Effect of symmetry energy in the hadron-quark phase transition in the neutron star

Collaborators

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Outline

1. Prologue

- We are in a golden era of nuclear many-body physics.
- Two axes that lead to golden age
- Astronomical observations: Neutron star mass-radius (NICER), Gravitational waves (LIGO, Virgo, KAGRA), X-ray burst, Supernova …
- Rare isotope facilities: RAON, RIBF, TRIUMF, GANIL, FRIB …
- Physics covered by nuclear many-body physics
- Length: 10^{-15} – 10^{10} m
- Density: 0 – 10^{15} g/cm^3
- Temperature: 0 – 10^{12} K
- Energy: 10^3 – 10^{12} eV

- **In this talk**

- Constrain the EoS of baryon matter by using both nuclear and astronomical data: KIDS (Korea-IBS-Daegu-SKKU) functional
- Determine the EoS of deconfined quark matter by applying the nuclear matter and neutron star constaints: MIT bag with vector repulsion
- Examine the effect of uncertainties in the baryon and quark EoS in the neutron star properties
- Determine the range of uncertainties allowed by the data
- Which quantity is most critical to distinguish the existence of quark matter core

2. Model for baryon matter

- **KIDS density functional**

- **Unified description of infinite nuclear matter and finite nuclei**
- **Formulation of the theory based on established rules: Effective field theory + Density functional theory**
- **Quantification and systematic control of uncertainties**

- **Rules**

1. **Expand the energy per nucleon in uniform nuclear matter in the ascending order of k_F ($\propto \rho^{1/3}$)**
2. **Determine the coefficients α_i, β_i to reproduce a given EoS or neutron star properties**

Energy per nucleon in uniform nuclear matter

$$\mathcal{E}(\rho,\delta) = \sum_{i=0}^{n} (\alpha_i + \beta_i \delta^2) \rho^{1+i/3}$$

$$\rho = \rho_n + \rho_p$$
$$\delta = (\rho_n - \rho_p)/\rho$$

3. Transform the energy density functional to Skyrme force: Parameters determined from EoS (t_0, y_0, t_{3n}, y_{3n}), New parameters (t_1, y_1, t_2, y_2, W_0)

Skyrme type energy density functional

$$\begin{aligned}
\mathcal{H} = & \frac{\hbar^2}{2m}\tau + \frac{3}{8}t_0\rho^2 - \frac{1}{8}(t_0 + 2y_0)(\rho_n - \rho_p)^2 \\
& + \sum_{n=1}^{N-1}\left[\frac{1}{16}t_{3n}\rho^{2+n/3} - \frac{1}{48}(t_{3n} + 2y_{3n})\rho^{n/3}(\rho_n - \rho_p)^2\right] \\
& + \frac{1}{8}(2t_1 + y_1 + 2t_2 + y_2)\rho\tau - \frac{1}{8}\left[(t_1 + 2y_1) - (t_2 + 2y_2)\right]\sum_{q=n,p}\rho_q\tau_q \\
& + \frac{1}{64}(9t_1 - 5t_2 - 4y_2)(\nabla\rho)^2 - \frac{1}{64}(3t_1 + 6y_1 + t_2 + 2y_2)\sum_{q=n,p}(\nabla\rho_q)^2 \\
& + \frac{1}{2}W_0\left[\mathbf{J}\cdot\nabla\rho + \sum_{q=n,p}\mathbf{J}_q\cdot\nabla\rho_q\right].
\end{aligned}$$

4. Fit the new parameters to nuclear properties, e.g. mass, radius …

★ Nuclear matter EoS is not affected by fitting to nuclear data, so we can describe nuclear matter and finite nuclei accurately and simultaneously

- **KIDS-A, B, C, D**

[1] Nuclear matter EoS

[2] Fitting to nuclear data

[3] Consistency with neutron star data

[1] Nuclear matter EoS

$$\begin{aligned}\mathcal{E}(\rho,\delta) &= E(\rho) + S(\rho)\delta^2 + O(\delta^4),\\ E(\rho) &= E_0 + \frac{1}{2}K_0x^2 + \frac{1}{6}Q_0x^3 + \cdots,\\ S(\rho) &= J + Lx + \frac{1}{2}K_{\rm sym}x^2 + \frac{1}{6}Q_{\rm sym}x^3 + \cdots\end{aligned}$$

$$x = (\rho - \rho_0)/3\rho_0$$

$$\mathcal{E}(\rho,\delta) = \sum_{i=0}^{n}(\alpha_i + \beta_i\delta^2)\rho^{1+i/3}$$

$$\rho = \rho_n + \rho_p$$
$$\delta = (\rho_n - \rho_p)/\rho$$

- **Determine three α_i and four β_i to reproduce: [E_0, ρ_0, Q_{sym} are fixed to single values]**

