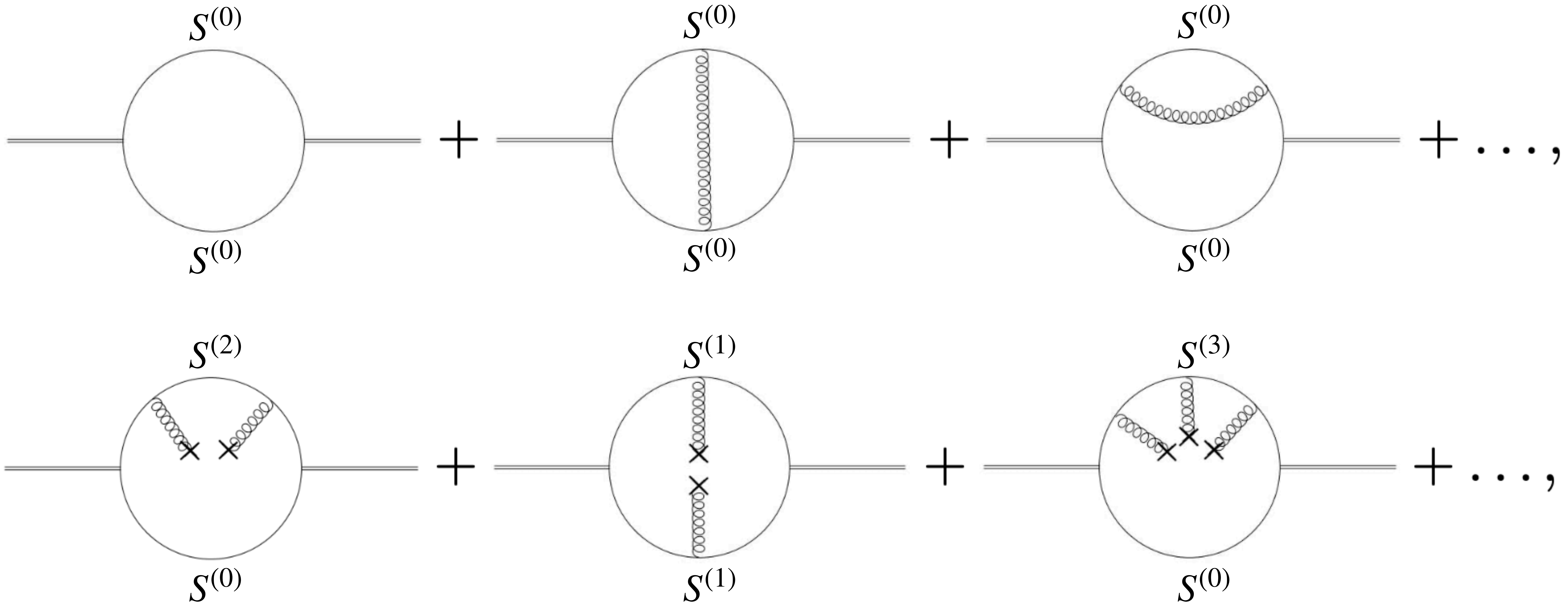

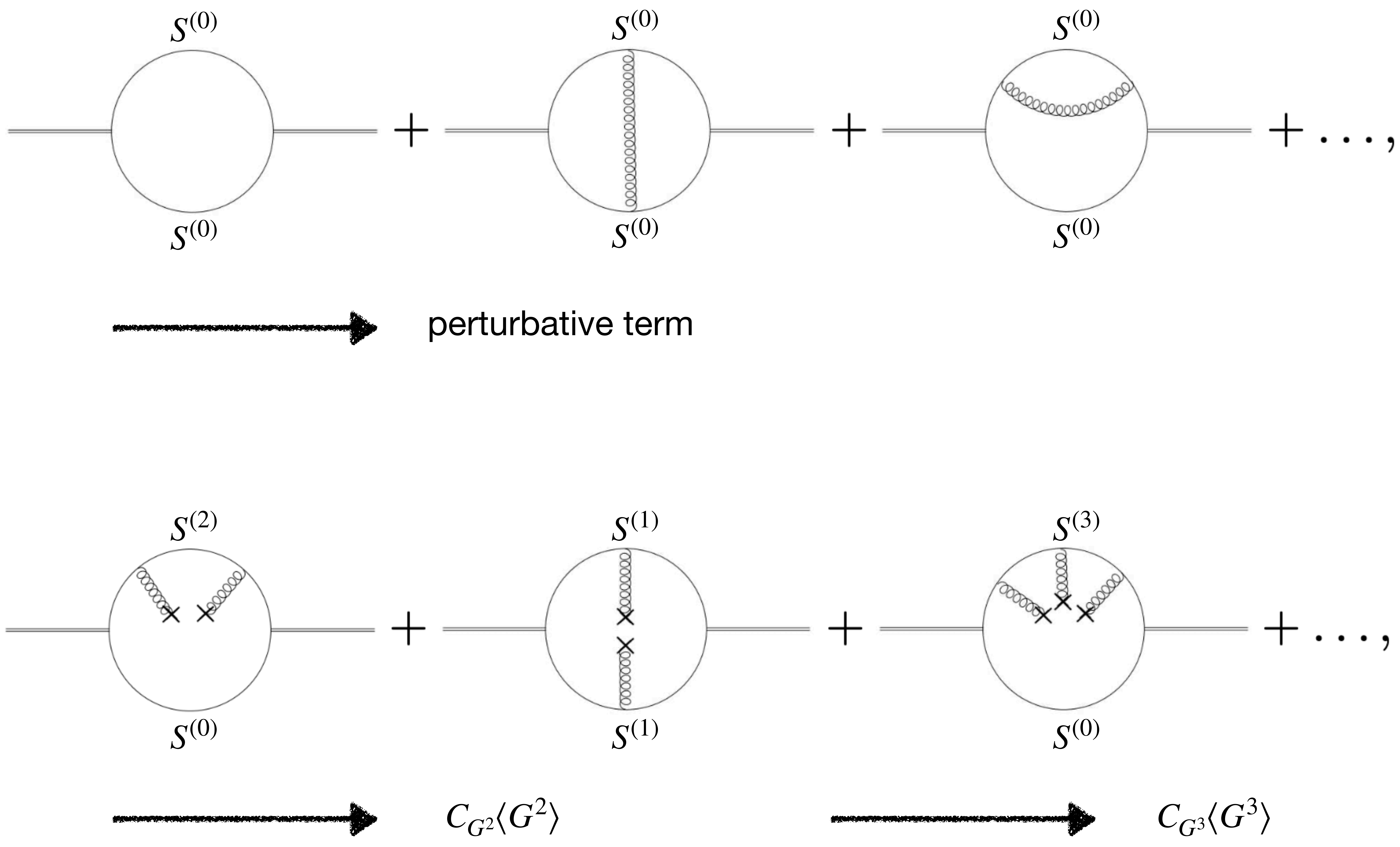
$S^{(0)}$
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$S^{(0)}$
$S^{(0)}$
$S^{(0)}$
$+ \dots,$
perturbative term
$S^{(2)}$
$S^{(0)}$
$S^{(1)}$
$S^{(1)}$
$S^{(3)}$
$S^{(0)}$
$+ \dots,$
$C_{G^2}\langle G^2\rangle$
$C_{G^3}\langle G^3\rangle$

QCD Sum Rules

$$\Pi_{\mu\nu}(q) = i\int d^4x e^{iq\cdot x}\langle T\{j_\mu(x)j_\nu^\dagger(0)\}\rangle$$

$$\Pi^{OPE}(Q^2) = \sum_n C_n(Q^2)\,\langle \mathcal{O}_n\rangle$$

$$\frac{1}{\pi}\mathrm{Im}\,\Pi(s) = f\,\delta(s - m_{J/\psi}^2) + \theta(s - s_0)\,\frac{1}{\pi}\mathrm{Im}\,\Pi^{\mathrm{pert}}(s)$$

$$\Pi^{OPE}(q^2) = \frac{1}{\pi}\int ds \frac{\mathrm{Im}\,\Pi(s)}{s - q^2 - i\epsilon}$$