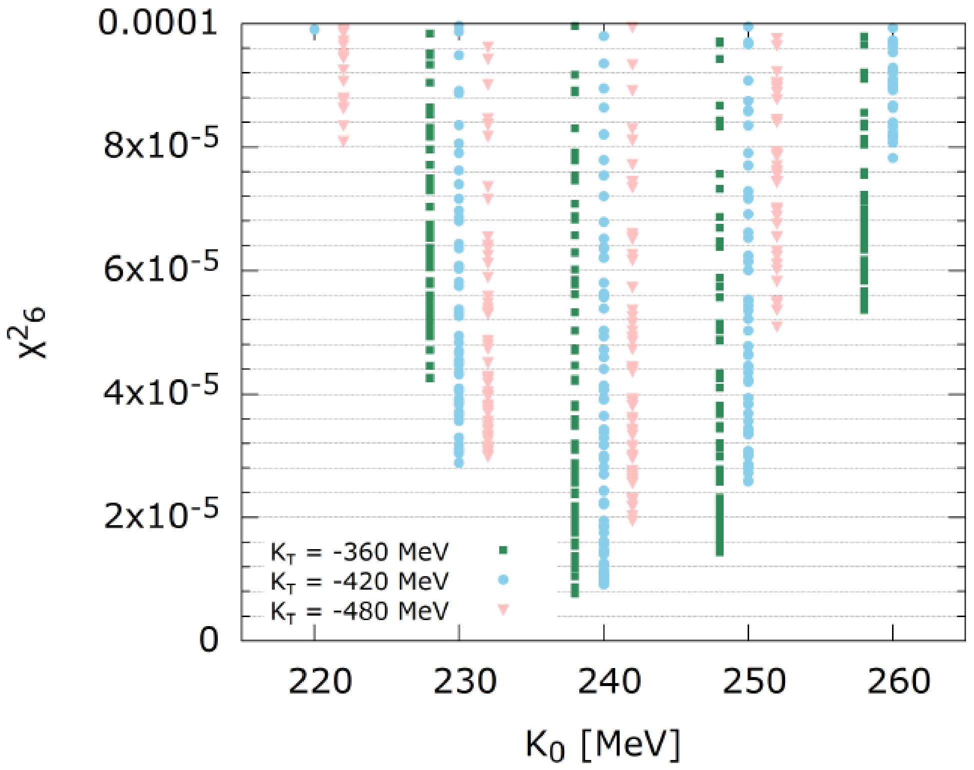
	K_0	J	L	K_τ
Min	220	30	40	-480
Max	260	34	70	-360
Δ	10	1	1	60
# of cases	5	5	30	3

$$K_\tau \equiv K_{\rm sym} - \left(6 + \frac{Q_0}{K_0}\right)L,$$

Total 2,250 EoSs

[2] Fitting to nuclear data

- Least parameters: t_1, W_0 ($t_2 \propto t_1$, $y_1=y_2=0$) [2 parameters]
- Least input data: mass and radius of 40Ca, 48Ca, 208Pb [6 data]
- Measure of accuracy: $\chi_6^2 \equiv \sum_{n=1}^{6} \left(\frac{O_n^{\mathrm{exp}} - O_n^{\mathrm{calc}}}{O_n^{\mathrm{exp}}} \right)^2$
- Result

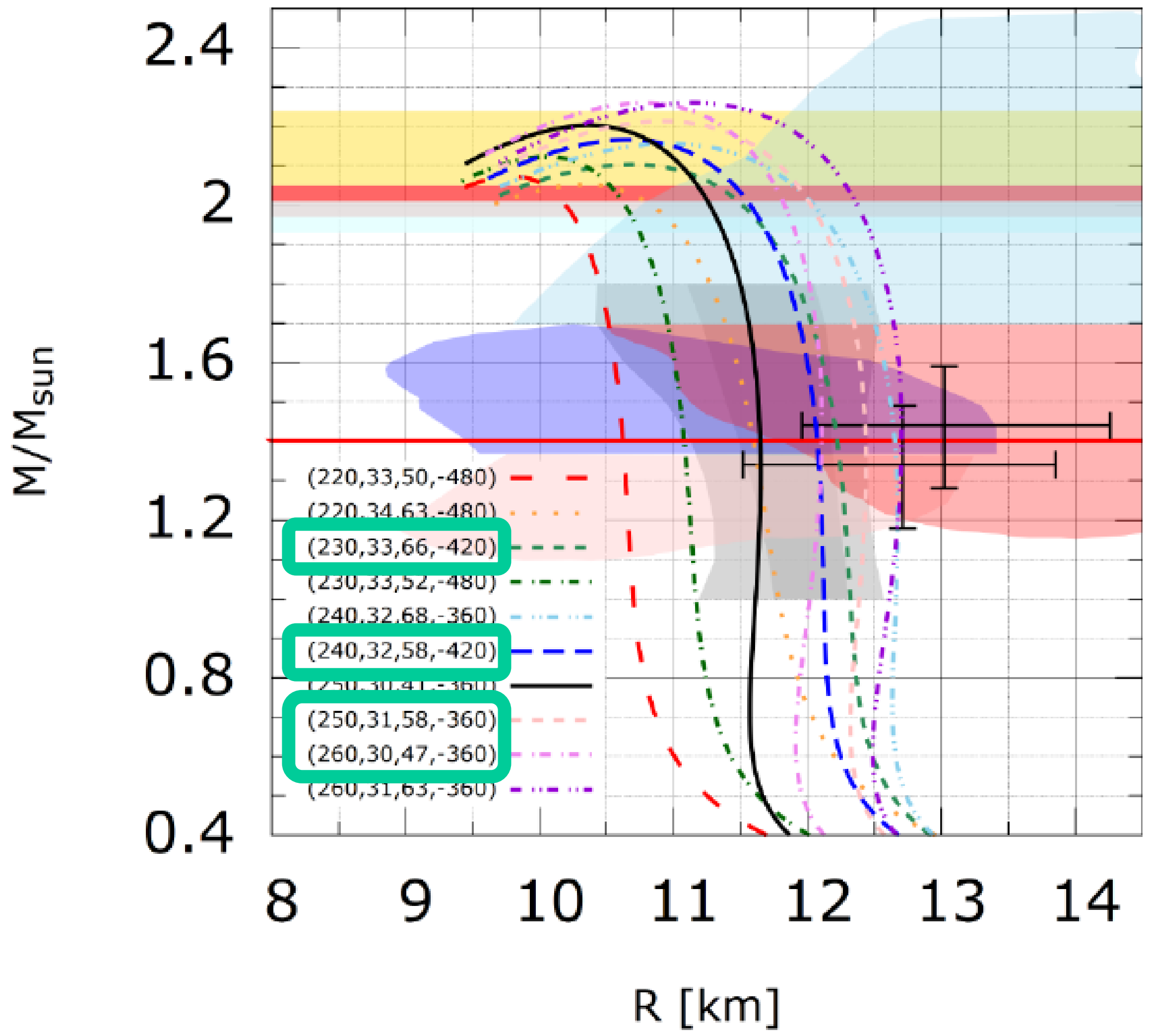


- Two best at each K_0

K_0	$(J,\ L\ K_\tau)$	$\chi^2(\times 10^{-5})$
220	(33, 50, −480)	9.45
	(34, 63, −480)	8.61
230	(33, 66, −420)	3.04
	(33, 52, −480)	3.01
240	(32, 68, −360)	0.75
	(32, 58, −420)	0.89
250	(30, 41, −360)	1.50
	(31, 58, −360)	1.43
260	(30, 47, −360)	5.55
	(31, 63, −360)	6.03

[3] Consistency with neutron star data

- **Neutron star mass radius with 10 best**



- **KIDS-A, B, C, D models**

	KIDS-A	KIDS-B	KIDS-C	KIDS-D
K_0	230	240	250	260
J	33	32	31	30
L	66	58	58	47
K_{sym}	-139.5	-162.1	-91.5	-134.5

Astronomical data

- X-ray burst (gray)
- GW (wings)
- NICER (cross bars)
- Large mass (bands)