QCD Sum Rules

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$$\Pi^{OPE}(q^2) = \frac{1}{\pi}\int ds\frac{\mathrm{Im}\,\Pi(s)}{s-q^2-i\epsilon}$$

quark gluon parameter $\longleftrightarrow$ hadronic quantity

QCD Sum Rules

Borel transform

$$\mathscr{M}^{\mathrm{OPE}}(M^2) = \lim_{\substack{n,\, Q^2 \to \infty \\ Q^2/n = M^2}} \frac{(Q^2)^{n+1}}{n!} \left(-\frac{d}{dQ^2}\right)^n \Pi(Q^2)$$

$$\Pi^{OPE}(q^2) = \frac{1}{\pi} \int ds \frac{\mathrm{Im}\,\Pi(s)}{s - q^2 - i\epsilon} + \underline{P_n(q^2)}$$

$$\Big\downarrow$$

$$\mathscr{M}(M^2) = \frac{1}{\pi} \int ds\, e^{-s/M^2}\, \mathrm{Im}\,\Pi(s)$$

1. It **removes** the polynomial part

2. It **suppresses** higher-state contributions through an exponential weight

QCD Sum Rules

Borel transform

$$\mathscr{M}^{\mathrm{OPE}}(M^2) = \lim_{\substack{n, Q^2 \to \infty \\ Q^2/n = M^2}} \frac{(Q^2)^{n+1}}{n!} \left(-\frac{d}{dQ^2}\right)^n \Pi(Q^2)$$

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$$\downarrow$$

$$\mathscr{M}(M^2) = \frac{1}{\pi}\int ds\, e^{-s/M^2}\, \mathrm{Im}\,\Pi(s)$$ with $\frac{1}{\pi}\mathrm{Im}\,\Pi(s) = f\,\delta(s - m_{J/\psi}^2) + \theta(s - s_0)\frac{1}{\pi}\mathrm{Im}\,\Pi^{\mathrm{pert}}(s)$

$$\downarrow$$

$$\mathscr{M}^{\mathrm{OPE}}(M^2) = f_0\, e^{-m_{J/\psi}^2/M^2} + \int_{s_0}^{\infty} ds\, e^{-s/M^2}\, \rho^{\mathrm{pert}}(s)$$

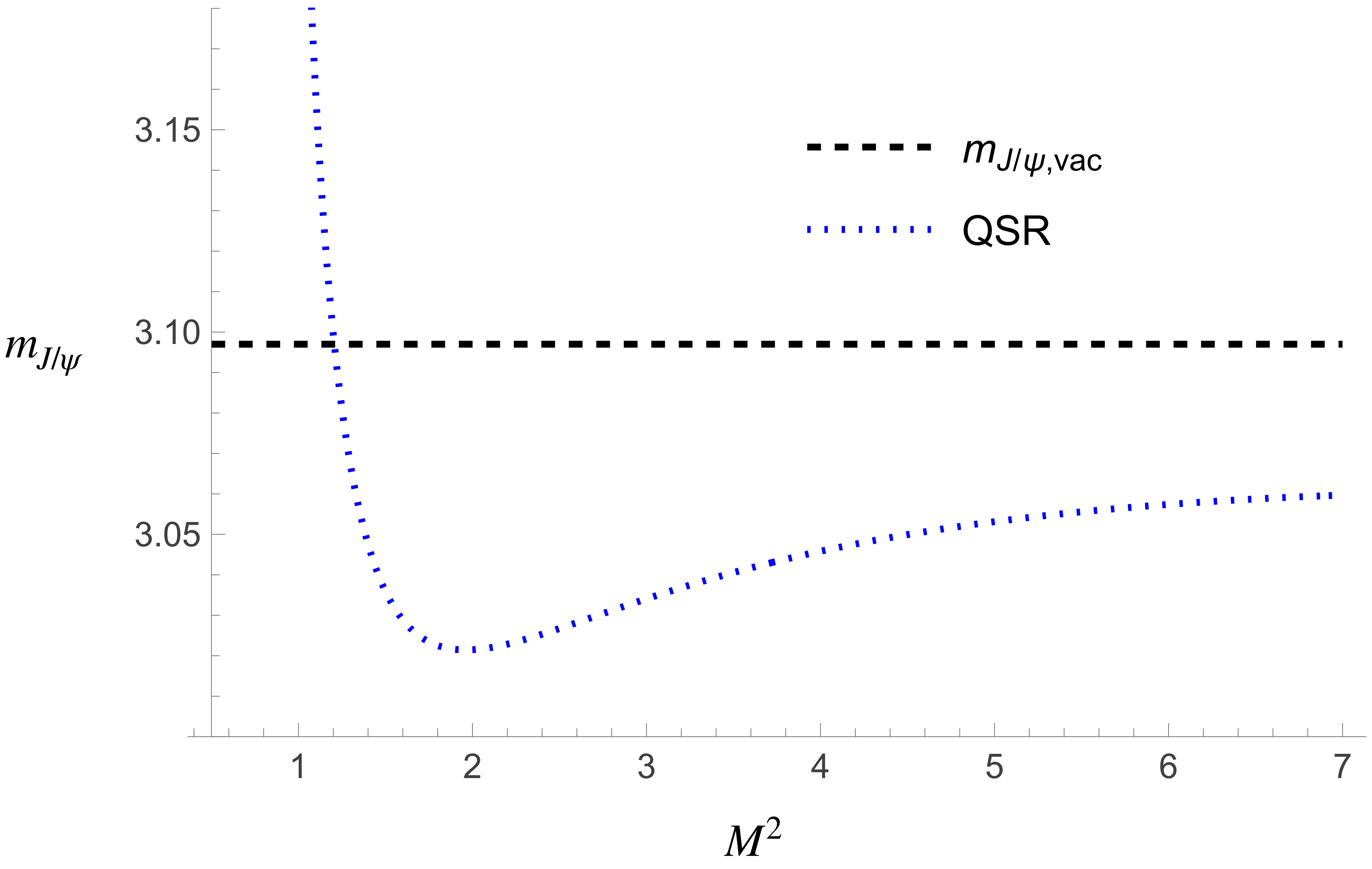
QCD Sum Rules

$$\mathscr{M}^{\mathrm{OPE}}(M^2) = f_0\, e^{-m_{J/\psi}^2/M^2} + \int_{s_0}^{\infty} ds\, e^{-s/M^2} \rho^{\mathrm{pert}}(s)$$

$$\longrightarrow \quad \boxed{m_{J/\psi}(M^2; s_0) = \sqrt{\frac{-\frac{\partial}{\partial(1/M^2)} \overline{\mathscr{M}}(M^2; s_0)}{\overline{\mathscr{M}}(M^2; s_0)}}}$$

where $\overline{\mathscr{M}}(M^2; s_0) = \mathscr{M}^{\mathrm{OPE}}(M^2) - \int_{s_0}^{\infty} ds\, e^{-s/M^2} \rho^{\mathrm{pert}}(s)$

QCD Sum Rules



$$\mathscr{M}^{\mathrm{OPE}}(M^2) = f_0\, e^{-m_{J/\psi}^2/M^2} + \int_{s_0}^{\infty} ds\, e^{-s/M^2} \rho^{\mathrm{pert}}(s)$$

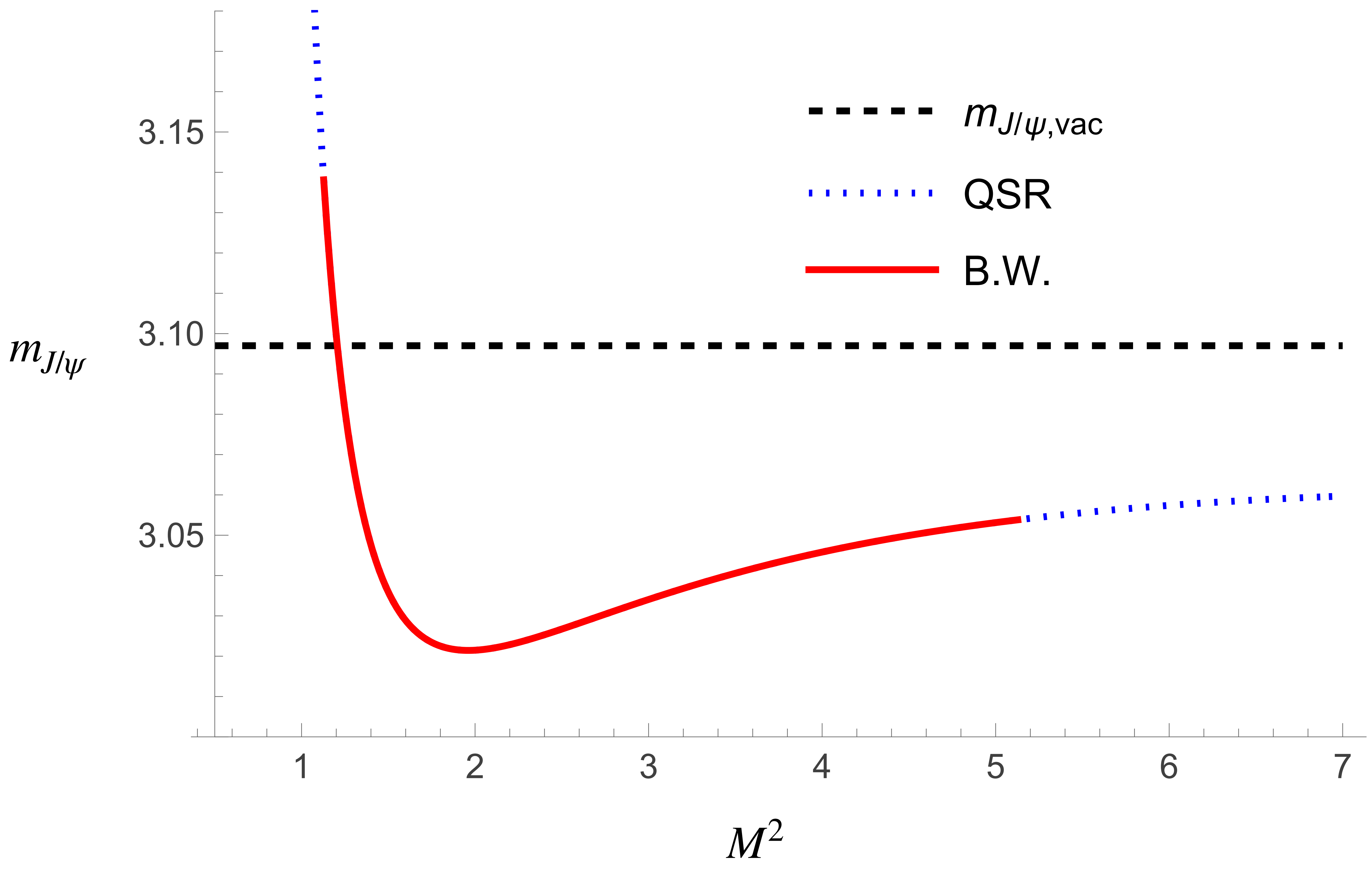
$$\downarrow$$

$$\sim \frac{\langle O_d \rangle}{M^d}$$

- Large M^2 : better OPE convergence, weaker pole dominance
- Small M^2 : stronger pole dominance, poorer OPE convergence

A reliable region: the Borel window

QCD Sum Rules



$$\frac{m_{exp} - m_{QSR}}{m_{exp}} \simeq 1.7\,\%$$

Finite-$\vec{q}$ dependence of heavy quarkonium near T_c

- At sufficiently low temperature, the vacuum dispersion relation can still be used as a good approximation

- The medium introduces an additional four-momentum, increasing the number of possible Lorentz scalars

$$u^\mu = (1,\mathbf{0})$$

$$q^2 \to q^2, \ \ u \cdot q$$

$$ex) \ \left\langle \frac{\alpha_s}{\pi} G^a_{\mu\alpha} G^{a\,\alpha}_{\nu} \right\rangle = \frac{1}{4} g_{\mu\nu} \underbrace{G_0}_{\text{scalar}} + \left(u_\mu u_\nu - \frac{1}{4} g_{\mu\nu} \right) \underbrace{G_2}_{\text{nonscalar}}$$

Finite-$\vec{q}$ dependence of heavy quarkonium near T_c

$$ex)\quad \left\langle \frac{\alpha_s}{\pi} G^a_{\mu\alpha} G^{a\,\alpha}_{\nu} \right\rangle = \underbrace{\frac{1}{4} g_{\mu\nu} G_0}_{\text{scalar}} + \underbrace{\left(u_\mu u_\nu - \frac{1}{4} g_{\mu\nu} \right) G_2}_{\text{nonscalar}}$$

$$G_0 = \left\langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{a\,\mu\nu} \right\rangle$$

$$G_2 = \frac{4}{3} \left[u^\mu u^\nu \underbrace{\left\langle \frac{\alpha_s}{\pi} G^a_{\mu\alpha} G^{a\,\alpha}_{\nu} \right\rangle}_{\text{tensor structure}} - \frac{1}{4} G_0 \right]$$

Finite-$\vec{q}$ dependence of heavy quarkonium near T_c

$$\Pi^J(q) = i\int d^4x\, e^{iq\cdot x}\langle T\{j^J(x)j^J(0)\}\rangle$$

$$j^P = i\bar{h}\gamma_5 h, \qquad j^S = \bar{h}h, \qquad j^V_\mu = \bar{h}\gamma_\mu h, \qquad j^A_\mu = \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right)\bar{h}\gamma_5\gamma^\nu h$$

$$\tilde{\Pi}_{P,S}(\omega^2, \mathbf{q}^2) = \frac{1}{q^2}\Pi_{P,S}$$

$$\tilde{\Pi}^{V,A}_L(\omega^2, \mathbf{q}^2) = \frac{1}{\mathbf{q}^2}\Pi^{V,A}_{00}$$

$$\tilde{\Pi}^{V,A}_T(\omega^2, \mathbf{q}^2) = -\frac{1}{2}\left(\frac{1}{\mathbf{q}^2}\Pi^{V,A\,\mu}_\mu + \tilde{\Pi}^{V,A}_L\right)$$

Finite-$\vec{q}$ dependence of heavy quarkonium near T_c

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$$\mathscr{M}^J_{\mathrm{OPE}}(M^2, \mathbf{q}^2) = e^{-\nu}A_J(\nu)\left[1 + \alpha_s a_J(\nu) + b_J(\nu)\phi_b + \left(c_J(\nu) + \frac{\mathbf{q}^2}{m_h^2}d_J(\nu)\right)\phi_c\right]$$

with $\nu = \dfrac{4m_h^2}{M^2}, \qquad \phi_b = \dfrac{4\pi^2}{9(4m_h^2)^2}G_0, \qquad \phi_c = \dfrac{4\pi^2}{3(4m_h^2)^2}G_2\,.$

Finite-$\vec{q}$ dependence of heavy quarkonium near T_c

- We probe an **integral** of the spectral function, not the full spectrum itself

- Therefore, especially near T_c, the qualitative **trend** is more meaningful than the exact numerical mass shift

Finite-$\vec{q}$ dependence of heavy quarkonium near T_c

- In medium, u^μ provides an additional four-vector, allowing extra Lorentz scalars

- Using $Q^2 = -\omega^2 + \mathbf{q^2}$,
  the momentum dependence can be separated into:
  - trivial part absorbed into Q^2 $\quad E^2 - \mathbf{q}^2 = m^2$
  - nontrivial part from explicit $\mathbf{q}^2$ $\quad E^2 - \mathbf{q}^2 = m^2(\mathbf{q}^2)$

- For spin-1 states, the nontrivial part may depend on polarization:
  - longitudinal
  - transverse