3. Model for quark matter

- **MIT bag model with vector repulsion**

$$\mathcal{L} = \sum_{f=u,d,s} \left[\bar{\psi}_f\{\gamma^\mu(i\partial_\mu - g_{qqV}V_\mu) - m_f\} - B\right]\Theta(\bar{\psi}_f\psi_f)$$
$$+ \frac{1}{2}m_V^2 V_\mu V^\mu - \frac{1}{4}V_{\mu\nu}V^{\mu\nu} + \overline{\psi}_e(i\gamma_\mu\partial^\mu - m_e)\psi_e,$$

- **Standard MIT bag**

- Only kinetic and bag constant
- Good at reproducing baryon mass spectrum
- Soft EoS in the neutron star: cannot support $2M_\odot$

- **Vector repulsion**

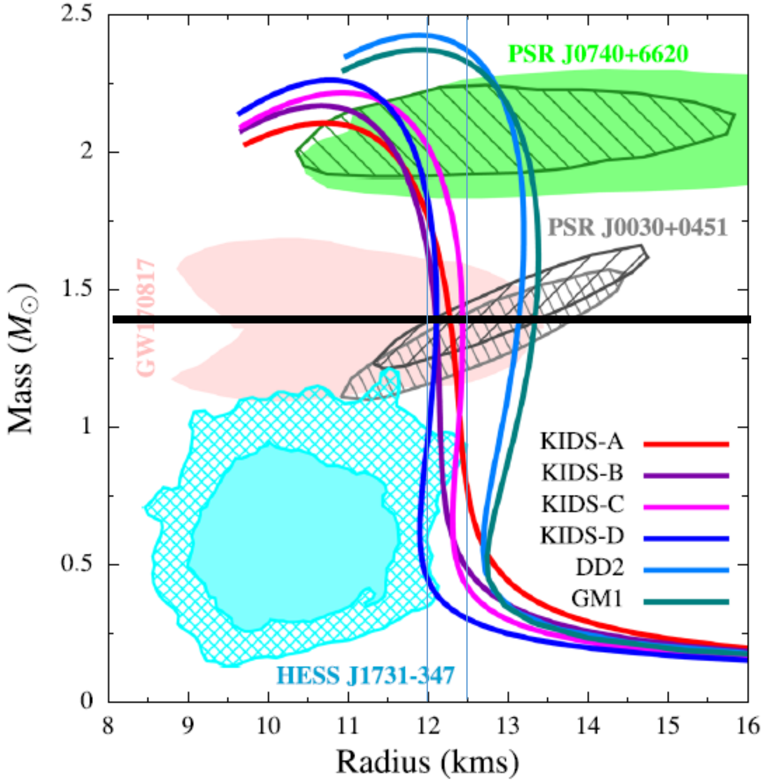
- $g_{uuV}=g_{ddV}$
- $X_V=g_{ssV}/g_{uuV}=0.4$
- B and g_{uuV} are dominant in EoS and X_V is marginal

4. Neutron star properties

- **Phase transition**

- Maxwell construction
- Local charge neutrality
- Transition conditions: $\mu^H_B = \mu^Q_B$, $P^H = P^Q$ (H: hadron phase, Q: quark phase, B: baryon)

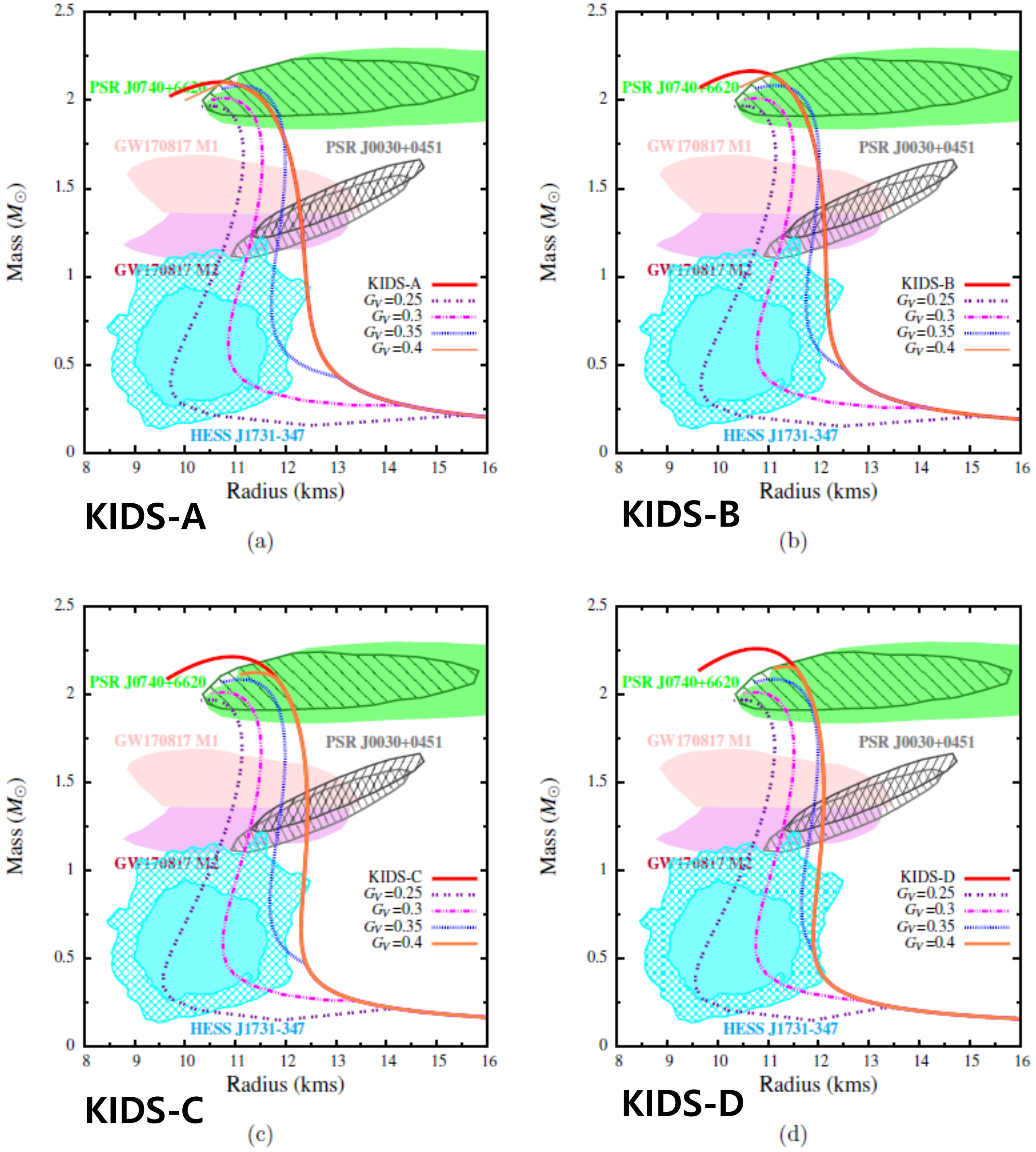
- **M-R of nucleon matter**



$R_{1.4}$ = 12.0 – 12.5 km

C>A>B=D

- **M-R with hadron-quark transition**

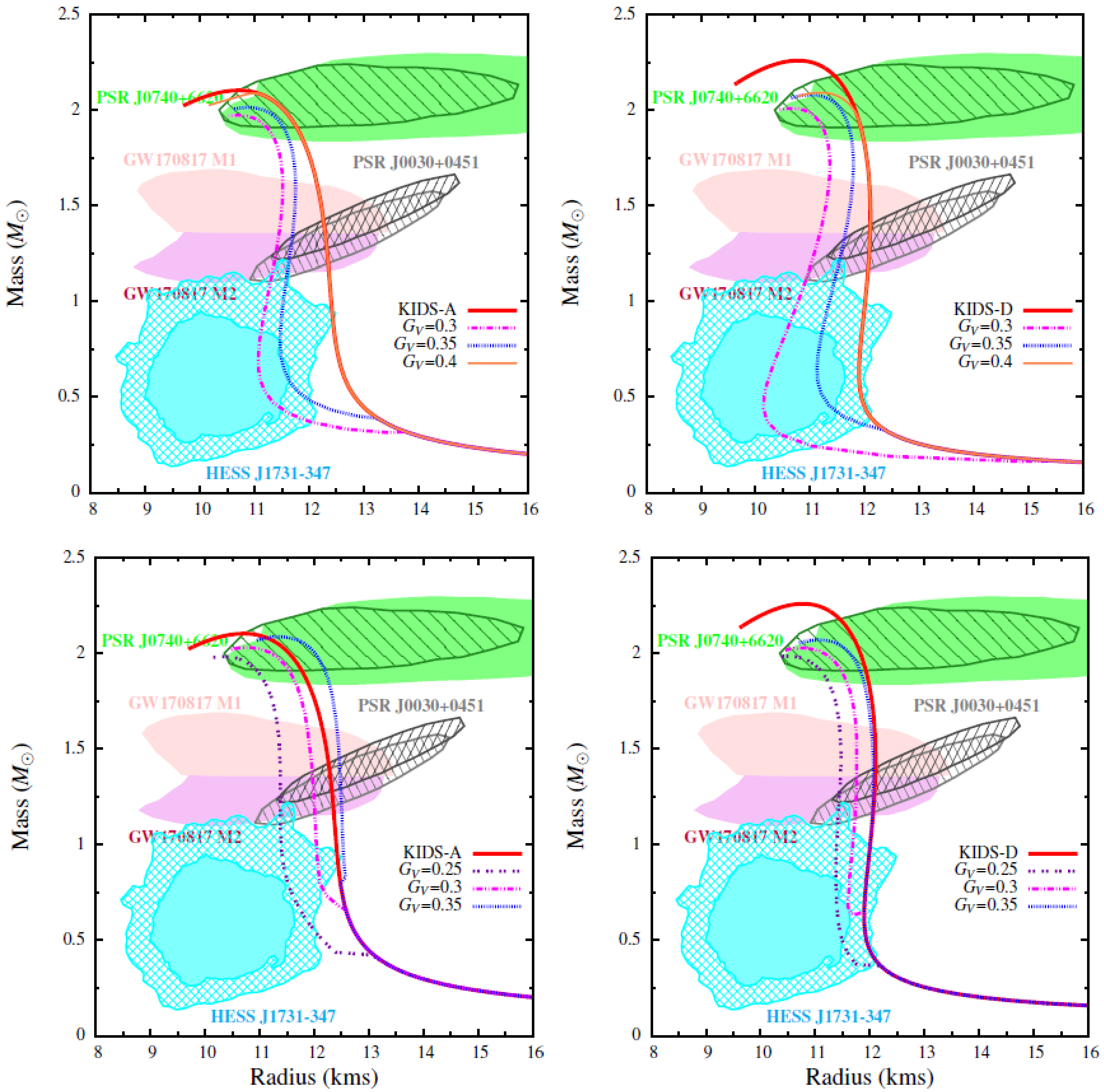


- Gv=0.25 incompatible with PSR J0030+0451
- Great effect to radius with Gv=0.25, 0.3
- Gv=0.4 no significant effect by transition
- Transition delayed at higher masses from A to D