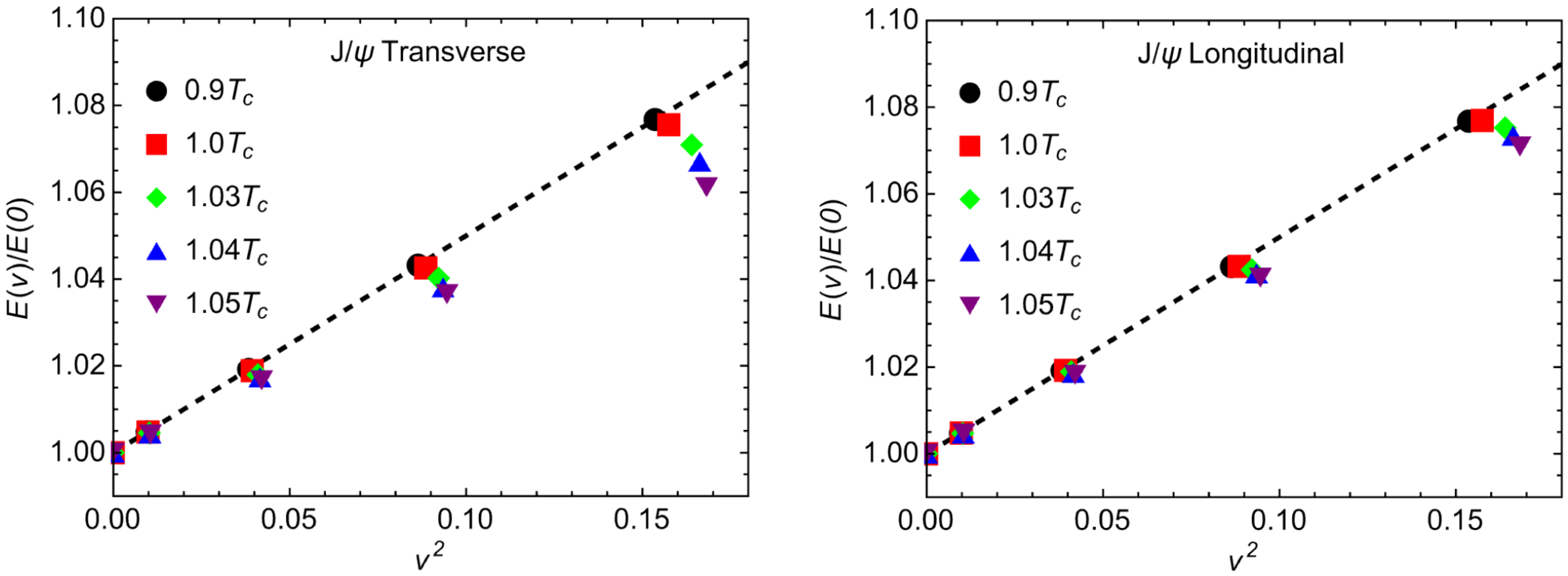
Finite-$\vec{q}$ dependence of heavy quarkonium near T_c

$$E^2 - \mathbf{q}^2 = m_g^2(T, \mathbf{q}^2) \simeq m_g^2(T,0) - \alpha(T)\mathbf{q}^2 + \cdots$$

$$\frac{E(v)}{E(0)} \simeq 1 + \frac{1}{2}v^2 + \mathcal{O}(v^4)$$

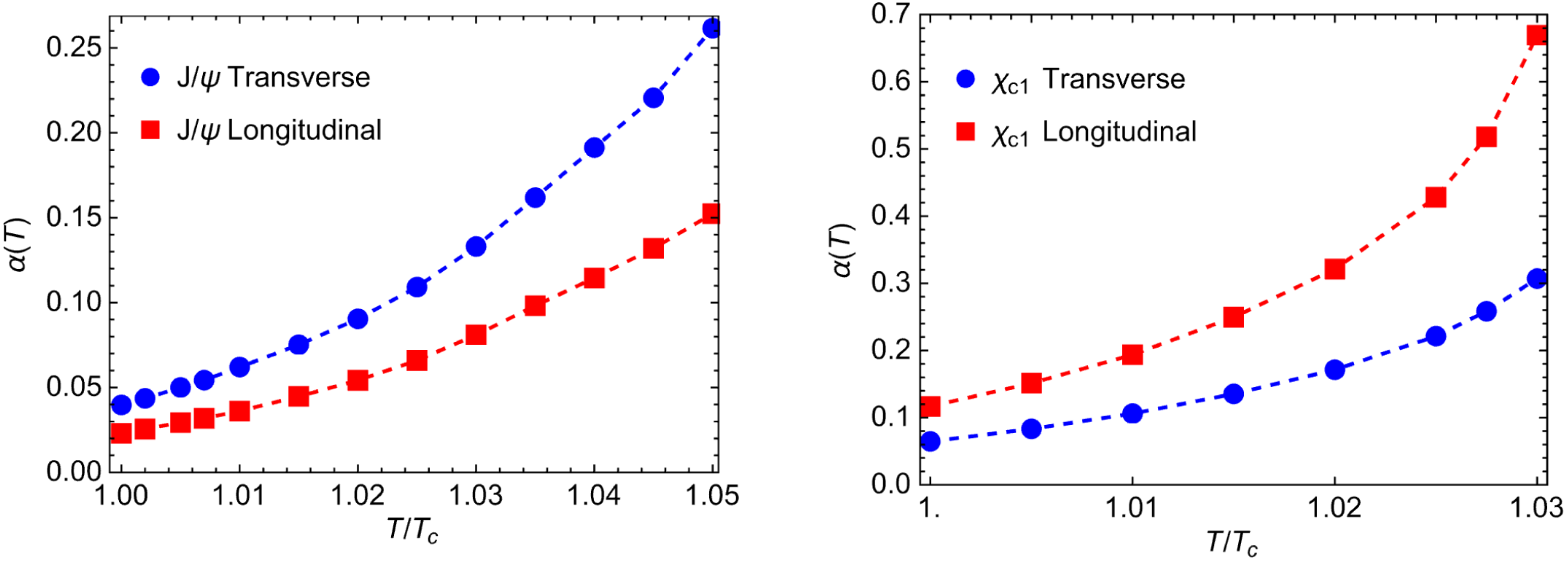
$$\longrightarrow \quad \frac{E(v)}{E(0)} \simeq 1 + \frac{1}{2}\Big(1 - \alpha(T)\Big)v^2 + \mathcal{O}(v^4), \qquad v = \frac{|\mathbf{q}|}{E(0)}$$