- **Dependence on the bag constant**

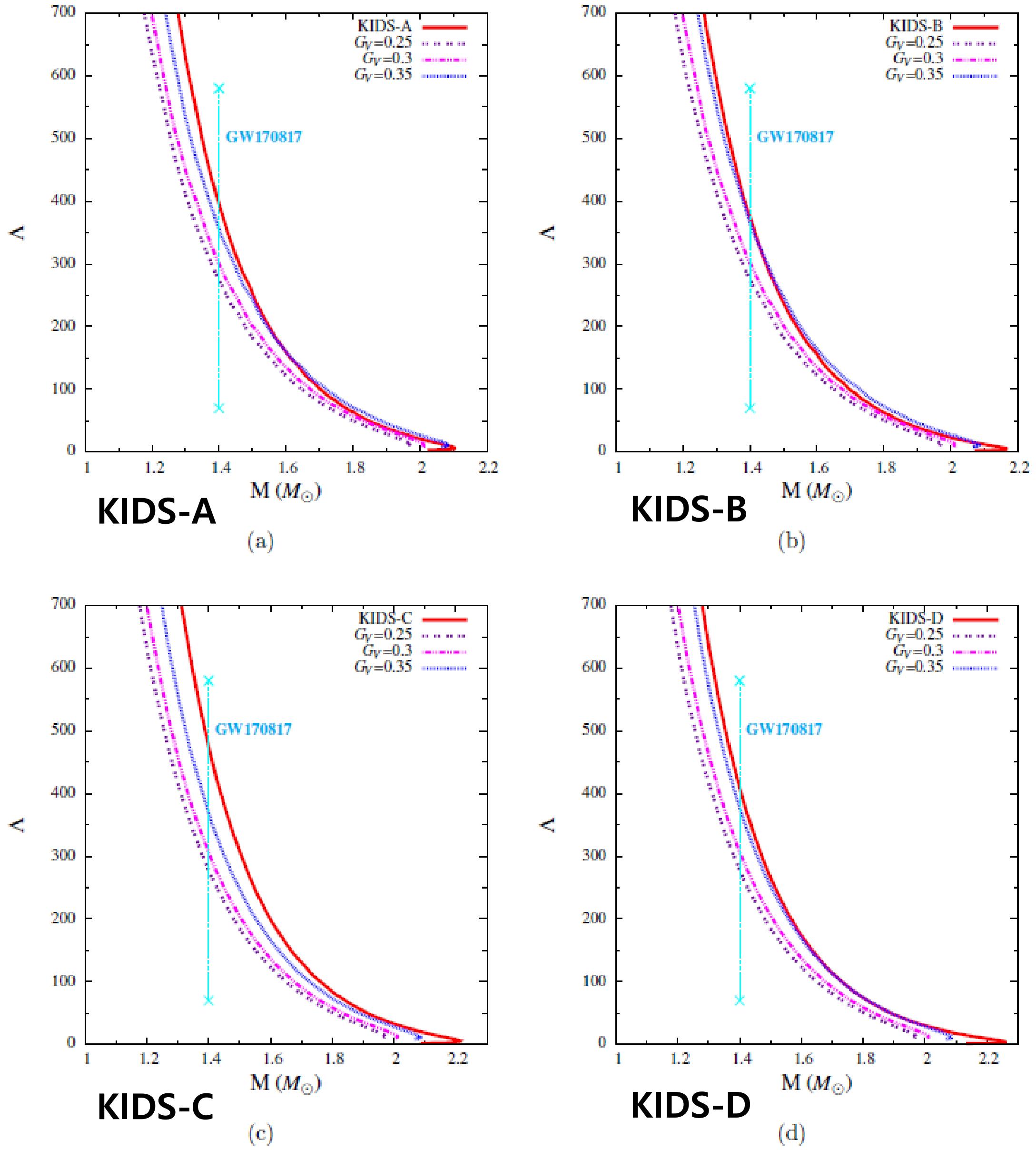
$B^{1/4}$=145 MeV

$B^{1/4}$=160 MeV



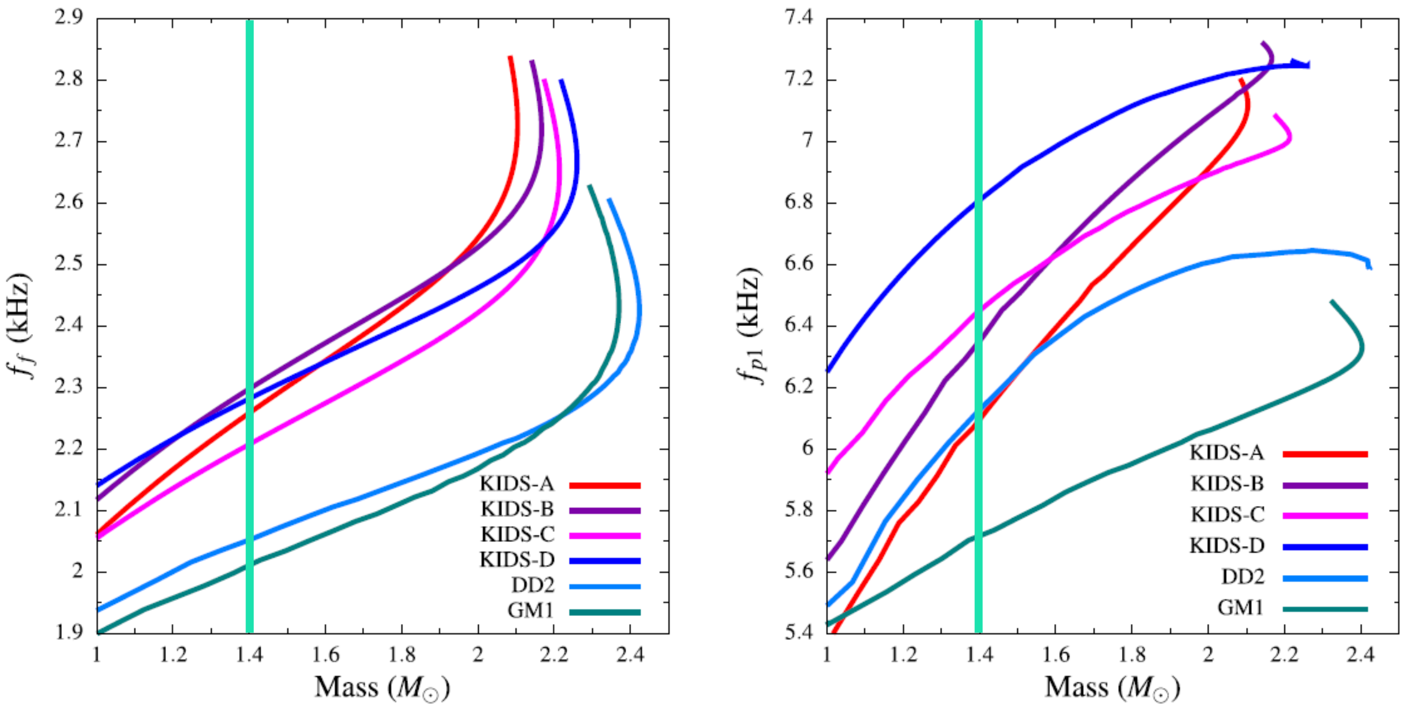
- Look very different at a glance
- Transition delayed to higher masses with large bag constant
- Radius fluctuates weakly with large bag constant