Finite-$\vec{q}$ dependence of heavy quarkonium near T_c



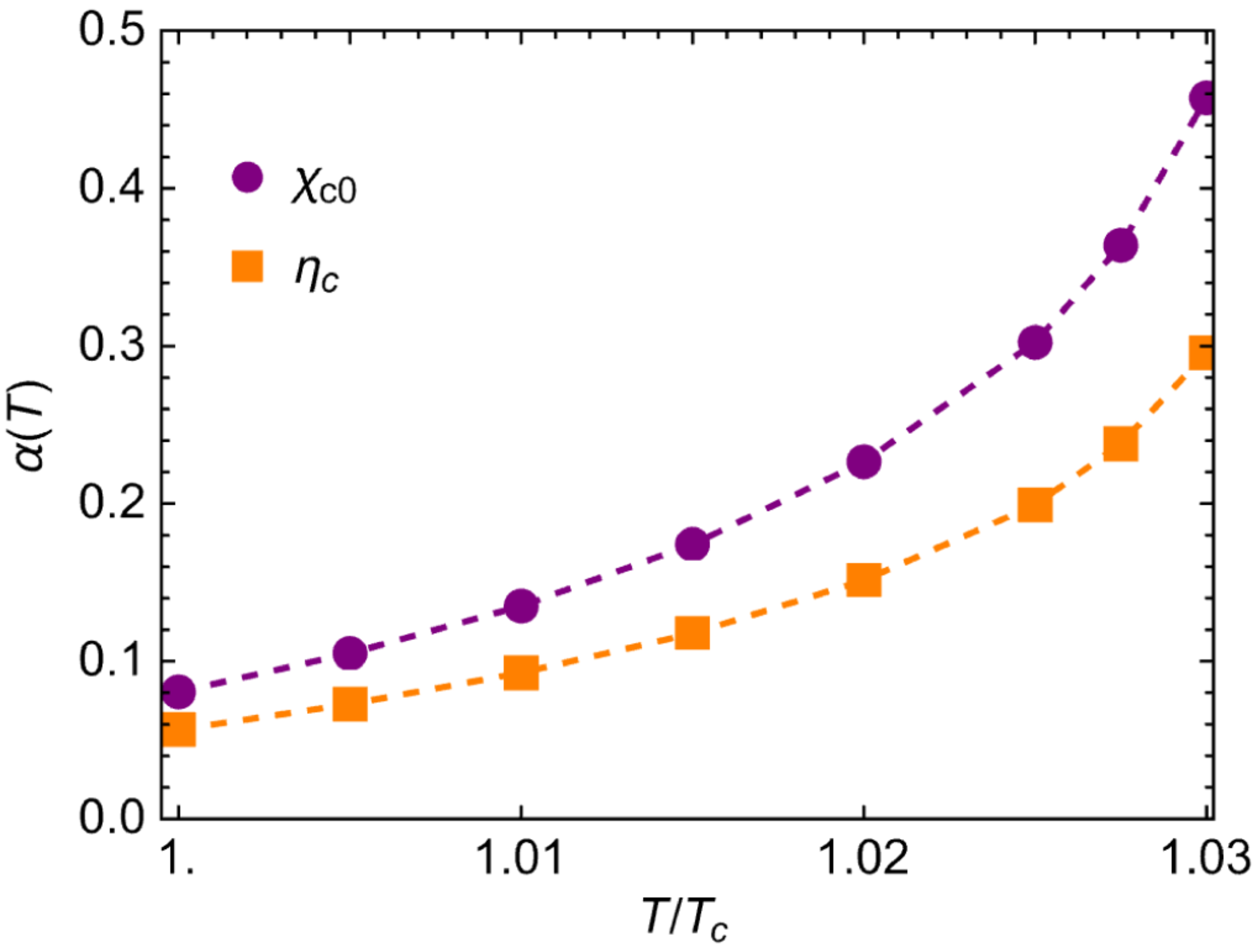
$$\frac{E(v)}{E(0)} \simeq 1 + \frac{1}{2}\big(1 - \alpha(T)\big)v^2 + \mathcal{O}(v^4), \qquad v = \frac{|\mathbf{q}|}{E(0)}$$

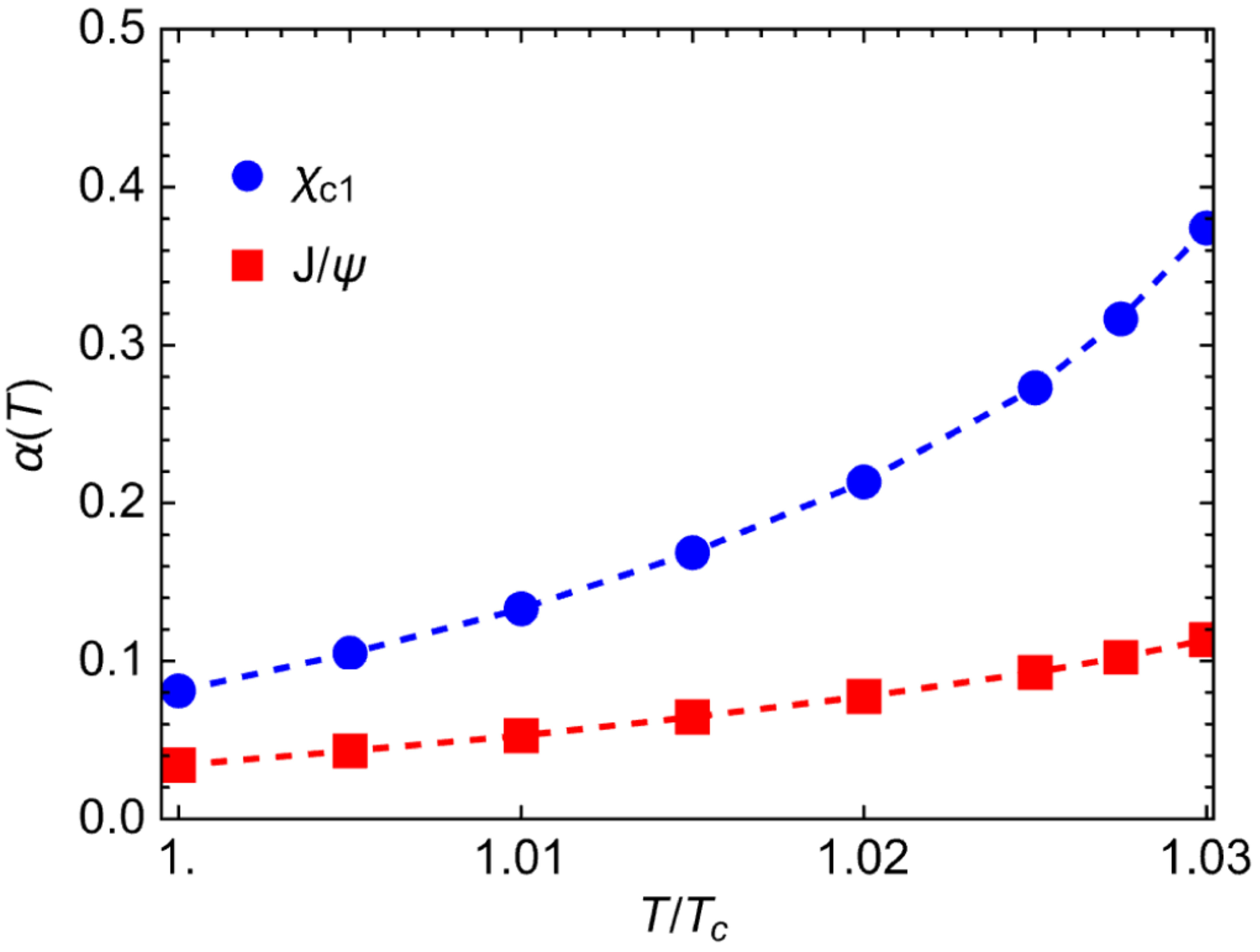
Finite-$\vec{q}$ dependence of heavy quarkonium near T_c