- **GW related observables: Tidal deformability**



- Gv=0.25, 0.3 are weakly dependent on the model: $\Lambda_{1.4} \approx 300$
- Gv= 0.35 exhibits clear discrimination from 0.25, 0.3 and evident dependence on the model
- KIDS-C model shows the widest range 250-500 for $1.4M_\odot$
- Improvement in the accuracy needed in the future measurement

- **GW related observables: Non-radial oscillation frequency**



- **Oscillation modes of star-quake**
- **f mode**

- $f_{1.4}$ = 2.2-2.3 kHz

- C<A<B=D: Exact reverse of $R_{1.4}$

- **p mode**

- p1.4 = 6 – 6.8 kHz

- A<B=C<D: Exact reverse of L

- **Transition density**

$B^{1/4}$ (MeV)	Hadronic Model	G_V	ρ_t^H/ρ_0	ρ_t^Q/ρ_0	M_{max} ($M_\odot$)
145	KIDS-A	0.3	0.821	1.875	1.972
		0.35	1.273	1.963	2.014
		0.4	5.800	6.238	2.088 (unstable)
	KIDS-D	0.3	1.006	1.900	2.009
		0.35	1.533	1.983	2.081
		0.4	3.845	4.638	2.092
155	KIDS-A	0.25	0.737	1.955	1.969
		0.3	1.093	2.218	2.011
		0.35	1.444	2.468	2.083
		0.4	6.500	7.262	2.103 (unstable)
	KIDS-D	0.25	0.606	1.937	1.969
		0.3	1.219	2.401	2.011
		0.35	1.756	2.897	2.084
		0.4	4.563	5.137	2.156
160	KIDS-A	0.25	0.794	1.973	1.983
		0.3	1.146	2.344	2.031
		0.35	1.500	2.508	2.087
	KIDS-D	0.25	0.671	1.997	1.985
		0.3	1.269	2.088	2.028
		0.35	1.803	2.179	2.087

ρ^H_t: below which only nucleons

ρ^Q_t: above which only quarks

- High transition densities favored with
  - Large bag constants
  - Large Gv values
  - Small L values for Gv=0.3, 0.35
- Condition: Quark matter absent at ρ_0
  - Gv=0.25 ruled out regardless of symmetry energy
  - Gv=0.3 is conditional on symmetry energy
  - Gv=0.35 looks most reasonable
  - Gv>0.4 no transition in all the model

	KIDS-A	KIDS-B	KIDS-C	KIDS-D
K_0	230	240	250	260
J	33	32	31	30
L	66	58	58	47
$K_{\rm sym}$	−139.5	−162.1	−91.5	−134.5

5. Epilogue

- ★ Baseline EoS: KIDS-A, B, C, D
- ★ Quark EoS: MIT bag with a vector repulsion
- ★ Uncertainties investigated with $B^{1/4}$=145, 155, 160 MeV, and Gv=0.25, 0.3, 0.35, 0.4
- ★ Transition density depends heavily on Gv > Symmetry energy > Bag constant
- ★ Range of Gv consistent with normal nuclear matter condition and neutron star data: 0.3<Gv<0.4: Consistent with the result obtained with NJL model
- ★ Transition effect is more dominant in the properties of low-mass stars
- ★ Accurate measurements of radii for all masses are crucial to identify the existence of the quark core and constrain the strength of vector coupling
- ★ Multi-messenger observations are indispensable to reduce the uncertainties in the nucleon matter EoS