$$m_g^2(T, \mathbf{q}^2) \simeq m_g^2(T,0) - \alpha(T)\mathbf{q}^2$$

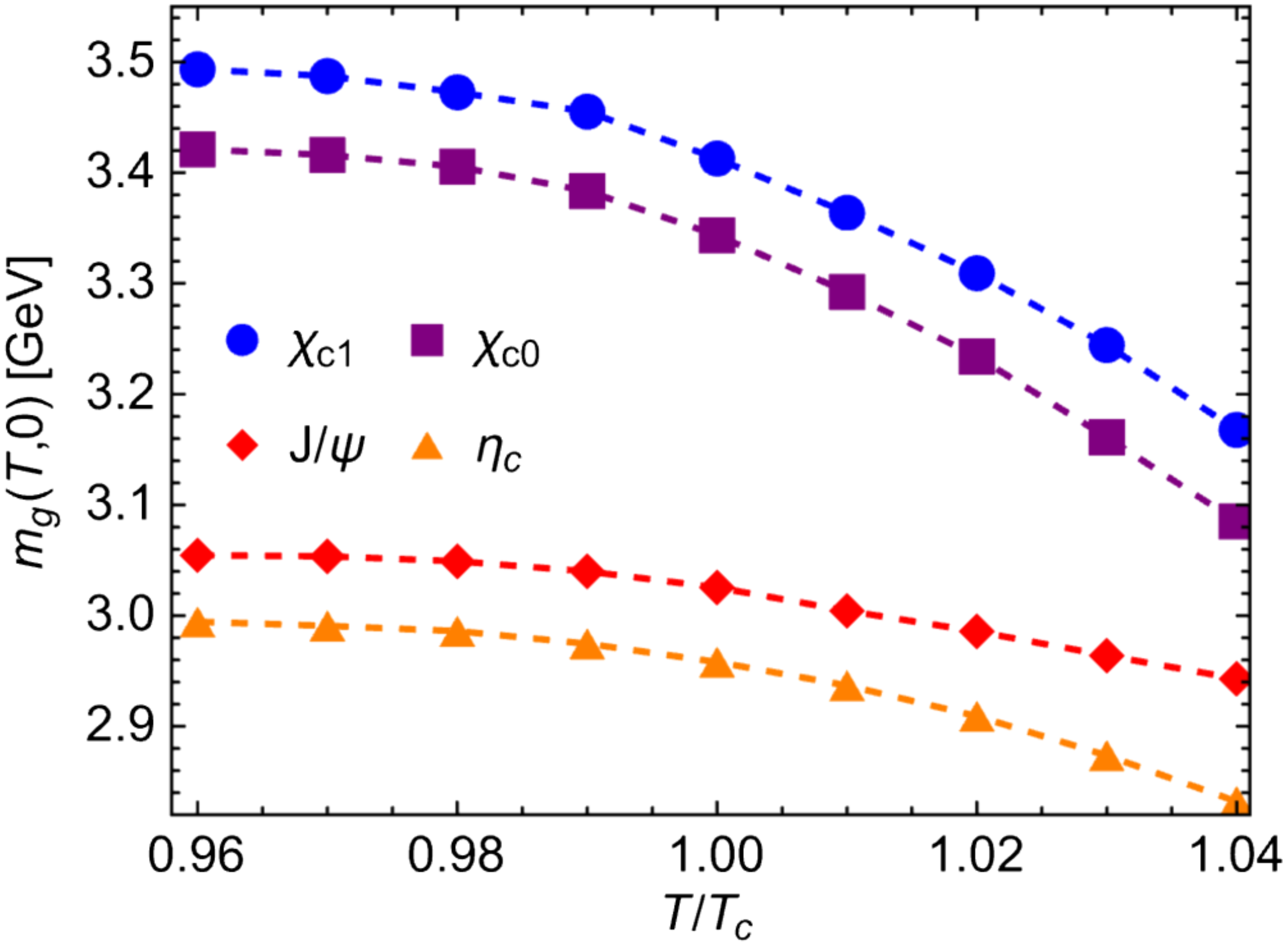
Finite-$\vec{q}$ dependence of heavy quarkonium near T_c





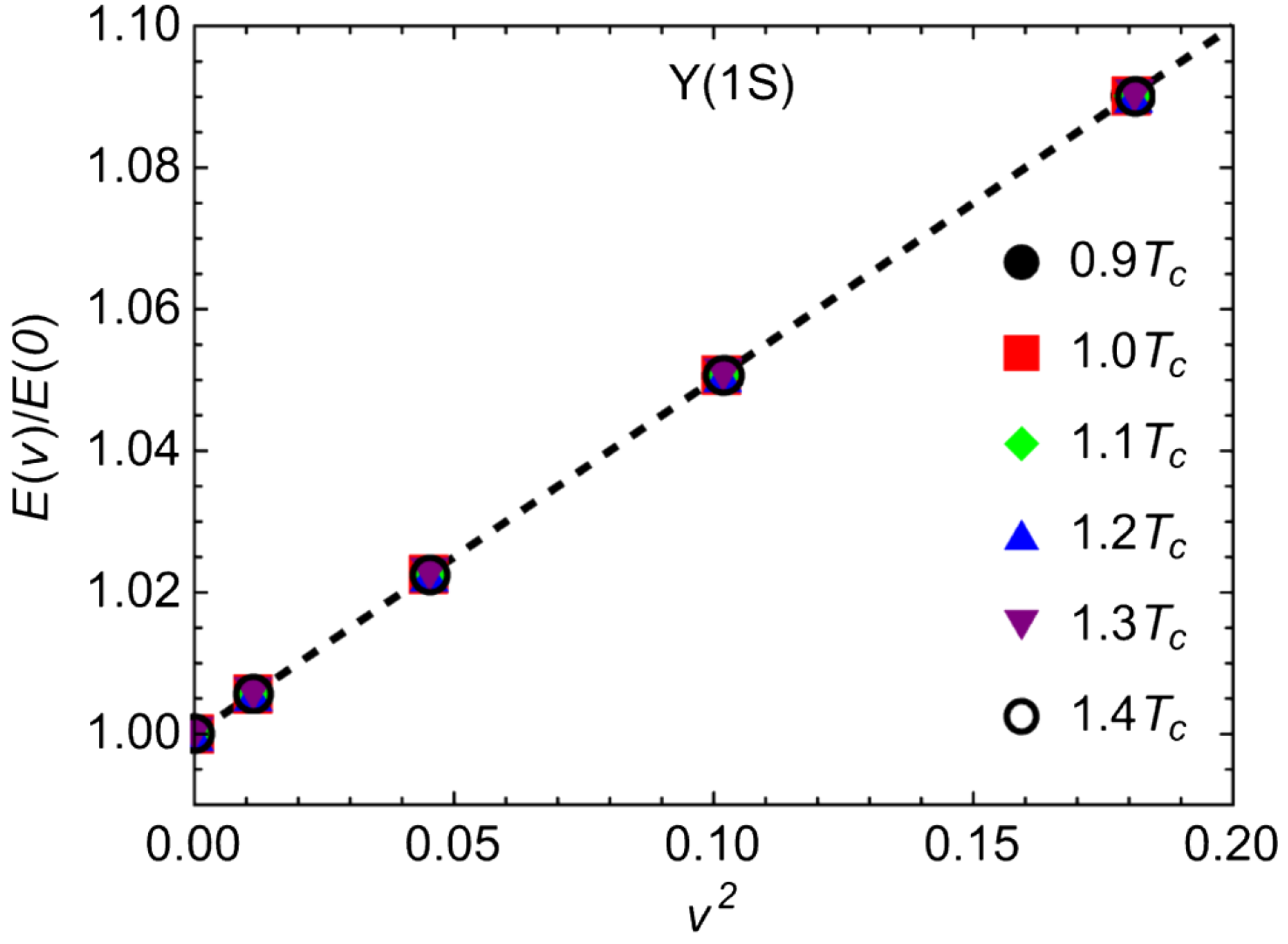
P-wave is more sensitive

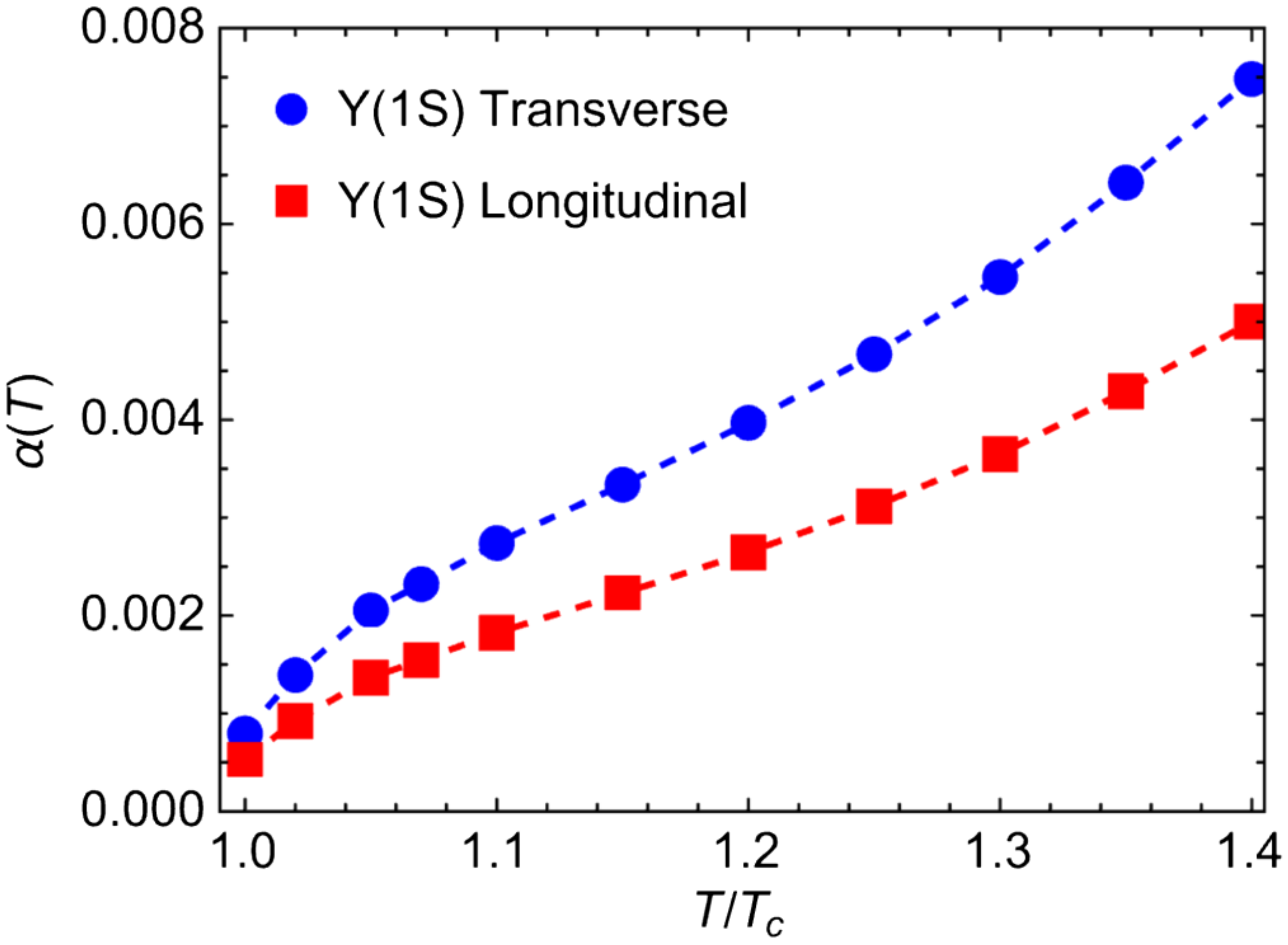
Finite-$\vec{q}$ dependence of heavy quarkonium near T_c



momentum dependence at $\vec{q} = 0$

Finite-$\vec{q}$ dependence of heavy quarkonium near T_c





1. $\frac{\mathbf{q}^2}{m_h^2} d_J(\nu)\phi_c = \frac{\mathbf{q}^2}{m_h^2} \frac{4\pi^2}{3(4m_h^2)^2} d_J(\nu) G_2$

2. more compact

Next topic: K_1

$K_1^{\pm}$ mesons moving in nuclear matter

K_1 is a open strange meson $\longrightarrow$ $K_1^{+}(u\bar{s}),\ K_1^{-}(\bar{u}s)$

They are degenerate in vacuum,

but not in nuclear matter, where

charge-conjugation symmetry is broken.

$K_1^{\pm}$ mesons moving in nuclear matter

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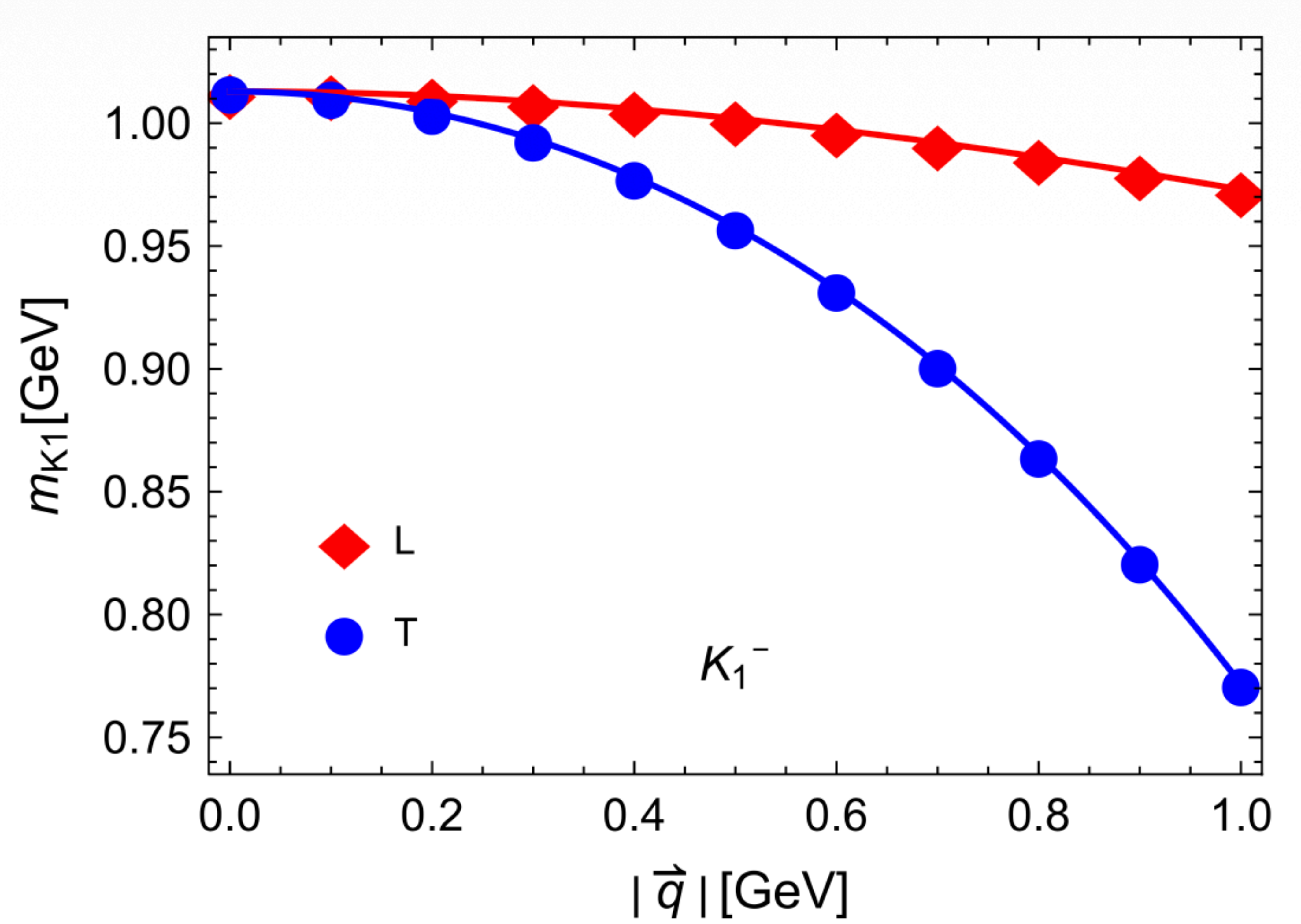
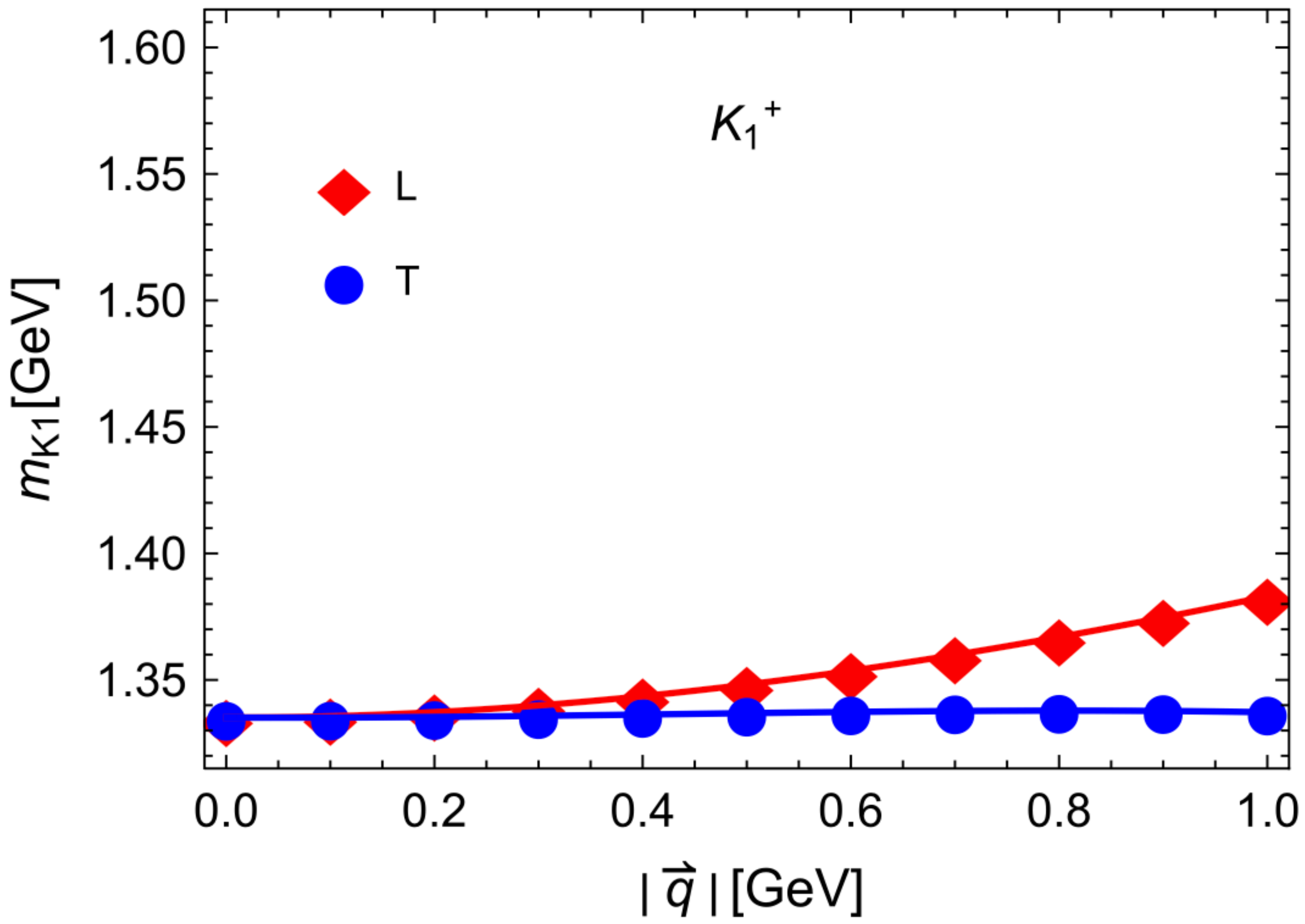
$$\Pi_{\mp}(Q^2, \vec{q}^2) = \Pi^e(Q^2, \vec{q}^2) \pm q_0 \Pi^o(Q^2, \vec{q}^2) \longrightarrow$$

$$\mathrm{Re}\left(\Pi^e \mp \sqrt{m_{K_1^{\mp}}^2 + \vec{q}^2}\,\Pi^o\right) = \frac{2}{\pi}\int \frac{\mathrm{Im}\left(\Pi^e \mp \sqrt{m_{K_1^{\mp}}^2 + \vec{q}^2}\,\Pi^o\right) ds}{s + Q^2}.$$

$K_1^{\pm}$ mesons moving in nuclear matter

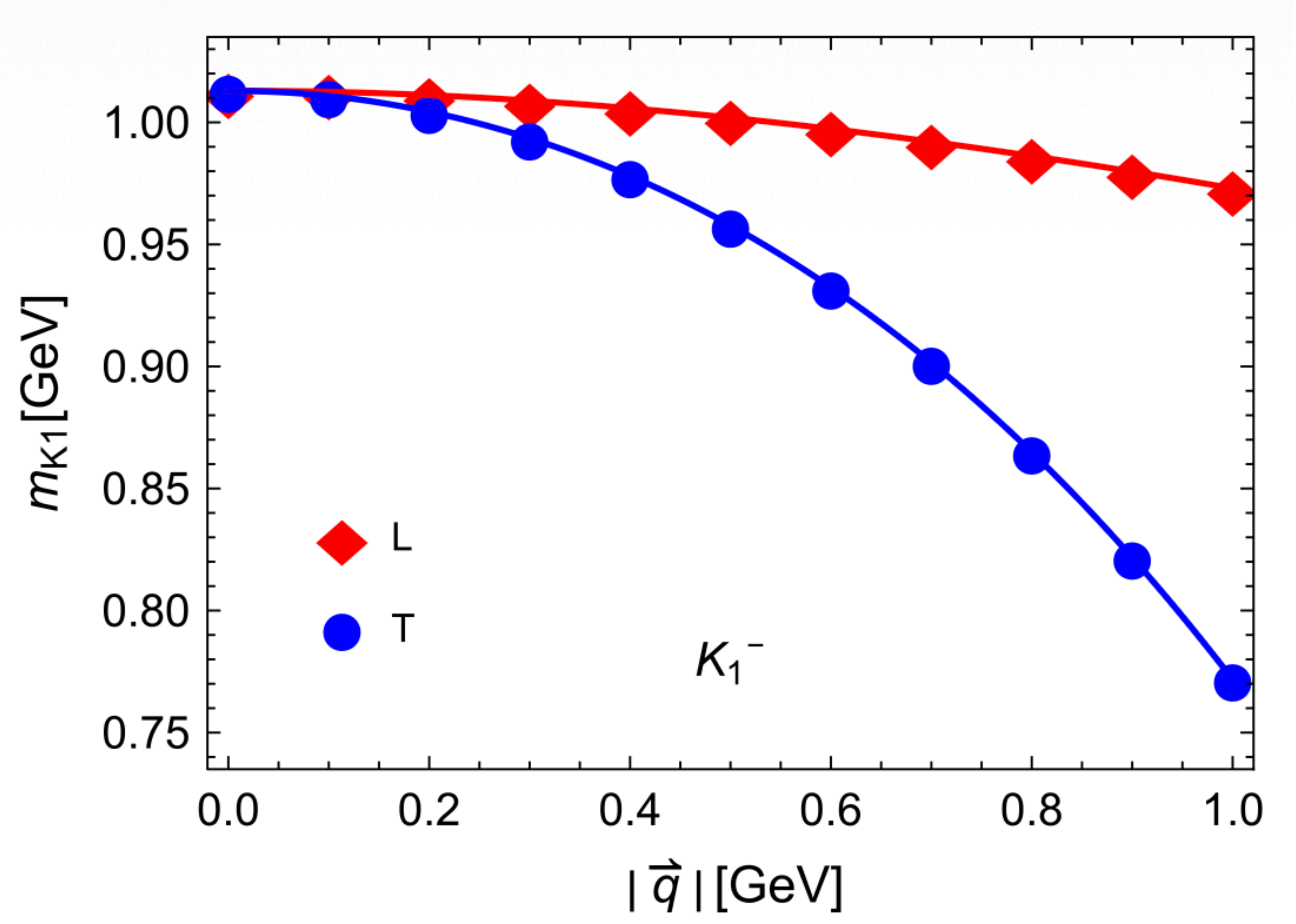


$|m_L - m_T| = 44$ MeV at $|\vec{q}| = 0.5$ GeV



$|m_L - m_T| = 11$ MeV at $|\vec{q}| = 0.5$ GeV

$K_1^{\pm}$ mesons moving in nuclear matter



$K^{*-}(s\bar{u})$

chiral partner $\uparrow$

$\boxed{K_1^-(s\bar{u})}$

isospin $\downarrow$

$\bar{K}_1^0(s\bar{d})$

chiral partner $\downarrow$

$\bar{K}^{*0}(s\bar{d})$

$\longrightarrow$ $(K^{*-}, K_1^-, \bar{K}_1^0, \bar{K}^{*0})$ similar pattern

Summary

- Nontrivial finite-$\mathbf{q}$ effects appear near T_c through medium-specific OPE structures.

- Polarization splitting occurs in spin-1 quarkonia; P-wave states are more sensitive than S-wave states, while $\Upsilon(1S)$ is much less affected.

- We performed the same calculation for K_1 in nuclear matter and found that the mass shift of K_1^- is large.

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Thank you